WORLD HEIGHT SYSTEM
DEFINITION AND IMPLEMENTATION

Richard H. Rapp
Nagarajan Balasubramania

The Ohio State University
Department of Geodetic Science and Surveying
Columbus, OH 43210

December 1993

Final Report
9 September 1992 - 11 December 1993

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

PHILLIPS LABORATORY
Directorate of Geophysics
AIR FORCE MATERIEL COMMAND
HANSCOM AIR FORCE BASE, MA 01731-3010
94-05755
Best Available Copy
This report has been reviewed by the ESC Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

qualified requestors may obtain additional copies from the Defense Technical Information Center (DTIC). All others should apply to the National Technical Information Service (NTIS).

If your address has changed, if you wish to be removed from the mailing list, or if the addressee is no longer employed by your organization, please notify PL/TSI, 29 Randolph Road, Hanscom AFB, MA 017310-3010. This will assist us in maintaining a current mailing list.

Do not return copies of this report unless contractual obligations or notices on a specific document requires that it be returned.
WORLD HEIGHT SYSTEM
DEFINITION AND IMPLEMENTATION

Richard H. Rapp
Nagarajan Balasubramania

Ohio State University
Department of Geodetic Science and Surveying
Columbus, OH 43210

December 1993

Final Report

ERRATA

Cover: In the title, the word "Defintion" should read "Definition."

PHILLIPS LABORATORY
Director of Geophysics
AIR FORCE MATERIEL COMMAND
HANSCOM AIR FORCE BASE, MA 01731-3010
# World Height System Definition and Implementation

Richard H. Rapp and Nagarajan Balasubramania

Department of Geodetic Science and Surveying
The Ohio State University
Columbus, Ohio 43210

Phillips Laboratory
29 Randolph Road
Hanscom Air Force Base, MA 01731-3010

Contract Manager: Anestis Romaides/GPEG

Approved for public release; distribution unlimited

Height systems of the world are inconsistent at the ±1 m level of accuracy. This report discusses two procedures that attempt to determine an ideal equipotential surface for heights and the connection of regional vertical datums. One method uses 18 stations whose precise geocentric coordinates are known along with the orthometric or mean sea level height with respect to a regional vertical datum. This information is combined with gravity data (for geoid undulation determination) in a general least squares adjustment. The results include the separation between a reference ellipsoid and the ideal (geoid) surface as well as the connection of the regional datums to the geoid. One finds, for example, the origin of the NAVD88 system (U.S.) is 40 ±4 cm below the geoid while the ODN (Newlyn) (England) is 26 ±23 cm below the geoid.

A second procedure to examine the difference between vertical datums used Doppler positioned stations on various vertical datums. Using a degree 360 potential coefficient model, for geoid undulation determination, the separation between several datums was determined. For example, the NN datum used in Germany appears to be 91 cm higher than the reference surface of the ODN (Newlyn) datum used in England.

The methods and results described in this report form a foundation for the development of a world height system.
# TABLE OF CONTENTS

1. INTRODUCTION ........................................................................................................... 1

2. RESEARCH ACTIVITIES AND FINDINGS ....................................................................... 1
   2.1 Research Activities of N. Balasubramania .................................................................. 1
       2.1.1 Studies in Height Bias Model Conversion .......................................................... 2
       2.1.2 Studies in the World Height System Definition .................................................. 10
   2.2 Research Activities of R.H. Rapp ............................................................................. 19
       2.2.1 Separation Between Reference Surfaces of Selected Vertical Datums ................ 20

3. PERSONNEL .................................................................................................................. 27

4. OTHER ACTIVITIES ...................................................................................................... 27

5. TRAVEL .......................................................................................................................... 27

6. PRESENTATIONS .......................................................................................................... 27

7. FUTURE ACTIVITIES .................................................................................................... 28

REFERENCES .................................................................................................................... 28
1. Introduction

This report is the final report under this project that was started on September 11, 1992. The research has been carried out under Grant F19628-92-K-0026 from the Phillips Laboratory at the Hanscom AFB, MA. The funding was awarded in response to a proposal submitted to the Lab on May 21, 1992. The original ending date of the grant was 8 August 1993. It was later extended to 11 December 1993.

There were two main areas of study that were outlined in the proposal. These were:

A. Height Bias Model for Conversion of GPS/Geoid Information to Local Vertical Datum
   1. Select two geographic regions in the U.S. and/or Canada for study. Acquire needed data from appropriate organizations.
   2. Develop height bias model after examining height inconsistencies assuming all data is correct.
   3. Carry out estimation of height bias model for two cases: one using absolute height determinations and another using relative height determinations.
   4. Discuss applications of modeling bias function with dependency on data distribution.
   5. Define recommended procedures for bias modeling under different data scenarios.

B. World Height System Definition Through Space Positioning
   1. Extend global simulation studies to incorporate local areas in which no global network stations are available.
   2. Implement a test calculation to provide a vertical datum connection between two well defined vertical datums. We would propose that the two systems be UELN73 and NAVD88.
   3. Develop recommendations for the acquisitions and use of data for the determination of a World Height System.

During this project time period, substantial research has been carried out for the two main areas of study noted above. In addition, a study was carried out that used a large station position data set to attempt a vertical datum analysis. The results of these studies will be reported in this final technical report of the project.

2. Research Activities and Findings

The research under this project was carried out by Richard H. Rapp and Nagarajan Balasubramania. Some of this research followed ideas developed in the report by Rapp and Balasubramania (1992). For convenience, this final report has been divided into sections prepared by each author.

2.1 Research Activities of N. Balasubramania

In this project research has been carried out in the two areas described in Section 1. The research in the topics of Section A were previously described in Status Report No. 1 of this project which was prepared in January 1993. For completeness, the material presented in the previous report will also be presented here. The research under Item B has not been previously reported so that the information to be given here is entirely new.
2.1.1 Studies in Height Bias Model Conversion

The initial studies in the project were related to the development of a height bias model using GPS heights, leveling information, and precise geoid undulation information. The research was concentrated mainly on the development of a height bias model that can facilitate the conversion of geoid/GPS information to local vertical datum. Since the regional vertical datums that are realized and used today do not have the ideal geoid as reference surface, the orthometric heights computed on a regional vertical datum will be different from the orthometric height that is computed with respect to an ideal geoid. The difference may simply be a bias or it may be a difference driven by a number of factors including distortion in a local vertical network (Rapp, 1992). Assuming a linear relation between the orthometric height computed in a local vertical datum $H_D$ and the orthometric height $H_I$ computed using the relation,

$$H_I = h - N$$  \hspace{1cm} (1)

where $h$ is the ellipsoidal height of the station measured along the ellipsoidal normal passing through the station and $N$ the geoidal undulation which is the separation between the defined ellipsoid and the ideal geoid, we can write

$$H_D = H_I + c(\phi, \lambda)$$  \hspace{1cm} (2)

where $c(\phi, \lambda)$ can be considered as a correction term, dependent on position in the datum, to correct orthometric heights referred to an ideal reference surface to the regional datum orthometric height. If sufficient points are available, the value of ‘$c$’ can be mapped and represented in some functional form depending on the size of the area and the behavior of ‘$c$’.

Development of an height-bias model or setting procedures for mapping the bias function value ‘$c$’ in some functional form requires GPS observations (height determination specifically), precise geoid undulation model and consistent orthometric heights in a local vertical datum. Currently such information is available in the United States, Canada and in some part of Europe. On our request, the Geodetic Survey of Canada (A. Mainville) provided the GPS data for four traverses: South Alberta (106 stations), North Alberta (51 stations), Central Alberta (52 stations), and Great Slave Lake area (91 stations). In addition, the orthometric heights for all the GPS stations in the Canadian Geodetic Vertical Datum (CGVD-28) and the high resolution geoid model GSD 91 were obtained. For the United States, the National Geodetic Survey (D. Milbert) provided the data for GPS traverses in Florida (52 stations), Virginia, G105 (62 stations), and in Oregon (44 stations). They also supplied the orthometric heights for all the stations in National Geodetic Vertical Datum NGVD-29 and the high resolution Geoid Height model GEOID90 (Milbert, 1991b).

Based on the data received, efforts to develop a suitable height bias model were made during the period under report. The procedure followed is as follows:

1. Since the GPS station coordinates in the Canadian GPS traverses were given in the WGS 84 coordinate system, they were first transformed to the ITRF 90 geocentric coordinate system using the transformation parameters given in Boucher and Altamimi (1991), to make the origin of the coordinate system compatible with the geopotential model used. In the case of GPS traverses in the United States, the station coordinates were already in the ITRF 90 geocentric system as reported by Milbert (1991a) and hence no transformation was required. The geoidal undulation at these stations were then interpolated using the respective geoid models.
2. The orthometric height bias ‘c’ was then computed using the relation,

\[ c(\phi, \lambda) = (h_{\text{GPS}} - N_{\text{Geoid Model}}) - H_{\text{local datum}} \]  

(3)

Statistical analysis of the height bias values for US and Canadian traverses are given in Table 1. An example of an orthometric height bias table is given in Table 2 for the Slave Lake Area.

Table 1: Statistics of the Difference in Orthometric Heights Derived from GPS/GSD91 and CGVD28 Heights in case of Canadian Traverses and from GPS/GEOID90 and NGVD29 Heights for GPS Traverses in the United States. Units are in cm.

<table>
<thead>
<tr>
<th>Traverse</th>
<th>United States</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oregon</td>
<td>Florida</td>
</tr>
<tr>
<td># of Stations</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>52</td>
</tr>
<tr>
<td>Min. Ht. Bias Value</td>
<td>-36</td>
<td>-77</td>
</tr>
<tr>
<td>Max. Ht. Bias Value</td>
<td>+58</td>
<td>+49</td>
</tr>
<tr>
<td>Mean</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>22</td>
<td>39</td>
</tr>
</tbody>
</table>

3. Three different procedures were attempted to map the height bias function value ‘c’.

They were:

- global surface fitting using deterministic polynomial functions,
- minimum curvature fitting using cubic spline functions, and
- height bias modelling using least squares collocation technique.

Since the polynomial functions provide unpredictable errors in the interpolated values away from the stations and also the minimum curvature technique do not provide the estimated accuracy of predicted height bias function values, the least squares collocation technique was attempted for modelling the height bias function value ‘c’. Assuming a correlation length of 40 km and using a second-order Markov model covariance function fitted to local data, the height bias value along with its estimated accuracy were predicted for any other location in the area using least squares collocation technique. Contour plots showing predicted height bias function in the area and its estimated accuracy one each for U.S. traverse (in Oregon) and Canadian traverse (in South Alberta) are shown in Figures 1 (a, b) and 2 (a, b) respectively with a contour interval of 5 cm. From Figures 1(b) and 2(b), we see that the estimated accuracy of predicted height bias function is of the order of 20 cm if we have well distributed GPS stations in the area. The advantage of developing such an height bias model is that, by knowing the orthometric height with respect to an ideal geoid using Geoid/GPS observations using the relation (1), its orthometric height in the local vertical datum can be computed using the interpolated bias function ‘c’. Also, the plot showing the estimated accuracy of predicted ‘c’ values, serves as a reliability diagram intimating the user the accuracy of the predicted ‘c’ value and hence the orthometric height in local vertical datum.

Considering Figure 1a, one sees a clear large signature that is associated with the higher elevations in the western part of Oregon. These larger bias terms may be associated with geoid undulation errors in Geoid 90 associated with the indirect effect.
Table 2: Computation of Orthometric height bias function in Great Slave Lake Area, Canada. Heights in meters

<table>
<thead>
<tr>
<th>Lat (deg)</th>
<th>Lon (deg)</th>
<th>Ellip.ht (h)</th>
<th>geoid.ht (N)</th>
<th>(h-N)</th>
<th>Ortho.ht (H)</th>
<th>Ht.error (in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.475</td>
<td>245.559</td>
<td>183.350</td>
<td>-25.905</td>
<td>209.255</td>
<td>209.229</td>
<td>2.5</td>
</tr>
<tr>
<td>60.038</td>
<td>243.117</td>
<td>279.969</td>
<td>-19.840</td>
<td>299.809</td>
<td>299.902</td>
<td>-9.2</td>
</tr>
<tr>
<td>60.800</td>
<td>243.408</td>
<td>244.827</td>
<td>-21.935</td>
<td>266.762</td>
<td>267.028</td>
<td>-26.5</td>
</tr>
<tr>
<td>60.709</td>
<td>244.995</td>
<td>216.481</td>
<td>-24.809</td>
<td>241.290</td>
<td>241.269</td>
<td>2.0</td>
</tr>
<tr>
<td>60.038</td>
<td>243.117</td>
<td>279.969</td>
<td>-19.840</td>
<td>299.809</td>
<td>299.902</td>
<td>-9.2</td>
</tr>
<tr>
<td>60.800</td>
<td>243.408</td>
<td>244.827</td>
<td>-21.935</td>
<td>266.762</td>
<td>267.028</td>
<td>-26.5</td>
</tr>
<tr>
<td>60.709</td>
<td>244.995</td>
<td>216.481</td>
<td>-24.809</td>
<td>241.290</td>
<td>241.269</td>
<td>2.0</td>
</tr>
<tr>
<td>60.038</td>
<td>243.117</td>
<td>279.969</td>
<td>-19.840</td>
<td>299.809</td>
<td>299.902</td>
<td>-9.2</td>
</tr>
<tr>
<td>60.800</td>
<td>243.408</td>
<td>244.827</td>
<td>-21.935</td>
<td>266.762</td>
<td>267.028</td>
<td>-26.5</td>
</tr>
<tr>
<td>60.709</td>
<td>244.995</td>
<td>216.481</td>
<td>-24.809</td>
<td>241.290</td>
<td>241.269</td>
<td>2.0</td>
</tr>
<tr>
<td>60.038</td>
<td>243.117</td>
<td>279.969</td>
<td>-19.840</td>
<td>299.809</td>
<td>299.902</td>
<td>-9.2</td>
</tr>
<tr>
<td>60.800</td>
<td>243.408</td>
<td>244.827</td>
<td>-21.935</td>
<td>266.762</td>
<td>267.028</td>
<td>-26.5</td>
</tr>
<tr>
<td>60.709</td>
<td>244.995</td>
<td>216.481</td>
<td>-24.809</td>
<td>241.290</td>
<td>241.269</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>62.759</td>
<td>244.193</td>
<td>158.507</td>
<td>-23.318</td>
<td>181.825</td>
<td>181.830</td>
<td>-0.4</td>
</tr>
<tr>
<td>60.456</td>
<td>245.675</td>
<td>229.981</td>
<td>-25.583</td>
<td>255.564</td>
<td>255.514</td>
<td>4.9</td>
</tr>
<tr>
<td>60.283</td>
<td>245.955</td>
<td>245.985</td>
<td>-27.566</td>
<td>271.699</td>
<td>271.624</td>
<td>7.4</td>
</tr>
<tr>
<td>61.185</td>
<td>246.301</td>
<td>132.292</td>
<td>-20.580</td>
<td>158.818</td>
<td>159.663</td>
<td>19.4</td>
</tr>
<tr>
<td>61.313</td>
<td>242.400</td>
<td>138.238</td>
<td>-20.420</td>
<td>287.709</td>
<td>287.873</td>
<td>-16.3</td>
</tr>
<tr>
<td>60.106</td>
<td>243.242</td>
<td>269.264</td>
<td>-20.821</td>
<td>290.082</td>
<td>290.243</td>
<td>-16.0</td>
</tr>
<tr>
<td>60.343</td>
<td>243.555</td>
<td>269.673</td>
<td>-21.223</td>
<td>290.896</td>
<td>291.060</td>
<td>-16.3</td>
</tr>
<tr>
<td>60.625</td>
<td>243.939</td>
<td>190.459</td>
<td>-22.742</td>
<td>213.201</td>
<td>213.289</td>
<td>-8.7</td>
</tr>
<tr>
<td>60.684</td>
<td>244.067</td>
<td>170.230</td>
<td>-22.473</td>
<td>193.330</td>
<td>193.446</td>
<td>-11.6</td>
</tr>
<tr>
<td>60.739</td>
<td>244.482</td>
<td>165.580</td>
<td>-24.253</td>
<td>191.573</td>
<td>191.611</td>
<td>-3.8</td>
</tr>
<tr>
<td>61.039</td>
<td>246.312</td>
<td>135.807</td>
<td>-27.525</td>
<td>163.332</td>
<td>163.270</td>
<td>6.1</td>
</tr>
<tr>
<td>60.838</td>
<td>244.220</td>
<td>138.667</td>
<td>-27.710</td>
<td>162.377</td>
<td>162.663</td>
<td>-28.6</td>
</tr>
<tr>
<td>60.000</td>
<td>247.395</td>
<td>197.123</td>
<td>-27.534</td>
<td>224.657</td>
<td>224.418</td>
<td>23.8</td>
</tr>
<tr>
<td>60.047</td>
<td>247.190</td>
<td>203.371</td>
<td>-27.345</td>
<td>230.716</td>
<td>230.463</td>
<td>25.2</td>
</tr>
<tr>
<td>60.046</td>
<td>246.745</td>
<td>242.739</td>
<td>-26.620</td>
<td>269.359</td>
<td>269.141</td>
<td>21.7</td>
</tr>
<tr>
<td>60.329</td>
<td>245.851</td>
<td>241.294</td>
<td>-26.623</td>
<td>266.917</td>
<td>266.865</td>
<td>5.2</td>
</tr>
<tr>
<td>60.670</td>
<td>245.404</td>
<td>232.053</td>
<td>-25.382</td>
<td>257.435</td>
<td>257.380</td>
<td>5.5</td>
</tr>
<tr>
<td>60.698</td>
<td>245.256</td>
<td>230.009</td>
<td>-25.197</td>
<td>255.206</td>
<td>255.157</td>
<td>4.8</td>
</tr>
<tr>
<td>60.825</td>
<td>245.629</td>
<td>190.050</td>
<td>-26.062</td>
<td>216.112</td>
<td>216.110</td>
<td>0.2</td>
</tr>
<tr>
<td>60.865</td>
<td>245.759</td>
<td>179.217</td>
<td>-26.381</td>
<td>205.598</td>
<td>205.594</td>
<td>0.4</td>
</tr>
<tr>
<td>60.928</td>
<td>245.883</td>
<td>165.257</td>
<td>-26.669</td>
<td>191.926</td>
<td>191.923</td>
<td>0.2</td>
</tr>
<tr>
<td>60.986</td>
<td>246.180</td>
<td>139.620</td>
<td>-27.235</td>
<td>166.855</td>
<td>166.851</td>
<td>0.4</td>
</tr>
<tr>
<td>61.132</td>
<td>246.370</td>
<td>151.929</td>
<td>-27.653</td>
<td>159.582</td>
<td>159.459</td>
<td>12.3</td>
</tr>
<tr>
<td>60.965</td>
<td>246.045</td>
<td>137.190</td>
<td>-27.013</td>
<td>164.203</td>
<td>164.180</td>
<td>2.346</td>
</tr>
<tr>
<td>60.731</td>
<td>244.844</td>
<td>178.432</td>
<td>-24.599</td>
<td>203.031</td>
<td>203.107</td>
<td>-7.581</td>
</tr>
<tr>
<td>60.742</td>
<td>244.260</td>
<td>151.468</td>
<td>-23.571</td>
<td>175.039</td>
<td>175.221</td>
<td>-18.233</td>
</tr>
<tr>
<td>60.766</td>
<td>244.148</td>
<td>148.863</td>
<td>-23.434</td>
<td>172.317</td>
<td>172.526</td>
<td>-20.886</td>
</tr>
<tr>
<td>60.559</td>
<td>243.860</td>
<td>236.930</td>
<td>-22.382</td>
<td>259.312</td>
<td>259.488</td>
<td>-17.575</td>
</tr>
<tr>
<td>60.001</td>
<td>243.018</td>
<td>276.084</td>
<td>-19.586</td>
<td>295.670</td>
<td>295.814</td>
<td>-14.376</td>
</tr>
<tr>
<td>60.985</td>
<td>242.753</td>
<td>171.685</td>
<td>-20.924</td>
<td>192.609</td>
<td>192.884</td>
<td>-27.462</td>
</tr>
<tr>
<td>60.931</td>
<td>243.082</td>
<td>226.397</td>
<td>-21.475</td>
<td>247.872</td>
<td>248.167</td>
<td>-29.489</td>
</tr>
<tr>
<td>60.936</td>
<td>242.933</td>
<td>214.140</td>
<td>-21.195</td>
<td>235.335</td>
<td>235.559</td>
<td>-22.448</td>
</tr>
<tr>
<td>61.173</td>
<td>242.469</td>
<td>152.332</td>
<td>-20.634</td>
<td>172.966</td>
<td>173.093</td>
<td>-12.733</td>
</tr>
</tbody>
</table>
FIGURE 1B
ACCURACY OF ESTIMATE OF PREDICTED HEIGHT BIAS FUNCTION WITH THE OREGON DATA SET
FIGURE 2B
ACCURACY ESTIMATE OF PREDICTED HEIGHT BIAS FUNCTION WITH THE SOUTH ALBERTA DATA SET

\[ c_1 = 5 \text{ cm} \]
2.1.2 Studies in the World Height System Definition

Basically four kinds of data are essential for modelling the World Height System (WHS). They are: accurately determined free-air gravity anomalies, precise orthometric heights of the stations above the regional vertical reference datums, ellipsoidal heights of the stations above adopted geocentric ellipsoid and accurate geopotential models. Since such data availability over the world are neither uniform nor to the required precision, different modelling procedures are needed to be developed to accommodate the available heterogeneous data to come up with an elegant observation equation for estimating the parameters defining a WHS. As understandable, it may require several iterations before realizing an ideal WHS which can meet the present day requirements of WHS established to an accuracy of ±10 cm.

The two different data scenarios anticipated in developing the procedure for setting up the observation equations for defining the WHS are:

(a) Given the free-anomalies in the Molodensky sense and normal heights of the stations, and
(b) Given the free-air anomalies in the classical sense and orthometric heights (or normal orthometric heights) of the stations.

Detailed discussion on the definition and comparison of free-air anomalies obtained in Molodensky sense and Classical sense can be found in Sideris and Forsberg (1991) and for various height definitions given in Rapp and Balasubramania (1992).

Fig. 3 Geometric representation of various equipotential surfaces

In Figure 3, Qi is the fundamental benchmark serving as the origin for the regional vertical datum 'i'. The potential of the reference surface for WHS is \( W_0 \). We define \( \mathcal{C}_{Q_0} = W_0 - W_0^i \) as the potential difference between the reference equipotential surfaces for datum 'i' and the WHS reference surface. Introducing a reference ellipsoid where \( U_0 \) is the potential on the surface, we define \( \Delta W_0 = W_0 - U_0 \). The distance between the equipotential
surfaces \( U = U_0 \) and \( W = W_0 \) is also shown in the figure, and depends on the disturbing potential at \( P' \) and \( \Delta W_0 \).

Procedure for setting up the observation equation for case (a) is as follows:

Let \( H^* \) be the normal height of the point \( P \) with respect to the WHS (to be defined) and \( H_i^* \) the known normal height of the same point referred to \( i^{\text{th}} \) regional vertical datum. Then from equation (2-122) in Pavlis (1988), we can write the relation:

\[
\Delta g_{M} = \Delta g_{M}^{(i)} - \frac{1}{\gamma_0} \left[ \left( \frac{\partial \gamma}{\partial h} \right)_{q_0} + \left( \frac{\partial^2 \gamma}{\partial h^2} \right)_{q_0} - H_i^* \right] \cdot C_{q,0} \tag{4}
\]

where \( \Delta g_{M}, \Delta g_{M}^{(i)} \) are the gravity anomalies computed with normal gravity \( \gamma_Q \) determined using normal heights \( H^*, H_i^* \) of the point \( P \) referring to WHS and regional vertical datum \( i \) respectively.

From Moritz (1980, eq. 48-1), the computation of height anomaly \( \zeta_p \) by analytical continuation to point level can be done using the formula:

\[
\zeta_p = \zeta_0 + \frac{R}{4\pi \gamma} \int (\Delta g_{M}^{c} + g_1) S(\psi) d\sigma \tag{5}
\]

where \( \zeta_0 = \frac{\Delta \gamma M}{\gamma} - \frac{\Delta W_0}{\gamma} \)

\( \Delta g_{M}^{c} \) is the free-air anomaly defined in Molodensky sense and corrected for systematic effects as discussed in Rapp and Pavlis (1990),

\( g_1 \) is the Molodensky correction term,

\( R \) is the mean radius of earth, \( \gamma \) the mean gravity, and

\( S(\psi) \) the Stokes' function.

If Faye anomalies are available equation (5) can be replaced with eqn. (24) in Wang (1993) as:

\[
\zeta_p = \zeta_0 + \frac{R}{4\pi \gamma} \int (\Delta g_{M}^{c} + C) S(\psi) d\sigma - \frac{\Delta g_{M}}{\gamma} \frac{\pi G\rho H_p^*}{\gamma} - \frac{\pi G\rho}{\gamma} \delta h^2 \tag{6}
\]

where \( H_p^* \) is the normal height of point \( P \) and

\[
\delta h^2 = 0.453 - 0.018 \sin \phi + 0.087 \cos \phi \cos \lambda + 0.204 \cos \phi \sin \lambda (\text{km}^2)
\]

Combining equations (4) and (6), we get
\[ \zeta_p = \zeta_0 + \frac{R}{4\pi y} \int \left( \Delta g_{s_i}^x + C \right) S(\psi) d\sigma - \frac{R}{4\pi y} \sum_{i=1}^{n} q_i C_{0,0} \int S(\psi) d\sigma, \]

where \( q_i = \frac{1}{\gamma} \left( \frac{\partial \gamma}{\partial h} \right)_{\psi_i} + \frac{1}{\gamma} \left( \frac{\partial^2 \gamma}{\partial h^2} \right)_{\psi_i} H_i^\gamma \).}

From equations (2-121) and (2-122) in Heiskanen and Moritz (1967) we can write

\[ q_i = -\frac{2}{R} \]  \hspace{1cm} (8)

From Figure 3 we note that:

\[ H_p = H_i + \frac{C_0,0}{\gamma} \] and \( h_p = H_p + \zeta_p \) or \( \zeta_p = h_p - H_p \) \hspace{1cm} (9)

where \( h_p \) is the ellipsoidal height of the point \( P \) and \( \zeta_p \) its height anomaly. Comparing eqns. (7) and (9) and rearranging known and unknown terms we write:

\[ Y = h - H_i - \zeta_i = -\Delta W_0 + \frac{C_0,0}{\gamma} + \frac{2}{\gamma} \sum_{i=1}^{n} C_{0,0} \left\{ \frac{1}{4\pi \sigma_i} \int S(\psi) d\sigma_i \right\} \]  \hspace{1cm} (10)

where \( \zeta_i = \frac{R}{4\pi \sigma_i} \int \left( \Delta g_{s_i}^x + C \right) S(\psi) d\sigma - \frac{\Delta g_{s_i}^x H_i^\gamma}{\gamma} - \frac{\pi Gp H_i^\gamma}{\gamma} - \frac{\pi Gp}{\gamma} \delta h^2 \) is the gravimetric height anomaly (without \( \zeta_0 \) term) and referred to regional vertical datum \( i' \).}

Similarly for case (b) when the free-air anomalies in classical sense are given along with orthometric heights of stations, modelling procedure given in Rapp and Balasubramania (1992) can be followed to come up with the observation equation:

\[ Y = h - H_i - N_i = -\Delta W_0 - \frac{C_0,0}{\gamma} + \frac{2}{\gamma} \sum_{i=1}^{n} C_{0,0} \left\{ \frac{1}{4\pi \sigma_i} \int S(\psi) d\sigma_i \right\} \]  \hspace{1cm} (11)

where \( N_i = \frac{R}{4\pi \gamma} \int (\Delta g_{s}^x + C) S(\psi) d\sigma - \frac{\pi Gp H_i^\gamma}{\gamma} \) is the gravimetric geoid undulation (without \( N_0 \) term) and referred to regional datum \( i' \).

Equations (10) and (11) are the key observation equations for solving the unknown parameters \( \Delta W_0 \) and \( C_{0,0} \) (for \( i = 1, 2, \ldots, n \)). The parameters can be obtained using a least-squares adjustment procedures. The adjustment process discussed by Rapp and Balasubramania (1992, Section 4.4) can be used to estimate the parameters defining the World Height System.

The solution of the parameter vector is given by
\[
\hat{X} = (A^T \Sigma_y^{-1} A + K^T K)^{-1} A^T \Sigma_y^{-1} Y
\]  
(12)

and the error covariance matrix of the estimated parameters would be

\[
\Sigma_{\hat{X}} = (A^T \Sigma_y^{-1} A + K^T K)^{-1} A^T \Sigma_y^{-1} A \left( A^T \Sigma_y^{-1} A + K^T K \right)^{-1}
\]  
(13)

where \( Y \) is the nx1 observation vector from equation (10),
\( A \) is the nxm coefficient matrix of observation,
\( X \) is the mx1 parameter vector,
\( K \) is the lxm coefficient-matrix of the constraints,
\( \Sigma_y \) is the variance-covariance matrix of observations.

Equation (12) and (13) correspond to equations (4-33) and (4-34) in Rapp and Balasubramania (ibid.).

Numerical Investigation

A first iteration test computation to estimate the parameters defining the WHS was carried out with available data from six regional vertical datums. The different vertical datums considered are: North American Vertical Datum 88 (NAVD88), ODN (UK), IGN69 (France), NN (Germany), AHD71 (Australia) and Scandinavian datum. Though there is no single regional vertical datum known as the Scandinavian datum, the individual datums RH70 (Sweden), N60 (Finland) and NN1954, NNN1957 (Norway) were combined to serve as a single regional datum for computational convenience.

The ITRF91 coordinate system was selected as the consistent coordinate system for defining the space geodetic station coordinates used in the realization of WHS. An ideal ellipsoid with the following parameters was adopted as the reference ellipsoid for the study:

\[
a_e = 6378136.3 \text{ m} \\
f = 1/298.257222101 \\
GM = 3986004 \times 10^8 \text{ m}^2\text{s}^{-2} \\
\omega = 7292115 \times 10^{-11} \text{ rad S}^{-1}
\]

The last three parameters are the same as the parameters of the GRS80 reference ellipsoid.

Table 3 below lists the distribution of space stations in various regional vertical datums and Figure 4 shows their locations.

<table>
<thead>
<tr>
<th>DATUM</th>
<th>NAVD88</th>
<th>Scandinavian</th>
<th>NN</th>
<th>IGN69</th>
<th>ODN</th>
<th>AHD71</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Stations</td>
<td>10</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

The accuracy and grid size of terrestrial gravity data used varied from one regional datum to another. For North American datum, 3° x 3' atmospherically connected Helmert anomalies around each of the U.S. stations received from D. Milbert, NGS were used. As quoted by Milbert (1991) these 3° x 3' mean anomalies have a random error of ±1.5 mgal. For the Scandinavian datum, Faye anomalies received from Dr. René Forsberg were used. For the other stations in England, Germany, France and Australia, the 6° x 10' and 2° x 2' atmospherically...
corrected free-air anomalies referring to OSU GRS (a = 6378136.0, f = 1/298.257222101) used by Despotakis (1983) in his thesis work were used. These gravity data sets are not corrected for terrain and as given in Torgé (1983), they have a random error of ±7 mgal.

The elevation of the space stations above their local vertical reference datum were obtained from NASA space geodesy program - catalogue of site information (NASA Tech Memo 4482). For the stations in USA, their heights referred to NGVD29 height system. Using the VERTCON software received from Zilskoski at NGS, the heights were converted to NAVD88 heights. For stations in England and Australia, the heights given were normal orthometric heights and for stations in Germany, France and Scandinavian countries, they are normal heights referred to their regional vertical datum. The global geopotential model OSU91A to degree 360 was used as the reference field in the study.

Three different techniques were used to compute the gravimetric height anomaly $\zeta_i$ and the gravimetric undulation $N_i$ in equations (10) and (11) respectively. They were:

(i) Modified Stokes' technique
(ii) Least squares collocation technique (LSC)
(iii) Interpolation from regional geoid height models developed using Fast-Fourier transform technique.

Comparison of gravimetric undulations computed using different techniques at the U.S. space geodetic stations are given in Table 4. The interpolated geoid height values from recent GEOID93 Geoid Height model received from Milbert at NGS, are also included for better comparison. The differences between the gravimetric undulation and geometric undulation are also shown. In Table 5, gravimetric height anomaly/undulation computed at the different European space stations are given. Since we had only the regional geoid height model NKG-89 for Scandinavian countries, only for stations in that area, the interpolated geoid height are included in Table 5. The gravimetric undulation at the Australian stations computed using the first two techniques and their comparison with geometric undulation are given in Table 6.

From Tables 4 and 5 we see that the rms difference between the geometric undulation and the gravimetric undulation computed using different techniques are almost the same but in Table 6 we see they differ by about 60 cm. Also from Table 6 we note the difference between the gravimetric and geometric undulations computed at the Yarragadee station (located in Western Australia) and at Canberra (located in Eastern Australia) differ by about 1.1 meters though both the stations are considered to lie in the same regional vertical datum (AHD71). This difference may be due to the suggested rise of mean sea level up the East Australian coast of about 1.5 m by Morgan (1992) and/or the same extent due to the long wave length error in the global geopotential model used.
Table 4: Comparison of gravimetric undulation computed using different techniques at the U.S. space geodetic stations. Units are meters.

<table>
<thead>
<tr>
<th>Station</th>
<th>Undulations</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSC Technique</td>
<td>Stokesian Approach</td>
</tr>
<tr>
<td>7051 Quincy</td>
<td>-22.796</td>
<td>-22.938</td>
</tr>
<tr>
<td>7062 San Diego</td>
<td>-34.112</td>
<td>-33.635</td>
</tr>
<tr>
<td>7082 Bear Lake</td>
<td>-13.733</td>
<td>-14.803</td>
</tr>
<tr>
<td>7091 Westford</td>
<td>-28.768</td>
<td>-28.429</td>
</tr>
<tr>
<td>7086 Ft. Davis</td>
<td>-21.632</td>
<td>-21.641</td>
</tr>
<tr>
<td>7105 GSFC5</td>
<td>-33.089</td>
<td>-32.934</td>
</tr>
<tr>
<td>7110 Monument P</td>
<td>-32.064</td>
<td>-31.488</td>
</tr>
<tr>
<td>7234 Pietown</td>
<td>-21.127</td>
<td>-21.460</td>
</tr>
<tr>
<td>7069 Patrick AF</td>
<td>-29.415</td>
<td>-29.430</td>
</tr>
<tr>
<td>7204 Green Bank</td>
<td>-31.814</td>
<td>-31.092</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>0.489</td>
</tr>
<tr>
<td></td>
<td>rms</td>
<td>0.495</td>
</tr>
</tbody>
</table>
Table 5: Comparison of gravimetric height anomaly computed using different techniques at the European space geodetic stations. Units are meters.

<table>
<thead>
<tr>
<th>Station</th>
<th>LSC technique $\zeta_{LSC}$</th>
<th>Stokesian $\zeta_{STOKES}$</th>
<th>NKG-89 Model $\zeta_{89}$</th>
<th>Geometric height anomaly $\zeta_{GEOM}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metsahovi</td>
<td>19.603</td>
<td>19.388</td>
<td>18.431</td>
<td>19.751</td>
<td>0.148</td>
</tr>
<tr>
<td>Onsala</td>
<td>35.938</td>
<td>35.854</td>
<td>35.359</td>
<td>37.223</td>
<td>1.285</td>
</tr>
<tr>
<td>Tromso</td>
<td>30.977</td>
<td>30.799</td>
<td>31.484</td>
<td>32.289</td>
<td>1.312</td>
</tr>
<tr>
<td>Wetzell</td>
<td>46.699</td>
<td>46.764</td>
<td>-</td>
<td>47.556</td>
<td>0.857</td>
</tr>
<tr>
<td>Grasse</td>
<td>51.203</td>
<td>51.932</td>
<td>-</td>
<td>52.120</td>
<td>0.917</td>
</tr>
<tr>
<td>RGO-UK</td>
<td>45.360</td>
<td>45.545</td>
<td>-</td>
<td>46.365</td>
<td>1.005</td>
</tr>
</tbody>
</table>

Table 6: Comparison of gravimetric undulation computed using different techniques at the Australian space geodetic stations. Units are meters.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yarragadee</td>
<td>-24.995</td>
<td>-25.337</td>
<td>-24.517</td>
<td>-24.883</td>
<td>0.438</td>
<td>0.820</td>
</tr>
<tr>
<td>Canberra</td>
<td>18.727</td>
<td>18.090</td>
<td>20.105</td>
<td>18.957</td>
<td>1.378</td>
<td>2.015</td>
</tr>
</tbody>
</table>

Using the data at 18 space geodetic stations in 6 regional vertical datums, and with 7 unknowns, observation equations and normal equations were formed. From equations (12) and (13) the parameters defining the WHS and their accuracy estimates were computed for different techniques used in computing gravimetric height anomaly/undulation. The estimated parameters are given in Figures 5 and 6.
From Figures 5 and 6 it is clear that the different regional vertical surfaces used in the definition of WHS show the same pattern of their relative positions to one another in spite of using two different techniques in their estimation but the numerical values are different. The Scandinavian datum which is 61 cm above the defined WHS in Figure 6 is only 38 cm above the same in Figure 6. This may be due to using limited terrestrial gravity data ($\psi = 1^\circ$) for computing the gravimetric height anomaly at the Scandinavian stations. The Australian height datum AHD71 which is about 106 cm above the defined WHS in Figure 5 is only 54 cm in Figure 6. The reason is that gravimetric undulation computed at the Australian space station using Modified Stokes' technique and LSC technique differs by about 70 cm. This may be due to
error in the gravity anomaly data used around these stations. The datum differences between DHNN and IGN69 as intimated by Boucher (private communication) is about 50 cm which agrees well with the differences shown in Figure 5 and 6. Except for the Australian Height datum, the differences between different regional datums as shown in Figure 5 compare well with the estimates given by Rapp (1993).

2.2 Research Activities of R.H. Rapp

During this grant Professor Rapp utilized existing Doppler positioned stations to study several vertical datums with the aid of gravimetric geoid undulations derived from a merger of new JGM-2 potential coefficient model (degrees 2-70) and the OSU91A model (degrees 71 to 360). This research led to a paper that has been accepted for publication in the Bulletin Geodesique or manuscripta geodaetica. This paper is made part of this report and starts on the following page.
2.2.1 Separation Between Reference Surfaces of Selected Vertical Datums

Richard H. Rapp

Department of Geodetic Science and Surveying, The Ohio State University, Columbus, OH 43210

November 1993

Abstract

This paper discusses the separation between the reference surface of several vertical datums and the geoid. The data used includes a set of Doppler positioned stations, transformation parameters to convert the Doppler positions to ITRF90, and a potential coefficient model composed of the JGM-2 (NASA model) from degree 2 to 70 plus the OSU91A model from degree 71 to 360. The basic method of analysis is the comparison of a geometric geoid undulation derived from an ellipsoidal height and an orthometric height with the undulation computed from the potential coefficient model. The mean difference can imply a bias of the datum reference surface with respect to the geoid. Vertical datums in the following countries were considered: England, Germany, United States, and Australia. The following numbers represent the bias values of each datum after adopting an equatorial radius of 6378136.3m: England (-87 cm), Germany (4 cm), United States (NGVD29 (-26 cm)), NAVD88 (-72 cm), Australia AHD (mainland, -68 cm); AHD (Tasmania, -98 cm). A negative sign indicates the datum reference surface is below the geoid. The 91 cm difference between the datums in England and Germany has been independently estimated as 80 cm. The 30 cm difference between AHD (mainland) and AHD (Tasmania) has been independently estimated as 40 cm. These bias values have been estimated from data where the geometric/gravimetric geoid undulation difference standard deviation, at one station, is typically ±100 cm, although the mean difference is determined more accurately.

The results of this paper can be improved and expanded with more accurate geocentric station positions, more accurate and consistent heights with respect to the local vertical datum, and a more accurate gravity field for the Earth. The ideas developed here provide insight on the determination of a world height system.

Introduction

This paper is written to estimate the linear separation between the origin reference surface of several vertical datums. The process for doing this inherently implies a concept of a world height system as discussed by Rapp and Balasubramania (1992). This paper is not meant to be an extensive analysis of theoretical procedures involved in vertical datum definition and connection. Recent papers in this area include those by Xu and Rummel (1991) and Heck and Rummel (1990). This paper is not meant to show results of the latest measurement procedures. Examples of such papers are those of Rizos, Coleman and Ananga (1991) and Pan and Sjöberg (1993). This paper is designed to examine some earlier ideas (Rapp, 1983) using the latest gravity field models and terrestrial reference frame transformations.

Principles

Let $N$ be the geoid undulation referred to a geocentric reference frame, an ellipsoid of a specified flattening, and an equatorial radius considered to be an optimum estimate. (Optimum may be a matter of opinion so this term is loosely used here.) Our calculation of $N$ will be done through a potential coefficient model of degree $L$. 

---

20
Given the fully normalized coefficients $C_{nm}$, $S_{nm}$, one has

$$N(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=2}^{\infty} \frac{(a/r)^n}{n!} \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda)P_{nm}(\cos \theta) + c$$

(1)

where the usual definitions (Rapp and Pavlis, 1989) apply. The $c$ term represents the downward continuation effects from an evaluation of the series at a valid point in space to a point on the geoid. We will assume that $c$ is zero in this analysis. The even degree zonal harmonics in (1) refer to the adopted reference ellipsoid.

Let $h$ be the ellipsoidal height of a point computed from rectangular coordinates in a geocentric reference frame in a true scale system. Let $H$ be the orthometric height with respect to the geoid that is described by the $N$ values of eq. (1). Then:

$$h = H + N$$

(2)

If one were given $h$ and $N$ then $H$ can be computed. The accuracy of the determination of this $H$ will depend on the accuracy of $h$, $N$ and the optimum equatorial radius.

Consider next an orthometric height ($H_D$) referred to a specific vertical datum $D$. The ideal situation is that the vertical datum is defined by an equipotential surface, near mean sea level, at one specific point. In practice, vertical datums may have been defined by fixing mean sea level to zero orthometric elevation. In this case, the reference surface and the $H$ values have some distortion with respect to the ideal case. In this paper, we will assume that a vertical datum is defined by a unique reference surface. This surface may not and actually will not coincide with the ideal reference surface, the geoid.

Let $B$ be the bias between the ideal system and a specific system such that:

$$H = H_D + B$$

(3)

where $H$ is the orthometric height in the ideal system and $H_D$ is the corresponding height in the datum system. A positive $B$ indicates the specific datum reference surface is above the ideal reference surface. We can estimate $B$ by substituting (3) into (2):

$$B = (h - H_D) - N$$

(4)

The evaluation of $B$ should be done using numerous stations connected to the datum $D$. Averaging individual values can reduce the noise inherent in the estimate of the three quantities on the right hand side of eq. (4). A comparison of $B$ from different datums will yield information on the relative positions of these vertical datums.

Data

The stations to be analyzed for this paper have been positioned through Doppler observation techniques during the time period 1971-1986. A description of some of the processing techniques used in the estimation of the precise positions of an analogous station set may be found in Weigel (1993, p. 14). With this station set, "heights" above mean sea level are generally available. These heights are not precisely defined in terms of datum reference, type of orthometric height, or in fact they may be normal heights. Although definitive results of this type of investigation require such information, this paper is considered a demonstration study and we will assume that the mean sea level heights are orthometric heights connected to the vertical datum of the region or country. Ultimately, we will restrict our analysis to stations that can be associated with a specific vertical datum.

Before any analysis is carried out, the ellipsoidal height given in the original data record is transferred to the ITRF90 system (Boucher and Altamimi, 1992, Table 4 (NWL 9D)) and referred to an ellipsoid whose equatorial radius is 6378136.3m and flattening is 1/298.257.

Analyses such as these are made more feasible as the knowledge of the Earth's gravity field improves. One of the recent models is JGM-2 (Lerch et al., 1993) which is a combination model complete to degree 70. This model has been augmented by the OSU91A model (Rapp, Wang, Pavlis, 1991) from degree 71 to degree 360. Although all tests to be described in this paper have been carried out with both the OSU91A and JGM-2 (augmented) model only (with one exception) results with the most current model (JGM-2) will be given. The geoid undulation can be computed from this data with an estimated accuracy of ±57 cm (Rapp, Wang, Pavlis, p. 64, ibid.) globally, which may be considerably poorer in areas lacking adequate gravity information.

Results - Global

The first computation involved the examination of all Doppler positions independent of their geographic location. Undulation comparisons were made with both the OSU91A and JGM-2 (augmented) potential coefficient models. In this analysis, stations with residuals exceeding 2m in absolute value were rejected from the statistical computations. This 2m criteria is
tighter than the 4m criteria used in past studies, but it is consistent with the criteria adopted for the regional studies to be reported shortly. The 2m criteria is basically twice the standard deviation of the geometric/gravimetric difference and thus is a 2 sigma criteria. The statistics on the differences are given in Table 1 where the mean difference is defined through the geometric undulation minus the potential coefficient implied undulation. The total number of stations considered was 2033. The results indicate little preference for one geopotential model although the use of the 91A model allows 11 more stations to be considered. If the comparisons were made being more selective on Doppler stations to be used (at least 35 passes are observed after June 1977) the mean differences given in Table 1 become more positive by 15 cm and the standard deviation decreases to ±87 cm. Assuming, somewhat incorrectly, that the undulation differences are independent, the standard deviation of the mean difference is approximately ±3 cm.

Table 1

<table>
<thead>
<tr>
<th>Geopotential Model</th>
<th>Mean Diff.</th>
<th>SD Diff.</th>
<th>No. Stat.</th>
<th>Accepted</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU91A</td>
<td>-9 cm</td>
<td>±100 cm</td>
<td>1428</td>
<td></td>
</tr>
<tr>
<td>JGM2/OSU91A</td>
<td>-15 cm</td>
<td>99 cm</td>
<td>1417</td>
<td></td>
</tr>
</tbody>
</table>

Results - England/Germany

The next step was to examine stations that could be closely associated with two regions with different vertical datums. The first area was England with stations selected in the region 50° to 60° N, 355° to 2°E. We assume that the orthometric heights we have are given in the Newlyn Datum (Ashkenazi et al., 1990). The second area was Germany with stations selected in the region 48° to 54° N, 6° to 13°E. We assume that the orthometric heights we have refer to the vertical datum described in Torge (1980, Section 6.2.3).

The results of the comparisons, based on the 2m rejection criteria, are given in Table 2. The mean differences imply that the Newlyn Datum is -87 cm below the geoid while the German datum is 4 cm above the geoid. The difference implies that the German vertical datum reference surface is 91 cm above the Newlyn Datum.

Table 2

<table>
<thead>
<tr>
<th>Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in England and Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Mean Diff.</td>
</tr>
<tr>
<td>SD Diff.</td>
</tr>
<tr>
<td>No. Stat.</td>
</tr>
</tbody>
</table>

An independent estimate of the above can be obtained as follows. Willis et al (1989) estimate (on the basis of several sources of information) that the origin of the French vertical datum (IGN69) is 30 cm ±8 cm above the origin surface of the Newlyn Datum (ODN). On the basis of the comparison of heights near the French/German border (Boucher 1993, private communication), the German system origin is 50 cm above the IGN69 system (i.e. H (Germany) = H (IGN69) - 50 cm). This numerical value can be caused not only by vertical datum differences, but also differences in normal heights and orthometric heights. The latter we will ignore for this paper. Taking the 30 and 50 cm into account would imply that the German origin surface is 80 cm above the Newlyn Datum surface. This number can be contrasted with the 91 cm obtained through the Doppler station analysis.

Results in the United States

The orthometric heights for the Doppler stations in the United States are given with respect to the National Geodetic Vertical Datum of 1929 (NGVD29). This datum was defined through a general adjustment in 1929 with height constraints imposed at 26 tide gauge stations. The orthometric heights were computed using normal corrections since no gravity observations were available along the leveling lines.

A new adjustment of leveling data has led to the North American Vertical Datum of 1988 (Zilkoski, Richards, and Young, 1992). This datum is based on geopotential numbers fixing the height at a primary tidal benchmark. Although we did not have heights for the Doppler stations directly in the NAVD88 system, a datum conversion software program (VERTCON, version 1.0, February 1993) has been developed at the National Geodetic Survey. This software was used to convert the NGVD29 heights into NAVD88 heights using a transformation procedure in lieu of the direct adjustment of the points in NAVD88.

The stations to be used in this analysis were initially selected in the region: 30° to 45°N, 240° to 290°E. This station set was then thinned to eliminate stations geographically close so that a file containing 350
The results of the undulation comparison with the JGM-2/OSU91A potential coefficient model and a 2m rejection criteria are given in Table 3.

Table 3
Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in the United States

<table>
<thead>
<tr>
<th>Vertical Datum</th>
<th>NGVD29</th>
<th>NAVD88</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Diff.</td>
<td>-26 cm</td>
<td>-72 cm</td>
</tr>
<tr>
<td>SD Diff.</td>
<td>±83 cm</td>
<td>±100 cm</td>
</tr>
<tr>
<td>No. Stations</td>
<td>321</td>
<td>321</td>
</tr>
</tbody>
</table>

The mean differences for NGVD29 and NAVD88 differ by 46 cm. Somewhat surprising is the somewhat poorer fit (100 cm vs 83 cm) when the NAVD88 datum is used. One would suspect that the NAVD88 heights should be more consistent with the ellipsoidal heights/gravimetric undulations than the NGVD29 heights which are given in a distorted datum.

Figure 4 of (ibid.) show the height differences between NAVD88 and NGVD29. This map shows that the heights with respect to the two datums are quite consistent in the eastern half of the country. Due to this, additional tests were made restricting the analysis to stations east of 254°E. The results of this analysis are given in Table 4. We see a substantial change (from Table 3) of the mean difference for stations on NAVD88 (-72 cm (Table 3) to -38 cm (Table 4)). The corresponding change in the NGVD29 difference is only 8 cm. The standard deviation of the undulation differences in going from NGVD29 to NAVD88 is smaller than the corresponding case in Table 3.

Table 4
Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in the Eastern United States

<table>
<thead>
<tr>
<th>Vertical Datum</th>
<th>NGVD29</th>
<th>NAVD88</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Diff.</td>
<td>-18 cm</td>
<td>-38 cm</td>
</tr>
<tr>
<td>SD Diff.</td>
<td>±86 cm</td>
<td>±92 cm</td>
</tr>
<tr>
<td>No. Stations</td>
<td>180</td>
<td>180</td>
</tr>
</tbody>
</table>

These computations were repeated restricting the analysis to stations where the number of satellite passes was greater than 35 and the observation data was after June 15, 1978. The standard deviations would typically be reduced by about 10%. The mean differences would change by 7 to 12 cm.

The results of Table 3 make it unclear as to the appropriate mean difference to be associated with the two vertical datums. One can argue that a single parameter can not be used to represent the difference between the NGVD29 reference surface and the geoid because of the manner in which NGVD29 was adjusted. However, there is only a 8 cm difference between the total station set and the eastern region data. On the other hand, one would expect the greatest consistency with the NAVD88 system. Unfortunately the mean difference differs by 34 cm from the total station set in the eastern station set. With no other information available, one perhaps should use the results given in Table 3 understanding the value for NAVD88 is subject to change when more accurate heights in the NAVD88 system are used in the analysis. Specifically from Table 3 one estimates that the NGVD29 reference surface is located 26 cm below the geoid while the NAVD88 reference surface is 72 cm below the geoid.

Results in Australia

The heights available at the Doppler stations are approximate orthometric heights referred to the Australian Height Datum (AHD). The development of this datum is described by Leppert (1974) and Rizos et al. (1991). The levelling adjustment was carried out in 1971 using height constraints at 30 tide gauge stations around the coastline of Australia.

The Doppler stations were well distributed around Australia. The results of the analysis are given in Table 5 (AHD (M)). The mean difference of -68 cm suggests that the average AHD reference surface is located 68 cm below the geoid as defined by the JGM-2/OSU91A potential coefficient model. This magnitude conflicts with estimates of dynamic topography which suggests that mean sea level in the vicinity of Australia is above the geoid. (For example, see Figure 1 in Rapp, Wang, Pavlis (1991)).

Table 5
Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in Australia and Tasmania

<table>
<thead>
<tr>
<th>Vertical Datum</th>
<th>AHD (M)</th>
<th>AHD (CD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Diff.</td>
<td>-68 cm</td>
<td>-98 cm</td>
</tr>
<tr>
<td>SD Diff.</td>
<td>96</td>
<td>66</td>
</tr>
<tr>
<td>No. Stations</td>
<td>85</td>
<td>4</td>
</tr>
</tbody>
</table>

An attempt was made to determine the difference between the AHD as used in the mainland and the AHD used in Tasmania. This test was a follow-up to the study described by Rizos, Coleman and Ananga (1991). The first step was to use four Doppler stations in Tasmania that yielded a mean difference of -98 cm with
a standard deviation of ±66 cm (see Table 5 AHD(T)). This result implies that the AHD on the mainland has its reference surface 30 cm higher than the AHD in Tasmania. This value is similar to an updated (from Rizos, Coleman, Ananga (1991)) estimate of 40 cm (Coleman, 1993, private communication). A caveat here is that this analysis uses data distributed over all of Australia while the Rizos, Coleman, Ananga study considered the difference between the AHD (Victoria coast) and AHD (Tasmania).

Conclusions

This paper has demonstrated a procedure that estimates the separation between reference surfaces defined by several vertical datums and the geoid that is defined by geoid undulations computed from the JGM-2 (degree 2 to 70) and OSU91A (degree 71 to 360) potential coefficient model. The ellipsoidal heights of the stations are determined from Doppler positioning techniques after a transformation to the ITRF90 system. Although the accuracy of the vertical position derived from the Doppler positions is on the order of ±80 cm, the large number of stations on some vertical datums makes these tests meaningful.

In the analysis we analyzed data that can be associated with vertical datums in England, Germany, Australia, and the United States. We assume that the difference between the geoid and the reference surface of the vertical datum can be modeled by a single parameter. This would be appropriate if the datums were uniquely defined by a reference surface through a specified point. However, the adjustment of several of the datums was carried out by fixing heights at tide gauges distributed around the borders of the country. This fixing introduces a distortion in the reference surface (Laskowski, 1983) with respect to a single point related to the reference surface. Our procedure then calculates a separation for some average reference surface representative of the area in which the stations are given.

The results for the differences in the origins of the vertical datums studied for this paper are shown in Figure 1. Not shown are the results for AHD (Tasmania) because of the few stations available. Two results for NAVD88 (U.S.) are shown because of the large differences in the result. One estimate is based on the complete station set after conversion from NGVD29 while the second estimate is based only on stations in the eastern part of the U.S. From Figure 1 one clearly sees the differences of the reference surfaces of a few vertical datums. It should be emphasized that the numerical value shown in tables and Figure 1 are dependent on many factors (station reference frame, geoid undulation model, equatorial radius, etc.).

The results (Table 3/4) for the two vertical datums in the U.S. are somewhat troubling. Why should the standard deviation of fit be poorer when the NAVD88 (a more rigorous datum) heights are used as opposed to the NGVD29 values? Why should the mean differences change so much (44 cm) when the results for the complete station set vs the eastern US station set of NAVD88 are considered? Additional study with more information on NAVD88 heights is needed.

Another area of concern is the magnitude of the geoid/datum separation for the Australian Height Datum. On the basis of oceanographic determinations of mean sea level one would expect the mean differences to be somewhat (~20 cm) positive instead of the -68 cm found in these results. On the encouraging side was the good agreement between the AHD(mainland)/AHD(Tasmania) difference between the 30 cm (AHD (M) higher) and the recent estimate by Coleman of 40 cm.

To resolve some of these questions, more accurate computations are needed that would use more precise geocentric positions of stations on various vertical datums and more refined computation for geoid undulation values at the stations. Procedures to use this data for vertical datum definition and connections are described in Rapp and Balasubramania (1992).

And finally, the results of this study imply that height systems can be inconsistent by one or two meters. This is not new information as it is well known from studies of dynamic height variations in the oceans. There seems to be a clear need to develop a unique height system that can be tied to the ideal reference surface, the geoid. Given a sufficiently accurate representation of geoid undulations, orthometric heights can be derived from an ellipsoidal height computed from space positioning. Additional consideration is needed to define the type of orthometric height wanted, or to implement the determination of a normal height through a height anomaly calculation instead of a geoid undulation computation. Whatever the case, we are approaching, although not yet there, the situation of having sufficient gravity field information and ellipsoidal height values to define a world height system at the 50 - 100 cm accuracy level.

Acknowledgment

This study was supported by the Defense Mapping Agency and sponsored under Phillips Lab contract F19628-92-K-0026.
Fig. 1. Summary of Reference Surface Differences with Respect to the Geoid. Units are cm.

References


Torge, W., Geodesy, de Gruyter, Berlin, New York, 1980.

Willis, P. et al., Connection of the two levelling system datum IGN69 and ODN through the Channel by using GPS and other techniques,
presented at the First International Workshop on Geodesy for the Europe-Africa fixed link feasibility studies in the Strait of Gibraltar, Madrid, 8-10 March 1989.


3. Personnel

The principal investigator of this project is Richard H. Rapp, Professor Emeritus, Department of Geodetic Science and Surveying. Mr. Nagarajan Balasubramania, Graduate Research Associate, also participated in the research under this project. Mr. Balasubramania is a Ph.D. candidate at The Ohio State University.

4. Other Activities

During this project, Dr. M. Kumar, of the Defense Mapping Agency, visited us to learn about the research activities taking place. The first visit was on October 22, 1992 while the second visit was on March 23, 1993. For both visits, presentations were made to Dr. Kumar. Copies of the presentations were sent to Anestis Romaides at Phillips Laboratories (Code LW).

5. Travel

In August 1993, Dr. Rapp traveled, with support from this project, to the NASA/Goddard Space Flight Center for a meeting with NASA and DMA representations. This meeting concerned the possible joint activities of NASA, DMA, and Ohio State, in the production of a new gravity model that could be used for precise geoid determination. This travel was the only travel supported by the project. A follow-up meeting on the joint gravity field venture was held in October 1993 at NASA/Goddard. This meeting was also attended by Dr. Rapp. No travel funds from this project were used for this travel.

6. Presentations

Dr. Kumar and Dr. Rapp submitted an abstract for a presentation at the 1993 Fall Meeting of the American Geophysical Union. The abstract, as it appears in EOS, Trans. American Geophysical Union, Vol. 74, No. 43, October 26, 1993, is as follows:

**Unification of World Height Systems**

M Kumar (Defense Mapping Agency, Fairfax, VA, 22031; tel: 703-487-8151)
R H Rapp (Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, OH 43210; tel: 614-292-6005; e-mail; RHRAPP@OHSTMVSA.ACS.OHIO-STATE.EDU)

There are several hundred vertical datums used worldwide and in some areas there is no information available that can be used to determine a height in the local system. Consequently, there is a substantial need to improve our vertical positioning capability for geodetic, mapping, and charting purposes. Numerous theories have been developed that enable connection and establishment of vertical datums. Such theories require data given in a specific datum with accurate space positioning techniques combined with dense gravity information. An alternative procedure proposed in this paper is to introduce the geoid as the unifying global vertical reference surface.

To locate the geoid with respect to a reference ellipsoid, geoid undulations must be accurately determined. Today a global geoid undulation accuracy estimate is about 26 cm in ocean areas. On land, the achievable accuracy is definitely poorer and in areas with scarce gravity data the accuracy may be even 2-5 m. To achieve an accurate determination of the geoid for vertical datum purposes improvement in our knowledge of the Earth's gravity field is needed.

Thus, as a first step, efforts are being made to develop a new degree 360 potential coefficient model combining all the available satellite and terrestrial gravity data. This new model should then enable the determination of geoid undulations to an accuracy of 50 to 100 cm in land areas
with substantially better accuracy over ocean areas. The new geoid is then intended to be implemented as the vertical reference surface for the World Geodetic System (WGS) 1984. These geoidal undulations would also enable the determination of orthometric heights from space positioning systems, e.g., GPS, with respect to a consistent zero reference surface globally. In effect, the improved geoid will result in the realization of WGS84 as a complete three-dimensional reference system. The connection of this global height system to local vertical datums can also be then easily achieved. The technical approach to develop such a united world height system will be described in this presentation.

7. Future Activities

Most of the research goals defined in the original proposal have been accomplished during this contract. This report has described two aspects of the research carried out. Additional details of the research by Balasubramania will be given in his dissertation which should be completed by the 2nd quarter of 1994.

The results of this research clearly show the need to more clearly define our vertical datums. The numerical results demonstrate the origin inconsistencies that exist in a number of current datums. Although these inconsistencies are at the ±1 m level, they are of increasing concern in today's high precision geodetic positioning. We have seen that one of the difficulties in connecting our datums in the lack of a sufficiently accurate gravity model for the Earth. Current models can be used to derive geoid undulations that have an accuracy of ±0.5 to ±5 m. With improved data both satellite and surface gravity data, an improvement in our representation of the Earth's gravitational potential can be anticipated. This improvement could lead to the ideal reference surface, the geoid, to be used for a global vertical reference system. Ultimately, this surface could be the reference for a World Height System.

References


Despotakis, V.K., Geoid Undulation Computation at Laser Tracking Stations, Report No. 383, Geodetic Science and Surveying, The Ohio State University, Columbus, 1983.


Rapp, R.H., Computation and Accuracy of Global Geoid Undulation Models, presented at the Sixth International Geodetic Symposium on Satellite Positioning, Columbus, Ohio, 17-20 March 1992.


Rapp, R.H., Separation Between Reference Surfaces of Selected Vertical Datums, November 1993, to appear in Bulletin Geodesique/manuscripta geodaetica (also see Section 2.2).


Wang, Y.M., Comment on Proper Use of the Terrain Correction of Height Anomalies, The Ohio State University, Columbus, 1993.