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"Can the Compressive Response of Fiber-Reinforced Composites be Modelled by Means of Layered Arrays?"

By

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Can the Compressive Response of Fiber-Reinforced Composites be Modelled by Means of Layered Arrays?*

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Abstract

The validity of some basic premises employed in modelling of the compressive response of fiber-reinforced composites is examined in the present article.

It is shown that the deformation fields associated with the compression of layered media yield inadmissible results when applied to hexagonal and square arrays of circular fibers. Consequently, predictions of compressive strength, which derive from the prevailing representations of layered geometry, may not apply to realistic circumstances.

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Introduction

With very few exceptions the modelling of the compressive response of fiber-reinforced composites is based upon the proposition that fiber and matrix regions can be considered as layered plates. In addition, the deformation of the soft matrix material is assumed to vary linearly between the boundary values imposed upon the matrix layer by the neighboring fibrous plates.

In this article it is shown that the latter assumption yields inacceptable deformation fields within the matrix when employed in conjunction with realistic fiber geometries, such as hexagonal or square arrays of circular fibers. This observation casts serious doubts about the validity of the basic premise employed in the modelling of the compressive response of composites.

Review of Current Modelling

Let X, Y, z denote rectilinear Cartesian coordinates and consider a composite material consisting of periodic layers of fiber and matrix materiais, with a cross section shown in Figure 1.

Let N denote a compressive load acting parallel to the z axis and assume that the composite deforms in a "shear mode"[1] as shown in Figure 2. Denote by W_m and W_f the components of displacement in the direction of z in the matrix and fiber layers, respectively. The basic assumptions employed in the modelling of the compressive response of composite are expressed by

$$W_m(X, z) = \frac{W_f(c, z) - W_f(-c, z)}{2c} X \tag{1}$$

and

$$U_m(X, z) = U_f(c + h, z) \tag{2}$$

Equation (2) derives from the assumption that the transverse translation of fiber and matrix regions is independent of X.

An essential simplification occurs when the fiber is assumed to rotate rigidly about its mid-plane, in which case we have

$$W_f(-c, z) = - W_f(c, z) \tag{3}$$

A most significant consequence of expressions (1), (2) and (3) is that the displacements in the matrix region take the forms

$$W_m(X, z) = \frac{W_f(c, z)}{c} X = F(z)X \tag{1a}$$

and

$$U_m(X, z) = P(z) \tag{2a}$$

Thereby, the shear strain in the matrix depends on the coordinate z alone, namely

$$\gamma_{xz}^{m} = F(z) + P'(z) = Q(z) \tag{4}$$

Obviously, we also have $\gamma_{yz}^{m} = 0$.

It will be shown in the next section that displacements of the forms given in equation (1), or (1a), are inacceptable for realistic fiber geometries because they yield singular values for γ_{yz}^{m}. Those singular values give rise to unbounded terms in the strain-energy of the matrix, and therefore render the layered geometry unsuitable for modelling the compressive response of fiber-reinforced composites.

<u>Hexagonal Fiber Arrays</u>

Consider the hexagonal array of circular fibers with a cross section shown in Figure 3. Let all the rows of fibers parallel to the Y axis undergo rigid rotations which are analogous to the rotations of the fiber layers shown in Figure 2.

To focus ideas consider the representative cross-sectional area depicted in Figure 4, and refer to Cartesian coordines x, y translated to the center of that area.

Let C_1 and C_2 denote the arcs which separate the fibers and matrix regions in the representative area, and divide the matrix into three sub-regions A, B, and A$'$. Also, let a denote the radius of the fibers centered at $\left(\frac{\sqrt{3}}{2}b, \frac{b}{2}\right)$ and $\left(-\frac{\sqrt{3}}{2}b, -\frac{b}{2}\right)$, as shown in Figure 4.

We have

$$\text{On } C_1: \quad \left(x - \frac{\sqrt{3}}{2}b\right)^2 + \left(y - \frac{b}{2}\right)^2 = a^2 \tag{5}$$

hence

$$x(C_1) = \frac{\sqrt{3}}{2}b - \sqrt{a^2 - \left(y - \frac{b}{2}\right)^2} \tag{5a}$$

$$\text{On } C_2: \quad \left(x + \frac{\sqrt{3}}{2}b\right)^2 + \left(y + \frac{b}{2}\right)^2 = a^2 \tag{6}$$

hence

$$x(C_2) = -\frac{\sqrt{3}}{2}b + \sqrt{a^2 - \left(y + \frac{b}{2}\right)^2} \tag{6a}$$

In analogy with the circumstance shown in Figure 2 we have

$$W_f\left(\pm\frac{\sqrt{3}}{2}b, y\right) = 0 \tag{7}$$

However, we now also have

$$W_m\left(\pm\frac{\sqrt{3}}{2}b, y\right) = 0 \tag{8}$$

In addition, in view of the presumed rigid rotations of the fibers we obtain

On C_1:

$$W_m = W_f = W_1 = F(z)\left(x(C_1) - \frac{\sqrt{3}}{2}b\right) = -F(z)\sqrt{a^2 - \left(y - \frac{b}{2}\right)^2} \tag{9a}$$

while on C_2:

$$W_m = W_f = W_2 = F(z)\left(x(C_2) + \frac{\sqrt{3}}{2}b\right) = F(z)\sqrt{a^2 - \left(y + \frac{b}{2}\right)^2} \tag{9b}$$

Assume that the matrix in sub-regions A, B, and A′ deforms in accordance with expressions (1) and (1a). In this case we obtain the following expressions for W_m:

In sub-region A:

$$W_m = W_1 \frac{x + \frac{\sqrt{3}}{2}b}{x(C_1) + \frac{\sqrt{3}}{2}b} = -F(z)\left(x + \frac{\sqrt{3}}{2}b\right)\frac{f(y)}{\sqrt{3}b - f(y)} \tag{10a}$$

In sub-region B:

$$W_m = W_2 + (W_1 - W_2)\frac{x - x(C_2)}{x(C_1) - x(C_2)}$$

$$= F(z)\left[g(y) - (f(y) + g(y))\frac{x - g(y) + \frac{\sqrt{3}}{2}b}{\sqrt{3}\,b - (f(y) + g(y))}\right] \tag{10b}$$

In sub-region A$'$:

$$W_m = W_2 \frac{\frac{\sqrt{3}}{2} b - x}{\frac{\sqrt{3}}{2} b - x(C_2)} = F(z)\left(\frac{\sqrt{3}}{2} b - x\right)\frac{g(y)}{\sqrt{3}\, b - g(y)} \qquad (10c)$$

In equations (9): $f(y) = \sqrt{a^2 - \left(y - \frac{b}{2}\right)^2}$ and $g(y) = \sqrt{a^2 - \left(y + \frac{b}{2}\right)^2}$.

From equations (10) it is noted that, in clear contradiction with expressions (1) or (1a), a linear variation of W_m with x in hexagonally arrayed circular fibers must take the form

$$W_m = F(z)\left(R(y)x + r(y)\right) \qquad (11)$$

Instead of $W_m = F(z)x$.

The qualitative distinction between the displacement fields which correspond to expressions (1) and (10) is exhibited in Figure 5.
By hypothesis $V_m = 0$, therfore upon differentiation of expressions (10) with respect to y, one obtains the following expressions for γ_{yz}^{m} :

In sub-region A:

$$\frac{\partial W_m}{\partial y} = F(z)\, \sqrt{3}\, b\left(x + \sqrt{3}\, b\right)\frac{y - \frac{b}{2}}{f(y)\left(\sqrt{3}\, b - f(y)\right)^2} \qquad (12a)$$

In sub-region B:

$$\frac{\partial W_m}{\partial y} = \frac{\sqrt{3}\, b\, F(z)}{\left[\sqrt{3}\, b - \left(f(y) + g(y)\right)\right]^2}\left\{\frac{y + \frac{b}{2}}{g(y)}\left(f(y) + x - \frac{\sqrt{3}}{2} b\right)\right.$$

$$\left. + \frac{y - \frac{b}{2}}{f(y)}\left(\frac{\sqrt{3}}{2} b + x - g(y)\right)\right\} \qquad (12b)$$

Since $f\left(\frac{b}{2} - a\right) = g\left(a - \frac{b}{2}\right) = 0$, expression (12b) yields unbounded values, singular of power 1/2, along the lines $y = \pm\left(a - \frac{b}{2}\right)$ which separate sub-region B form subregions A and A'. The above singular terms result in a non-intergrable (power −1) singular expression for the strain-energy within the matrix. Consequently, the basic premises employed in the "layered formulation" in equations (1) or (1a) fail to provide a meaningful approximation to the response of fiber-reinforced composites.

It should be noted that Figures 4 and 5 correspond to the case $a > \frac{b}{2}$, namely for fiber volume fractions $V_f > 0.25$.

The case of $V_f < 0.25$ corresponds to the configuration shown in Figure 6, in which case the expressions for W_m in sub-regions A and A' are still given by equations (10a) and (10c) while in region B $W_m = 0$. In this case we again obtain singular values for $\frac{\partial W_m}{\partial y}$ along $y = \pm\left(\frac{b}{2} - a\right)$ with unbounded strain energy.

Square Fiber Arrays

Square fiber arrays correspond to the configuration shown in Figure 6, except that the fibers are centered at points $\left(\frac{b}{2}, \frac{b}{2}\right)$ and $\left(\frac{-b}{2}, \frac{-b}{2}\right)$. The displacement W_m in regions' A and A' is given by equations (10a) and (10c) upon replacing $\frac{\sqrt{3}}{2}b$ by $\frac{b}{2}$.

In addition W_m vanishes region B. The singular nature of $\frac{\partial W_m}{\partial y}$ and the unboundedness of the strain-energy follow in the same manner as in hexagonal arrays with $V_f < 0.25$.

Conclusions

It has been shown that the replacement of hexagonal or square arrayed fiber geometries by an equivalent, plate-like, layered geometry results in a displacement field which is inappropriate to determine the compressive strength of fiber-reinforced composites. A model based upon a plate-like geometry obscures the three dimensional features of the displacement field and may give highly unrealistic predictions.

Reference:

[1] B. W. Rosen: "Mechanics of Composite Strengthening" in Fiber Composite Materials, ASM, 1965, pp. 37-75.

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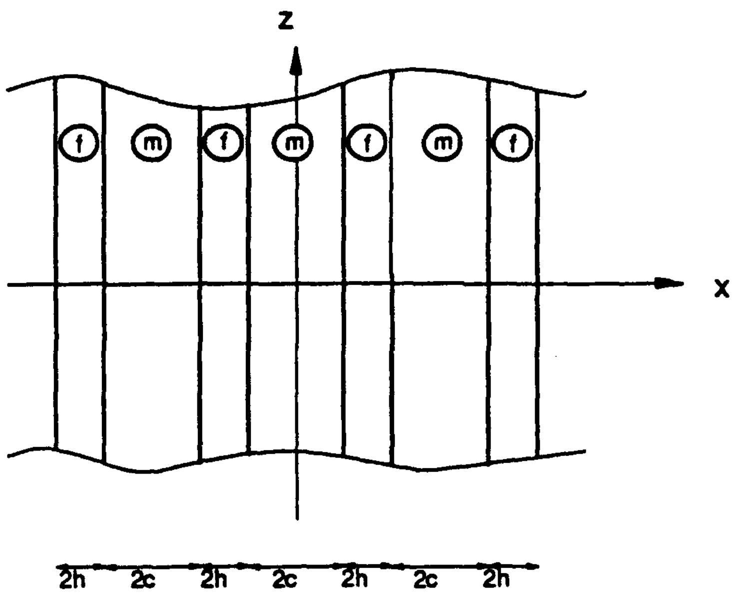


Fig. 1. A fiber reinforced composite medelled as a two dimensional lamellar region consisting of fiber and matrix plates.

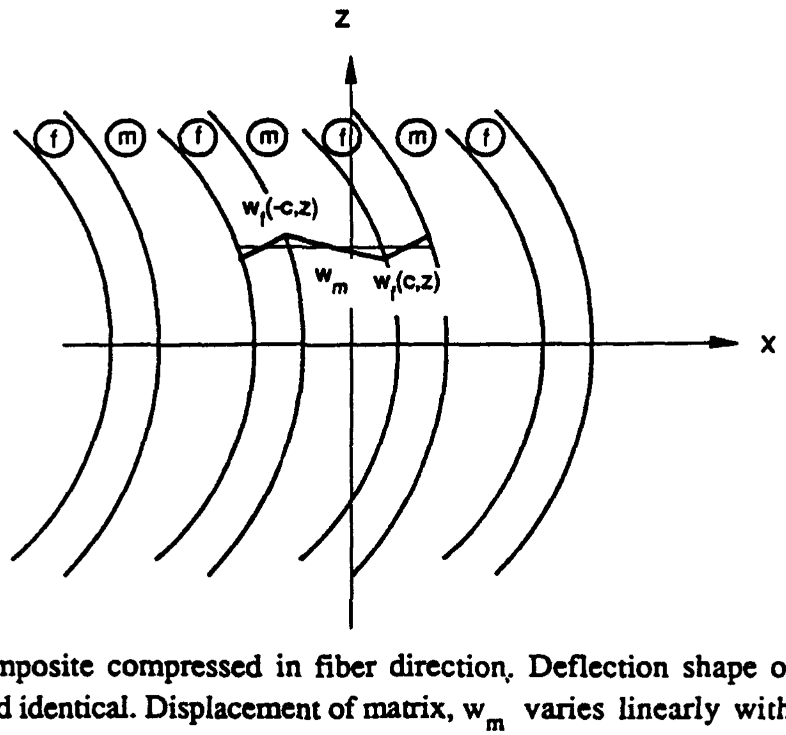


Fig. 2. Composite compressed in fiber direction. Deflection shape of all fibers are assumed identical. Displacement of matrix, w_m varies linearly with respect to x.

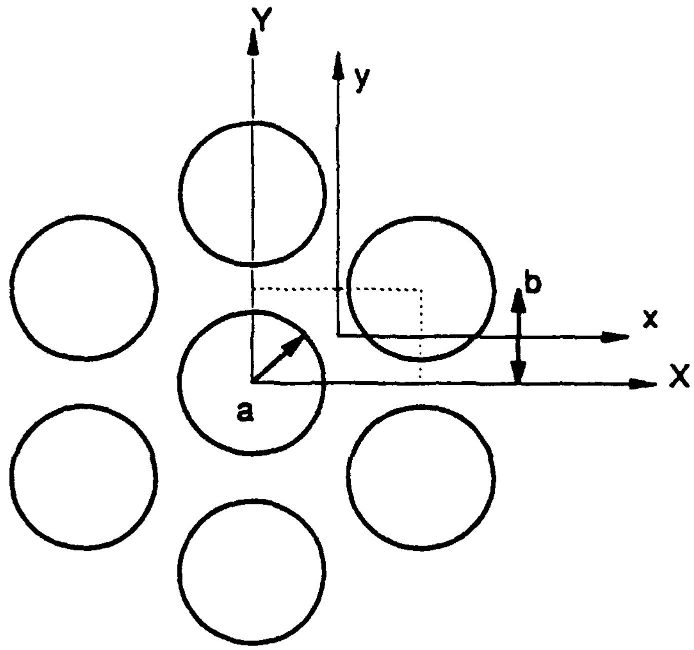
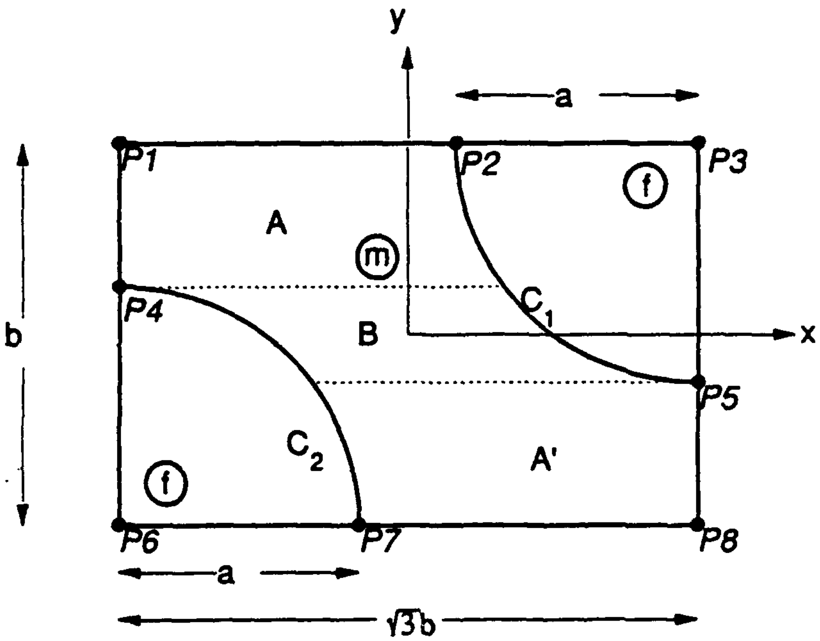
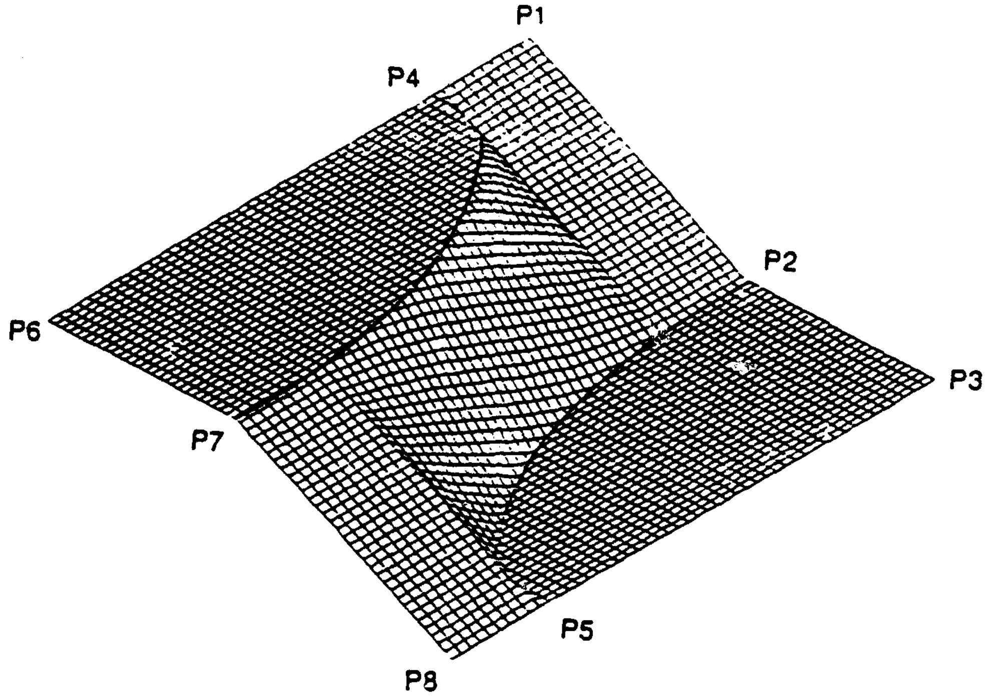


Fig. 3. Hexagonally arrayed fibers in composite. Part in dotted rectangle is representative volume element.

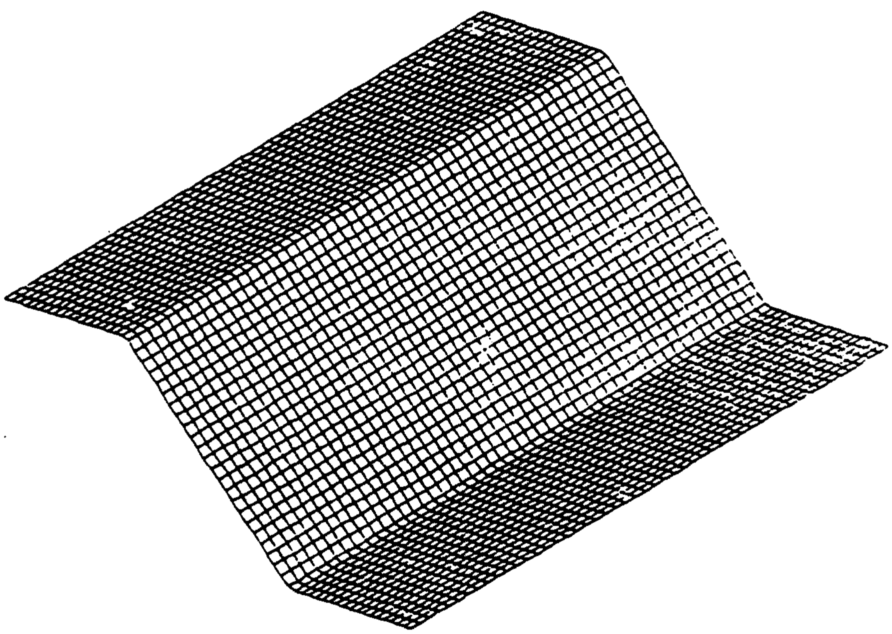


P3; $\left(\frac{\sqrt{3}b}{2}, \frac{b}{2}\right)$ P4; $\left(-\frac{\sqrt{3}b}{2}, a-\frac{b}{2}\right)$ P5; $\left(\frac{\sqrt{3}b}{2}, \frac{b}{2}-a\right)$ P6; $\left(-\frac{\sqrt{3}b}{2}, -\frac{b}{2}\right)$

Fig. 4. Representative volume element for hexagonally arrayed composite with $V_f >$ 0.25.

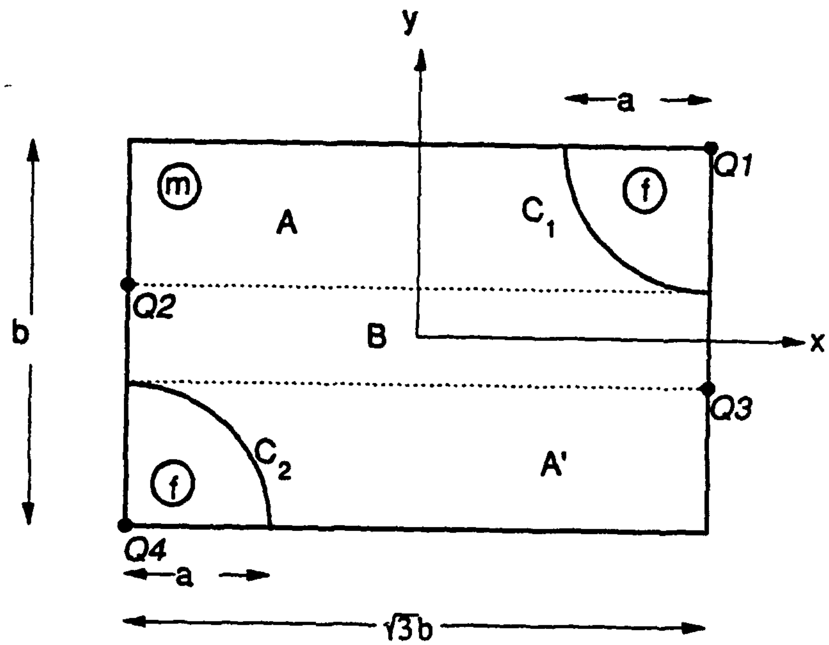


(a)



(b)

Fig. 5. Out-of-plane displacement, w, of (a) RVE in Fig. 4 under compression (Pi (i=1,...,8) indicates location corresponding to Fig. 4), and (b) two dimensional model in Fig. 2.



Q1; $\left(\frac{\sqrt{3}b}{2}, \frac{b}{2}\right)$ Q2; $\left(-\frac{\sqrt{3}b}{2}, \frac{b}{2} - a\right)$ Q3; $\left(\frac{\sqrt{3}b}{2}, a - \frac{b}{2}\right)$ Q4; $\left(-\frac{\sqrt{3}b}{2}, -\frac{b}{2}\right)$

Fig. 6. Representative volume element for hexagonally arrayed composite with $V_f <$ 0.25.