IMPROVEMENT OF THE OPTIMAL CONTROL OF INERTIAL OBJECTS
IN THE PRESENCE OF PERTURBATIONS

By V. V. Kazakevich
and
A. P. Turkevich

- USSR -

DISTRIBUTION STATEMENT A,
Approved for public release
Distribution Unlimited

LIBRARY COPY

JUN 9 1961

Distributed by:
OFFICE OF TECHNICAL SERVICES
U.S. DEPARTMENT OF COMMERCE
WASHINGTON 25, D. C.
Best Available Copy
FOREWORD

This publication was prepared under contract by the UNITED STATES JOINT PUBLICATIONS RESEARCH SERVICE, a federal government organization established to service the translation and research needs of the various government departments.
IMPROVEMENT OF THE OPTIMAL CONTROL OF INERTIAL OBJECTS IN THE PRESENCE OF PERTURBATIONS.

Following is a translation of an article by V. V. Kazakevich and A. P. Yurkevich, entitled "Ob uluchshenii kachestva ekstremal'nogo regulirovaniya inertsionnykh ob'ektov pri nalogii vozmushcheniy" (English version above), in Doklady Akademii nauk SSSR (Reports of Academy of Sciences SSSR), Vol. 136, No. 4, 1961, pp. 783-786.

The solution of the problem of the optimal control of inertial objects in the presence of low frequency external perturbations demands the application of special systems (1, 2). In the cases of application of optimal controllers of the usual type, for example with a memory storage of the optimum, it is necessary to lower the speed of hunting because of the inertial effect of the object, which, in its turn, deteriorates the interference immunity of the system.

In (2) a method of optimal control is suggested and theoretically proven. It permits, in principle, to remove the harmful inertial effect of the object of any order, in the absence of dynamic links in the object, located before its optimal characteristic. The system is distinctive in that at the control input is applied a definite, generally non-linear, combination of signals in accordance with the derivatives of the controlled value and by means of a non-linear link there is also
introduced an action in accordance with the change of
the function.

The system has better characteristics with respect
to the quality of the process of controlling inertial
objects, for low-frequency perturbations even as com-
pared with a system of the usual type with a non-iner-
tial object, since the effect of the low-frequency per-
turbations to a certain extent is filtered out. How-
ever, in the presence of strong perturbations, the level
of the optimum of the function changes very rapidly and
at a not very rapid change of the executing organ, the
losses in hunting may still be considerable.

Let us consider the frequently encountered case
of optimal control of an object of the first order. Let
the characteristic of the optimal element have the form

\[ y_1 = -K_1 x^2, \quad (1) \]

where \( x \) and \( y_1 \) are correspondingly the values of the in-
put and output of the optimal element. Let also
\( |\dot{x}| = V_x = \text{const} \) and the relation of the output of the
object \( y_2 \) with \( y_1 \), have a form

\[ y_2 = \frac{\kappa}{2\tau + 1} y_1, \quad (2) \]

where \( \kappa \) and \( \tau \) are constants.

Using the method of control shown in \((2)\), if the
action of the non-linear link is not taken into consideration, the signal at the input of the optimal controller will be \( z = K_2y \). Denoting \( y_iK_2K_1/\tau = y' \), we obtain

\[ z = \frac{\rho \tau_0}{\rho \tau_0 + 1} y'. \]  

(3)

The transient conditions in the control system under consideration is described by a system of equations analogous to that which describes the transient conditions in a non-inertial object with the application of a dynamic converter of the input signal.

The dynamics of a system of optimal control with a dynamic converter in the form of eq. (3) is considered in (3,4), in which the determination of parameters of the transient conditions and limiting cycles, taking into account the external perturbations, is obtained by the method of construction of phase trajectories of the system on a multi-leaf surface (5) in coordinates \( x = x'V_1\tau_0 \), \( \beta = \beta/2K_1V_1^2\tau_0 \).

We will apply this method here. Within the limits between reversals of the executing organ we have

\[ \beta = (\beta_0 + 1)e^{-\Delta z} - 1, \]

where \( \Delta z = z - a_0; \beta_0 \) and \( a_0 \) are the coordinates which define the points for \( t = 0 \). The reversals of the operating organ occur each time after curve \( \beta \) intersects the abscissa axis and the change in the area between curve \( \beta \) and
The abscissa axis reaches the value \( S_n = \Delta w/2w_n \).

Where \( \Delta w = z_n - y_n \) is the zone of insensitivity of the controller, \( y_n \) is the value of the controlled quantity at the optimal point, \( \eta = Kx^2_{\text{max}}/y_n \), \( v_{ir} = \Delta w/\Delta x \).

At the reversal points \( \beta \) changes by a jump by \( \pm 2\tau \) depending on the value of \( i \).

We will assume that starting at some moment of time \( t = 0 \) for \( x = 0 \) and \( y = 0 \), the action of the external perturbation is expressed by a linear time function. Then instead of (1) we have

\[
y_t = -K_x x^2 + K_f t,
\]

and at the initial moment (for \( t = 0 \)) \( \beta = \beta_0 = K_1/2K_x V_x x \).

The process of control corresponding to the given case, in terms of projection on the phase surface \( \alpha, \beta \) for system parameters \( \delta = 0.5\% \), \( \lambda = K_1/K_v x^2 X_{\text{max}} = 1 \), \( v_{ir} = 3 \), is shown in Fig. 1. It may be seen that there is an oscillation about the optimum of a diminishing amplitude which contrasts to the limiting cycle with an amplitude \( \alpha = \alpha_p \).

For the given parameters the variation in the process of oscillation is equal to 0.11 (to which corresponds \( \psi = x/x_{\text{max}} \approx 0.55 \)) and the timing is equal to \( \approx 3\tau \). The introduction of a switch circuit which periodically reverses the system
every period $t_k$ decreases the shown variations to the value

$$\psi_k = V_t / x_{\text{max}}.$$  

In the presence of a strong perturbation the effect of which appears to be a quadratic function of time giving

$$y_1 = -K_1 x^2 + K_0,$$

the deviation of the system from the optimum in the limiting cycle, in the absence in the controller of an insensitivity zone, can be described by a simplified formula

$$\psi_p = -\frac{v_x}{1.15} \ln(1 - \lambda_p).$$  (5)

where

$$\lambda_p = K_1 / K_0.$$  

These deviations for a large value of $v_x$ and a limited velocity $V_x$ can be appreciable. For example, for $\lambda_p = 0.2$ and $v_x = 5$, $\psi_p = 1$. For a reduction in the change time of the executing organ, there is a reduction of the time of the transient condition and of the amplitude $\Delta \alpha$ for the case of either a linear or a quadratic perturbation.

The quality of the optimal control of inertial objects in the presence of perturbations can be improved by application of a combination system for conversion of the input signal as shown in Figure 2.

Into the structural diagram of the system enter the following links: ye. ye. - optimal element with a characteristic $y_1 = -K_1 x^2$, 0. - dynamic part of the object which is described by the differential equation

$$y_1 + \psi(y_1, y_2, ..., y^{n-1}) + \psi(y) = k_0 y_1,$$

u. f. s. - the generating device which develops the sig-
nal $y$, the dynamic converter and the optimal controller $ye. r$. Thus, the system contains the device described in (2), generating a signal in accordance with the derivatives and changes of function $y_2$, which removes the effect of the inertia of the controlled object and besides it has a dynamic converter, representing a proportional link, encompassing an integrating feedback with an insensitive zone $z_0$. The dynamic converter at its optimal value for a constant time $t$ in a number of cases practically eliminates or lowers the unfavorable effect of low-frequency perturbations. The introduction into the feedback of the insensitive zone $z_0$ increases the precision of control on the sloped portions of the optimal characteristic near the optimum for the action of monotonic external perturbations, which change $x$ and at slow speed of the executing organ. The given system permits a great increase in the sensitivity

Figure 2
of the controller (6).

For determination of parameters of transient conditions and limiting cycles of the system also in this case it is convenient to apply the method of construction of its phase trajectories in the coordinates $\alpha$ and $\beta$ on a multi-leaf surface.

In the ideal case, with the absence of instrument distortions in operation of the installation, the excluding effect of the inertia of the object can be taken as $y = k_n y_n$.

Conversion of signal $y$ into $z = u$ is determined by equations

$$u = \frac{p^2 r}{p_r + 1} y \text{ at } -z_0 > z > z_0;$$

$$u = py \text{ at } -z_0 < z < z_0. \tag{7}$$

The transition from eq. (6) to eq. (7) when the external perturbations are absent can be accomplished by $y_k = 0$, thereby $\dot{i} = \dot{y}$, to which corresponds $\beta = \beta_n = \dot{x}$.

In the presence of an external perturbation which is a linear function of time

$$\beta_n = \lambda/2\nu - \nu \text{ sign } i,$$
where \( v_t = tV_e / x_{\text{max}} \).
The operation of the system within the limits of the insensitivity zone of the feedback $z_0$ corresponds to some change $\beta_n$, whereby the area between the portion of the straight line $\beta_n$ and its projection on the abscissa axis is equal to $S = \delta_n^2/2\eta V_0^2$, where $\delta_n = z_0/y_0$. The phase trajectories corresponding to equation (6) have the same form as for equation (3).

On Fig. 3 are given the projections on the phase plane of the phase trajectories of the optimal control for $v_c = 0.15$ and the same external conditions as in the case corresponding to Fig. 1. As can be seen on Fig. 3 deviations of the executing organ from the position corresponding to the optimum of the function diminished and the process of the setting up the limiting cycle was considerable speeded up; under this condition the transient period $\psi$ does not exceed 0.33.

In the case of reaction of external perturbations, which are quadratic function of time, the precision of control is also determined by eq. (5), but is determined by relation $\tau V_0/x_{max}$ which can be many times smaller than $\tau_0 V_0/x_{max}$. The value $\psi_0$ will be correspondingly smaller.

The introduction into the control system of a switching circuit permits to reduce greatly the indicated momentary deviations of the systems.

The Moscow aviation Institute Sergo Ordahonikidze Received
July 12, 1960.
BIBLIOGRAPHY

1. V. V. Kazakevich, DAN, 126, No. 3 (1959)
2. V. V. Kazakevich, DAN, 133, No. 4 (1960)
3. A. P. Yurkevich, DAN, 133, No. 6 (1960)
5. V. V. Kazakevich, DAN, 49, No. 7 (1945); 74, No. 4, (1950).