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DRA Haslar, Gosport, Hants PO12 2AG

APPROXIMATION TO SOUND RADIATION AND SCATTERING
BY SIMPLY SUPPORTED LAYERED CYLINDER

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APPROXIMATION TO SOUND RADIATION AND SCATTERING
BY SIMPLY SUPPORTED LAYERED CYLINDER

by J H James

ABSTRACT

A theoretical approximation is obtained for calculating the acoustics of a finite multilayered anisotropic cylinder which is excited either by a mechanical force or by a plane wave at oblique incidence. The cylinder dynamics is obtained by a semi-analytical theory, and the far field acoustics by standard cylindrical baffle theory. A numerical example of steady state and transient sound radiation demonstrates an application of the model.
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1. Introduction

Approximate theories for calculating sound radiation from and sound scattering by a simply supported cylindrical shell, which is contained between cylindrical rigid baffles, are now basic to acoustics, see, for example, the standard text [1] on fluid-structure interaction and the reports by James [2-3]. A report by Jones [4] gives the theory of a simply supported shell which is covered by a coating modelled as fluid. Many assumptions have been made in order to render tractable the mathematics of the simply supported shell vibrating in an acoustic fluid, among which is the assertion that modal coupling of the in-vacuo structural modes by the surrounding fluid can be neglected, and for certain modes, structural loss factors are greater than radiation loss factors. Thus, while recognizing that no more than qualitative numerical values or physics can be anticipated from these models, their primary role is to help contribute valuable insight into the physical mechanisms that control the acoustics of finite shells.

An increasing use of composite materials makes desirable analytical methods for predicting their acoustic characteristics. Recent work by Skelton & James [5] has provided a substantial theoretical basis for calculating the acoustics of infinite cylindrically layered media comprising anisotropic solids and acoustic fluids. A semi-analytical approach is used: axial and circumferential variations are represented by Fourier transforms and series, respectively; through thickness variations by the finite element method. The aforementioned work is used as the foundation of the theory which is given here with a minimum of explanation. The geometry is shown in Figure 1: a simply supported cylinder, comprising layers of anisotropic solid and acoustic fluid, is contained between rigid baffles. The objective is to give an approximate theory which can be used to predict the far field sound radiation when the excitation is a mechanical force and the target strength when the excitation is a plane wave at oblique incidence.

The work described herein was done under item AS02EH14 of the Strategic Research Programme, "acoustics of novel materials and hull configurations", it contributes to the third Milestone.
2. Mathematics

2.1 Fourier Expansions

Letting $u_z(r,\phi,z)$, $u_\phi(r,\phi,z)$, $u_r(r,\phi,z)$ denote axial, tangential and radial displacements, then the following expansions satisfy the simply supported boundary conditions at the ends of the cylinder, $z=-L$ and $z=L$,

$$u_z(r,\phi,z) = \sum_{m,n} u_z(r,n,m) \cos\left[\frac{m\pi(z+L)}{2L}\right] \exp\left(\imath n\phi\right),$$

$$u_\phi(r,\phi,z) = \sum_{m,n} u_\phi(r,n,m) \sin\left[\frac{m\pi(z+L)}{2L}\right] \exp\left(\imath n\phi\right),$$

$$u_r(r,\phi,z) = \sum_{m,n} u_r(r,n,m) \sin\left[\frac{m\pi(z+L)}{2L}\right] \exp\left(\imath n\phi\right),$$

where the sums are understood to extend over values of $m$ ranging from 1 to $\infty$, and values of $n$ ranging from $-\infty$ to $+\infty$. The time variation factor $\exp(-\imath \omega t)$ is suppressed in these and subsequent equations. The spectral quantities are given by the inverse transforms, for example,

$$u_z(r,n,m) = \frac{1}{2\pi L} \int_{-L}^{2L} \int_{0}^{2\pi} u_z(r,\phi,z) \cos\left[\frac{n\pi(z+L)}{2L}\right] \exp\left(-\imath n\phi\right) d\phi dz.$$  \hspace{1cm} (2.1.2)

2.2 Dynamic Stiffness of Elastic Element

The procedure closely follows previous work [5], except that a Fourier series represents axial variations rather than a Fourier integral. The element has three nodes through its thickness, the first coincides with the outer surface $r=b$, the second node is at the mid-surface $r=(a+b)/2$, and the third node is at the inner surface $r=a$. Omitting details, the $9 \times 9$ spectral dynamic stiffness matrix, when the variations of the displacements in the radial coordinate are approximated by quadratics, is found to be

$$[S(n,m) - \omega^2 M]u(n,m) = F(n,m),$$  \hspace{1cm} (2.2.1)

where

$$S(n,m) = (A^{-1})^T \int_a^b B^*(r,n,m)^T D B(r,n,m) r dr A^{-1},$$  \hspace{1cm} (2.2.2)

$$M = (A^{-1})^T \int_a^b \rho N^*(r)^T N(r) r dr A^{-1},$$
with the asterisk denoting the operation of complex conjugate. The integrals are evaluated numerically by using a two-point Gaussian quadrature formula.

The $9 \times 1$ displacement and excitation matrices, which are the double Fourier series coefficients of the nodal displacements and nodal stress excitations, are

$$u(n,m) = [u_x(b,n,m), u_x(b,n,m), u_x(b,n,m), u_x(c,n,m), u_x(c,n,m), u_x(c,n,m), u_x(a,n,m), u_x(a,n,m), u_x(a,n,m)]^T,$$

and

$$F(n,m) = [F_x(b,n,m), F_x(b,n,m), F_x(b,n,m), F_x(c,n,m), F_x(c,n,m), F_x(c,n,m), F_x(a,n,m), F_x(a,n,m), F_x(a,n,m)]^T,$$

with $c = (a+b)/2$. The stress excitations are defined as positive when they act in the positive directions of the coordinate axes. The $9 \times 9$ matrix connecting spectral displacements and unknowns is

$$A = \begin{bmatrix}
1 & b & b^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & b & b^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & b & b^2 \\
1 & c & c^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & c & c^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & c & c^2 \\
1 & a & a^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & a & a^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & a & a^2 & 0
\end{bmatrix}.$$
The 6x9 matrix connecting spectral strains and unknowns is

\[
B(r, n, m) = \begin{bmatrix}
-a & -ar & -ar^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \text{in}/r & \text{in} & \text{inr} & 1/r & 1 & r \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2r \\
in/r & \text{in} & \text{inr} & a & ar & ar^2 & 0 & 0 & 0 \\
0 & 0 & 0 & -1/r & 0 & r & \text{in}/r & \text{inr} & 1 \\
0 & 1 & 2r & 0 & 0 & 0 & a & ar & ar^2
\end{bmatrix}
\] (2.2.6)

were \( \alpha = \frac{m\pi}{2L} \). This matrix differs from the corresponding matrix of the infinite cylinder [5], as it reflects a Fourier series expansion in the axial direction rather than a Fourier integral transform. The 3x9 polynomial matrix is

\[
N(r) = \begin{bmatrix}
1 & r & r^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & r & r^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & r & r^2
\end{bmatrix}
\] (2.2.7)

The inverse of the 6x6 constitutive matrix when the material is orthotropic is

\[
D^{-1} = \begin{bmatrix}
1/E_{xx} & -\nu_{xy}/E_{yy} & -\nu_{xy}/E_{yy} & 0 & 0 & 0 \\
-\nu_{xy}/E_{yy} & 1/E_{yy} & -\nu_{xy}/E_{yy} & 0 & 0 & 0 \\
-\nu_{xy}/E_{yy} & -\nu_{xy}/E_{yy} & 1/E_{yy} & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{xz} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{xz} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{xz}
\end{bmatrix}
\] (2.2.8)

where the constants have their customary meanings and

\[
\nu_{xy}/E_{yy} = \nu_{yz}/E_{zz}, \quad \nu_{xy}/E_{yy} = \nu_{yz}/E_{zz}, \quad \nu_{xy}/E_{yy} = \nu_{yz}/E_{zz}
\] (2.2.9)

For fibre reinforcement at a helical winding angle along the cylindrical surface, the matrix transformations to convert a constitutive matrix in a local axes set, in which it is orthotropic, to a global axes can be found elsewhere [6]. Damping is simulated by allowing the elastic constants to be complex, viz \( E = E(1-i\eta) \) etc where \( \eta \) is the hysteretic loss factor.
After taking the inverse of the $9 \times 9$ dynamic stiffness matrix in equation (2.2.1), deleting rows and columns 4, 5, 6 and then inverting the resulting receptance matrix, the $6 \times 6$ dynamic stiffness matrix relating spectral displacements and excitations at the outer and inner boundaries, $r=b$ and $r=a$, is obtained for the element as

$$[S_e(n,m)]u_e(n,m) = F_e(n,m),$$

where

$$u_e(n,m) = [u_z(b,n,m), u_y(b,n,m), u_r(b,n,m), u_z(a,n,m), u_y(a,n,m), u_r(a,n,m)],$$

$$F_e(n,m) = [F_z(b,n,m), F_y(b,n,m), F_r(b,n,m), F_z(a,n,m), F_y(a,n,m), F_r(a,n,m)].$$

are the surface spectral displacements and excitations.

### 2.3 Dynamic Stiffness of Fluid Element

For an acoustic fluid layer of density $\rho$, sound speed $c$, inner radius $a$ and outer radius $b$, the $2 \times 2$ dynamic stiffness matrix is obtained, after expanding the pressure in a Fourier sine series as

$$\frac{\rho \omega^2}{\gamma W} \begin{bmatrix} H_{|n|}(\gamma b)J'_{|n|}(\gamma a) & 2i/\pi \gamma b \\ J_{|n|}(\gamma b)H'_{|n|}(\gamma a) & 2i/\pi \gamma a \\ \end{bmatrix} \begin{bmatrix} u_z(b,n,m) \\ u_r(b,n,m) \\ \end{bmatrix} = \begin{bmatrix} F_z(b,n,m) \\ F_r(b,n,m) \\ \end{bmatrix},$$

where

$$W = J'_{|n|}(\gamma b)H'_{|n|}(\gamma a) - J_{|n|}(\gamma a)H'_{|n|}(\gamma b),$$

and $\gamma = \omega^2/m^2 - \pi^2/4L^2$, with $k = \omega/c$. 

5
For an exterior acoustic fluid of density $\rho$, sound speed $c$, and outer cylinder radius $b$, the $1 \times 1$ dynamic stiffness matrix is obtained as

$$\rho \omega^2 [H_{|a|}(\gamma b)/\gamma H'_{|a|}(\gamma b)] u,(b,n,m) = F_r(b,n,m).$$

(2.3.3)

For an interior acoustic fluid of density $\rho$, sound speed $c$, and inner cylinder radius $a$, the $1 \times 1$ dynamic stiffness matrix is obtained as

$$-\rho \omega^2 [J_{|a|}(\gamma a)/\gamma J'_{|a|}(\gamma a)] u,(a,n,m) = F_r(a,n,m).$$

(2.3.4)

### 2.4 Assembly of Elements

Assembly of $M$ element matrices, together with matrices for the exterior and interior fluids, is a straightforward finite element procedure which results in the system matrix equation

$$Z(n,m) U(n,m) = E(n,m)$$

(2.4.1)

where $Z$ is a $(3M+1) \times (3M+1)$ matrix of bandwidth 11. $U$ is a column vector of the $3(M+1)$ spectral displacements at the interfaces, and $E$ is a column vector of the spectral excitations which are positive when acting in the positive directions of the coordinate axes.

A $2 \times 2$ fluid dynamic stiffness matrix can be assembled by treating it as $6 \times 6$ elastic dynamic stiffness matrix whose elements are zero except for the $(1,1)$, $(1,4)$, $(4,1)$ and $(4,4)$th elements which are obtained from equation (2.3.1). For the special case in which fluid layers form consecutive elements it will be necessary to set to zero rows and columns corresponding to axial and tangential displacements at the common interface, the diagonal elements being set to unity to allow numerical inversion without the chore of condensing out displacements associated with these null rows and columns.

### 2.5 Point Force Excitation

A point force applied to the $j$th interface at coordinate $(r_0, \phi_0, z_0)$ is represented by the $3 \times 1$ excitation vector

$$F(\phi,z) = F_0 \delta(z-z_0) \delta(\phi-\phi_0)/r_0,$$

(2.5.1)

where $F_0 = (F_z, F_\phi, F_r)^T$ are the point force amplitudes. The spectral representation of these forces is
The elements of the force vector are added to the \(3j-2\), \(3j-1\) and \(3j\)th elements of the excitation.

The spherically spreading far field sound radiation, in spherical polar coordinates \((R, \theta, \phi)\), in a fluid of density \(\rho\) and sound speed \(c\), is obtained from cylindrical baffle theory [1-3] as

\[
p_{\gamma}(R, \theta, \phi) = -i\rho \omega^2 \exp(iR) \sum_{n,m} \frac{(-i)^{|n|} L A_m(\alpha) u_r(b,n,m) \exp(in\phi)}{\sigma_n'(k\sin \theta)},
\]

where

\[
A_m(\alpha) = \exp(i\alpha L)(m\pi/2L^2)[1-(-1)^n \exp(-2\alpha L)]/(m^2 \pi^2/4L^2 - \alpha^2)
\]

is the inverse Fourier integral transform, divided by \(L\), of the radial displacement function \(\sin[m\pi(z+L)/2L]\) which is assumed to be zero outside the interval \(z=(-L,L)\). This function must be evaluated at the stationary phase wavenumber \(\alpha = k\cos(\theta)\). The displacement \(u_r(b,n,m)\) is the third element in the solution vector \(U(n,m)\) of equation (2.4.1), ie it is the spectral amplitude of the radial displacement of the outer surface of the layered cylinder.

### 2.6 Plane Wave Excitation

A plane wave arriving from angles \(\theta = \theta_i\) and \(\phi = \phi_i\),

\[
p_i = P_0 \exp(-ikz\sin \theta_i \cos \phi_i - ikz\sin \theta_i \sin \phi_i - ikz\cos \theta_i),
\]

is expressed in cylindrical polar coordinates as

\[
p_i = P_0 \exp(-ikz\cos \theta_i) \sum_{n} (-i)^{|n|} \exp(in(\phi - \phi_i)) J_0(kr\sin \theta_i).
\]

The pressure scattered from an infinite rigid cylinder, of radius \(b\), is

\[
p_r = -P_0 \exp(-ikz\cos \theta_i) \sum_{n} (-i)^{|n|} \exp(in(\phi - \phi_i)) \frac{J_0'(k\sin \theta_i) H_0'(k\sin \theta_i)}{H_0'(k\sin \theta_i)}.
\]
The excitation on the cylinder surface is taken as the blocked pressure, \(-p_b-p_r\), evaluated at \(r=b\),
\[
E(b,\phi,z) = P_0 \exp(-ikz\cos\theta) \sum_n \frac{-2i(-i)^n \exp(in(\phi-\phi_0))}{\pi kbsin\theta H'_n(kbsin\theta)}
\] (2.6.4)

The Fourier series coefficients of this excitation confined to the axial coordinates \(z=(-L,L)\), are obtained from equation (2.1.2), using the axial part of the transform alone, as
\[
E(b,n,m) = \frac{-2P_0 A_{n,m}(\alpha=kcose)i(-1)^n \exp(-in\phi)}{\pi kbsin\theta H'_n(kbsin\theta)}
\] (2.6.5)

This excitation is added to the 3rd element of the excitation vector.

The pressure scattered to the far field in the exterior fluid is the sum of two terms, viz
\[
p(R,\theta,\phi) = p_s(R,\theta,\phi) + p_e(R,\theta,\phi)
\] (2.6.6)

where \(p_s\) is the pressure scattered by a rigid cylinder of length \(2L_b\), as given by James [3],
\[
p_s(R,\theta,\phi) = 2iL_b P_0 \sin\theta [\sin[kL_b(\cos\theta + \cos\phi)]]/kL_b(\cos\theta + \cos\phi) \times
\]
\[
\exp(ikR) \sum_n \frac{(-1)^n \sin[kL_b(\cos\theta + \cos\phi)]}{\pi kbsin\theta H'_n(kbsin\theta)}
\] (2.6.7)

and \(p_e\) is in the far field pressure radiated by surface vibrations confined to \(z=(-L,L)\). This pressure is given by Equation (2.5.3) in which \(u_{b,n,m}\) is the third element in the solution vector \(U(n,m)\) of equation (2.4.1). Normally the length, \(2L_b\), for calculating the "rigid" body scattering is taken as the length, \(2L\), of the vibrating cylinder. The pressure \(p_s\) does not obey the reciprocity principle as \(\theta\) and \(\theta_i\) are not interchangeable. This is a defect of cylindrical baffle theory which requires that caution should be exercised when interpreting scattering levels as \(\theta=180^\circ\) is approached.
3. Numerical Examples

3.1 General

A Fortran program (the listing is not given here) has been written to calculate transfer functions of far field sound level at selected observation angles \((\theta, \phi)\) when the excitation is a point force; and transfer functions of both monostatic and bistatic target strength when the excitation is a plane wave at arbitrary incidence. For the case of a thin steel shell, with or without a fluid coating, excellent numerical agreement has been obtained with results calculated from thin shell theories \([2-4]\), but these comparisons are not included here. Transient responses are readily calculated from the time-harmonic transfer functions by using an auxiliary program for which the time variation of the source can be specified by a library of excitations; see the report by James \([7]\) for details.

3.2. Radiation

In order to illustrate an application of the program, consider a plastic cylinder which is reinforced circumferentially by carbon fibre, and covered by a compliant coating. The constants of the CFRP cylinder are \(E_{zz}=E_{rr}=7.4\), \(E_{\phi\phi}=202.0\), \(v_{z\phi}=0.32\), \(v_{rz}=0.427\), \(v_{r\phi}=0.0117\), \(G_{rr}=2.74\), \(G_{z\phi}=G_{rz}=2.59\), \(\rho=1520\), outer radius \(b=1.0\), inner radius \(a=0.95\) and loss factor \(\eta=0.02\). The compliant coating constants are \(E=0.0028\), \(v=0.40\), \(G=0.001\), \(\rho=1100.0\), outer radius \(b=1.05\), inner radius \(a=1.00\) and loss factor \(\eta=0.1\). All values are in SI units with the Young's and shear moduli being in \(\text{GN/m}^2\). The composite cylinder is immersed in water for which \(\rho=1000\) and \(c=1500\), and contains a light fluid whose reaction on the shell is negligible. The excitation is a unit radial point force located at the centre of the cylinder on its inner surface, at \(\phi=0^\circ\); the far field sound radiation is measured at beam aspect \(\phi=0^\circ\), \(\theta=90^\circ\).

In Figure 2(a) is shown the transfer function of the far field sound level in the absence of the compliant coating. The strong resonances can be identified by the integer wavenumbers \((m,n)\), where \(m\) is the number of half-axial wavelengths and \(n\) the number of full circumferential wavelengths, though this is not done here. In Figure 2(b) is shown the transient response when the excitation is a single cycle of a 200 Hz sine wave. The small response up to relative time \(t=10\ \text{ms}\) is non-causal due to the abrupt termination of the radiated sound transfer function at 500 Hz where it is large; formally this is the constraint that a function cannot be both time limited and bandwidth limited. There follows a rapid resonant build-up and a slowly decaying ringing dominated by the 185 Hz discrete in the transfer function spectrum.

In Figure 3(a) is shown the transfer function of the far field sound level when the CFRP cylinder has an external compliant coating. The isolating properties and large damping of
the coating have reduced overall sound levels and blunted sharp resonances. In Figure 3(b) is shown the transient response when the excitation is a single cycle of a 200 Hz sine wave. The reduction in amplitude and much faster decay of the transient far field pressure, due to the coating, is most evident when this plot is compared with Figure 2(b).
4. Conclusions

Formulae have been given for calculating sound radiation from and sound scattering by a laminated composite cylinder whose ends are simply supported. Numerical results of both steady-state and transient acoustics have demonstrated an application of the model, which is believed to be a useful tool for investigations of the physical mechanisms that control the acoustics of finite fibre reinforced shells.
References


Figure 1. Layered cylinder, coordinate systems and excitations. The cylinder is located between rigid baffles which are not shown.
Figure 2. Beam aspect sound radiation from a circumferentially reinforced CFRP cylinder in water: (a) transfer function of a unit time-harmonic point force; and (b) transient sound radiation when the excitation is a single cycle of a 200 Hz force. The cylinder is uncoated.
Figure 3. Beam aspect sound radiation from a circumferentially reinforced CFRP cylinder in water: (a) transfer function of a unit time-harmonic point force; and (b) transient sound radiation when the excitation is a single cycle of a 200 Hz force. The cylinder is covered externally by a soft, highly damped, coating.
Abstract
A theoretical approximation is obtained for calculating the acoustics of a finite multilayered anisotropic cylinder which is excited either by a mechanical force or by a plane wave at oblique incidence. The cylinder dynamics is obtained by a semi-analytical theory, and the far field acoustics by standard cylindrical baffle theory. A numerical example of steady state and transient sound radiation demonstrates an application of the model.

Descriptors