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CONTROL OF NONLINEAR SYSTEMS
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Summary

This report gives an overview of recent work by the PI and his graduate students on: State-Space and I/O Stabilization, Systems with Saturated Control, Discrete-Time Control, Identification of Nonlinear Systems, and I/O Equations.

1 Introduction

This report deals with the following topics:

1. State-Space and I/O Stabilization

2. Systems with Saturated Control

3. Discrete-Time Control

4. Identification of Nonlinear Systems

5. I/O Equations

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It is an updated version of the Progress and Forcast Report submitted 6 months ago.

2 State-Space and I/O Stabilization

This work deals with stability of nonlinear systems, and the effect of input perturbations, and is a long-standing direction of research pursued by the PI.

2.1 Stabilization With Respect to Sets

This research was done with the participation of graduate student Yuandan Lin, and resulted in his Ph.D. Thesis, which was completed in the late summer. (Dr. Lin subsequently became an Assistant Professor at Florida Atlantic University.) It dealt with issues related to the stabilization of nonlinear systems with respect to (not necessarily compact) sets. In this we were motivated by potential applications to a wide variety of areas.

As an illustration, consider problems of output feedback. One common definition of "detectability" (e.g. as done in the work of Vidyasagar) involves the existence of an observer for which the error satisfies Lyapunov estimates which depend only on the difference $\|x(t) - z(t)\|$, where $x(t)$ is the state of the plant and $z(t)$ is the state of the observer. For the joint plant/observer system, detectability becomes stabilization with respect to the set

$$A := \{(x, z) | x = z\}.$$

In many applications, one is interested in stabilization of an output variable (as opposed to the complete state). Consider the following two-dimensional system:

$$\dot{z} = z, \tag{1}$$

$$\dot{y} = -y + ux, \tag{2}$$

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with the variable y taken as the output. Observe that when $u \equiv 0$ the y variable converges exponentially to zero, uniformly on the initial state $(x(0), y(0))$. However, for nonzero u, no matter how small, the output diverges if $x(0) \neq 0$. Indeed, if $u \equiv \epsilon$ and $x(0) = x_0$, one has

$$y(t) = y_0 e^{-t} + \epsilon x_0 \sinh t \to \infty .$$

This is in marked contrast to the case of state-space stability, where at least for small controls and small initial states, bounded states result, if the system was asymptotically stable for $u \equiv 0$. One of the results in Lin's thesis, also presented in [11], applies in particular to give a feedback stabilizer so that the closed-loop system obtained for this example after applying that feedback law does have suitable stability properties even for nonzero u. This is useful when dealing with input perturbations, and continues research carried out under this grant but which in the past dealt only with state-space stability.

Another example arises in adaptive control, where one does not usually obtain convergence of parameters but only of states; that is, one has to study in effect stability of the adaptive control system with respect to the set $\mathcal{A} := \{(0, \lambda), \lambda \in \Lambda\}$ where λ is the vector of unknown parameters and "0" stands for the zero state. As yet another motivation, many problems involving tracking and regulation can be expressed as partial stabilization problems (of an error signal). Finally, systems in which derivatives of controls appear can be reduced, adding integrators, to systems in which such derivatives do not appear, but at the cost of extra state variables which are not to be controlled.

Lin's thesis and the paper [11] concentrated on some basic questions related to set stability. We gave there a converse Lyapunov theorem that does not assume compactness of the attracting set. After establishing the converse theorem, we gave a result on input to state stabilization that generalizes that available in the case of stablization to equilibrium points (given by the PI a couple of years back and published in the IEEE Transactions in Automatic Control).

In practice, control systems are very often affected by noise, expressed for instance as perturbations on controls and errors on observations. Thus, it is desirable for a system not only to be stable, but also to display so-called "input/state" stability properties. Essentially, this means that the behavior of the system should remain bounded when its inputs are bounded, and should tend to equilibrium when inputs tend to zero. This is closely related to the topic of stability under perturbations, studied in the classical dynamical systems literature. In very classical work, Lefschetz discussed stability with respect to equilibria under perturbations (referred by the author as quasi-stability). Later, Seibert studied local stability under perturbations with respect to closed sets. In the late 1980s, the PI introduced a particular precise definition of *input/state stability*, and established a series of results in this area. Inspired by that work in the equilibrium case, we investigated with Lin input/state notions of stabilizability for control systems with respect to closed sets, and generalized many results from the equilibrium case to the set case. A major qualitative difference between the equilibrium case and the *general case of closed sets is* caused by the lack of compactness of a general closed set, and because of this, some properties for the equilibrium case do not hold in the set case. Even passing from equilibria to compact sets is not entirely trivial, and we will give a few results that apply only in the compact case.

A closely related concept to input/state stability is *input/output* stability, or operator stability. In the classical linear case, one relates external stability, in the sense of boundedness of operators in L_∞ —or more general L_p— norms, to Lyapunov stability for realizations, under appropriate conditions of reachability and observability. For nonlinear systems, a notion of input/output stability was introduced by the PI a few years back, based on comparison functions, and a generalization was provided of the above classical facts. In the work with Lin, we further extended this latter input/output stability notion, and then established that internally stable systems give rise to input/output stable ones, as well as the converse implication provided that "well-posed" reachability and observability hold. The extension of the connection between internal and external stability, in particular, required much care, due to the use of possibly noncompact attracting sets.

In the work with Lin, we introduced a notion of robust uniform set stability for parameterized systems, and provided Lyapunov-theoretic characterizations. For equilibria, Lyapunov functions for parameterized families as in our definition are routinely assumed in an ad hoc manner in robust control studies. Our results provided a conceptual characterization of this assumption.

Another application of the set stability concepts which we discussed in the work with Lin was to the observer design problem. It is well known that in the linear case, if a system is stabilizable and

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observable, then one can always stabilize the system using the state estimates generated by an observer; this is a deterministic version of the "certainty equivalence" principle. The problem is, in the nonlinear case, much less trivial. Some authors, starting with Vidyasagar in the early 1980s, have studied the problem, and partial results have been obtained. In this part of the project, we compared Vidyasagar's definition of observers using Lyapunov functions with Tsinias' definition in terms of set stability.

Many open problems remain in this area, and the PI expects further progress.

2.2 Systems with No Drift

One of the major developments in nonlinear stabilization during the last year was the work by Coron on the existence of time-varying smooth feedback. This work followed an idea introduced by the PI and Hector Sussmann in a paper in the IEEE Conference on Decision and Control, 1981, but Coron carried out his work in great generality, proving that at least for systems with no drift, the usual Lie algebra condition suffices to guarantee the existence of the above type of feedback. His results gave rise to a large number of recent preprints on applications of these ideas, especially in France.

In [5], which was submitted in early January 1992 and quickly accepted for publication, the PI was able to give a simple and direct proof of the main technical lemma needed for Coron's proof (in the analytic case; Coron's result is a bit more general, as it requires just differentiability, but this should be enough for engineering applications). Moreover, the PI's proof shows that the main difficulty in applying the approach, namely that of finding certain open-loop controls, can be avoided by making use of a class of "random" oscillations. A paper describing a numerical technique is given in [7]. Unfortunately, we did not have time to pursue realistic applications examples, but we still hope to do so in the near future.

3 Systems with Saturated Control

Substantial progress has been achieved in this area, in joint work with the PI's colleague Sussmann and graduate student Yudi Yang.

Linear systems with saturated controls can be stabilized continuously —and even smoothly— as long as certain natural controllability and eigenvalue conditions hold. This was shown in our previous paper with Sussmann in the 1990 IEEE Conference on Decision and Control. The construction there relied on a complicated and far from explicit inductive procedure. On the other hand, in particular cases, such as one- and two-dimensional systems, it is easy to obtain stabilizing laws using a saturated linear feedback $u = \sigma(Fx)$, where σ is a saturation function. It is therefore natural to ask if such simple control laws can also be used in the general case. Unfortunately, Fuller established in 1969 that this is already false for *triple integrators*, at least for certain classes of functions σ which include most usual saturating nonlinearities. One is thus motivated to search for other simple control rules, such as *linear combinations* and *compositions* of saturation nonlinearities.

Recently, A. Teel showed that, in the particular case of single-input multiple integrators of any dimension, such combinations of saturations are indeed sufficient to obtain stabilizing feedback controllers. (Roughly, $n/2$ saturations are needed.) In our work [9], we show how to extend the results of Teel to obtain an analogous solution for the general case. While based on Teel's ideas, the technical details become far more complicated, because of the need to consider nonzero eigenvalues and to deal with multiple inputs. We remark that the approach is heavily based on the notion of CICS ("convergent-input implies convergent state") stability, which was studied in previous work by the PI.

During the last few months, we have made progress as follows: (1) the theorem has been extended to allow essentially arbitrary saturation nonlinearities; (2) a detailed example involving the stabilization of an airplane was developed in detail, with simulations of the obtained control law (a paper is in preparation); (3) initial progress was achieved in the more interesting question of tracking (as opposed to stabilization); (4) initial results are being obtained on operator stability questions for linear systems with saturation. The are some overlaps with our work in neural nets, because of the use of saturation functions, of course.

4 Discrete-Time Control

Modern control techniques typically result in complex regulation mechanisms, which must be implemented using digital computers. Consequently, an important area of research is that of studying the constraints imposed by computer control. The behavior of continuous-time plants under digital control can be modeled by discrete-time systems. Since the mid-seventies the PI had been interested in problems of control for nonlinear discrete time systems, and during the early 1980s the focus shifted largely to questions of accessibility and controllability. A major paper was published in early 1990, jointly with B. Jakubczyk, in the SIAM J. of Control and Optimization. In subsequent work with graduate student Francesca Albertini, we were able to generalize considerably the results in the paper with Jakubczyk. We submitted a paper last year, and it has already been accepted for publication, after some revisions which we made during this grant period, also in the SIAM J. of Control; see [3].

Current work with Albertini has succedeed in extending substantially even the results in [3]. For instance, we now know that a very weak recurrence assumption is sufficient to guarantee a "Chow's Lemma" valid for discrete time systems, and this applies to the analysis of Markov processes modeled by systems forced by white noise. Moreover, an analysis of the "noninvertible" case, one of the objectives of this part of the project, is begining to take shape. Currently Albertini is concentrating on writing up all of this for her Ph.D. thesis, but in the meantime we have just submitted [4], and another paper is being prepared. (Independently of all this, Albertini, who is a truly exceptional student, has been making great progress on different topics, related to our neural networks project.)

5 I/O Representations

Work continued together with the PI's former graduate student Y. Wang on the relation between realizability and the existence of "input output equations" of the type

$$E\left(w(t), w'(t), w''(t), \ldots, w^{(r)}(t)\right) = 0 \tag{3}$$

where

$$w(\cdot) = (u(\cdot), y(\cdot))$$

are the i/o pairs of the system. Understanding this relation is essential when dealing with issues of identifiability. The area generalizes to nonlinear systems the equivalence between autoregressive representations and finite dimensional linear realizability. Work under the previous grant proved that, for input/output operators that are defined by generating series, such an operator F satisfies an algebraic i/o equation (i.e., all pairs $(u, F[u])$ do) *if and only if* it the operator is realizable by a singular polynomial system. An analytic version is also available, and two papers, [1] and [2] finally appeared in print this year.

More recent work, the first part of which was reported at the 1991 Conference on Decision and Control, deals with the replacing of differential algebraic equations by *integral* equations, in so far as possible. The latter are far less sensitive to noise and more suitable for identification algorithms that involve prefiltering (as done in the linear case). In the linear case, differential equations are always equivalent to integral ones, but that is seen not to be so in the nonlinear case. This work has also resulted in a more complete understanding of the relations between our approach and that pursued using differential-algebraic techniques; see [6] for details. The newer paper [12] deals with the same issues for discrete-time, bringing out new phenomena that were not observed previously in that context.

6 Identification of Nonlinear Systems

In the work [8], with M.A. Dahleh, D.N.C. Tse, and J.N. Tsitsiklis, the problem of asymptotic identification for a class of nonlinear fading memory systems in the presence of bounded noise was studied. For any experiment, the worst-case error was characterized in terms of the diameter of the worst-case uncertainty set; optimal inputs that minimize the radius of uncertainty were characterized. Finally, a

convergent algorithm that does not require knowledge of the noise upper bound was furnished. The methods as well as the results are quite general and are applicable to a larger variety of settings. A journal paper is about to be submitted.

This is a somewhat new direction of research for the PI, motivated by trying to understand how the nonlinear techniques developed by the PI and others can be applied in the context of identification. We expect to carry out some more work along these lines, but not to the exclusion of the other topics reported above.

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