The report describes work by several investigators on a number of projects in the general area of dynamical systems, nonlinear partial differential equations, delay equations and continuum physics.
Dynamical Systems & Nonlinear Partial Differential Equations

FINAL REPORT

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WORK BY C.M. DAFERMOS

Dafermos worked in the general area of hyperbolic systems of conservation laws and their applications to continuum physics. He developed the method of generalized characteristics for BV solutions of hyperbolic systems of conservation laws [1] and used it in the study of solutions of the system of the equations of electrophoresis [2] (joint work with X. Geng) and a system of two equations with coinciding shock and wave curves in one field [3] (also jointly with X. Geng). Also he used generalized characteristics to study the large time behavior of solutions of genuinely nonlinear systems of two equations under periodic initial data [4]. Dafermos also established [5] the equivalence of the Lagrangian and the Eulerian formulation of the balance laws of continuum physics under very general conditions.

References


WORK BY JOHN MALLET-PARET

Our work has dealt with infinite dimensional systems, in particular dissipative systems such as differential delay equations and parabolic PDE's. Both linear and nonlinear equations have been considered, and the problems studied have strong connections with the applications.

In [N1], with R. Nussbaum, a class of differential delay equations, arising in various models in physiology and biology, was studied. The outstanding feature of these equations is a variable time delay which is state-dependent. A general theory of singularly perturbed state-dependent delay equations was initiated in a subsequent paper [N2]. While such equations have been proposed in a wide variety of models, and have been the subject of much heuristic and numerical analysis, very little theoretical work has been done until recently. Our studies have served to confirm and explain much of the observed dynamical behaviour. A student, Panos Paraskevopoulos (Ph.D. expected 1993), is also studying such problems, while another student, Fatihcan Atay (Ph.D. expected 1994), is considering time-delay problems with implicitly defined derivatives.

In [CL], with S.-N. Chow and K. Lu, asymptotic spectral properties of linear, time-periodic, parabolic equations were studied. In particular, an infinite dimensional Floquet theorem was proved. Such questions of theoretical interest were stimulated by our earlier work on infinite dimensional Poincare-Bendixson theory. In addition, such results lay the foundation for the practical application of homotopy methods to the numerical calculation of eigenvalues in infinite dimensional problems, a project of current interest. A former student, Ying Huang (Ph.D. 1991), has considered analogous problems for differential delay equations in her thesis [H], and with her we are pursuing extensions of this. With Chow and Lu we are also pursuing non-periodic extensions of [CL]. With George Sell and Zhoude Shao [SS] we have obtained general spectral conditions which preclude the existence of inertial manifolds for infinite dimensional dynamical systems. In particular, a class of nonlinear parabolic equations in a four-dimensional spatial domain, for which no normally hyperbolic inertial manifold exists, was exhibited. This result complements earlier work with Sell on sufficient conditions for the existence of inertial manifolds.
References


WORK BY P.E. SOUGANIDIS

Souganidis worked on a number of projects relating to the Propagation of Fronts. In collaboration with Barles he showed the convergence of a wide class of approximation schemes to the solution of fully nonlinear second-order elliptic or parabolic, possibly degenerate, partial differential equations.

References


WORK BY WALTER STRAUSS

W. Strauss completed his monograph on nonlinear wave equations, which has become the primary reference work on the subject. In his work on kinetic theory, he and Glassey have investigated the effect of very fast-moving particles in a plasma. They have also studied collision effects. In particular, they have a proof of the asymptotic stability of the equilibrium in the Boltzmann equation with relativistic effects. He and his student, Y. Guo, have studied the effects of boundaries. In most cases the natural boundary conditions are mathematically incompatible, which means that they will generate singularities.

The stability of solitary waves continued with the general study of Grillakis, Shatah and Strauss and particular studies together with Souganidis on the BBM equation and with Strauss' student, Y. Liu on the Boussinesq equation. Precise conditions for the instabilities were found.

Work of Craig, Kappeler and Strauss was begun on the disappearance of singularities of dispersive systems. They showed that dispersive smoothing can occur even for fully nonlinear systems, for equations of KdV type. Investigation of general systems in continuing.

Papers appeared or completed in the period 89-92:

References


