SOME POTENTIAL ERRORS IN SATELLITE WIND ESTIMATES
USING THE GEOSTROPHIC APPROXIMATION
AND THE THERMAL WIND

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**Title:** SOME POTENTIAL ERRORS IN SATELLITE WIND ESTIMATES USING THE GEOSTROPHIC APPROXIMATION AND THE THERMAL WIND

**Authors:** James Cogan

**Abstract:**
Current and planned passive sensors on meteorological satellites measure radiances that are converted directly into temperature and humidity soundings. In turn, wind velocity is derived from the temperature soundings and other derived variables (for example, geopotential heights). Current operational methods involve either the geostrophic wind or the thermal wind approximation. This report presents some information and sample calculations on the types of errors that may be expected when either approximation is used to estimate the actual wind from data gathered by passive satellite sensors. The assumption is that no additional data are available (that is, only satellite sounding data from sensors of the type found on present day environmental satellites). This report does not treat the improvements that should occur when data from other sources of data (for example, radar profilers, ground-based radiometers, unmanned aerial vehicles) are combined with the satellite data.
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1. INTRODUCTION

Passive satellite sounders do not directly measure wind velocity; instead they derive it indirectly from the satellite temperature profile. The temperature profile is computed from radiances (infrared or microwave) emitted by volumes of atmosphere, each defined by a horizontal resolution area, and a vertical thickness according to a weighting function centered at some pressure level. The temperature profile is used to compute geopotential heights for the several pressure levels. The height field from many soundings for each pressure level is used to compute the geostrophic wind. The geostrophic wind is an approximation to the real wind that assumes frictionless flow parallel to "straight" height contours (along a great circle), where the pressure force balances the coriolis force. An alternative method involves the shear of the geostrophic wind, commonly referred to as the "thermal wind." For a given layer, thermal wind may be derived by using the gradient of mean layer temperature or the gradient of layer thickness. Since the thermal wind is really a shear, the wind velocity at some baseline level must be measured or computed (for example, geostrophic or measured wind at 800 hPa). Figure 1 illustrates the meaning of the thermal wind through a simple example. In the example the assumption is that the geostrophic wind at 700 hPa (1 hPa = 1 mbar) has a speed and direction of $20 \text{ m s}^{-1}$ and $240^\circ$, and at 500 hPa the values are $20 \text{ m s}^{-1}$ and $300^\circ$. The thermal wind speed and wind direction are $20 \text{ m s}^{-1}$ and $360^\circ$, respectively. In this case the isotherms run from north to south, with the warm air to the west. Geostrophic wind and thermal wind are defined and derived in standard texts (Haltiner and Martin 1957, Holton 1979).

![Diagram](image)

Figure 1. Illustration of the thermal wind for a layer from 700 to 500 hPa (see text).
This report presents some information and sample calculations on the types of errors that may be expected when geostrophic wind or thermal wind is used to estimate the actual wind from data gathered by passive satellite sensors. The assumption is that no additional data are available (that is, only satellite sounding data from sensors of the type found on present day environmental satellites). This report does not treat the improvements that should occur when data from other sources of data (for example, radar profilers, ground-based radiometers, unmanned aerial vehicles) are combined with the satellite data.

2. GEOSTROPHIC WIND

2.1 Basic Derivations

The equation for geostrophic wind along a constant pressure surface is given by

\[ V_g = \frac{1}{f} \frac{\partial \phi}{\partial n} \]  

(1)

where \( f = 2 \Omega \sin \theta \) (\( \Omega \) is the angular velocity of the earth about the local zenith at latitude \( \theta \), and \( \Omega \) is the angular velocity at the poles), \( n \) is the horizontal distance, \( \phi \) is geopotential height, and \( V_g \) is the geostrophic wind component perpendicular to the \( n \) direction. In general, geopotential height is \( \phi = \frac{f}{g} \int_0^z \frac{dz}{z} \) where \( z \) is height and \( g = \) gravitational acceleration. For \( z \) in meters \( \phi = 0.98z \).

The geostrophic wind also may be computed for a constant level (height) surface where

\[ V_h = \frac{1}{p} \frac{\partial \phi}{\partial n} \]  

where \( \frac{1}{p} = \frac{R T_v}{\bar{p}} \) (\( T_v \) = mean virtual temperature along the distance \( n \) on the surface, \( \bar{p} \) = mean pressure along that same path, and \( R \) is the gas constant for dry air). The virtual temperature accounts for moisture in the air, which having a lower density than dry air causes \( T_v > T \). However, at temperatures normally found above 4 or 5 km \( T = T_v \) and for the purposes of this report one may consider \( T = T_v \) at satellite heights >2 km (generally true over land except in a moist summer or tropical atmosphere). This second form of the geostrophic wind equation can be used to check for changes caused by errors in \( \bar{p} \) or \( T_v \) (or \( T \)).

A similar type of "equilibrium" wind for curved parallel contours is referred to as the gradient wind. In this case, the tangential acceleration \( = 0 \). The scaler version of the equation is

\[ V_{gr} = \frac{f}{2} \left( -1 + \left( 1 + 4 \frac{V_g}{rf} \right)^{\frac{1}{2}} \right) \]  

where \( r = R \tan \alpha \) (\( R \) = radius of earth, \( \alpha \)
= angular radius of the equivalent small circle for \( r \) as seen from the center of the earth, and \( r = \text{radius of curvature.} \) The equation may be rewritten for \( V \) and \( V_g \) in knots and nautical miles, or kilometers per hour and kilometers.

\[
V = 1800rf \left( -1 + \left( 1 + \frac{V_g}{3600rf} \right)^2 \right) \tag{2}
\]

The parameter \( rf \) may be computed or extracted from standard tables to a sufficient accuracy (List 1984).

Heights of pressure levels are normally computed from the hydrostatic equation

\[
z - z_o = - \frac{RT}{g} \ln \left( \frac{P}{P_o} \right) \tag{3}
\]

where the subscript \( o \) refers to the surface or bottom of a layer. As before, \( T \) may be substituted for \( T' \). Here the mean value is along a vertical path for a layer of atmosphere. Computation begins at the surface or a reference \( p \) level. The height differences or layer thicknesses \( (z - z_o = \Delta z) \) are "stacked" upon each other from bottom to top.

The real wind may be defined as the geostrophic wind plus an ageostrophic component. The horizontal part of this component consists of a pressure tendency term and a term involving the change in spacing of the contours (or isobars). To a sufficiently reasonable approximation (Haltner and Martin 1957), the pressure tendency \( \left( \frac{\partial P}{\partial t} \right) \) term is given by

\[
(V - V_g)_t = \frac{1}{\rho f^2} \frac{\partial P}{\partial \xi} = - \frac{1}{\rho f^2 \Delta s} \frac{\Delta P}{\partial \xi} \tag{4}
\]

where \( \Delta s \) consists of normal and tangential distances. The term for the change in contour spacing may be found by \( (V - V_g)_s = \frac{V \partial V}{f \partial s} \) where \( V \) is the real wind. To a fairly reasonable approximation, \( V \) on the right side may be approximated by \( V_g \), the mean value along the distance \( s \) (in this case along a path equidistant from neighboring contours).
One may compute values that roughly estimate "typical" magnitudes of the possible errors in wind caused by the geostrophic approximation and errors in the geostrophic wind caused by errors in input values (for example, mean layer temperature). As a first step, wind speed only is considered.

### 2.2 Errors In \( V_g \) Caused By Mean Profile Temperature Error

The geostrophic wind for contours at 100 geopotential meters (gpm) in units of knots (List 1984) may be stated as

\[
V_g = \frac{0.01712}{\Delta n}
\]

where \( \Delta n \) is in degrees of latitude. At \( 45^\circ \) latitude and a contour interval of 30 gpm, and \( \Delta n \) expressed in kilometers,

\[
V_g = \frac{5.701 \times 10^3}{\Delta n} \text{ in kn or } V_g = \frac{2.9387 \times 10^3}{\Delta n} \text{ in } \text{ms}^{-1}
\]

For other contour intervals multiply by the ratio of "new interval"/30 gpm. For a typical midlatitude atmosphere (Jursa 1985), for the surface (assume at 1000 hPa) to 500 hPa layer, the possible error can be computed from net temperature profile error by using equations (3) and (6). The mean layer temperature for the April atmosphere at \( 45^\circ N \) (Jursa 1985) is used to compute the height of the 500 hPa level.

\[
\Delta z = \frac{-287 \text{ K}^{-1} \text{m}^2\text{s}^{-2}(265 \text{ K})(-0.693)}{9.81 \text{ m} \text{ s}^{-2}} = 5373.6 \text{ m} = 5266.1 \text{ gpm}
\]

Values are computed for differences of \( \bar{T} \) of 1 and 2 K, or \( \bar{T} = 264 \) and 263 K.

\[
\Delta z_{264} = 5353.3 \text{ m} = 5246.2 \text{ gpm}
\]

\[
\Delta z_{263} = 5333.1 \text{ m} = 5226.4 \text{ gpm}
\]

For the case of a difference over the 1000 to 700 hPa layer for the same atmosphere,

\[
\Delta z_{268} = \frac{-287 \text{ K}^{-1} \text{m}^2\text{s}^{-2}(268 \text{ K})(-0.356675)}{9.81 \text{ m} \text{ s}^{-2}} = 2797.0 \text{ m} = 2741.1 \text{ gpm}
\]

\[
\Delta z_{267} = 2786.6 \text{ m} = 2730.9 \text{ gpm}
\]

\[
\Delta z_{266} = 2776.1 \text{ m} = 2720.6 \text{ gpm}
\]
Similar computations may be made for other layers or sublayers, for example, in table 1. We now assume a "true" $\Delta \phi$ of 30 gpm between soundings 200 km apart. The differences in $\bar{T}$ errors between the soundings are assumed to be 1 and 2 K (for example, the first sounding has a $\bar{T}$ error of 2 K, and the second has an error of 3 K, for a difference in error of 1 K).

### TABLE 1. DIFFERENCES IN GPM COMPUTED FOR THE LISTED LAYERS FOR THE LISTED DIFFERENCES IN $\bar{T}$ ERROR BETWEEN TWO SOUNDINGS 200 KM APART.

<table>
<thead>
<tr>
<th>$\bar{T}$ Difference (K)</th>
<th>Layer (hPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000-900</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Equation (6) can be used to compute the $V_\phi$ values for the "measured" difference (30 gpm) over 200 km, and $V_\phi$ arising from the two $\bar{T}$ error differences (1 and 2 K), for both the entire 1000 to 500 hPa layer and the several sublayers. Table 1 shows the gpm differences and table 2 shows values of $V_\phi$.

### TABLE 2. VALUE OF $V_\phi$ IN $ms^{-1}$ COMPUTED FOR THE LISTED LAYERS AND DIFFERENCES IN $\bar{T}$ ERROR BETWEEN TWO SOUNDINGS 200 KM APART. $\phi$ DIFFERENCES WERE ADDED TO THE "MEASURED" 30 GPM TO OBTAIN THE VALUES SHOWN.

<table>
<thead>
<tr>
<th>$\bar{T}$ Difference (K)</th>
<th>Layer (hPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000-900</td>
</tr>
<tr>
<td>0</td>
<td>14.7</td>
</tr>
<tr>
<td>1</td>
<td>16.2</td>
</tr>
<tr>
<td>2</td>
<td>17.7</td>
</tr>
</tbody>
</table>
Table 2 shows that the $V_e$ differences are about 9.7, 5.0, and 1.5 $ms^{-1}$ respectively for the layer and, for example, the 1000 to 700 hPa and 1000 to 900 hPa sublayers for a $T_e$ error difference of 1 K. For a 2-K difference, the $V_e$ differences are about 19.4, 10.0, and 3.0 $ms^{-1}$, respectively.

These differences in $V_e$ are not small and may even result in a 180-degree change in direction (for example, subtracting 39.7 gpm from 30 gpm gives -9.7 ms$^{-1}$ (reversed direction of the gradient) resulting in a small $V_e$ in the opposite direction).

2.3 Errors Caused By Curved Flow

One may compute the difference in wind speed caused by the flow being curved instead of straight (along a great circle). For this comparison, the gradient wind for moderate geostrophic winds (15 and 30 $ms^{-1}$) is computed. From tables in List (1984) an $r_f$ parameter of 0.062 at a latitude of 45° is obtained for a radius of curvature (r) of about 600 nm ($= 1110$ km). For $V_e = 15$ $ms^{-1}$,

$$V = 111.6sh^{-1}nms^{-1}\left[-1 + \left(1 + \frac{4(29.1nmh^{-1})}{3600sh^{-1}0.062nms^{-1}}\right)^{1/3}\right] = 26.1 \text{ kn} = 13.5 \text{ ms}^{-1}.$$  

Also, for $V_e = 30$ $ms^{-1}$,

$$V = 47.9 \text{ kn} = 24.7 \text{ ms}^{-1}.$$  

The two differences ($V - V_e$) computed above are 1.5 and 5.3 $ms^{-1}$.

2.4 Errors Caused by Mean Temperature Error

Another possible cause of error is an incorrect value of $T$ but no difference in $T$ error between the two soundings (same $T$ error at both locations). The constant level surface version of equation (1) is used; that is, $V_e = \frac{\nabla \bar{T} \cdot \Delta \mathbf{p}}{\bar{p} \cdot \Delta n}$, where the assumption is that

$$\Delta P = 20 \text{ mbar}, \Delta n = 1000 \text{ km}, \bar{p} = 700 \text{ mbar}, \text{ lat} = 45°.$$  

For $\bar{T} = 270$ K,

$$V_e = \frac{(287 \text{ K}^{-1} m^2 s^{-2}) 270 \text{ K} \left(\frac{20 \text{ mbar}}{1.03 \times 10^{-4} 	ext{ s}^{-1} \text{ 700 mbar}}\right)}{10^6 \text{ m}} = 21.5 \text{ ms}^{-1}.$$
and for $T = 267$ K,

$$\nu_e = 21.3 \text{ ms}^{-1}.$$  

The difference in these moderate values of $\nu_e$ is about 0.2 ms$^{-1}$. The likely error of a few tenths of a meter per second for moderate $\nu_e$ is small relative to the other possible errors.

2.5 Errors From Ageostrophic Deviations

A significant potential source of error arises from not considering certain deviations from the geostrophic wind, which may be roughly approximated by the horizontal ageostrophic wind components. Equation (4) may be used to approximate the ageostrophic component arising from the pressure tendency.

Here a pressure tendency normal to the wind of -1 hPa over 1 h near 45° latitude is considered. One hPa = 10 kg m$^{-1}$ s$^{-2}$ and the air density ($\rho$) = 1 kg m$^{-3}$.

\[
(V - \nu_e)_r = \frac{-1}{1 \text{ kg m}^{-3}(1.03 \text{ s}^{-1})^2} \left( \frac{1}{2 \text{ m}} \right) \left( -10 \text{ kg m}^{-1}\text{s}^{-2} \right) = 1.3 \text{ ms}^{-1}
\]

If the tangential component was the same size, the rough magnitude of the total pressure tendency term would be about 1.7 ms$^{-1}$.

Equation (5) expresses the ageostrophic component arising from a change in the spacing of contours on a constant pressure surface (or isobars on a constant height surface). A change of geostrophic wind of 2 ms$^{-1}$ over a distance of 200 km is not unreasonable. For geostrophic wind speeds of 15 ms$^{-1}$ and 30 ms$^{-1}$, at 45° latitude,

\[
(V - \nu_e)_s = \frac{15 \text{ ms}^{-1}}{1.03(10^{-4})\text{s}^{-1}} \left( \frac{2 \text{ ms}^{-1}}{2(10^3)\text{m}} \right) = 1.5 \text{ ms}^{-1}
\]

and $(V - \nu_e)_s = 2.9 \text{ ms}^{-1}$

Higher values of $\nu_e$ or $\Delta \nu_e$ can occur, leading to higher values of this "contour spacing" component. When both components are combined, the ageostrophic part of the "real" horizontal wind may exceed 5 ms$^{-1}$ (or < 1 ms$^{-1}$ if the two terms tend to cancel or $\nu_e$ is small).
2.6 Apparent Error From Direction Deviations

So far the error caused by incorrect orientation of the gradient has not been explicitly considered. Jedlovec (1985) reported on differences between gradients derived from data of the visible infrared spin scan radiometer (VISSR) atmospheric sounder (VAS) carried on the geostationary operational environmental satellite (GOES) and those computed using data from a special dense net of rawinsondes (50-km spacing). The differences ranged from a few degrees (almost parallel) to about 90° (nearly perpendicular). In the absence of other factors, an error in direction of the gradient (or contours) of around 30° could lead to a difference in \( V_e \) along the expected wind direction of about 2.0 and 4.0 \( ms^{-1} \) for \( V_e \) values of 15 and 30 \( ms^{-1} \), respectively. A realistic high deviation in gradient near 45° could produce \( V_e \) differences of about 4.4 and 8.8 \( ms^{-1} \) for the aforementioned \( V_e \) values. While even good quality data from rawinsondes may contain wind velocity errors, the deviations in direction reported by Jedlovec (1985) may reflect real differences from the true direction of the gradient (or contours).

2.7 Net Error Values

Table 3 lists some of the potential causes of error in satellite-derived estimates of wind speed that use the geostrophic wind, along with possible magnitudes of those errors. A moderate wind speed is assumed (10 to 30 \( ms^{-1} \)).

<table>
<thead>
<tr>
<th>Table 3: Possible Magnitudes of Error in Wind Speed Arising From Listed Causes for Satellite-Derived Geostrophic Winds.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cause</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

*(Component in expected direction of wind)*

*For the \( T \) sounding error (item 3) a layer of at least 300 hPa was assumed (for example, near sea level surface to about 3 km AGL, the region of largest satellite temperature error over land).*
These values may tend to augment or cancel one another. For example, items 3 and 4 may result in errors of over 20 ms$^{-1}$ or only 1 or 2 ms$^{-1}$. Miers et al. (1992) and their references report satellite geostrophic wind errors from ±4 to ±14 ms$^{-1}$, generally compared with wind speeds measured by rawinsondes. Rawinsonde data also may have errors, ranging from < 1 ms$^{-1}$ for high quality equipment to > 2.5 ms$^{-1}$ for older (now mostly obsolete) equipment (Fisher et al. 1987). Reasonable "middle of the road" errors (1 to 4 of table 3) from this analysis may lead to a "total error of about 15 to 20 ms$^{-1}$ where the individual errors sum together or perhaps only 2 or 3 ms$^{-1}$ where they largely cancel.

Deviations in direction of the gradient or contours from the actual direction (Miers et al. 1992, Jedlovec 1985) could result in apparent errors of up to 10 ms$^{-1}$. This "error" would be important for computing the wind speed for a specified direction. An example would be the cross or along trajectory wind speed for an artillery application. The apparent error (but a real component error) could either increase or decrease the "true" error. For example, an error in wind speed of 10 ms$^{-1}$ combined with a deviation in gradient direction could result in a down range error of approximately 15 or 5 ms$^{-1}$.

The net errors shown in table 3 represent a range of "typical" values for moderate wind speeds (10 to 30 ms$^{-1}$). However, the reader may compute other values of potential errors for greater or lesser wind speeds, as well as for other parameters (for example, different radii of curvature, or differences in $\bar{T}$ error less than 0.5 K or greater than 2 K). The reader may elect to calculate new potential errors for one or more of the causes of table 3 to arrive at additional net error values. Appendix A contains a selection of tables from List (1984) that should suffice, along with the equations presented herein, for nearly all additional calculated errors.

3. THERMAL WIND

3.1 Basic Derivations

A number of texts provide information on formulations of the thermal wind (Haltiner and Martin 1957, Holton 1979). The form derived in Holton (1979) is used here. Given the horizontal components of the geostrophic wind in geopotential form,

$$u_x = -\frac{1}{f} \frac{\partial \phi}{\partial y} \quad \text{and} \quad v_y = \frac{1}{f} \frac{\partial \phi}{\partial x}$$

and

$$\frac{\partial \phi}{\partial \rho} = -\frac{1}{\rho} = \frac{RT}{\rho}$$

(7)

(8)
When equation (7) is differentiated with respect to $p$ and equation (8) is applied, the results are

\[
\frac{\partial v}{\partial p} = \frac{\partial v}{\partial \ln p} = -\frac{R}{f} \frac{\partial T}{\partial x},
\]

\[
\frac{\partial u}{\partial p} = \frac{\partial u}{\partial \ln p} = \frac{R}{f} \frac{\partial T}{\partial y},
\]

or in vector form \( \frac{\partial \mathbf{v}}{\partial \ln p} = -\frac{R}{f} k x (\mathbf{v} T) \).

and \( V_T = V_T(p_1) - V_T(p_o) = -\frac{R}{f} \int_{p_o}^{p_1} (k x \mathbf{v} T) d\ln p \)

where $T$ = temperature, $p$ = pressure, $\rho$ = density, $R$ = gas constant for dry air, $\Phi$ = geopotential, and $V_g$ = geostrophic wind. $f$ is the coriolis parameter \( (= \Omega \sin \theta) \)

where $\Omega$ is the angular velocity of the earth at the poles and $\theta$ is latitude). In component form

\[
u_T = -\frac{R}{f} \frac{\partial T}{\partial x} \ln \left( \frac{P_o}{P_1} \right)
\]

\[
u_T = -\frac{R}{f} \frac{\partial T}{\partial y} \ln \left( \frac{P_o}{P_1} \right)
\]

where $u_T = u_T(p_1) - u_T(p_o)$ and $v_T = v_T(p_1) - v_T(p_o)$ are the thermal wind components defined as the differences in the geostrophic components for the upper $p$ surface, denoted with subscript 1, and the lower $p$ surface, denoted by subscript 0. $\bar{T}$ is the mean temperature for the layer from $p_o$ to $p_1$, and $x$ and $y$ are distances in the east-west and north-south directions, respectively. An alternative method is to use the geopotential form, or the equivalent "height" form.

\[
u_T = -\frac{1}{f} \frac{\partial (\Phi_1 - \Phi_o)}{\partial y} = -\frac{g}{f} \frac{\partial (z_1 - z_o)}{\partial y}
\]

\[
u_T = -\frac{1}{f} \frac{\partial (\Phi_1 - \Phi_o)}{\partial x} = -\frac{g}{f} \frac{\partial (z_1 - z_o)}{\partial x}
\]
Since this paper is concerned with errors in satellite temperature soundings, equations (9) and (10) will be used.

### 3.2 Errors From Incorrect Mean Temperatures

Equations (9) and (10) may be approximated in finite difference form to a sufficient accuracy.

\[
\begin{align*}
    u_T &= - \frac{R}{f(\Delta y)} \left( \frac{\Delta T}{\Delta y} \right) \ln \left( \frac{P_o}{P_1} \right) \\
    v_T &= \frac{R}{f(\Delta x)} \ln \left( \frac{P_o}{P_1} \right)
\end{align*}
\]

Taking differences in errors in $T$ between soundings 200 km apart of 0.5, 1, and 2 K, one can compute potential discrepancies in thermal wind for given layers of atmosphere. The 700 to 300 hPa layer will be considered as a whole and subdivided into 100 hPa sublayers. Isotherms orientation will be assumed to be NE-SW (45°) and latitude to be 43°. For the entire layer, for a 2-K error in $\bar{T}$

\[
\begin{align*}
    u_T &= - \frac{287 \, K^{-1} \, m^2 \, s^{-2}}{10^4 \, s^{-1}} \left( \frac{1.414 \, K}{141.4 \, (10^3) \, m} \right) (0.8473) = - 28.7(0.8473) \\
    &= - 24.3 \, ms^{-1} \\
    v_T &= 24.3 \, ms^{-1}
\end{align*}
\]

The magnitude (wind speed) from the standard formula is

\[
V_T = \left( u_T^2 + v_T^2 \right)^{\frac{1}{2}} = 34.4 \, ms^{-1}
\]

For a 1-K error in $\bar{T}$,

\[
\begin{align*}
    u_T &= 12.15 \, ms^{-1} \\
    v_T &= 12.15 \, ms^{-1} \\
    V_T &= 17.2 \, ms^{-1}
\end{align*}
\]
For 0.5-K error in $\overline{T}$,

$$V_T = 8.6 \text{ ms}^{-1}$$

Table 4 shows the values for individual sublayers (I) and the cumulative error (sum of errors from lowest to given layer, C). Slight differences between last cumulative values and values for entire 700 to 300 hPa layer appear because of round off of table values.

**TABLE 4. THERMAL WIND VALUES (ms$^{-1}$) FOR LISTED LAYERS AND DIFFERENCES IN ERRORS IN MEAN TEMPERATURE ($\overline{T}$) BETWEEN SOUNDINGS 200 KM APART.**

<table>
<thead>
<tr>
<th>$\overline{T}$ Difference</th>
<th>Layers (hPa)</th>
<th>700-600</th>
<th>600-500</th>
<th>500-400</th>
<th>400-300</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>C</td>
<td>I</td>
<td>C</td>
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<td>9.1</td>
</tr>
</tbody>
</table>

### 3.3 Net Potential Error

The results shown in table 4 do not include possible errors in the baseline wind field (for example, wind at 700 hPa), errors arising from not considering curvature, and ageostrophic components. Use of measured wind as the baseline value will reduce the total possible error. However, since the thermal wind itself makes use of the geostrophic approximation, the other sources of error may have an increasingly greater effect as height from the baseline level increases.

As a first estimate of potential errors, one can use the values of table 3, with the range of values from table 4 replacing item 3 (varying error in $T$ sounding causing incorrect contour gradient). Table 5 repeats part of table 3 along with new results from table 4 for a 700 to 300 hPa layer. The values of item 1 of table 5 would be smaller for thinner layers (for example, a range of $<1 \text{ ms}^{-1}$ to around $12 \text{ ms}^{-1}$ for differences in $\overline{T}$ error of a few tenths of a degree to about 2 K for a 100 hPa thick layer).

These errors may tend to augment or cancel one another. For example, items 1 and 3 may result in errors of over $35 \text{ ms}^{-1}$, or only 1 or 2 $\text{ ms}^{-1}$. For thinner layers (for example, 700 to 500 hPa) the value of item 1 would be much smaller (table 4), reducing
the overall potential error. However, the numbers in table 2 assume a "perfect" measured wind velocity at 700 hPa. The wind speed error of measured wind (rawinsonde or radar profiler) is likely to have a value near 1 to 2 $\text{ms}^{-1}$ (Miers et al. 1992, Fisher et al. 1987). This error also may augment or diminish the "thermal wind" error. Some rough idea of potential error may be obtained by assuming "middle of the road" errors from table 5 (items 1 to 3). The potential total error may reach values over 25 $\text{ms}^{-1}$ where the individual errors sum together, or perhaps less than 10 $\text{ms}^{-1}$ where they largely cancel.

**TABLE 5. POTENTIAL MAGNITUDES OF ERROR IN THERMAL WIND GIVEN POSSIBLE DIFFERENCES IN $\overline{T}$ ERROR BETWEEN SOUNDINGS 200 KM APART.*

<table>
<thead>
<tr>
<th>Cause</th>
<th>Potential Wind Speed Error ($\text{ms}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $\overline{T}$ gradient of layer (700-300 hPa) incorrect</td>
<td>5 to 35</td>
</tr>
<tr>
<td>2 Gradient instead of Geostrophic (curved flow)</td>
<td>1 to 5</td>
</tr>
<tr>
<td>3 Ageostrophic component</td>
<td></td>
</tr>
<tr>
<td>Pressure tendency</td>
<td>0.5 to 2</td>
</tr>
<tr>
<td>Contour spacing change</td>
<td>1 to 5</td>
</tr>
<tr>
<td>4 Incorrect direction of temperature gradient</td>
<td>Ne: r 0 to 10</td>
</tr>
</tbody>
</table>

*The assumption is that the difference in $\overline{T}$ error does not exceed 2 K for the 700 to 300 hPa layer. Baseline wind measured at 700 hPa.

Deviations in the temperature or height gradient from the actual direction (Miers et al. 1992, Jedlovec 1985) could result in apparent errors (but a real component error) of up to 10 $\text{ms}^{-1}$ in geostrophic wind. These same deviations could cause errors in estimates of thermal wind of about the same size for thicker layers (for example, 700 to 300 hPa). Since thinner layers (for example, 700 to 600 hPa) normally would have smaller thermal wind values, the error from deviations from the true gradient direction would be smaller, perhaps less than 3 $\text{ms}^{-1}$. If a measured baseline wind was not available, the error could be considerably larger. For example, for a 700 to 300 mbar layer use of the geostrophic wind as the baseline wind could lead to errors of over 40 $\text{ms}^{-1}$ if the net thermal wind error and the error in the baseline geostrophic wind tended to sum together.
4. CONCLUSION

Errors in satellite estimates of the real wind that use the geostrophic approximation or thermal wind may reach significant values. Net differences between the actual and geostrophic wind from a few to over 20 $\text{ms}^{-1}$ are possible, with reasonable middle values of 10 or 12 $\text{ms}^{-1}$. The apparent wind speed error caused by differences in orientation of the geopotential height gradient may increase (or decrease) the component of the geostrophic wind in a specific direction, resulting in an augmentation or reduction in the effect of the other "real" errors. This effect may be especially important for artillery or aviation.

Net differences between the real wind and that derived from the thermal wind may range from a few to over 30 $\text{ms}^{-1}$ for thick layers (for example, 700 to 300 hPa), with reasonable middle values around 15 to 18 $\text{ms}^{-1}$. For a thin layer of approximately 100 hPa thickness, the errors may range from 1 to over 10 $\text{ms}^{-1}$ with reasonable middle values near 3 to 6 $\text{ms}^{-1}$. The apparent wind speed error caused by differences in temperature gradient orientation may increase (or decrease) the component of the estimated wind in a specific direction. As with the geostrophic wind, this apparent error may augment or reduce the effect of the other "real" errors.

The potential errors described in this report may occur when satellite data are used alone, without any additional data to "tie" down the appropriate satellite values. Some improvement should be possible through the use of new sources of data (Miers et al. 1992) such as radar wind profilers and accompanying radio acoustic sounding system (RASS), ground-based radiometers, and sensors on unmanned aerial vehicles, combined with processing techniques currently under development. In addition, new satellite sensors will eliminate many of the difficulties associated with current instruments. For example, new active sensors planned for early in the next century (for example, the laser atmospheric wind sounder) will measure wind velocity directly for cloud-free lines of sight, and new passive sounders (for example, atmospheric infrared sounder) will improve temperature profiles. In the interim the combination of additional sources of data, new processing techniques, and satellite sensors coming on line by the end of the 1990's (for example, special sensor microwave imager/sounder on Defense Meteorological Satellite Program satellites, and the advanced microwave sounding unit-A on National Oceanic and Atmospheric Administration satellites) may provide the best solution. Descriptions of these sensors may be found in NASA (1991), Patel (1992), and Swadley and Chandler (1992).
REFERENCES


List, R. (Ed), 1984, Smithsonian Met Tables, Smithsonian Institute Press, Washington, DC.


APPENDIX A. TABLES FOR CALCULATING GEOSTROPHIC AND GRADIENT WINDS

This appendix presents a series of tables from List that may be used as an aid in calculating geostrophic and gradient winds. Accompanying explanations are included. For further details see List (1984),1 Haltiner and Martin (1957),2 and Holton (1979).3

3R. List (Ed), 1984, Smithsonian Met Tables, Smithsonian Institute Press, Washington, DC.
The scalar equation for the geostrophic wind on a constant pressure surface is

\[ \mathbf{V}_g = \frac{f \, \mathbf{S}}{\frac{\partial z}{\partial x}} \]

where \( \phi \) is the geopotential in a constant-pressure surface, \( \mathbf{S} \) is distance measured in the horizontal direction, \( f \) is the coriolis parameter, and \( \mathbf{V}_g \) is the component of the geostrophic wind normal to the direction in which \( \mathbf{S} \) is measured.

On a constant pressure surface with contours drawn for intervals of 100 geopotential meters (gpm), this reduces to

\[ \mathbf{V}_g(\text{knots}) = \frac{0.017120}{\tan} \]

(continued on next page)

![Table 37: Geostrophic Wind, Constant Pressure Surface](image)

(To convert knots to other measures of speed see Table 35.)

(continued)

**TABLE 37**

**GEOSTROPHIC WIND, CONSTANT PRESSURE SURFACE**

100 geopotential meter contours

The scalar equation for the geostrophic wind on a constant pressure surface is

\[ \mathbf{V}_g = \frac{f \, \mathbf{S}}{\frac{\partial z}{\partial x}} \]

where \( \phi \) is the geopotential in a constant-pressure surface, \( \mathbf{S} \) is distance measured in the horizontal direction, \( f \) is the coriolis parameter, and \( \mathbf{V}_g \) is the component of the geostrophic wind normal to the direction in which \( \mathbf{S} \) is measured.

On a constant pressure surface with contours drawn for intervals of 100 geopotential meters (gpm), this reduces to

\[ \mathbf{V}_g(\text{knots}) = \frac{0.017120}{\tan} \]

(continued on next page)

![Table 37: Geostrophic Wind, Constant Pressure Surface](image)

(To convert knots to other measures of speed see Table 35.)

(continued)
TABLE 37 (CONCLUDED)

GEOSTROPHIC WIND, CONSTANT PRESSURE SURFACE

100 geopotential meter contours

where \( \lambda \) is the contour spacing measured in degrees of latitude (i.e., one unit of \( \lambda \) has the length of one degree of latitude at the place for which the contour spacing is measured). Table 37 gives values of \( V_p \) in knots as a function of \( \lambda \) and latitude with auxiliary columns giving equivalents of \( \lambda \) in kilometers, statute miles, and nautical miles. If the latter are measured by a map scale true at some other latitude the value should be corrected to the latitude at which the measurements are taken (see Table 35).

Since the geostrophic wind is inversely proportional to the contour spacing and directly proportional to the contour interval (10 gpm.), values of \( V_p \) for 1/10 of the indicated spacing may be found by multiplying the tabular values by 10, etc., and for contour intervals that are multiples or submultiples of 10 gpm. by multiplying the tabular values by \( \lambda/100 \) (e.g., for 50 gpm. contours multiply by 2, for 50 gpm. contours multiply by 0.5, etc.).

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<th>Contour spacing</th>
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</table>

(To convert knots to other measures of speed see Table 31)
The equation for the gradient wind speed $V$ in cgs units is,

$$V = \frac{r}{2} \left( -1 + \sqrt{1 + \frac{4P}{r^2}} \right), \quad (1)$$

where

- $r$ = "radius of curvature" of the trajectory $= R \tan \alpha$ (see Table 166),
- $f$ = Coriolis parameter $= 2 \omega \sin \phi$ ($\omega$ = angular velocity of rotation of the earth, $\phi$ = latitude)
- $P$ = geostrophic wind speed (see Tables 37-39).

Equation (1) can be rewritten in the following form

$$V = \frac{3600 \sqrt{f}}{2} \left( -1 + \sqrt{1 + \frac{4P}{3600 \, r^2}} \right) \quad (2)$$

for any of the following consistent combinations of units:

- a. $V$ and $P$ in miles per hour, $r$ in statute miles,
- b. " " " = knots, $r$ in nautical miles,
- c. " " " = kilometers per hour, $r$ in kilometers.

$V$ is a function of the parameter $P$ and of the geostrophic wind speed $P$, only. In applying equations (1) and (2) the following sign convention is necessary: for cyclonic curvature $P > 0$, for anticyclonic curvature $P < 0$.

Table 40 A gives values of the parameter $P$ as a function of latitude, $\phi$, and $r$. Table 40 B gives values of $V$ for cyclonic curvature and Table 40 C for anticyclonic curvature as a function of the parameter $P$ and the geostrophic wind speed $P$.

To find the gradient wind speed at a given point:

1. Determine the latitude $\phi$ and the value $r$ of the trajectory. (Table 166 indicates a method for finding $r$ on a polar stereographic map projection; for other projections an estimate must be made.)

2. From Table 40 A find the parameter $P$. (This parameter is linear in $r$ so that values for other radii than those given may be readily determined from the table, e.g., if $r = 2300$, add the values for $r = 2000$ and $r = 3000$.)

3. Determine the geostrophic wind speed $P$, at the given point (see Tables 37-39). ($P$ and $r$ must be in one of the consistent combinations of units given above.)

4. Enter Table 40 B (cyclonic case) or Table 40 C (anticyclonic case) with the arguments $P$ and $P$. The corresponding tabular value is the gradient wind $V$ in the same units as $P$. 

(continued)
### Table 40 (continued)

#### GRADIENT WIND

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**Table 40 A.—Values of the parameter \( v_f \)**

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*Smithsonian Meteorological Tables*
### Table 40 (concluded)

#### GRADIENT WIND

**Table 40 C.—Anticyclonic curvature**

| $v$ | $f / 0.005$ | $f / 0.007$ | $f / 0.009$ | $f / 0.01$ | $f / 0.02$ | $f / 0.03$ | $f / 0.04$ | $f / 0.05$ | $f / 0.06$ | $f / 0.07$ | $f / 0.08$ | $f / 0.09$ | $f / 0.10$ | $f / 0.11$ | $f / 0.12$ | $f / 0.13$ | $f / 0.14$ | $f / 0.15$ | $f / 0.16$ | $f / 0.17$ | $f / 0.18$ | $f / 0.19$ | $f / 0.20$ | $f / 0.30$ | $f / 0.40$ | $f / 0.50$ | $f / 0.60$ |
|-----|--------------|--------------|--------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 5   | 12           | 11           | 10           | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         | 10         |
| 10  | 26           | 24           | 23           | 22         | 22         | 22         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         | 21         |
| 30  | 43           | 38           | 36           | 35         | 34         | 33         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         | 32         |
| 40  | 49           | 43           | 40           | 38         | 37         | 36         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         | 35         |
| 50  | 53           | 48           | 47           | 46         | 46         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         | 45         |
| 60  | 59           | 56           | 54           | 53         | 52         | 51         | 50         | 50         | 50         | 50         | 50         | 49         | 49         | 49         | 49         | 49         | 49         | 49         | 49         | 49         | 49         | 49         | 49         | 49         |
| 70  | 64           | 62           | 60           | 59         | 58         | 57         | 56         | 56         | 55         | 55         | 55         | 54         | 54         | 54         | 54         | 54         | 54         | 54         | 54         | 54         | 54         | 54         | 54         | 54         |
| 80  | 74           | 72           | 71           | 70         | 69         | 68         | 67         | 67         | 66         | 66         | 66         | 65         | 65         | 65         | 65         | 65         | 65         | 65         | 65         | 65         | 65         | 65         | 65         | 65         |
| 90  | 85           | 80           | 76           | 74         | 73         | 71         | 70         | 69         | 68         | 67         | 67         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         |
| 100 | 98           | 85           | 76           | 74         | 73         | 71         | 70         | 69         | 68         | 67         | 67         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         | 66         |
| 120 | 144          | 120          | 111          | 105        | 102        | 100        | 98         | 96         | 95         | 94         | 92         | 91         | 89         | 88         | 86         | 84         | 83         | 82         | 81         | 80         | 79         | 78         | 77         | 77         |
| 150 | 184          | 157          | 145          | 138        | 132        | 129        | 126        | 124        | 122        | 121        | 119        | 117        | 116        | 114        | 112        | 110        | 108        | 107        | 105        | 104        | 103        | 102        | 101        |

Note.—The critical value of $v$ in the anticyclonic case is defined as that value of $v$ for which the radical in equation (2) is equal to zero.

Values of $v$ are tabulated only when they are less than the critical value. For imaginary values of equation (2) no gradient wind can be specified; if such a condition existed, the wind would not flow along the isobars. At the critical value of $v$, the following apply:

\[
V_{\text{critical}} = 2v_0 \\
(f)_{\text{critical}} = -\frac{v}{900}
\]

CONTINUOUS METEOROLOGICAL TABLES
TABLE 166

RADIUS OF CURVATURE ON A POLAR STEREORGRAPHIC PROJECTION

In computing gradient wind speeds (Table 40) and in other problems it is necessary to determine a factor \( r \) which depends on curvature of the trajectory. This factor arises in taking account of the horizontal component of the centrifugal force acting on a particle. The problem is twofold: (1) to determine the trajectory of the particle on a map, and (2) to determine the required value of \( r \) if the trajectory on the map is known. The first problem is of such nature that it cannot be treated adequately here. (Note.—In many cases an approximation is made from the curvature of the isobars or streamlines.) The second problem has been solved for the case of a polar stereographic projection, since on this projection a "small circle" on the earth projects as a circle on the map. Table 40 provides a means for computing the desired \( r \) for trajectories on a polar stereographic projection.

Let \( R \) be the radius of the earth, \( r' \) the true radius of the "small circle" on which the particle is assumed to be traveling at a given instant, and \( a \) its angular radius (as seen from the center of the earth). Then \( r' = R \sin a \). Since we are concerned with the horizontal component of the centrifugal force, the effective horizontal radius of the curvature required in the gradient wind equation is given by \( r = r' \sec a = R \tan a \). If an arc on a map representing the instantaneous trajectory of a particle of air is determined, this arc may be regarded as a portion of a "small circle."

To determine \( r \) for a given arc of a trajectory on the map:

1. Complete the circle by extending the arc (a set of circular templates will prove very useful).
2. Find the meridian which passes through the center of this circle.
3. Determine the latitudes \( \phi_1 \) and \( \phi_2 \) of the points where this meridian intersects the circle (extend the meridian across the pole if necessary).
4A. If the circle found in step 1 does not contain the pole, find the difference between \( \phi_1 \) and \( \phi_2 \) and enter part A of the table with this difference as the argument. The corresponding tabular value is the required radius \( r \) in statute miles, from the formula \( r = R \tan \frac{1}{2}(\phi_1 - \phi_2) \).
4B. If the circle found in step 1 contains the pole, find the sum \( (\phi_1 + \phi_2) \) and enter part B of the table with this sum as the argument. The corresponding tabular value is the required radius \( r \) in statute miles, from the formula \( r = R \tan \left( 90^\circ - \frac{1}{2}(\phi_1 + \phi_2) \right) \).

(continued)
### TABLE 166

**RADIUS OF CURVATURE ON A POLAR STEREGRAPHIC PROJECTION**

#### A. Circle not including pole.

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