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AND THEIR INTERPRETATION

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### Key Words

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- Photometry
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SUMMARY

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INTRODUCTION

It would be desirable to have available a photometric function for planetary surfaces that would describe the brightness of a surface in reflected sunlight as function not only of the angles of incidence, emission, and phase, but also as function of the physical properties of the soil that forms the surface. The commonly-used, indeed universally-used, photometric function for correlating and analyzing data on planetary surfaces and for describing the reflection from
(or brightness of) planetary surfaces is the well-known Minnaert function. This function attempts to describe the brightness as it depends on the geometry of the situation; namely, the angles. The Minnaert function, however, has very insufficient basis in theory, is not derived from first principles, is restricted in validity to certain classes of scattering geometries, and, of considerable importance, can not be used to interpret photometric behavior in terms of the physical properties of the reflecting surface.

This undesirable situation has now been improved. W. E. Meador and W. R. Weaver of the Langley Research Center have derived a new photometric function in an as yet unpublished paper entitled "A Photometric Function for Diffuse Reflection by Particulate Materials." This photometric function is not only grounded in theory but has been verified for several soils in laboratory experiments. It is applicable to all geometries of scattering, it takes into account single scattering, multiple scattering, and interparticle shadowing, and it also contains parameters the values of which depend on the physical properties of the surface. The investigation on which this new photometric function is based is not yet complete, inasmuch as interpretation of all but one of these parameters in terms of physical properties of the soil is not yet on a very exact basis, but the investigation is at a stage that permits qualitative and semi-quantitative statements about the size and the albedo of the particles that make up the soil and the degree of packing of the particles, or the porosity of the soil.

The purpose of this paper is to use this new photometric function together with the latest and apparently the best data on the photometric properties of Mars to obtain both a complete statement of the whole-disk photometry of the
planet (at one wavelength) and some at least qualitative information on the
effective average properties of the soil.

THE PHOTOMETRIC FUNCTION

The photometric function derived by Meador and Weaver is

\[ \phi(i, \varepsilon, \alpha) = \frac{\cos \theta}{(1 + a_0 + a_1)(\cos \theta + \cos \varepsilon)} \left[ (1 + a_0 \cos \alpha) f(i, \varepsilon, \alpha; a_2) + a_1 (\cos \theta + \cos \varepsilon) \right] \]

(1)

where \( \phi(i, \varepsilon, \alpha) \) is the brightness (normalized to unity at \( i = \varepsilon = 0 \)); \( i \) is the angle of incidence; \( \varepsilon \) is the angle of emission; \( \alpha \) is the phase angle; and \( a_0, a_1, a_2 \) are empirical parameters that contain information about the surface. The factor \( f(i, \varepsilon, \alpha; a_2) \) is given by

\[ f(i, \varepsilon, \alpha; a_2) = e^{\mu - \nu} \]

\[ + \int_0^1 \exp \left\{ \mu - \frac{\nu}{6\pi g} \left[ 3\pi(2g-1)x + 6x\sin^{-1}x + 2(2+x^2)(1-x^2)^{1/2} \right] \right\} dx \]

(2)

where

\[ \mu = \frac{4a_2 (1 + \cos \alpha)}{3 \sin \alpha} \]

\[ \nu = \frac{\pi a_2 (\cos \theta + \cos \varepsilon)[\cos \gamma + \cos (\alpha - \gamma)] \cos \psi}{\sin \alpha \cos \theta \cos \varepsilon} \]

\[ \cos \alpha = \cos \theta \cos \varepsilon + \sin \theta \sin \varepsilon \cos \phi \]

\[ g = \frac{(\cos \theta + \cos \varepsilon) \cos \gamma \cos (\alpha - \gamma) \cos \psi}{[\cos \gamma + \cos (\alpha - \gamma)] \cos \theta \cos \varepsilon} \]

and \( \psi \) is the angle between the surface normal and the scattering plane, \( \phi \) is the azimuthal angle between the plane of incidence and the plane of emission, and \( \gamma \) is the angle between the incidence direction and the projection of the
surface normal on the scattering plane. The complete normalized photometric function is the combination of equation (1) with the f-factor of equation (2).

THE DATA

In reference 1, Thorpe reported the results of a statistical comparison of more than 2500 Mariner 9 television-camera pictures of Mars that was made to determine the integral (as opposed to regional) photometric properties of the planet. He reported his results in the form of the Minnaert parameters \( B_0 \) and \( k \). At this point we remind the reader that the Minnaert function is

\[
\Phi = B_0 (\cos i)^k (\cos \varepsilon)^{k-1}
\]

and that a Minnaert plot is a plot of \( \ln(\Phi \cos \varepsilon) \) against \( \ln(\cos i \cos \varepsilon) \).

Thorpe obtained the values of the Minnaert parameters for a portion of the mission that followed abatement of the dust storm (revolutions of Mariner 9 nos. 220 to 260). The values of these parameters are shown in Thorpe's Table I. The Minnaert coefficient \( B_0 \) has been plotted in the present paper's figure 1. Also shown in figure 1 is the \( B_0 \) obtained from measurements in the laboratory by Meador and Weaver on Colorado basalt (mafic latite porphyry) of size 150 to 300 \( \mu \text{m} \). The values of \( B_0 \) for both curves are relative, as there is no means for normalizing them to the same standard other than by simple matching at some point. The curve for Colorado basalt was adjusted to agree with the Mariner 9 curve at a phase angle of 50°. These curves indicate that the trends in the plot of Thorpe's data are in reasonable agreement with laboratory results.

Thorpe's values for the Minnaert exponent \( k \) have been plotted in figure 2. Also in figure 2 are plots of \( k \) for two sizes of Colorado basalt as determined by Meador and Weaver in their laboratory investigation. They are shown in order to indicate whether the magnitude and the trend of Thorpe's \( k \) appear to
be in reasonable agreement with laboratory results. They do. Also shown in figure 2 are values of \( k \) from Binder and Jones (reference 2) (corrected from an erroneous plot) for regions at and near Chryse, Tharsis, and Amazonis, and values from Young and Collins (reference 3) for Ophir, Aeolis, and Solis Lacus.

**DERIVATION OF PARAMETERS**

The theoretical expression of Meador and Weaver for \( \phi \) and Thorpe's photometric data on Mars were used together to derive numerical values for the parameters \( a_0, a_1, \) and \( a_2 \) in the Meador-Weaver photometric function. An iterative procedure was followed that makes use of the fact, derived by Meador and Weaver, that the Minnaert function is a good approximation to the Meador-Weaver function for three geometries. One of these is a geometry for which the angular deviation from the mirror point is sufficiently small that the corresponding plot is linear. The constant part of the slope of a Minnaert plot of equation (1), which is \( k \), is calculated from the equation

\[
k(a_0, a_1, a_2, \alpha) = \lim_{\xi \to 0} \frac{d[\ln \left( \phi \cos \epsilon \right)]}{d[\ln (\cos i \cos \epsilon)]} = \text{constant}
\]

(3)

where \( \xi = \) angular deviation from mirror geometry. The iterative procedure is:

1. A value for \( a_0 \) between -1 and +1 is selected.
2. Values of \( a_1 \) and \( a_2 \) are determined such that equation (3) agrees with Thorpe's data for \( k \) as a function of \( \alpha \).
3. Thorpe's values for brightness are then normalized to match \( \phi \) for \( \alpha = 50^\circ \) and \( \epsilon = 0^\circ \).
4. Thorpe's values of normalized brightness are graphically compared with the \( \phi \) of equation (1) for the scattering geometries \( \epsilon = 0^\circ \) and varying \( \alpha \).
(Brightnesses corresponding to these geometries have been shown to be well approximated by Minnaert functions.)

5. Then other values of \( a_0 \) are chosen and the entire procedure is repeated until a plot of \( \Phi \) vs \( \alpha \) agrees closely with a plot of Thorpe's normalized brightness vs \( \alpha \).

The results of this matching of theory and experiment are that

\[
\begin{align*}
a_0 &= 0.55 \\
a_1 &= 0.60 \\
a_2 &= 0.18
\end{align*}
\]

To see that equation (1), when used with these values for the parameters, provides good agreement with Thorpe's data, figure 3 may be examined. The \( \Phi \) of equation (1) is shown by the curve, Thorpe's data by the circles. Figure 4 shows the agreement between the effective Minnaert exponent from equation (3) and the values of Thorpe.

The above values of the parameters \( a_n \) together with equation (1) permit calculations of the brightness of Mars for any geometry. They also provide information for use in designing cameras for photography of Mars.

**INTERPRETATION OF VALUES OF PARAMETERS**

The development of the photometric function of Meador and Weaver has lead to the ability to make qualitative, and even approximate quantitative, interpretations of the numerical values of the three parameters in the photometric function in terms of the physical properties of the surface. Because, however, further theoretical and experimental investigation of the exact meaning of each of these parameters is needed, and because of the inadequacy of data about Mars, deductions presented herein about the physical properties of the surface mater-
ial should be considered to be tentative and to be relative rather than absolute indications of properties of the surface material of Mars.

The parameter $a_0$ is permitted to have any value between the limits of -1 and +1. An excess of forward scattering, as could occur by means of transmission through small particles, is indicated by negative values of $a_0$. An excess of backscattering is indicated by positive values of $a_0$. Isotropic scattering is indicated by $a_0$ equal to zero. The determination of $a_0$ from experimental data should yield some concept of mean particle size through the relation of that property to the particle transparency.

In interpreting the values of the parameters for Mars, it is helpful to compare the values with those measured for other materials in the laboratory. In Table I are shown the results of some measurements by Meador and Weaver on two size ranges of each of two different materials. Notice that as the size of the laboratory materials increases the value of $a_0$ increases. The value of $a_0$ for Mars of 0.55 indicates that the particles are sufficiently large that backscattering predominates over forward scattering. If the material on the surface of Mars has values of permittivity, permeability, and electrical conductivity that are not too different from those of the laboratory materials, then the large positive value of $a_0$ implies a mean particle size considerably larger than 225 μm.

The parameter $a_1$ is proportional to the ratio of multiple to single scattering and thus relates to the single-particle albedo. The limits of $a_1$ are 0 and $\infty$, corresponding, respectively, to the limit of very dark, single-scattering materials and to the limit of very bright, Lambert surfaces. Generally, as particle size is decreased some of the radiation that would be absorbed by the larger particles is transmitted through the smaller ones to be scattered.
again. This process enhances multiple scattering and is the explanation for the increase in $a_1$ with decrease in particle size shown in Table I. Since the transmitted and rescattered light partially emerges as reflected light, this process also accounts for the common observation that, for sufficiently small particles, surfaces are brighter for smaller particles of the same composition than they are for larger. The value of $a_1$ of 0.6 indicates only that multiple scattering is a significant contributor to the brightness. Thus any light-scattering model of the Martian surface must take into account both single and multiple scattering to be adequate.

The parameter $a_2$ is a packing factor. It has the value $n(cd)^3$ where $d$ is the mean particle diameter and $n$ is the particle number density. To the extent that radiation passing through the gaps between particles is undiffracted, the value of $c$ lies between 0.5 and 1. We choose for use here the value 0.75. Thus, with $a_2 = 0.18$ for Mars, the mean intercenter particle distance between nearest neighbors, which is the cube root of $1/n$, is indicated to be $4/3$ of the mean diameter of the particles. This result should be regarded as a lower limit because of the expectation that $c$ is an increasing function of wavelength throughout the visible and infrared range.

The lower limit of $a_2$ approaches zero and the upper limit approaches unity. The smaller the value of $a_2$ the more porous is the structure of the surface. That the value of $a_2$ for Mars of 0.18 is approximately midway in the range of $a_2$ for the laboratory materials indicates that the surface of Mars is packed to about the same extent as the surface of the laboratory materials. The laboratory materials were applied by sifting, or airdropping the particles to the surface from a height of about 0.5 m. The degree of packing that occurred is not inconsistent with a Martian surface formed by strong
Aolian forces. A cautionary note, however, is needed. The measure of packing derived herein applies only to that layer of surface material that contributes to photometry—probably only a few particle diameters. The use of radiation of much longer wavelength than that of visible light; e.g., radar wavelengths, is needed to determine the bearing strength of the soil.

OTHER INDICATIONS OF PARTICLE SIZE

Reference 4 reports the results of an investigation of the thermal properties of portions of the surface of Mars by an infrared radiometer that was carried on Mariner 9. Figure 5 of this reference shows that as the effects of the dust storm decreased, the best-fit planet-wide value of the thermal inertia approached the value $0.007 \pm 0.001 \text{ cal cm}^{-2} \text{ K}^{-1} \text{ sec}^{-1/2}$. Figure 11 of the same reference shows that, on the basis of the observations made with the radiometer at a latitude of $-30^\circ$ and with a reasonable assumption about the value of the volume specific heat, the above value of thermal inertia indicates a mean particle diameter of $360 \pm 180 \mu m$. The statement is also made that these results are not strong functions of latitude, date, or the predawn time of day at which the surface temperatures were measured.

There is one further piece of evidence about the size of the surface particles on Mars. As is well known, the minimum value of the speed of the wind on Mars, at the top of the boundary layer, that is required to initiate the motion of surface particles is rather high, about 50 m/s (ref. 5). Instances where such motion occurs seem to be rather frequent. One would expect, therefore, that the initial motion of particles would be that of particles for which the minimum value of wind speed is sufficient and that a significant portion of the
soil is composed of such particles. As is shown in reference 5, the particle
diameter for which the required wind speed is a minimum is about 450 μm.

There is thus no disagreement between other indications of particle size
and the implication from the value of $a_0$ that the size should be considerably
greater than 225 μm.

REFERENCES

1. Thorpe, T. E.: Mariner 9 Photometric Observations of Mars from November
<table>
<thead>
<tr>
<th>Material</th>
<th>Size range, μm</th>
<th>Mean size, μm</th>
<th>a₀</th>
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</tr>
</tbody>
</table>
Figure 1. - Comparison of the Minnaert coefficient for Mars (Mariner 9, Thorpe) and a basalt.
Figure 2. - Comparison of the Minnaert exponent for Mars (derived from Earth-based and Mariner 6, 7, and 9 data) and a basalt.
Figure 3. - Comparison of the normalized brightness of Mars (Mariner 9, Thorpe) and the value given by equation (1).
Figure 4. Comparison of the Minnaert exponent for Mars (Mariner 9, Thorpe) and the value given by equation (3).