Significant progress was made in a number of aspects of nonlinear and stochastic systems. Important contributions in the adaptive control of finite state Markov chains under partial observations were solved, and significant progress was made along more general directions. A project in surveying the literature on the ergodic control problem for discrete-time controlled Markov processes was completed. This work presented a comprehensive account of the considerable research on this problem over the past three decades. Further extending this effort, we embarked on writing a research monograph entitled "Ergodic Control of Markov Chains and Stochastic Games" intended for publication as a volume in the series of "Applications of Mathematics" by Springer-Verlag. A controlled switching diffusion model was developed to study the hierarchical control of flexible manufacturing systems. This study led to significant results in optimal control of stochastic hybrid system in both the discounted and average cost cases. In the area of deterministic nonlinear systems, numerical aspect of approximation linearization were investigated.
Significant progress was made in a number of aspects of nonlinear and stochastic systems. Important contributions in the adaptive control of finite state Markov chains under partial observations were solved, and significant progress was made along more general directions. A project in surveying the literature on the ergodic control problem for discrete-time controlled Markov processes was completed. That work presents a comprehensive account of the considerable research on this problem over the past three decades. Further extending this effort, we embarked on writing a research monograph entitled "Ergodic Control of Markov Chains and Stochastic Games," intended for publication as a volume in the series "Applications of Mathematics" by Springer-Verlag. A controlled switching diffusion model was developed to study the hierarchical control of flexible manufacturing systems. This study led to significant results in optimal control of stochastic hybrid systems in both the discounted and average cost cases. In the area of deterministic nonlinear systems, numerical aspects of approximate linearization were investigated.
1. SUMMARY OF RESEARCH PROGRESS AND RESULTS

During the two years supported by this grant, we have made significant progress both in areas we proposed to investigate and in related areas. In this section, we summarize the progress in those areas that have resulted in publications.

1.1. Stochastic Control of Markov Processes.

We have continued our research program in adaptive estimation and control problems for stochastic systems involving either incomplete (or noisy) observations of the state. The first class of problems we have been studying involves finite state Markov chains with incomplete state observations and unknown parameters; in particular, we have studied certain classes of quality control, replacement, and repair problems. We have in the past considered a quality control problem in which a system, such as a manufacturing unit or computer communications network, can be in either of two states: good or bad. A finite set of control actions are available to the decision-maker. Under these actions, the system is either subject to Markovian deterioration, or is restored to the good state. The problem is modeled as a partially observed Markov decision process (POMDP). Furthermore, we assume that deterioration of the system depends on an unknown parameter, namely the probability of the state going from the good to the bad state in one time epoch. The adaptive stochastic control problem for this class of systems is fairly difficult, because the presence of feedback causes the system transitions to depend on the parameter estimates and introduces discontinuities. In [5] and [7], we have analyzed algorithms for this quality control problem, and also presented a general framework for the study of optimality of adaptive policies. Using the ODE method, we show that two algorithms, one based on maximum likelihood and another based on prediction error, converge almost surely to the true parameter value. In addition, we modify the method of Shwartz and Makowski to prove optimality of the resulting certainty equivalent adaptive policy, assuming only the existence of some sequence of parameter estimates converging almost surely to the true parameter value. Again, the discontinuities and partial observations in this problem preclude the direct use of previously existing methods, but we have been able to generalize the method to problems such as this. Also, we have avoided the very strong standard assumption that the parameter estimates converge almost surely to the true parameter value.
under any stationary policy. In more general directions, convexity properties were explored in [4] and [8].

A project in surveying the literature on the ergodic control problem for discrete-time control Markov processes was completed [3]. This was a major effort which puts together a comprehensive account of the considerable research on this problem over the past three decades. Our exposition ranges from finite to Borel state and action spaces, and includes a variety of methodologies to find and characterize optimal policies. We have included a brief historical perspective of the research efforts in this area and have compiled a substantial bibliography. In the process we have identified several important questions which are still left open to investigation.

We embarked on writing a research monograph entitled “Ergodic Control of Markov Chains and Stochastic Games,” intended for publication as a volume in the series “Applications of Mathematics” by Springer-Verlag. The principal investigator spent a two-month period as a visiting faculty at the Indian Institute of Science, Bangalore, where a major portion of this effort was completed.

Some interesting results on the vanishing discount method for partially observed Markov chains were obtained in [11]. The vanishing discount method is crucial in establishing solutions for the average cost optimality equation in controlled Markov chains. In our work we make use of generalized limits of functions to extend the results of Platzman and Ross. We study the cases of a finite state space with compact actions as well as countable state space with finite actions.

1.2. Hybrid Stochastic Systems.

A major part of our efforts was devoted to the study of hybrid stochastic systems. This was motivated from control problems of systems exhibiting multiple modes or failure modes, including the hierarchical control of flexible manufacturing systems. A flexible manufacturing system (FMS) consists of a number of workstations, with each workstation having a set of identical machines. The model used involves a hybrid process in continuous time whose state is given by a pair $\langle X(t), S(t) \rangle$. Here $X(t)$ denotes the downstream buffer stock of parts, which may have a negative value to indicate a backlogged demand. The continuous component $X(t)$ is governed by a controlled diffusion process with a drift vector which depends on the discrete component $S(t)$. Thus, $X(t)$ switches from one diffusion path to another as the discrete component $S(t)$ jumps from one state to another. On the other hand, the discrete
component $S(t)$, denoting the number of operational machines, is influenced by the inventory size and production scheduling, and can also be controlled by various decisions such as produce, repair, replace, etc. Hence, $S(t)$ evolves as a "controlled Markov chain" with a transition matrix depending on the continuous component. This model motivates the study of a stochastic optimization problem in a more abstract setting which is manifested in numerous other situations. For example, it is encountered in a hybrid model proposed for the study of dynamic phenomena in large scale interconnected power networks, in macroeconomic problems and in dynamic renewal problems in general. Our treatment of the optimization problem [2], [9] was based on a convex analytic approach, which is interesting in its own right and would be more flexible and powerful for certain other purposes, e.g., the pathwise average cost problem or problems with several constraints, where the analytic approach does not seem to be amenable.

Also, the study of the ergodic cost problem led to a number of very significant results in [6], [10], [12] and [13]. We have analyzed the optimal control of switching diffusions with pathwise average cost criterion. Under certain conditions we have established the existence of a stable Markov nonrandomized policy which is a.s. optimal in the class of all admissible policies. Also, the existence of a unique solution of the associated HJB equations is established in a certain class, and the optimal policy is characterized as a minimizing selector of an appropriate Hamiltonian. We have applied our results to a manufacturing model and have obtained an optimal production policy which is of hedging point type. By studying the recurrence and ergodic properties of switching diffusions we have obtained two new results in partial differential equations viz. the maximum principle and Harnack's inequality for a uniformly elliptic system.

In [15], we study a parameterized linear system perturbed by white noise. The parameters are randomly switching from one state to the other and are modeled as a finite state Markov chain; the values of the parameter and the state of the linear system are assumed to be known to the controller. The cost function is quadratic. Under certain conditions, we find a linear feedback control which is almost surely optimal for the pathwise average cost over the infinite planning horizon.

In [14] we investigate the weak and strong controllability of a class of stochastic systems, with a bounded Lipschitzian nonlinearity. The concepts of weak and strong
controllability are natural generalizations of nondegeneracy and positive recurrence — concepts well known in the theory of stochastic processes. Our results extend those known for linear problems and are stated in terms of verifiable conditions.

1.3. Stochastic Approximations.

The Ordinary Differential Equation (ODE) method is one of the most powerful tools for the study of convergence of stochastic approximations. The objective in this method is to associate to a given algorithm an "averaged" system described by a differential equation, through which the asymptotic behavior of the algorithm can be investigated. Quite often, the stochastic problem is such that the associated ODE has a discontinuous right hand side, rendering the analysis problematic. This situation is not adequately covered in the existing literature on the ODE method. The main reason that discontinuous dynamics are not treated in the stochastic averaging literature is due to the limitations inherited by the desire to apply the Ascoli-Arzela Theorem in the Picard-type iterations of the shifted piecewise-linear interpolants of the process. In [16] we obtain convergence results for the case of a Markovian noise with countable state. Both state-independent and state-dependent noises are considered.

1.4. Nonlinear Systems.

In the area of nonlinear deterministic systems we have investigated numerical issues of approximate linearization. Approximate linearization of nonlinear systems becomes important for systems where the nonlinearities are severe enough that exact linearization fails. An approximate method that linearizes the system up to a certain order was originally proposed by A. Krener. Since less restrictive conditions are required for approximate linearization, this technique offers the means of enlarging the class of nonlinear systems to which linearizing techniques are applicable. In [1] we studied the problem via differential forms. We show that this approach results in substantial computational savings. Furthermore the method is constructive and offers a simple solution to the problem.

1.5. Other Related Research.

We have started working on a particular state-estimation problem involving interconnected power systems. The objective here is to develop a systematic procedure for identifying the best measurement points to be used in estimating the location
of harmonic sources in power systems. Harmonic distortion in power-distribution systems is reaching detrimental levels and causes problems such as overheating and failure of equipment, malfunction of protective equipment, nuisance tripping of sensitive loads, and interference with communication networks. From an analytical point of view, we posed the problem of selecting the measurement locations that will minimize the expected value of the sum of squares of differences between estimated and true parameter variables. The key features of our contribution to the problem so far, are a) a model describing how the measurements are related to the variables to be estimated, (b) the criteria to be used in the estimation process, (c) a mathematical model of the uncertainties present in the problem, and (d) structural properties of the best set of measurements. Among the findings, thus far, we have been able to demonstrate why capacitor busses normally serve as the best locations for instrument placement. In addition we developed a simple sequential procedure for identifying the best measurement points and have shown through examples that it is nearly-optimal [17]. Finally, various generic network geometries have been studied analytically, and the model has been augmented to take into account the effect of switching capacitors and other uncertainties.
2. PUBLICATIONS

Journal Publications


3. RESEARCH PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

Principal Investigators.
(1) Aristotle Arapostathis

Research Assistants.
(1) Younseok Choo
(2) Hyungsuk Yoo

Other Professional Personnel.
(1) T. K. Konstantopoulos, Associate Investigator
(2) Mrinal K. Ghosh, Postdoctoral Fellow