The research goals for this grant were to obtain algorithms for control oriented system identification is to construct dynamical models of systems based primarily on measured data that are compatible with robust control design techniques. The research carried out under this grant has continued the research on control oriented identification originated by the PI and his collaborators, has extended control oriented identification methods to new classes of dynamical systems and has initiated a study unifying identification and control laws design. The research that has extended existing problem formulation concerns the construction of algorithms for linear shift invariant systems using a combination of apriori and experimental information. Algorithms for the identification of continuous time systems and efficient linear algorithms have been constructed. The research that has extended the existing problem formulation concerns the development of algorithms for the construction of parameterized linear families form a combination of apriori and measured information. Algorithms for this type of nonlinear system identification have been given that produce models suitable for gain scheduled controllers. Finally research into the integration control oriented identification and control for slowly time varying systems was initiated.
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Final Report

AFOSR Grant 91–0222
Control Oriented System Identification

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1 Summary

The research goals for this grant were to obtain algorithms for control oriented system identification. The goal of control oriented system identification is to construct dynamical models of systems based primarily on measured data that are compatible with robust control design techniques. The research carried out under this grant has continued the research on control oriented identification originated by the PI and his collaborators. It has extended control oriented identification methods to new classes of dynamical systems and has initiated a study unifying identification and control law design. The research that has extended the existing problem formulation concerns the construction of algorithms for linear shift invariant systems using a combination of apriori and experimental information. Algorithms for the identification of continuous time systems and efficient linear algorithms have been constructed. The research that has extended the existing problem formulation concerns the development of algorithms for the construction of parameterized linear families from a combination of apriori and measured information. Algorithms for this type of nonlinear system identification have been given that produce models suitable for gain scheduled controllers. Finally, research into the integration control oriented identification and robust control for slowly time varying systems was initiated under this grant.

2 Report of Research Status

The purpose of this section is to give brief explanations of the status of the research objectives of this grant and to summarize the research accomplishments. The discussion below contains a brief narrative of the accomplishments, a list of publications that stem from or were covered under the time frame of the grant and a list of participating professionals.

The research objectives of this grant can be summarized under the headings of control oriented system identification as (a) extension of existing idea, (b) development of identification for a class of nonlinear systems and (c) initiation of a new research thrust.

The research objective that extends the existing paradigm of control oriented identification was successful in producing new algorithms for identification. The research objective of worst case or control oriented identification of linear shift invariant systems is to produce not only a nominal model of a system but in addition produce a quantification of model error given a type of experimental information and certain apriori information concerning the unknown system. The quantification of uncertainty should be in a form that is suitable for a subsequent robust control design procedure, hence, the appellation control oriented identification. Research advances were made in this area by providing a series of algorithms and an assessment of the global optimality of these algorithms.

The algorithms and associated optimality studies were presented in the papers [1, 2, 3, 4, 7, 8]. The publications [1, 2, 3, 7, 8] deal with identification in the setting of $H_{\infty}$ and the paper [4] deals with the setting of $\ell_1$. The normed spaces $H_{\infty}$ and $\ell_1$ here denote the norm used for quantifying the error and these norms are appropriate for control oriented identification as they are the induced $L_2$ and $\ell_{\infty}$ norms, respectively, of linear systems, hence fit naturally into the setting of robust control theory that has recently been developed.

The research into $H_{\infty}$ worst case system identification has produced: (1) an original algorithm for discrete time systems that is (essentially) optimal, (2) a version of the algorithm for continuous time systems, (3) linear algorithms for the discrete time setting that offer a more efficient alternative and (4) algorithms that involve probabilistic apriori information for the discrete time setting. A short description of each contribution is now given.

The $H_{\infty}$ identification for discrete systems has the following problem formulation. The three components are (a) apriori information that delineates a class of systems, (b) experimental information assumed
available regarding the unknown system and (c) the metric used to quantify the identification error. The class of systems consists of linear shift invariant exponentially stable systems and the a priori information that is assumed available consists of an upper bound on the system gain and a lower bound on the decay rate of the system. Mathematically this information specifies the smoothness of the frequency response (the frequency response is assumed to lie in the disc algebra $A(D)$ rather than $H_\infty$), moreover, this class of systems delineated by the apriori information specifies a compact subset of $H_\infty$. The experimental information that is assumed available consists of a finite set of evaluations of the frequency response (values on the unit circle) of the known system with unknown but bounded in size corruption. The metric used for error quantification is the $H_\infty$ norm which allows, as mentioned above, the subsequent step of robust control to be carried out cleanly. In this setup two main results have been given: (1) an algorithm for identification and (2) a global lower bound for identification error that is algorithm independent. The tools of information based complexity have been introduced into the identification area to obtain the lower bound and the use is to show that the algorithm developed is essentially optimal (to within a factor of two). The algorithm is a two step procedure of spline interpolation followed by the solution of a Nehari problem for the "projection" of $L_\infty$ onto $H_\infty$. This procedure computationally requires an FFT of the order of the number of data points and a singular value decomposition of a larger matrix. The computations, while numerically intensive, are simple to implement.

The ability to relax certain assumptions of the discrete time setting motivated the papers [2, 3, 7, 8]. The paper [2] deals with the extension to continuous time systems using an additional piece of a priori information concerning the rate of decay of the frequency response (strictly proper). The paper [3] deals with the development of less computationally burdensome algorithms. These algorithms are based on least squares optimization methods and use results on the growth of analytic functions to derive error bounds. The disadvantage of these algorithms is the requirement that they be so called tuned, or, use a priori information explicitly. This paper raised the importance of obtaining correct a priori information as opposed to merely guaranteeing the existence which is of course more reasonable. Finally, in an attempt to reduce the dependence on the a priori information the papers [7, 8] explored the use of probabilistic a priori information. The thrust of this work is to add a degree of freedom to the identification setting and the main result is to show that this probabilistic framework only partially mitigates the dependence on prior information. The technical details are borrowed from similar work in information based complexity theory.

The research that developed worst case identification to different classes of dynamical systems under this grant concentrated on certain types of nonlinear systems. The research has been documented in the papers [10, 5] and the thesis of K. Perev [9] carried out under the supervision of the PI. These works contain the main results of (1) clarifying the approximation power or type of uncertainty in a parameterized linear family introduced by Rugh and coworkers and (2) developing identification algorithms for the construction of parameterized linearization families from measured data with error bounds in a form suitable for gain scheduled controller design. The results concerning approximation power of parameterized linearization families begin to clarify to what extent scheduled linear models can represent nonlinear difference equations over wide variations in the operating regime, that is, not small signal type operation. The results concerning identification build upon the linear shift invariant results previously developed and utilize a priori information of the nonlinear system concerning the smoothness or rate of variation of the nonlinear system along a prescribed trajectory. Identification of systems operating in closed loop and near Hopf bifurcation (small signal unstable) are considered briefly in the thesis of K. Perev as well.

A new research direction was begun under this grant. The integration of worst case or induced norm identification with robust control design techniques is a natural area to clarify given the claims of "control oriented" identification. The paper [6] presents preliminary results that illustrate the simplicity that worst case identification can bring to the analysis and systematic design of adaptive controllers. The paper presents results for the adaptive control of slowly time varying systems and the main result is a
quantification of an upper bound on the time variation of the system that may be tolerated to preserve a level of performance (tracking and disturbance rejection) based on an $H_\infty$ control design carried out on frozen time systems identified through recursive worst case identification techniques.

3 Publications Under Grant

The following list of references are publications that were completed under the time frame of this grant.

References


4 Participating Professionals

The research carried out under this grant involved collaboration with several individuals. A list is as follows.

- Arthur Helmicki, Assistant Professor, Department of Electrical Engineering, University of Cincinnati. The main area of collaboration was in the area of $H_\infty$ control oriented identification.
- Carl Nett, United Technologies Research Center. The main area of collaboration was $H_\infty$ control oriented identification.

- Jonathan Partington, Lecturer, University of Leeds. The main area of collaboration was in $\ell_1$ control oriented identification.

- Gilead Tadmor, Assistant Professor, Northeastern University. The main area of collaboration was in the integration of worst case identification with $H_\infty$ robust design techniques for slowly time varying systems.