This is a final report of the project titled “Problems related to conservation laws - Phase transitions and viscoelasticity”. In this project I considered the dynamical aspects of a phase transition problem and the stability issue of waves for viscoelastic materials with memory.
Final Report

In this grant I studied two subjects, a phase transition problem and equations related to viscoelastic materials with memory. In what follows, I summarize the results in both areas.

Phase Transition Problem
I mainly considered the equation

\[ u_{tt} = \sigma(u_x)_x + \nu u_{xxt} - \eta u_{xxxx} \]

with boundary conditions

\[ u(0, t) = 0, \quad \sigma(u_x(1, t)) + \nu u_{xt}(1, t) - \eta u_{xxx}(1, t) = P, \]
\[ u_{xx}(0, t) = 0, \quad u_{xx}(1, t) = 0. \]

Equation (0.1) models a bar which goes through a phase transition. The boundary conditions (0.2) show that the bar is under a soft loading device. The boundary conditions (0.3) are the natural boundary conditions for the corresponding variational problem. The terms with the coefficients \( \nu \) and \( \eta \) are called viscosity and capillarity terms, respectively. We assume that \( \sigma \) is a nonmonotone function of its argument. The main results that I obtained are summarized as follows:

1. the existence of global solutions,
2. the bifurcation and dynamic stability of steady state solutions,
3. the connecting orbit problem,
4. the existence of slow motions.

Viscoelasticity
I studied the existence and nonexistence of global solutions for a semilinear integral equation with memory which can be reduced to a system of quasilinear equations. I also considered the nonlinear stability of rarefaction waves and the travelling wave solutions. Also I discussed the existence of travelling wave solutions for systems with nonconvex constitutive relations.

List of Publications


8. The Korteweg theory of capillarity and the phase transition problems. (Transactions of the tenth army conference on applied mathematics and computing), 131-142.


List of participating scientific personnel
Hiroaki Fujimoto Master's degree