A method of reverberation suppression and characterization, called the Principal Components Inverse (PCI) method, has been used to mathematically characterize 87% to 95% of the reverberation observed during selected ARSRP experiments. The number of principal signal components needed to achieve such an explanation seldom exceeds four, illustrating the fact that, over a time slice of about 1.6 of a second, the observed reverberation is mathematically simple. As a result of its mathematical simplicity, it follows that the reverberation can be explained in terms of only a few acoustic components at any given time -- possible components which represent reflections from bottom facets. It is of interest to explain the mathematical components in terms of such physical components, and it is also of interest to be able to separate a beamformed acoustic signal into what the PCI method regards as "reverberation" and "other." We have, therefore, used the bistatic scattering strength model (BISSM) to estimate mean reverberation power, and then have used that information to refine the energy threshold used by the PCI method prior to producing such a separation. We have also injected a synthetic reflection from a rectangular plate into existing ARSRP hydrophone data. In this paper, we will first introduce the PCI method and then subsequently discuss our initial attempts at both acoustic component identification and at signal separation.
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REVERBERATION CHARACTERIZATION BY MEANS OF THE PRINCIPAL COMPONENTS INVERSE METHOD

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Abstract

A method of reverberation suppression and characterization, called the Principal Components Inverse (PCI) method, has been used to mathematically characterize 87% to 95% of the reverberation observed during selected ARSRP experiments. The number of principal signal components needed to achieve such an explanation seldom exceeds four, illustrating the fact that, over a time slice of about 1/6 of a second, the observed reverberation is mathematically simple. As a result of its mathematical simplicity, it follows that the reverberation can be explained in terms of only a few acoustic components at any given time — possibly components which represent reflections from bottom facets. It is of interest to explain the mathematical components in terms of such physical components, and it is also of interest to be able to separate a beamformed acoustic signal into what the PCI method regards as "reverberation" and "other". We have therefore used the bistatic scattering strength model (BISSM) to estimate mean reverberation power, and then have used that information to refine the energy threshold used by the PCI method prior to producing such a separation. We have also injected a synthetic reflection from a rectangular plate into existing ARSRP hydrophone data. In this paper, we will first introduce the PCI method and then subsequently discuss our initial attempts at both acoustic component identification and at signal separation.

Introduction

The singular value decomposition (SVD) of a matrix is given by a well-known theorem which states that a non-square matrix $A$ can be expressed as a product of three other matrices such that $A = U \Sigma V$, where the product matrices $U$ and $V$ are self-orthogonal, and $\Sigma$ is a diagonal matrix of so-called singular values. Expressed as a product of partitioned matrices, the SVD may also be written as:

$$A = (U_S \quad U_0) \begin{pmatrix} \Sigma_S & 0 \\ 0 & \Sigma_0 \end{pmatrix} \begin{pmatrix} V_S \\ V_0 \end{pmatrix}.$$  

The diagonal matrices indicated as $\Sigma_i$ contain the singular values of $A$ along their main diagonals, and these values are ordered from largest to smallest as one proceeds along the diagonal. This makes it easy to select only the largest (or smallest) singular values for the purpose of approximating the matrix $A$ by one of lower rank. If only the singular values in one partition — say, $\Sigma_S$ — are retained, then the Eckhart-Young theorem states that a matrix formed as the product $(A) = U_S \Sigma_S V_S$ is the best lower rank approximation that one can make to the original matrix. If the matrix $A$ was formed from digitized acoustic samples, then the approximation $(A)$ is also in some sense an approximation to the original acoustic signal, but one which has the meaning of its individual elements obscured. In this paper,
we will introduce a method of extracting the strongest signal component (the reverberation) from time-slices which are processed in a way suggested by the SVD theorem in order to separate the reverberant energy from the rest of the signal components.

Kirsteins and Tufts have considered the problem of rapid, adaptive signal detection when the noise plus reverberation matrix $A$ is approximately low rank, and only a limited amount of data is available $[1, 2, 3, 4]$. The resulting method has come to be called the Principal Component Inverse (PCI) method, and is the method which will be discussed throughout the remainder of this paper.

The major advantages of the PCI method are (1) the adaptation is rapid, allowing one to follow the fluctuations in reverberation frequency and direction, and (2) information for both reverberation suppression and signal detection are obtained from the same data set. No reverberation-only training data is needed.

Reverberation data from the Acoustic Reconnaissance Cruise of the Acoustic Reverberation Special Research Program (ARSRP) $[5]$ were analyzed using the PCI method, as part of an ongoing 6.2 research effort sponsored by the Office of Naval Research under its ASW Environmental Technology Block. The planned, high resolution, experiments along with the high resolution models already developed within SRP community afford an excellent opportunity to attempt to understand the physical mechanism(s) which give rise to the mathematical components identified by the PCI method. An improved physical understanding of the mathematical components should in turn lead to improvements in the PCI method itself.

It must be understood by the reader that, unless explicitly stated to the contrary, the discussion in this paper refers to basebanded, hydrophone-only data prior to matched filtering (passthroughs). Before discussing the data further, however, we return to the discussion of the PCI method itself.

**Discussion**

The PCI method essentially consists of repeatedly forming a low rank matrix out of overlapping time-slices of data samples, applying the SVD algorithm, and obtaining a lower rank matrix ($A$). The approximated data set is then reconstructed from the non-overlapping portions of the output. Typically, the time-slices are 20–30 samples in duration (the appropriate size depends upon the sampling rate).

The structure of the PCI estimate can be justified using some results of Claus et al $[6]$ and Liu and Nolte $[7]$ on detection of a signal of known form but random phase in correlated gaussian noise. Claus et al showed in their analysis that the optimum processor removes an estimate of the interference from the data. Those eigencomponents corresponding to large eigenvalues of the noise covariance matrix $R$ are scaled more in that removal process. The matrix filter that operates on a data vector $z$ to produce the estimate of the interference is given by:

$$P = \sum_{k=1}^{r} \frac{\lambda_k}{\lambda_k + \sigma^2} v_k v_k^H$$

The $\lambda_k$ and $v_k$ are the $r$ principal eigenvalues and eigenvectors respectively of the covariance matrix, and $\sigma^2$ is the variance of the background white noise component. When the interference is strongly low rank — as in the case of reverberation — then $\lambda_k \gg \sigma^2$, and $P$ is approximately the following projection matrix:
To form the low rank matrices used with the ARSRP data, time-slices were taken from basebanded, complex hydrophone data and then formed into matrices of high redundancy (low intrinsic rank). The matrix form chosen was that of a Toeplitz / Hankel structure because of its desirable properties [13, 14]. It is known that the reverberation estimate near the edges of each time-slice is poorer than than in the center [15]; for that reason, overlapping time-slices were used and the samples near the edges were discarded.

In the process of performing the preceding operations, one must obviously make a choice as to what portion of the singular values is to be retained in the matrix $\Sigma_S$. If nothing is included (a possibility), then the implication is that there is no signal of interest in the present time-slice, and so there is no need to estimate a signal. If everything is included in $\Sigma_S$, then the unlikely implication is that only reverberation is present. The method of making the optimum choice is still the subject of ongoing research, although for the following analysis an energy thresholding method was employed. In this case, the sum of the squared singular values within matrix $\Sigma_S$ is required to exceed a threshold energy which is in turn estimated from either the known background noise level or from an acoustic reverberation model. It is this partitioning which controls the accuracy of the splitting of the original acoustic signal into “reverberation” and “other”.

In the process of forming the estimation matrix $(\hat{A})$, the rank of the estimate is also noted, since this is equal to the number of principal components which explain the data mathematically to within some mean square error. If the number of components is small compared to the number of degrees of freedom of the data, then the method has achieved the desired simplification of the signal estimate. Given an acceptable estimate, it is possible to isolate the reverberation by means of a simple subtraction.

We give an example of such a process in Figures 1a and 1b. Figure 1a shows the output of a simple delay and sum beamformer, as applied to the basebanded signals from the receiver hydrophone array. The data are taken from ping 190 of the ARSRP Reconnaissance Cruise and represent the time-series which was seen shortly after the transmission of the initial two second, 210 to 280 Hz hyperbolic fm pulse. The projector array was steered downward at six degrees while the ship was moving at a speed of 3.3 knots and at a heading of 300 degrees true. Figure 1b shows the result of applying the PCI method to the (complex) input signal with a threshold which had been fixed throughout the analysis to a level which was based upon a background noise estimate. The first 10 samples of Figure 1b are zero due to a windowing phenomenon. Figure 2a repeats the plot of the original signal so that Figure 2b — the reverberation-free estimate — can be compared to it. Throughout the estimation procedure, the rank of the estimate ranged between three and five.

A second example is given in figure 3 in which the threshold significantly exceeded the estimate of the background. As a result, only the strongest parts of the signal are selected, the residual (Figure 3c) still contains strong reverberation components, but in the places in which a component is selected only a rank one matrix is needed to represent that signal.

Once a residual signal is obtained, one can then apply a matched filter in the conventional way for the purpose of examining only weaker responses. As we demonstrated in an earlier paper, it is possible to recover signals from the residual that would otherwise be masked by the strong interference [8]. However, that is not the primary goal of the present effort, so further elaboration of that particular topic is not included here.

What is of interest is twofold: (1) We are interested in examining the effects of even crude acoustic modeling upon the PCI procedures, and (2) we wish to identify, if possible, the
acoustic component(s) which give rise to a PCI component. Currently, acoustic models such as the Bistatic Scattering Strength Model (BISSM) cannot predict the detailed structure of the observed matched filtered signals, although they can do quite well at predicting the lower-frequency structure in the signals [9]. It is presently thought that bathymetric detail is lacking, and that small facets are probably responsible for the higher-frequency behavior. Therefore, it is presently difficult to accomplish (2), although there is no reason that current models cannot have an immediate impact on (1).

Model Predictions: A typical acoustic model predicts the received acoustic energy as a function of direction. The PCI method, however, works with the recorded hydrophone signals, and so there must be some additional processing performed in order to make use of a model's results. There are a number of possible approaches, but we have elected to use the matrix of singular values, $\Sigma$, as an indicator of locally averaged signal energy. The sum of the squares of the main diagonal of $\Sigma$ represents a locally averaged energy that can be compared to the output of most acoustic models. When enough singular values have been included in $\Sigma_s$ so that the local reverberation is accounted for, then the "rest" (those contained in $\Sigma_0$) are presumed to describe the non-reverberant energy. This is a different procedure than the one outlined earlier, because in the previous cases it was assumed that it was the noise that had been estimated a priori.

Figures 4a and 4b compare the output of the ARSRP receiving array to the PCI energy estimate as a function of time from the start of ping 190. As can be seen, the second figure is quite similar to the output of many acoustic models. In particular, it is congruent with the output of the BISSM model, and we can therefore use that model to do the reverse; that is, to predict the portion of the PCI energy that ought to be due to reverberation.

Figure 5 shows the initial output obtained from a high-resolution run of BISSM. The samples of interest lie between 135 and 255 and correspond to the time interval of the earlier figures. If the appropriate scaling is performed, and the predicted mean level is used to control the PCI separation, then Figure 6 shows the initial results that have very recently been obtained. These are encouraging, in the sense that the separated signal of Figure 6b has a pleasing visual appearance, but additional analysis will be required to decide upon the validity of the output.

Injected Signals: We have also made some very crude calculations of the reflected hfm pulse amplitude versus frequency to be expected from rectangular facets of $3\lambda$, $5\lambda$, and $8\lambda$ square dimension at 210 Hz. Figures 7a and 7b show the basebanded, ideal hfm pulse, versus its idealized reflection from a $5\lambda$ facet inclined at a 45 degree angle of incidence to the wavefront. As can be seen, there is a strong amplitude-dependence which is in turn the result of the changing frequency within the incident hfm pulse. This behavior which is typical for a wide range of incidence angles and plate sizes, and it would seem that the process of matched filtering the reverberation could be optimized in some sense by using a matched filter which more closely matches the actual return. Due to space limitations, it is not possible to discuss this fully here, although we would like to state that such matched filtering has been attempted, including the case in which an injected signal from a $5\lambda$ plate was present, and a corresponding matched filter was used.

Surprisingly, the hfm pulse is quite robust against such departures from ideality, and the outputs of several kinds of matched filters all gave essentially the same output. We have interpreted this to mean that it is unlikely that the presence of facets can be directly detected with suitably adjusted matched filters.

Conclusions

The PCI method appears to be combinable with the output of many acoustic models, al-
though at this point we have not demonstrated that the separation achieved is in fact optimal. The identification of PCI mathematical components with those predicted by acoustic models still remains to be accomplished, although the preliminary results given here are encouraging.

The PCI method can indeed separate most of the reverberant energy from an acoustic signal without the need for an acoustic model, and in that sense is a useful signal processing tool as it stands.
References


Broadside beam response from BISSM