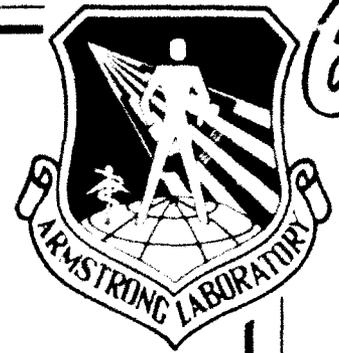


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ARMSTRONG

OPTICAL DISPLAYS: A TUTORIAL ON IMAGES AND IMAGE FORMATION

Herschel C. Self

CREW SYSTEMS DIRECTORATE
HUMAN ENGINEERING DIVISION

LABORATORY

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FOR THE COMMANDER



KENNETH R. BOFF, Chief
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PREFACE

This report was prepared in the Human Engineering Division, Crew Systems Directorate, of the Armstrong Laboratory (AL), Wright Patterson Air Force Base, Ohio. The work was performed under Project 7184, "Man-Machine Integration Technology," Task 718411, "Design Parameters for Visually-coupled Display Systems." Thanks are due to coworkers for encouragement in organizing notes into this report. The assistance of Miss Yolanda Crawford and Miss Sheila Radford in preparing the manuscript is sincerely appreciated.

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INTRODUCTION

Displays are at the heart of most man-machine systems, yet many display users, including many doing research on displays, do not understand the basic optical properties of displays and of object-image relationships. Those who, desiring a better understanding, turn to optics textbooks are often disappointed. Particular textbooks are frequently irrelevant to their needs, or are too general or too complex, or require mathematics that the reader finds either unfamiliar or too abstract. The present tutorial goes into some aspects of optics in considerable detail, but is intended to be easily followed in its arguments and derivations by readers who have had algebra and geometry. To permit the various sections to be somewhat independent, i.e., be understandable without having fully digested previous sections, there is some repetition of concepts and definitions.

This report is a primer on the geometrical optics characteristics of displays and how each optical or geometric characteristic is dependent upon other characteristics. Several sections discuss optical aspects of images and displays. Terms are defined and examples are given. The Gaussian Lens Equation $1/(\text{Object Distance}) + 1/(\text{Image Distance}) = 1/(\text{Focal Length})$, derived in Appendix 1, is applied repeatedly in the text. A section is devoted to optical and optically-related concerns of helmet-mounted displays, including trade-offs of characteristics.

Topics include angular subtense, apparent and instrument fields of view, real and virtual images, exit pupil size and distance, collimation, magnification, eyepieces, objectives, microscopes and telescopes, and helmet-mounted displays.

1.0 REAL AND VIRTUAL IMAGES

In optics, a real image is an image whose energy is located at the same place in physical space as the image. An energy pattern forming the image is located at the image. Unexposed photographic film, exposed at the image location for an appropriate time, when developed, would contain a copy of the image. A virtual image, in contrast to a real image, would not produce an image on sensitive photographic film exposed at the image location. No energy pattern that is in the form of the image is there. The most common example of a virtual image is an image seen in a mirror. Obviously, there is no energy behind or within the mirror. In contrast to a mirror, a camera lens forms a real image. In optics, "real" means "not virtual": it is not intended to imply "not imaginary."

With a telescope or with a helmet-mounted display (HMD), the objective forms or projects a real primary image which is viewed through an eyepiece which forms a virtual image for the observer. By adjusting the distance between the eyepiece and the real primary image formed by the objective, the displayed virtual image can be positioned anywhere in a range from optical infinity to within millimeters of the eye. This adjustment is called focussing. The adjustment or focussing range of the eyepiece of most optical instruments is not constructed to permit bringing the image to less than about 200mm from the eye. The virtual image appears to be out in front of the viewing device, but, obviously, no energy is emitted from the front of the device to form an actual energy pattern.

The lens system of the observer's eye can focus the virtual image produced by an eyepiece onto the retina of the observer's eye as a real or energy image whose energy cause chemical changes in the retina, making vision possible. It is worthwhile to note that, in the absence of cues from sources other than the image, an observer can not discriminate between real and virtual images. For example, with a clean perfect front-surface mirror viewed at an angle through a tube, one can not tell if an object is seen directly or by reflection.

2.0 FIELD OF VIEW, VIEWING DISTANCE, AND MAGNIFICATION

The objective field of view, external field of view, or field of view of an instrument, is the angular coverage of the scene or area that will be displayed. Field of view (FOV) is usually specified either in degrees or in radians, where $1 \text{ radian} = 180/\pi \text{ degrees} \approx 57.296^\circ$. For display systems that provide a round disc image, instrument field of view is the angular diameter of the area or space that the system can display. It is an angular measure of how much of the external world can be taken in by the instrument. When the display is rectangular, the external or instrument field of view may be given as either the angular subtense of the field diagonal or as the angular width and angular height of the field.

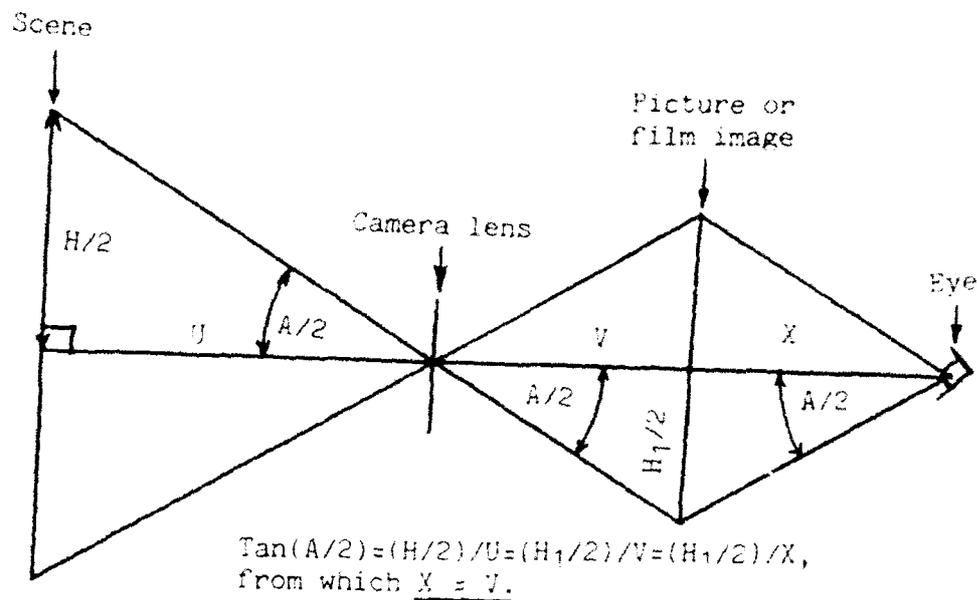
Apparent field-of-view is the angular subtense at the observer's eye of the display image. It is the angular size of the display. In many optical viewing devices, such as microscopes and telescopes, the angular size of the display is much larger than the instrument's field

of view. The ratio of apparent field of view to external or instrument field of view is angular magnification. As noted elsewhere in this report, angular magnification is often given as the ratio of the tangents of the two angles, rather than the ratio of the angles. When display magnification is unity, i.e., the ratio of apparent-to-instrument field of view is 1, the angles are equal. Unity display magnification is usually used on helmet-mounted displays used by aircraft pilots.

Most optical instruments that are binocular, i.e., that have a display for use by both eyes, have completely overlapping fields-of-view for the two eyes. However, in some binocular helmet-mounted displays, called panoramic displays, computer processing (predistortion) of an image source permits presentation of a display with divergent optical axes, so that right and left eye views do not completely overlap. The right eye field extends further to the right than the field of the left eye, and vice versa. In such a display, the horizontal field-of-view (FOV) is twice the eyepiece FOV minus the overlap, or, alternatively, eyepiece FOV minus the overlap for one eye. Melzer and Moffitt (1989) discuss, in detail, partial overlap in helmet-mounted binocular displays.

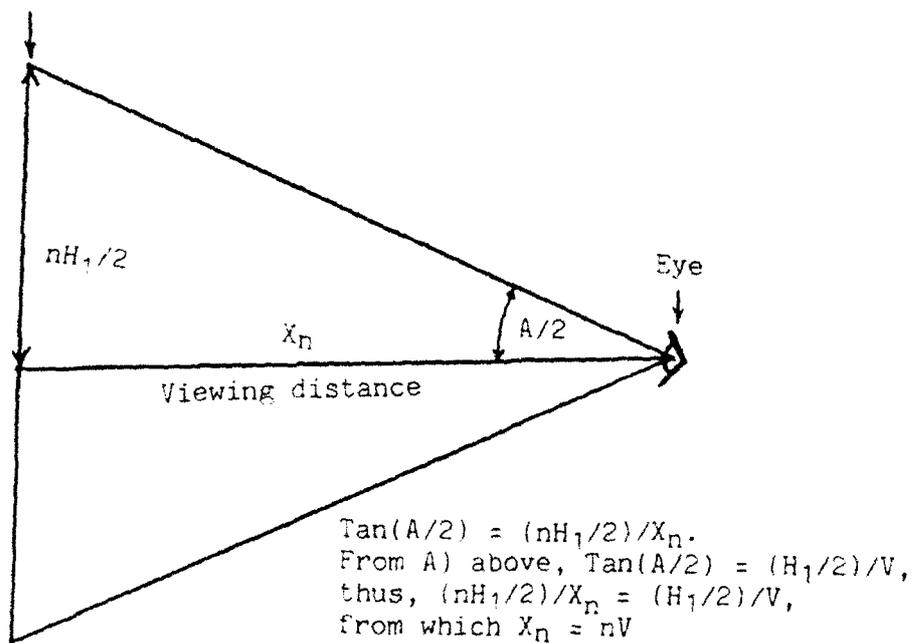
When viewing a display or a photograph, perspective appears normal when objects depicted on the display subtend, at the observer's eye, the same angle that the objects subtended at the camera lens, i.e., display magnification is unity. This principle can be used to determine the proper distance to view a photograph. Suppose that an object's image on the film has a linear height H_1 , while the object itself has a height H_0 . If the camera that took the picture was at a distance U from the object of height H_0 , then the subtended half angle A at the camera lens would be $(\text{Angle } A)/2 \approx \tan(A/2) = H_0/2U$. The geometry of the situation is shown in Figure 1. By corresponding parts of similar triangles being proportional, from the figure, $(\text{Image Height})/(\text{Object Height}) = (\text{Image Distance})/(\text{Object Distance})$, i.e., $H_1/H_0 = V/U$, which is linear magnification. From this equation, $H_1 = VH_0/U$. The camera lens obeys the Gauss Lens Equation $1/(\text{Object Distance}) + 1/(\text{Image Distance}) = 1/(\text{Focal Length})$, i.e., $1/U + 1/V = 1/F$. From this equation, $V = FU/(U-F)$. Linear magnification is then $V/U = H_1/H_0 = FH_0/(U-F)H_0 = F/(U-F)$. For an object at a distance from the lens of P focal lengths, linear magnification is $F/(U-F) = F/(PF-F) = 1/(P-1)$. For example, for $P = 26$, linear magnification is $1/(26-1) = 1/25$. In this example, image dimensions are corresponding object dimensions divided by 25.

Going back to proper viewing distance for a picture, substituting $V = FU/(U-F)$ from above for V , $H_1 = VH_0/U = [FU/(U-F)]H_0/U = FH_0/(U-F)$. Suppose that the film is enlarged n times or n diameters, then the image height of the object in the enlargement or picture is $H_2 = nH_1 = nFH_0/(U-F)$. When viewed from a distance D , half of this image height H_2 will subtend an $(\text{Angle } B)/2 = \tan(B/2) = H_2/2D = nFH_0/2D(U-F)$. For proper perspective, this angle should be the same as $(\text{Angle } A)/2$, shown above to be given by $\tan(A/2) = H_0/2U$. Equating the two subtended angles, $H_0/2U = nFH_0/2D(U-F)$, from which viewing distance $D = nFU/(U-F) = nF/(1-F/U)$. When object distance U is many times larger than the focal length F of the camera lens, $(1 - F/U) \approx 1$,



A. Viewing the unenlarged picture or the negative.

n-Diameter enlargement



B. Viewing an n-times enlargement.

Fig.1. Viewing distance for pictures and enlargements to subtend the same angle at the eye as the scene subtended at the camera that took the picture.

so that $D = nF/(1 - 1/U) = nFU/(U - F)$. Thus, for photographs of objects at distances of many times the focal length of the camera lens, proper viewing distance is lens focal length times diameters of picture enlargement. Photography books, typically, do not mention the part of the rule about object distance being many focal lengths of the camera lens.

For a viewing distance example, suppose that a camera lens with a focal length of 50mm takes a picture of an object at a distance of 3 meters or 3,000mm from the camera lens. This is many focal lengths of the lens, i.e., $3000/50 = 60$, so that viewing distance for an unenlarged (or contact print) picture is approximately $D = nF$. For a contact print, which has image details of the same size as details on the film, n is 1. Here, proper viewing distance for correct or undistorted perspective is $D = nF = 1F = 50\text{mm}$, or about 2 inches. Since people can not focus their eyes on a picture that close to the eye, they can not view a contact print from a camera with a 50mm lens without appreciable perspective distortion. If the picture in this example were to be enlarged 8 times or diameters, i.e., $n = 8$, then proper viewing distance would be approximately $D = nF = 8 \times 50\text{mm} = 400\text{mm}$, or about 16 inches. Most people can easily view a picture this close to the eyes, hence they can easily view it at the distance that provides proper perspective. For a more exact viewing distance in this example, correct viewing distance $D = nFU/(U - F) = (8)(50)(3000)/(3000 - 50) = 407\text{mm}$. or approximately 16 inches.

A second example of viewing distance, one where the nF rule is not close enough, is as follows. Let the picture on the film be a close-up, or macrophoto, of a flower taken at a distance U of 150mm (about 6 inches) with, again, a 50mm camera lens. Let the picture to be viewed be, as before, an 8 times or 8 diameter enlargement ($n = 8$) of the film. For these conditions, object distance U is only $150/50 = 3$ focal lengths, so that proper picture viewing distance would be $D = nFU/(U - F) = (8)(50)(150)/(150 - 50) = 600\text{mm}$, or about 24 inches, not the 16 inches viewing distance for a picture taken at a distance of 3 meters. For close-up pictures, the picture-taking distance should be taken into account, using the $D = nFU/(U - F)$ rule, not the $D = nF$ approximation.

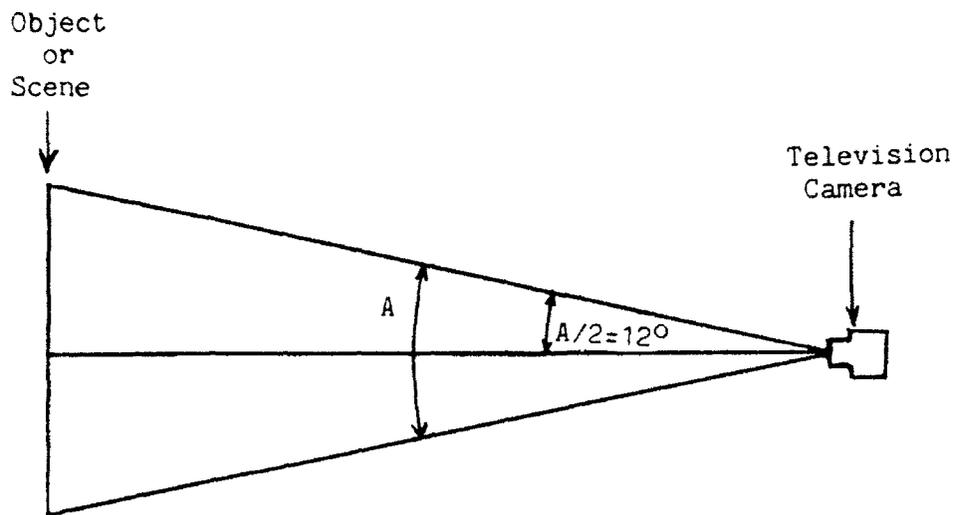
Pictures taken with long-focus or telephoto lenses also may present problems in proper viewing distance. As an example, suppose that a picture is taken with a telephoto lens with a focal length of 300mm (about 12 inches) of an object at a distance of 1524 meters (about 5000 feet). Again, let the camera film be standard 35mm film with a 24x36mm format, thus with a diagonal of 43.27mm. The picture to be viewed is an enlarged photographic print of the full film format with a picture diagonal of 200mm. What would be the proper viewing distance? In this example, the linear print magnification, or diameters of print enlargement is $n = 200/43.27 = 4.62X$. The viewing distance for unity magnification, relative to viewing the object from the picture-taking distance, hence obtaining correct perspective, for this long-distance picture, is approximately $nF = (4.62)(300) = 1386\text{mm}$, or about 54.6 inches. People do not ordinarily view a picture with a diagonal of 200mm, about 8 inches, from such a long distance.

For an example on field-of-view with binoculars, an observer with 7-power binoculars whose apparent or eyepiece field-of-view is 55° is viewing an outdoor scene. Here, magnification $M = 7x = (\text{apparent field-of-view}) / (\text{Instrument field of view}) = 55^\circ / (\text{Instrument field of view})$, from which instrument or external field of view is $55^\circ / 7 = 7.86^\circ$. By a second definition, magnification or magnifying power M is the ratio of the tangent of the apparent field of view to the tangent of the instrument field of view. By this definition, $M = 7 = \text{Tan } 55^\circ / [\text{Tan}(\text{Instrument Field-of-View})]$, from which $(\text{Instrument field of view}) = \text{ArcTan}(7/55) = 7.25^\circ$. This value is a little less than the 7.86° provided by the earlier definition, due to the 55 degree angle not being a small angle for which the tangent of the angle closely approximates the angle in radians. The smaller an angle, the closer the tangent of the angle approximates the angle in radians.

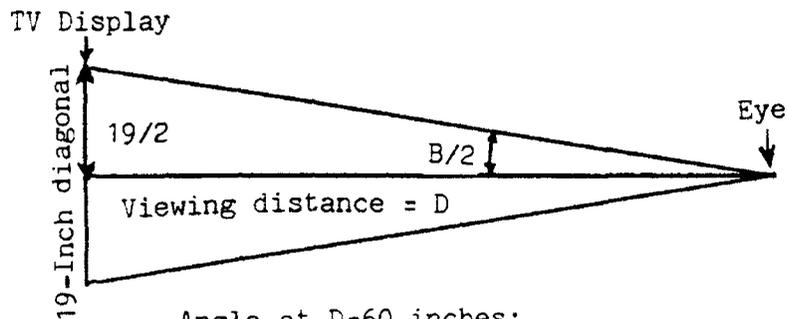
For still another example of field of view, an observer is watching, with unaided vision, from a distance of 60 inches, a TV screen with a diagonal of 19 inches. The TV picture is provided by a television camera whose diagonal coverage, i.e., angular subtense of the field-of-view, is 24 degrees. What is the display magnification in this example? The geometry of the observer-display situation is shown in Fig. 2. The TV screen diagonal of 19 inches subtends a half angle of $A/2 = \text{Tan}(A/2) = 19 / (2 \times 60) = .15833$, from which $A = 2 \text{ ArcTan}(.15853) = 17.994$ degrees. Since, at the TV camera, the field-of-view is 24° , the TV display subtends a smaller field of view than does the camera. If magnification M is defined as the ratio of apparent field of view to instrument field of view, then $M = 17.994 / 24 = .75$, usually given as .75X, .75 times, or .75 diameters. As noted above, for some purposes, magnifying power is defined as the ratio of the tangents of the angles. For the conditions of the example, $M = \text{Tan } 17.994^\circ / \text{Tan } 24^\circ = .73X$ only slightly different from the .75x obtained from the ratio of the angles.

In the example with the TV display, at what viewing distance D would the TV diagonal subtend the same 24° as the TV camera, yielding unity display magnification? Here, $(\text{camera subtense}) = 24^\circ$, so that $(\text{TV screen subtense}) = 24^\circ = 2 \text{ ArcTan}(19/2D)$, from which $D = 19/2 \text{ Tan } 12^\circ = 44.7$ inches from the screen. Any object displayed on the screen subtends the same angle at the eye as it did at the TV camera.

Display magnification is image size relative to object size. When the object being viewed is close, magnification is calculated as image subtense divided by angular subtense of the object when the object is viewed from a standard distance from the eye or 250mm or 10 inches. When the viewed object is distant from the observer, magnification is calculated as image angular subtense divided by angular subtense of the object when viewed by the unaided eye. These definitions do not quantify "near" and "far," so there is the problem of what definition to use when an object is at an intermediate distance. The quite different definitions of magnification are discussed elsewhere in this paper.



(a). Television camera with a 24° field of view.



Angle at $D=60$ inches:
 $\tan(B/2) = (19/2)/60 = .15833$
 $B = 2\text{ArcTan}.15833 = B = 17.994^\circ$
 Viewing distance D for 24°FOV :
 $D = (19/2)\tan(B/2) = 9.5\tan(24/2) = D = 44.7$ inches

(B). Field of view of a TV display and viewing distance that will provide unity display magnification.

Fig. 2 . Field of view and display magnification for a TV camera and a television display with a 19-inch diagonal.

3.0 DISPLAY FIELD OF VIEW AND THE SIZE AND COMPLEXITY OF OPTICAL ELEMENTS

The size or diameter of the image provided by the objective of a camera, a telescope, or an HMD depends upon its construction. Usually, a part of the optics housing obstructs (vignettes) light rays that are at a large angle off the optical axis. It is relatively easy to correct optical aberrations over a narrow field of view (FOV): only a few optical elements are required. As the FOV grows to wide or very wide, correction, especially near the edge of the FOV, becomes difficult: many elements, some having steep surface curvatures, may be required to provide an image of acceptable quality in all parts of the image. Additional optical elements permit a better correlation and balancing of the several optical aberrations that reduce image quality.

An iris or diaphragm may be closed down in some objectives, such as camera lenses, to improve image quality, especially for image areas at large off-axis angles. However, "stopping down" reduces image illuminance. When the overall system already is suffering from a dim image, stopping down may not be acceptable.

In binoculars, telescopes, microscopes, and most HMDs, the primary image formed by the objective is magnified by the eyepiece (or ocular) according to the equation (instrument field of view) = (apparent field of view / (magnification)), i.e., $FOV = AFOV/M$. When magnification is large, as it often is, the FOV of the objective will be small, hence easily corrected to obtain high optical quality. For example, a telescope with an eyepiece having a 60° AFOV and magnifying 20 times, has an objective or external FOV of only $60/20 = 3^\circ$. An objective covering such a narrow angle can supply high quality images with as few as 2 optical elements.

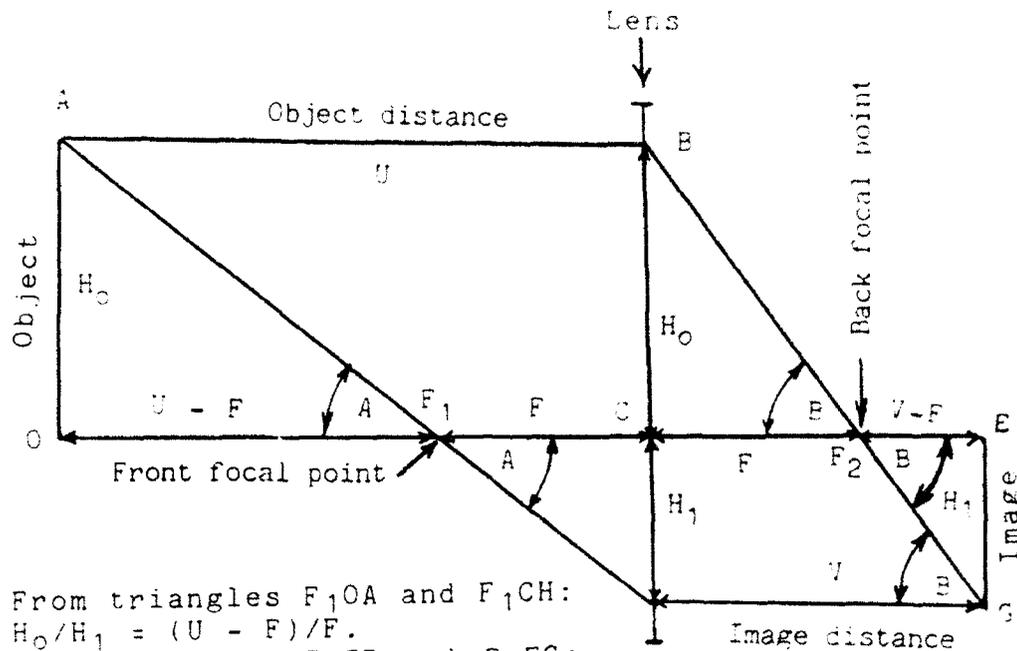
In contrast to the objectives of binoculars and telescopes, whose instrument or external field-of-view is usually quite small, eyepieces often have large to very large fields of view. Some eyepiece fields of view exceed 80 degrees. The larger the eyepiece field of view, the more complex, hence bigger and heavier, the eyepiece. In some optical systems, such as an HMD, an appreciable portion of users wear spectacles. There must be space between spectacle and eye and between a beam-splitter (or combiner) and spectacles. Thus, the eyepiece may have to be an appreciable distance from the observer's eye. If the display image is to subtend a large angle at the eye, the clear or effective aperture of the eyepiece will have to subtend a large angle at the eye. The result is a large eyepiece. A large AFOV usually requires multiple optical elements to provide good image quality over a wide AFOV. The eyepiece is thus both large and multi-element, resulting in a big, complex, heavy and expensive eyepiece.

4.0 FOCAL LENGTH

A basic concept in optics is that of focal length, for it relates the distance of an image from a reference point to the distance of the object being imaged, thus also relating relative sizes of objects and their images. Focal length may be defined as the distance from the rear nodal point of a lens to the rear focal point. The rear focal point is a point on the optical axis at which another point, also on the optical axis, but infinitely distant in front of the lens, is imaged, i.e., brought to a focus. For an object at infinity, the image distance V is one focal length F . Since focal length is usually different for different wavelengths, focal length is usually specified for the wavelength that is the mean of the two yellow "D" spectral lines of sodium vapor. Nodal points are reference points on the optical axis, frequently within the lens. Optical systems have a front and a rear nodal point. Nodal points have the property that an incident light ray passing through the first nodal point and making an angle with the optical axis will leave the second nodal point with the same angle to the axis. When focusing an object at optical infinity, because of this, the image doesn't move when the lens is pivoted from side to side above the second nodal point. A lens may be placed on a nodal slide, which has a screen to focus the image and a pivot point. The lens is aimed at a distant object or at a distant image provided by a collimator. Lens position along the lens axis over the pivot point is easily slid toward and away from the object until a position giving no image motion on the screen is found when the lens is pivoted to aim from side to side. Focal length is then the distance from the second nodal point (the pivot) to the screen image, as noted earlier, since image distance is one focal length for very distant objects.

An alternative definition or way of thinking about focal length is that focal length relates object and image distances and sizes. The geometry of the relationships is shown in Fig. 3, which also gives equations relating sizes and distances. The rationale for the drawing of Fig. 3 is given in Appendix 1 where the basic or Gauss lens equation is derived. This equation is $1/(\text{Object Distance}) + 1/(\text{Image Distance}) = 1/(\text{Focal Length})$, i.e., $1/U + 1/V = 1/F$. Solving for image distance, $V = FU/(U-F)$. In the figure, the equation $H_0/H_1 = (U-F)/F$, yields image size $H_1 = (\text{Object size } H_0)F/(U-F)$, i.e., $H_1 = H_0F/(U-F)$, from which $F = H_1U/(H_0 + H_1) = U/(1 + H_1/H_0)$. Thus, focal length may be defined in terms of the ratio of image size to object size at a specified object distance.

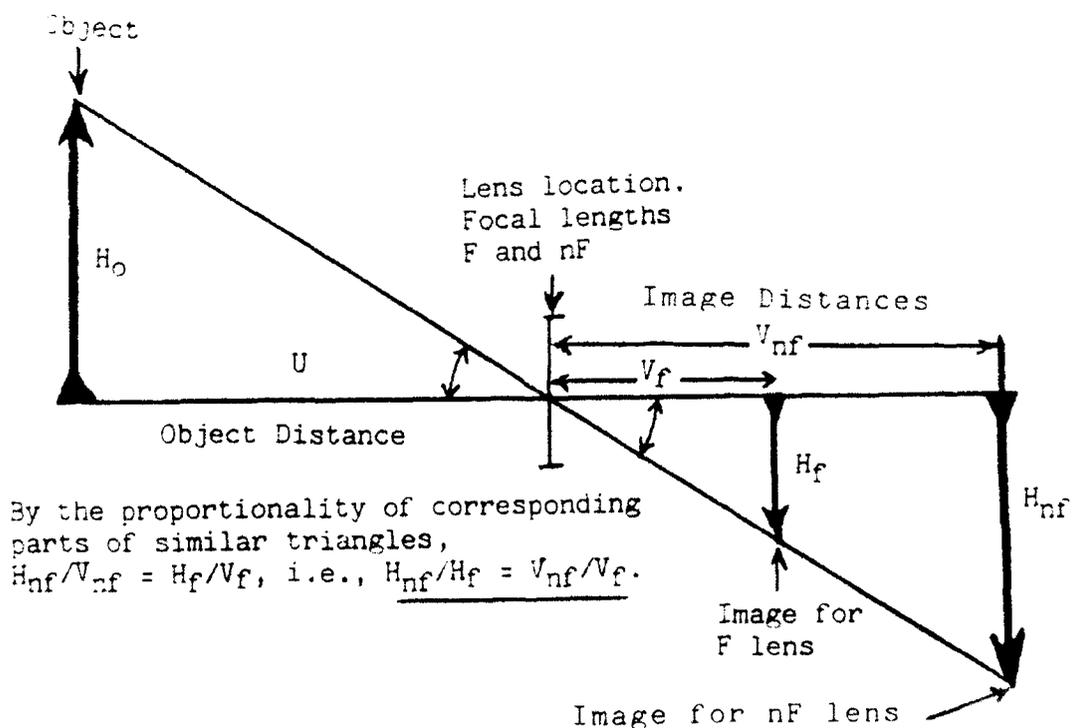
To examine some of the consequences or properties of the Gauss lens equation $1/U + 1/V = 1/F$, suppose that two lenses, one with a focal length F and one with a focal n times as large, i.e., with a focal length nF , each focuses on a screen an image of an object whose height is H_0 setting on the lens axis. Object distance U is the same for both lenses. The optical situation is shown in Fig. 4. Using the above image distance equation $V = FU/(U-F)$, the two image distances are $V_f = FU/(U-F)$ and $V_{nf} = nFU/(U-nf)$. The ratio of these two distances is



From triangles F_1OA and F_1CH :
 $H_o/H_1 = (U - F)/F$.
 From triangles F_2CB and F_2EG :
 $H_o/H_1 = F/(V - F)$.

Equating the two values of H_o/H_1 :
 $(U - F)/F = F/(V - F)$, from which
 $1/U + 1/V = 1/F$ (see Appendix 1)

Fig. 3. The role of lens focal length in relating image size and distance to object size and distance.



By the proportionality of corresponding parts of similar triangles,
 $H_{nf}/V_{nf} = H_f/V_f$, i.e., $H_{nf}/H_f = V_{nf}/V_f$.

Fig. 4. Images formed by lenses of focal lengths F and nF of the same object of height H_o at the same object distance U .

$V_{nf}/V_f = [nFU/(U - nF)]/[FU/(U - F)] = n(U - F)/(U - nF)$. In the figure, it is shown that the ratio of the image heights is $H_{nf}/H_f = V_{nf}/V_f$. Substituting the ratio of V_{nf}/V_f from above, $H_{nf}/H_f = V_{nf}/V_f = n(U - F)/(U - nF)$. When object distance U is very large compared to nF , $U - F$ and $U - nF$ are both approximately U , so that $H_{nf}/H_f = n$, and the ratio is exactly n when object distance U is infinity. Thus, for very distant objects, image linear dimensions, such as length or width, are multiplied by n when focal length is multiplied by n . Stated differently, image linear dimensions are proportional to focal length for distant objects. Now, in the equation $H_{nf}/H_f = n(U - F)/(U - nF)$, $(U - nF)$ is less than $(U - F)$, so that $(U - F)/(U - nF)$ is greater than 1. Thus, as object distance decreases from infinity to nF , the image size ratio increases from n to infinity. Dividing both the numerator and the denominator of the image size ratio by n , $H_{nf}/H_f = (U - F)/(U/n - F)$. Clearly, as n increases, $(U/n - F)$ decreases, so that the image size ratio becomes increasingly larger than n . The equation for relative image dimensions, i.e., the ratio of corresponding linear image dimensions, for a lens focal length F and a lens of focal length n times as large, i.e., nF , as a function of the focal length ratio, is plotted in Fig. 5 for object distances of 4, 8, 16, 32, and 64 focal lengths. Note the high curvature of the line on the log-log plot for the object distance U curve for $U = 4F$. For object distance $U = 64$ focal lengths, image size ratio as a function of focal length ratio is close to being a straight line: when object distance is as great as 64 focal lengths, image size is very nearly proportional to focal length.

For a numerical example of the effect of focal length upon image size, let the focal length be doubled, i.e., $n = 2$, and let object distance U be 20 focal lengths, i.e., $U = 20F$. Then, by the above equation, the ratio of image heights for the two focal lengths is $H_{2f}/H_f = n(U - F)/(U - nF) = 2(20F - F)/(20F - 2F) = 2.11$ (not 2.00). Similarly, for $n = 4$, $H_{4f}/H_f = 4.75$ (not 4.00), etc. When object distance U is decreased to $U = 10F$, the equation yields, for $n = 2$, $H_{2f}/H_f = 2.25$, and $n = 4$ yields $H_{4f}/H_f = 6.00$. Thus, with an increase in focal length, image size grows faster than growth in focal length, and image growth rate relative to focal length increase is greater at shorter distances, as noted earlier.

In summary, lens focal length relates object and image distances according to the Gauss lens equation $1/U + 1/V = 1/F$, where U and V are object and image distances, respectively, and F is focal length. Image distance behind a lens increases as focal length increases. In conjunction with object size and distance, focal length determines the linear dimensions of image details. For very distant objects, the linear dimensions of image details are directly proportional to focal length, e.g., doubling focal length doubles the size of images. For object distances less than infinity, image size is more than proportional to focal length: image size grows faster than focal length increase, and the disproportion grows with increase in focal length and with decrease in object distance. Thus,

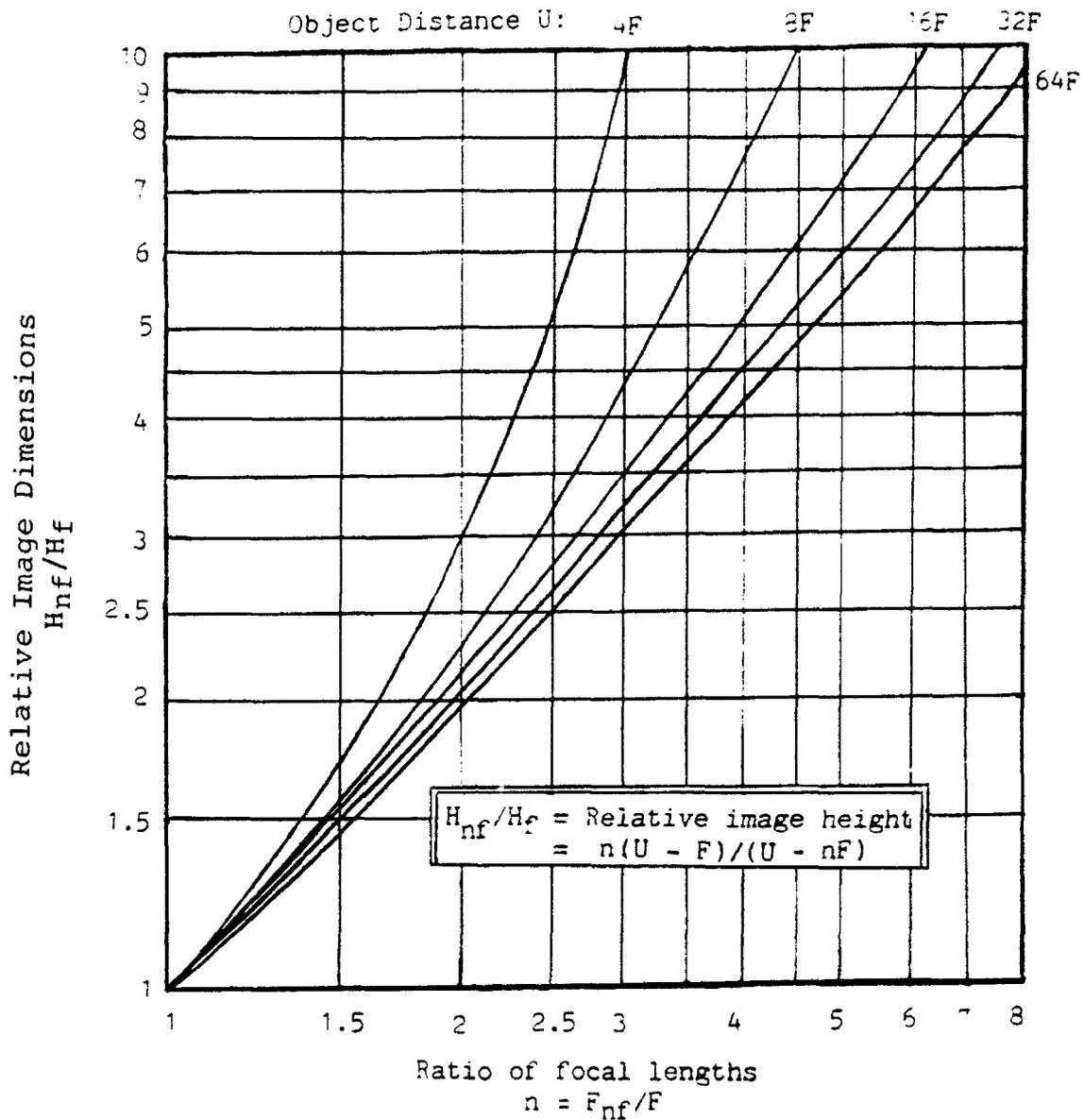


Fig. 5. Relative image dimensions as a function of focal length ratio for various object distances.

doubling focal length for objects nearer than optical infinity more than doubles the size of the image of an object, and the ratio is greater at closer distances.

5.0 MAGNIFICATION

When objects are small or distant, their details may be difficult or impossible to discern with the unaided eye. Adequate examination may require the assistance of optical viewing devices that appear to enlarge or magnify the object: to magnify means to enlarge. The size of an object or of an image can be specified by its linear dimensions: length, width, diagonal, diameter, etc. These are measures of linear size. A second way to specify object size is to give the size of the angle that covers an object from a reference point, such as the objective of a microscope or the entrance pupil of an observer's eye. The reference point is the vertex of the subtended angle. The angular extent or angular size is the angular subtense. Optical devices are said to magnify if they can provide an image on the observer's retina that is larger in linear dimensions than the retinal image obtained with the unaided eye. The angular subtense at the eye of the displayed image is larger than the angular subtense at the unaided eye of the object. Hand-held magnifying glasses, microscopes, and telescopes all provide an enlarged retinal image. They are viewing devices that optically magnify.

For some purposes, observation is improved by presenting an image whose angular subtense is smaller than the subtense of the object viewed with the unaided eye. For example, looking at the virtual image provided by the slightly-convex side-mounted mirror of an automobile, it is apparent that, compared to direct rearward vision, the mirror images of objects are reduced in angular size. The purpose of this minification is to provide a larger angular coverage or field-of-view of the rearward scene than would be available from a flat mirror of the same size located at the same distance from the eye. If the angular subtense of objects seen, for example, in an automobile rear-view mirror, is less than it would be by direct view, it can be said that the mirror minifies, rather than magnifies. However, in optics, the term "magnification" is used to cover the range from 0 to infinity. Thus, if a device provides images with angular subtenses 10% less the angle at the unaided eye, the device is said to have a magnification of $.9x$, $.9$ diameters, or $.9$ times, not a minification of 10% or $.1$. Magnification can be unity or larger or smaller than unity.

In describing the magnification of an optical device, an observer may remark that, for example, binoculars bring objects closer, that the image is bigger and closer than the object. Actually, the images provided by binoculars or telescopes are larger than the object in that they subtend a larger angle at the eye. However, depending upon the eyepiece focus adjustment, the virtual image displayed to the observer may be anywhere from a few centimeters to optical infinity distant from the observer. Thus, while a viewing device that provides increased angular

subtenses may give the impression that it "brings things closer," the displayed image is not necessarily optically closer than the object. Optical distance is the distance to which a viewing device must be focussed to obtain a clear display image. For example, if a small telescope is used to look at the image in the eyepiece of a second telescope, and the small telescope has to be focussed with the same eyepiece adjustment as when looking at a real object that is 300 feet away, then the image in the second telescope eyepiece is at an optical distance of 300 feet. In optics, bigger does not imply closer.

Magnification is a relative term: image size relative to object size. For an optical viewing device, linear magnification, which is sometimes called lateral magnification, is the ratio of the length H_1 of an image dimension to the length H_0 of the corresponding object dimension. Linear magnification $L.M. = H_1/H_0$. The concept of L.M. is illustrated in Figure 6. As an example of linear magnification, a camera lens projects, onto photographic film, an image of a circle painted on a wall. If the circle has a diameter H_0 of 80mm, and the image on the film has a diameter H_1 of 10mm, then linear magnification $L.M. = H_1/H_0 = 10/80 = .125X$, i.e., .125 times or diameters. The term "diameters, rather than "times," is used to indicate that it is linear, rather than area magnification, that is being used. As a second example to illustrate the meaning of linear magnification, an aircraft image 30mm long on transparency film is projected to focus, on a screen, an aircraft image 400mm long. Here, linear magnification $L.M. = H_1/H_0 = 400/30 = 13.3X$.

While linear magnification is the ratio of corresponding linear dimensions of the image and the object, angular magnification is the ratio of the corresponding angular subtenses, i.e., angular sizes. Angular magnification = (Image Angle)/(Object angle at the unaided eye) = (Angle B)/(Angle A). Angle A subtended at the unaided eye by the object depends upon the location of the eye. The eye may be assumed to be at the exit pupil of the viewing device, or at the objective lens of the viewing device, or at the standard viewing distance of 250mm or 10 inches. These different eye locations are used in different definitions of magnification. It is obvious that each of these assumed eye locations or reference points, in some situations, provides a different numerical value for angular magnification. For observing very distant objects, the eye location at the exit pupil or at the objective is, practically, the same distance from the object, so that angle A, hence angular magnification, is the same for both eye locations. As object distance U_0 from the objective decreases until telescope length is no longer trivial relative to object distance U_0 , angle A at the exit pupil becomes increasingly smaller than angle A at the objective.

Apparent angular magnification M (there is no standard nomenclature) is illustrated in Fig. 7. M_A is the ratio of the angle B at the eye subtended by the displayed image to the angle A subtended at the eye by the object with the optical device removed,

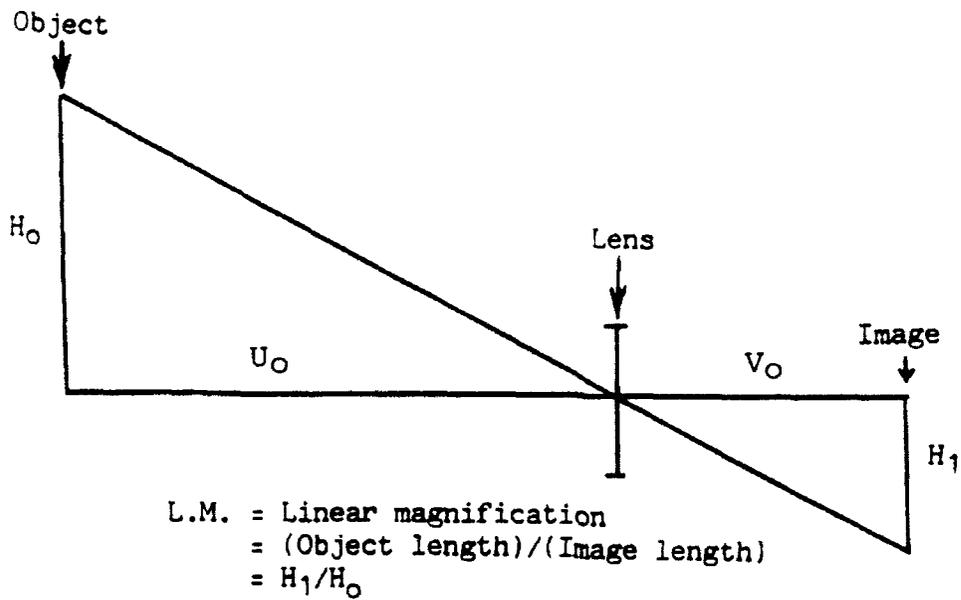
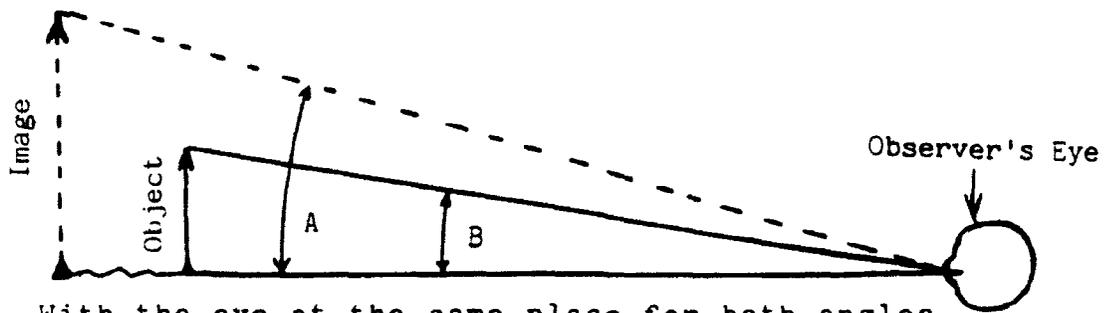


Fig. 6. Linear magnification.



With the eye at the same place for both angles,
 M_a = Apparent angular magnification
 = (Image subtense)/(Object subtense)
 = (Angle B)/(Angle A)
 $\approx \text{Tan B/Tan A}$

Fig. 7. Apparent angular magnification.

i.e., with the eye still at the place formerly occupied by the exit pupil:

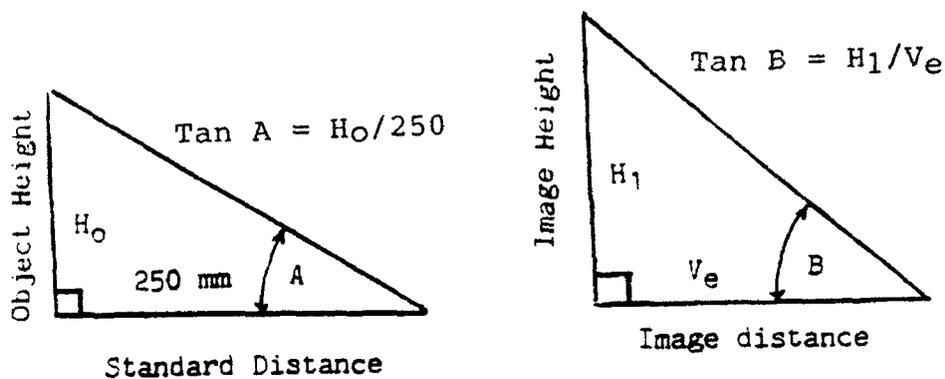
$$M_A = (\text{Image Subtense}) / (\text{Object Subtense}) = (\text{Angle B}) / (\text{Angle A}).$$

For small angles measured in radians (1 radian \approx 57.296 degrees), rather than in degrees, the angles are closely approximated by the tangents of the angles. Using tangents has the advantage that the sizes and distances of objects and their images can be used. Using tangents, apparent angular magnification $M_A = (\text{Angle B}) / (\text{Angle A}) \approx \text{Tan B} / \text{Tan A}$. This equation is used when the object is "standing" on the optical axis of the objective. For objects centered on the axis, half above and half below, half angles are used.

When using a small powerful magnifier, i.e., one with a short focal length, the eye is close to the magnifier and the magnifier is close to the object. The eye will be too close to focus on the object if the magnifier is removed and the eye is still in the same place, i.e., quite close to the object. In this case, apparent angular magnification M_A is not a practical measure of magnification, and it is worthwhile to have a measure of magnification that relates image angular subtense to the angular subtense of the object when viewed by the unaided eye at the closest distance that can be comfortably viewed by most people. Young adults with normal vision can comfortably view objects as close as 250mm or 10 inches. These two values are, by convention, the standard or average distance of closest distinct vision, and can be used in defining relative magnification.

Relative magnification M_r , called relative because it is relative to a standard viewing distance, is defined as the ratio of the angular subtense B of an image formed by an optical device at a distance U_o from an object to the angular subtense A of the object at a reference point (or at the eye) that is 250mm from the object: $M_r = (\text{Angle B}) / (\text{Angle A})$, as illustrated in Fig. 8. As with apparent angular magnification M_A , the small angles may be replaced by the tangents of the angles, so that $M_r = (\text{Angle B}) / (\text{Angle A}) \approx \text{Tan B} / \text{Tan A}$. If the distance from the entrance pupil of the eye to the image is V_e , object height is H_o , and image height is H_i , then, from the figure, $M_r = \text{relative magnification} \approx (\text{Tan B}) / (\text{Tan A}) = (H_i / V_e) / (H_o / 250) = (H_i / H_o) (250 / V_e) = [\text{Image Height}] / (\text{Object Height}] \times [(\text{The standard viewing distance}) / (\text{Image Distance})]$

Relative magnification may be illustrated by an example in which a view camera with a ground glass viewing screen for focussing and composition is used. The geometry of the viewing situation is shown in Fig. 9. In this example, a distant object is imaged on the ground glass by a camera lens with a focal length F_o of 250mm. Since the object is distant, object distance V_o is, for practical purposes, optical infinity, so that the camera lens is at its focal length $F_o = 250\text{mm}$ from the ground glass. From part A of the figure, it is clear that the angular subtense, angle A, of an object, if viewed by an unaided eye at the same location as the camera lens, is the same angular size, angle A, as the subtense at the eye viewing the image on the ground glass of the camera. Thus,



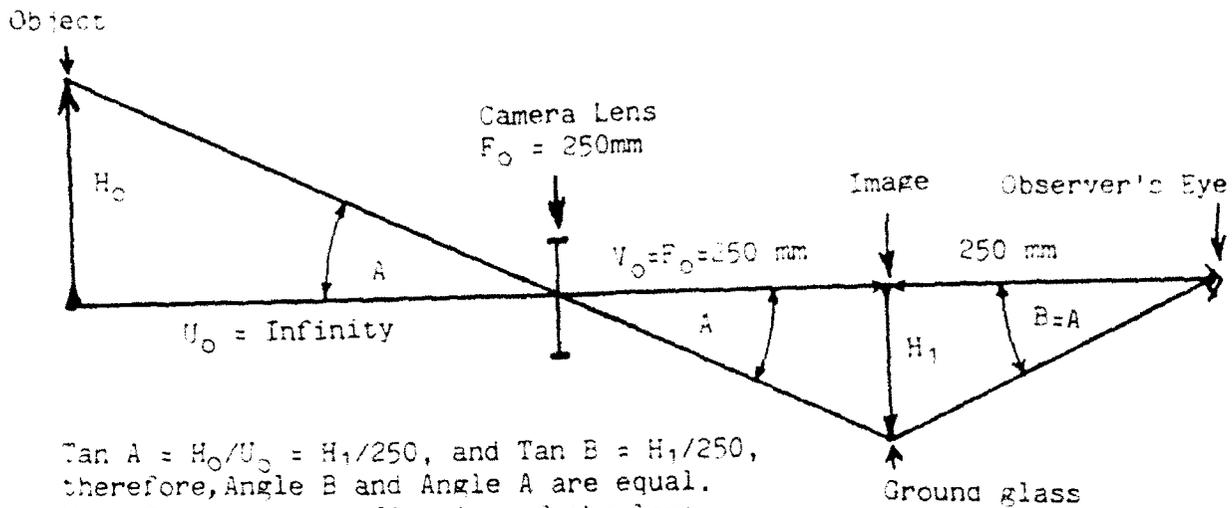
$$\begin{aligned}
 \text{Relative Magnification} &= (\text{Angle B})/(\text{Angle A}) \\
 &\approx \text{Tan B}/\text{Tan A} \\
 &= (H_1/V_e)/(H_o/250) \\
 M_r &= (H_1/H_o)/(250/V_e)
 \end{aligned}$$

Fig. 8. Relative magnification.

$$\begin{aligned}
 \text{angle A} &\approx \text{Tan A} \\
 &= H_o/U_o \\
 &= H_1/V_o \\
 &= H_1/250.
 \end{aligned}$$

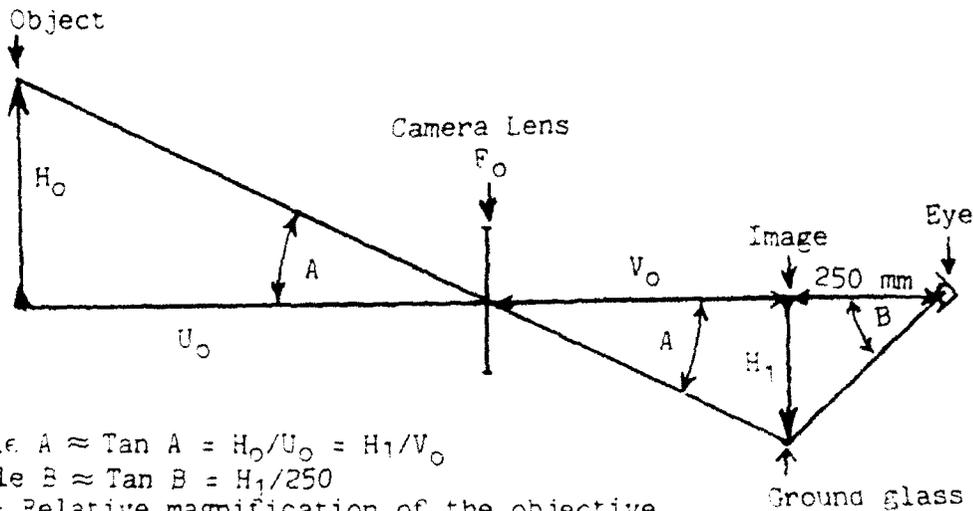
It is also clear that, if the camera lens had a focal length F_o of n times the standard viewing distance of 250mm, then angle $A \approx \text{Tan A} = nH_1/250$. In the general case, shown in part B of the figure, the viewing distance from the lens to the object is U_o , image distance from lens to image is V_o , and the eye is the standard viewing distance of 250mm behind the camera's ground glass. In this case, the image would subtend, at the eye, an angle B , and, from the figure, $(\text{Angle B}) \approx \text{Tan B} = H_1/V_o$. Thus, when an objective projects a real primary image, the relative magnification of the objective is the standard viewing distance of 250mm divided by the distance from the objective to the image. The linear magnification of the objective, in forming a real image, as noted earlier, is the ratio of image height to object height, or H_1/H_o .

In devices, such as helmet-mounted displays, magnification can be a confusing concept. Such devices may present a displayed image super-imposed on the directly-viewed external environment, a "see-through" capability. In such cases, it is sometimes desirable that images of objects displayed by the helmet-mounted display have the same angular subtense at the eye as the subtenses of the directly-viewed objects. When the angular subtenses are equal, the display is said to have unity display magnification, even though the HMD optics magnifies the picture on the small CRT many times, e.g., 7x,



$\tan A = H_o/U_o = H_1/250$, and $\tan B = H_1/250$,
 therefore, Angle B and Angle A are equal.
 M_r = Relative magnification of the lens
 = (Angle B at the eye)/(Angle A at the lens)
 = (Angle A)/(angle A) = 1, or unity.

A. Imaging a very distant object with a lens of 250 mm focal length.



Angle A $\approx \tan A = H_o/U_o = H_1/V_o$
 Angle B $\approx \tan B = H_1/250$
 M_r = Relative magnification of the objective.
 $M_r = (\text{Angle B})/(\text{Angle A}) \approx \tan B/\tan A =$
 $= (H_1/250)/(H_1/V_o) = V_o/250.$

B. The general case: Focal length F_o and object distance U_o .

Fig. 9. The relative magnification of an objective as illustrated by a camera lens forming an image on the ground glass of a view camera.

to provide a size-matching image. Thus, display magnification is relative to the scene, not relative to the CRT or other imaging device examined with the display optics. In some situations, to discern small details provided by a narrow-angle sensor, display magnification may be higher than unity relative to the outside environment.

In this section on magnification, magnification is defined as the ratio of image size to object size, a ratio with possible values from 0 to infinity. Linear magnification is defined as the ratio of an image linear dimension to the corresponding linear dimension of the object. Apparent angular magnification is the ratio of angular image subtense to object subtense with the viewing device removed with the eye at the same distance from the object as it was with the magnifier in place. Relative magnification is the angular subtense of the object at a distance of 250mm divided into image subtense. The tangents of the angular subtenses may be substituted for the angles, so that object and image distances and heights may be used instead of angles.

6.0 COLLIMATION AND ALIGNMENT

By one definition, an optical system is collimated when the image provided to the observer is a virtual image located at optical infinity. If the image is closer than infinity, the display is not collimated. There is a second and quite different definition of collimation which is applied to binocular devices. By this second definition, an optical device is collimated when the optical and mechanical axes are parallel. Thus, by this definition, a pair of binoculars can be collimated even if the eyepiece focus does not provide infinity images. When the optical and mechanical (hinge) axes are parallel, rotating the two halves of the binoculars about the central hinge permits adjusting interpupillary distance for use by different observers without loss of parallelism of the optical axes.

In the present discussion, collimation, defined as providing an image at optical infinity, will be discussed. When the distance of an eyepiece or magnifier from an examined object or from an examined primary image is changed, the distance from the eye to the virtual image provided by the eyepiece changes. A collimated eyepiece, sometimes called a collimating eyepiece, is not a special kind of eyepiece. It is, as noted, merely an eyepiece focussed to provide an image at optical infinity. Figure 10 shows the optical geometry for a collimating eyepiece (see page 28).

An eyepiece provides a virtual image at optical infinity of an examined object, whether the object is a real object or a real image, when the object is at distance of one focal length F_e of the eyepiece in front of the eyepiece. This follows from the basic Gauss lens equation for the eyepiece when it forms a virtual image: $1/(\text{Object Distance}) + 1/(\text{Image Distance}) = 1/(\text{Focal Length})$, i.e., $1/U_e - 1/V_e = 1/F_e$. When image distance V_e is infinity, $1/U_e - 1/V_e = 1/U_e - 0 = 1/F_e$, from which object distance $U_e = F_e$. When a lens projects a real image, image distance V_e is not negative, and the Gauss lens equation is $1/U_e + 1/V_e = 1/F_e$. For virtual images, since the image and the object are on the same side of the lens, image distance is negative. The basic or Gauss lens equation relating object distance, image distance, and focal length is derived in Appendix 1.

When observing distant objects with the unaided eye, the axes of the eyes are parallel. If the optical axes of a binocular viewing device or display are parallel, and are separated by the distance between the centers or the observer's eye pupils (the interpupillary distance or IPD), the images of objects occupy corresponding parts of the observer's retinas, and the images from the two eyepieces are easily fused. The observer sees only one object or scene. If the optical axes are not parallel, the images of objects do not fall upon corresponding retinal areas, and serious viewing problems can occur. Large misalignment prevents fusion, so that double images are seen. With only a little misalignment, fusion will occur, but visual fatigue and headache can occur. Alignment does not have to be perfect. There are tolerances within which extended use is comfortable. For binocular

devices there are also tolerances for rotation of one image relative to the other, and for differences in the sizes of the two images. Self (1986) reviewed the literature on optical tolerances for binocular devices.

When objects or images are examined, the eyes rotate horizontally in their sockets so that the same parts of the images in the two eyes are focused onto corresponding parts of the retinas of the eyes. For distant objects, this requires that the axes of binocular devices be approximately parallel. There is a little latitude, so that small error will not prevent fusion. For objects closer than infinity, the eyes must converge. Some binocular devices, such as stereomicroscopes, are constructed so that their optical axes converge to intersect at the optical distance of the displayed image. In the human visual system, eye focus control and eye convergence control systems cooperate or work together, so that any mismatch or conflict between the two can result in maladjustment and neuromuscular effort to resolve the conflict, and discomfort may follow. When one is discussing binocular viewing equipment, it is preferable to use the term "optically aligned," rather than the term "collimated."

7.0 OBJECTIVES AND EYEPIECES

THE OBJECTIVE

The objective in a compound optical system is the part of the instrument closest to the object being examined. It forms a real inverted image that is viewed through the eyepiece, the part next to the eye. The angular coverage of the objective is the angular coverage or angular field size of the instrument. The angular coverage of the instrument is referred to as the angular field of view, or, more commonly, just field of view (FOV). The angle covered by an objective, such as a camera lens, a telescope objective, or a microscope objective, depends upon its construction. Usually, a lens barrel, a circular disc with a hole, or a bellows obstructs or cuts off light that will not be recorded or displayed. In compound systems, defined as systems containing an objective and an eyepiece, the instrument's field of view (FOV) is the field of view in object space. The angle subtended at the eye by the field of view seen in the eyepiece is eyepiece field of view, referred to as apparent field of view. Magnification may be defined as apparent FOV B of the eyepiece divided by true or external or instrument FOV A, i.e., $M = (\text{Angle B}) / (\text{Angle A}) = B/A$. Thus, $A = B/M$: Instrument FOV is apparent FOV divided by instrument magnification. For example, binoculars with a 7° field of view A and a magnification M of 7x have an apparent FOV of $B = MA = (7)(7^\circ) = 49^\circ$.

All objectives have aberrations or faults in the images that they form or project: no lens is perfect. Even disregarding size differences of objects and their images, the images that they provide are not exact copies of the original object. Fortunately, few objectives, except those of research microscopes or professional astronomy telescopes, have to even approach optical perfection. It is more difficult to correct aberrations as angular coverage increases, as focal length increases beyond a few inches, as lens speed increases, and as greater corrections of aberrations are attempted. Lens speed is the ratio of aperture to focal length, i.e., lens speed is the inverse of lens f-ratio. Optical aberrations are corrected by varying the number of optical elements, the thicknesses, surface curvatures and spacing of elements and the optical materials of the elements. The number of optical elements tends to increase with the required optical quality of the image and with both lens speed and angular coverage.

A telescope objective that is slow (a high ratio of focal length to aperture), for example an f/15 objective, and covers a narrow field of only 2° or less, can produce excellent images with as few as two optical elements. In contrast to this, most f/1.4 50mm camera lenses for 35mm cameras contain 6 or 7 elements (pieces of glass), because they are both "fast" and cover a diagonal field of 46.8 degrees.

The objective of a helmet-mounted display (HMD) has to project a primary image in a short physical distance to save

weight and volume. Exit pupil diameters of 10mm or more require a large aperture objective because exit pupil diameter is effective objective diameter, i.e., entrance pupil diameter, divided by overall magnification. The HMD objective, then, has a short focal length and a large aperture. Since the angular coverage of an objective in an HMD or other compound optical system is apparent field of view B divided by system or instrument magnification M, the objective of an HMD does not have to cover a wide angle. For example, if magnification M of the HMD is 7x and apparent field of view B is 60° , then objective field of view, i.e., instrument field of view, viewing a CRT phosphor, is $A = 60^\circ/7 = 8.6^\circ$. Thus, the objective, though of short focal length and large aperture, hence being a "fast" lens, does not have to cover a wide angle. Because it is a fast lens, i.e., has a large aperture relative to its focal length, it may require several optical elements to supply a primary image with adequate image quality. An all-glass objective may be heavy so that optical quality plastics, which are much less dense, hence lighter, than glass may be used. In addition, the surfaces of some optical elements may have aspheric, rather than spherical, curvatures, to reduce the required number of elements. Other weight-saving methods employ aspheric reflecting mirrors, plastic and glass combinations, and holographic lenses.

THE EYEPiece

The primary image provided by the objective is presented to the eyepiece through which the observer looks. The eyepiece magnifies the primary image and presents a virtual image at a suitable optical distance from the eye. The eyepiece is a simple magnifier. It is called a simple magnifier to distinguish it from compound magnifiers, not to imply that it is structurally simple. In fact, some eyepieces are rather complex and contain as many as 9 optical elements. The total display image seen in an eyepiece is almost always circular. Its diameter subtends, at the eye, an angle called the apparent field of view (AFOV) or eyepiece FOV. Eyepiece field of view is less than 25° in some instruments, and as high as over 80° in others. A simple eyepiece containing only 2 or 3 optical elements may provide adequate image quality in an eyepiece with an apparent field of view of 30° , if of all glass construction. However, an eyepiece field of view of 80° usually requires 7-9 optical elements, especially if high image quality near the edge of the field of view is necessary. Such an eyepiece with a focal length of, for example, 2 inches and having all-glass optical elements (or optics) would weigh pounds. If part of an HMD, such an eyepiece alone could weigh as much as, or more than, the total allowable weight for the HMD. When light weight, moderate (2-3 inches) focal length, and a large apparent field of view is required for an eyepiece, the weight of an all-glass eyepiece may be prohibitive. When weight is a problem, plastics may be used for some optical elements. For systems utilizing a narrow bandwidth of light, holographic lenses contained in very thin optical films may be used. Instead of purely transmissive (refractive) elements, combinations of glass elements and curved mirrors may be used for eyepieces.

8. COMPOUND MAGNIFIERS

Microscopes, telescopes, binoculars, and helmet-mounted displays are compound magnifiers. A compound magnifier has two parts: an objective that forms or projects a real primary image, and an eyepiece that magnifies this image, projecting a virtual display image for the instrument user. The objective is the front part, the part closest to the object or viewed scene. The eyepiece is the part closest to the eye of the observer. The primary image projected by the objective is upside-down (inverted) and left-to-right (reversed) relative to the object. The eyepiece does not correct this image orientation. To invert and reverse the primary image, so that the displayed image is right-side-up and unreversed relative to the object, prisms or a lens or a system of mirrors may be placed between the objective and the eyepiece. When the viewed object is a CRT, the image on the CRT can be reversed and inverted by the CRT electronics, making it unnecessary to use additional optical devices between the objective and the eyepiece.

This section will examine only compound optical systems that do not contain optical devices for correcting image orientation. Appendix 3 briefly examines optical viewing systems that contain a relay lens between the objective and the eyepiece.

A. Magnification by Microscopes and Telescopes

Objects examined with a microscope are usually too small for an observer to see well, or to see at all, with the unaided eye, which is often called the naked eye. Objects are viewed with telescopes or binoculars when they are too far away to see well without the assistance of optical devices.

Microscopes are usually used with their objectives close to the examined object, while telescope objectives are usually at an appreciable distance from the object. However, some telescopes are constructed so that they can be used to examine nearby objects. The object distance at which a close-focussing telescope becomes a long distance microscope, or just a microscope, is arbitrary. While some people would regard a particular compound optical system as a microscope, others may classify it as a telescope. Although not customary or conventional, one way to distinguish between a microscope and a telescope is the linear size of the primary image projected by the objective. When the linear dimensions of the primary image are larger than the corresponding linear dimensions of the object, the instrument could be regarded as a microscope. When the linear dimensions of the primary image are smaller than the corresponding dimensions of the object, the device could be regarded as a telescope. Because of the differences in object distances for the two general classes of compound magnifiers, microscopes or binoculars and telescopes, and the different ways that magnification is defined for microscopes and telescopes, they will be treated separately in the present paper.

There are several different kinds of magnification, i.e., ways of defining and measuring magnification, depending upon the purpose to be served by the obtained numerical value. The different definitions usually lead to very different numerical values. As an example of a type of magnification, linear magnification is the ratio of a linear image dimension, such as length or width, to the corresponding object dimension. Angular magnification is the ratio of the angle subtended at some reference point, such as the eye of an observer, by an image dimension to the angle subtended by the corresponding object dimension at the same reference point or even at a different reference point. Relative magnification is the ratio of the angular subtense of a dimension of the displayed image to the angular subtense, at the eye, of the corresponding object dimension when the eye is a standard observation distance of 10 inches or 250mm from the object. Microscope magnification is calculated by multiplying the linear magnification of the objective by the relative magnification of the eyepiece. In commercial practice, these magnification factors are marked on the objectives and eyepieces and are valid when used on microscopes with standard optical tube lengths.

As an alternative to multiplying objective and eyepiece magnifications, the combined focal length of the objective and eyepiece, considered as a single system or a single magnifier, can be calculated, with eyepiece magnification being 10 divided by combined focal length in inches, or 250 divided by focal length in mm. Later on in this section, both methods will be used to derive equations for microscope magnification.

Telescope magnification is calculated by multiplying the relative magnifications of the objective and the eyepiece. An alternative method is to take the ratio of the image subtense at the eye to the angular subtense, at the eye or at the objective, of the object itself. For objects at long distances from the instrument, subtense at the eye and at the objective are essentially the same, but, close up, they may differ appreciably. This difference will be examined in detail later on in this section. By this angular ratio definition, the magnification of a telescope is the image angular subtense B divided by the object angular subtense A , i.e., $(\text{Angle } B)/(\text{Angle } A)$. For small angles measured in radians, where one radian is $180/\pi$, or approximately 57.296deg , the angles are closely approximated by the tangents of the angles. Substituting tangents for subtended angles, magnification $M \approx (\text{Tan } B)/(\text{Tan } A)$.

In some textbooks, the optical axis of the objective is shown as going through the center of the object, so that half angles, rather than whole angles, are used. In the present text, objects are usually pictured in the figures as "sitting" on the axis, and whole angles are used. As with the microscope, magnification may be obtained by calculating the combined focal length of the eyepiece and the objective, and treating the combination as a simple magnifier. In the following text, all three methods will be used to derive the same equation for telescope magnification.

B. Reference Points in Optical Devices

Optical components, such as objectives and eyepiece, have an appreciable thickness from front to back. This thickness poses a problem when specifying distances, such as object distance or image distance. The problem is "where are the points of reference within, but not always within, the components from which the distances are to be measured?" For example, image distance V_o to the image projected by an objective is not measured from the very back of the objective, nor from its center, but from a point that, often, is a short distance within the objective from its back. The reference point here is the rear nodal point of the objective. In similar fashion, distance from the objective to the object is the distance to the object measured from the front nodal point. For many objectives, both front and rear nodal points are within the objective. When using equations for calculating object and image distances, the thicknesses of the optical elements does not enter into the calculations. The distance between the front and back nodal points of, for example, an objective, adds a little to the physical length of the system, but does not introduce errors into calculations or conclusions based on them. In the present paper, objectives and eyepieces are treated as if they were, from front to back, thin optical devices. This thin lens assumption simplifies the presentation without invalidating the derived equations.

C. The Microscope

A microscope is a type of compound magnifier whose objective is close to the viewed object so that the real primary image projected or brought to a focus by the objective is larger in linear dimensions, such as length or width, than the corresponding dimensions of the object. This primary image is both upside-down (inverted) and backwards (reversed) relative to the object being imaged. The primary image is magnified by the eyepiece which projects or forms a secondary image which, like the image formed by the objective, is also inverted and reversed relative to the viewed object. In projecting its output image, an eyepiece does not invert or reverse its input image. Unlike the real primary image formed by the objective, the secondary image formed by the eyepiece is a virtual image. It is the image presented to the observer, i.e., it is the display image. The displayed image that is presented to an observer looking into the eyepiece appears to be in front of the eyepiece. Assume, in the derivations that follow, that the object being imaged, hereafter referred to as the object, has a small height H_o , and that it "sits" on the optical axis of the objective, i.e., its height is measured from the optical axis. The distance of the object from the objective is U_o , which will be called the object distance.

Let the primary image projected by the objective be at an image distance V_o behind the objective. The geometry of primary image formation by the objective is shown in part A of figure 10, which illustrates angles, sizes, and distances for microscopes and telescopes. Note how the light rays from the

object cross over the optical axis of the objective, forming an inverted image of the object. This cross-over of light rays from the object also occurs in a plane perpendicular to the plane of the paper, so that the image is reversed relative to the object. The linear magnification of the objective is the ratio of the height H_1 of the image to the height H_0 of the object, i.e., H_1/H_0 . Since the primary image is inverted and below the axis of the objective, by convention, image height H_1 is negative, so that linear magnification $L.M._0$ by the objective is negative:

Eqn. 1. $L.M._0 = - H_1/H_0$ Linear magnification
by the objective.

The objective, in projecting the primary image, obeys the Gauss lens equation $1/(\text{Object Distance}) + 1/(\text{Image Distance}) = 1/(\text{focal length})$, i.e.,

Eqn. 2. $1/U_0 + 1/V_0 = 1/F_0$ The Gauss lens equation
for the objective.

Since the Gauss lens equation, which relates object distance, image distance, and focal length, is used frequently in this tutorial, it is derived in Appendix 1. Hereafter, the Gauss lens equation will sometimes be referred to as the lens equation. The subscript o in equation 2 above indicates that the equation applies to the objective. For the eyepiece, the subscript would be e . From equation 2, $1/U_0 + 1/V_0 = 1/F_0$, object distance $U_0 = F_0 V_0 / (V_0 - F_0)$. From part A of Fig. 10, which shows primary image formation by the objective, $H_0/U_0 = H_1/V_0$, since corresponding parts of similar triangles are proportional. From this equation, $H_1/H_0 = V_0/U_0$, which, with a negative sign appended, is the linear magnification of the objective. Substituting $F_0 V_0 / (V_0 - F_0)$ from above, for U_0 ,

$$\begin{aligned} L.M._0 &= V_0/U_0 \\ &= V_0/[F_0 V_0 / (V_0 - F_0)] \\ &= (V_0 - F_0)/F_0. \end{aligned}$$

Since the image is inverted, by convention, magnification is regarded as negative, so that

Eqn. 3. $L.M._0 = - (V_0 - F_0)/F_0$ Linear magnification by
the objective.

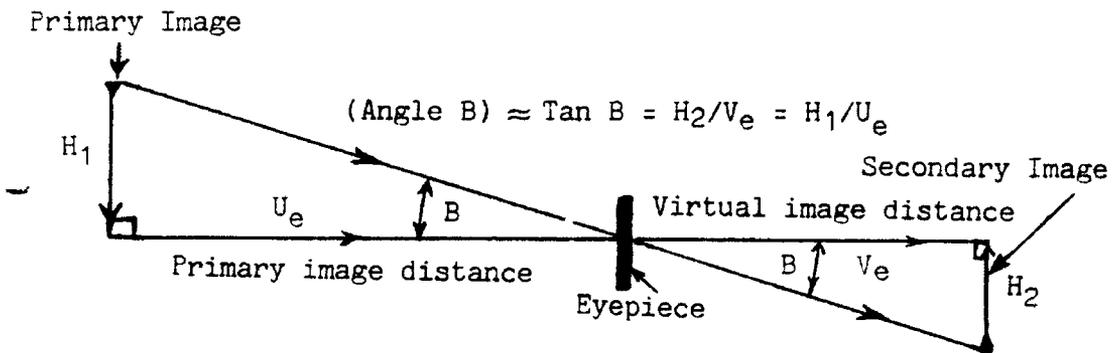
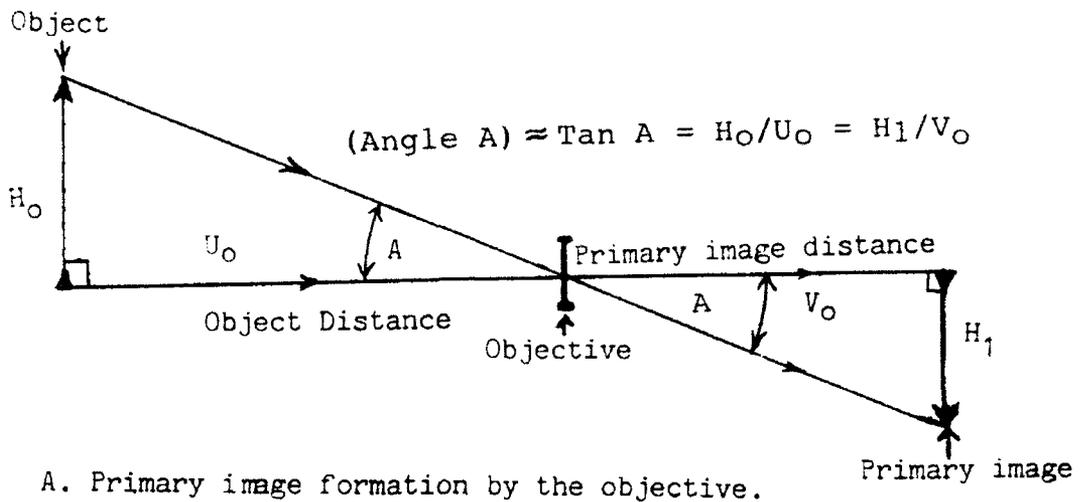
From equation 2, $1/U_0 + 1/V_0 = 1/F_0$, $V_0 = F_0 U_0 / (U_0 - F_0)$. From above, $L.M._0 = H_1/H_0 = V_0/U_0$. Substituting this for V_0 ,

$$\begin{aligned} L.M._0 &= V_0/U_0 \\ &= F_0 U_0 / [(U_0 - F_0)/U_0] \\ &= F_0 / (U_0 - F_0). \end{aligned}$$

By convention, this magnification is negative, so that

Eqn. 4. $L.M._0 = - F_0 / (U_0 - F_0)$ Linear magnification
by the objective.

Equating the linear magnifications given by equations 3 and 4, $-(V_0 - F_0)/F_0 = - F_0 / (U_0 - F_0)$. Cross multiplying, $F_0^2 = (U_0 - F_0)(V_0 - F_0)$. This is Newton's form of the basic lens equation.



Note: H_1 , H_2 actually on same side of eyepiece, no crossover or image inversion occurs.

Fig. 10. Geometry of angles, sizes, and distances for microscopes and telescopes.

The eyepiece, in forming a magnified virtual image from the primary image projected by the objective, obeys the basic lens equation, referred to as the Gauss lens equation:

Eqn. 5. $1/U_e - 1/V_e = 1/F_e$ The Gauss lens equation for the eyepiece when projecting a virtual display image.

By convention, a minus sign is used for the $1/V_e$ term of the equation, because the final or displayed image formed by the eyepiece is on the same side of the eyepiece as the primary image, which is the object or input for the eyepiece. When the eyepiece

is located at its own focal length F_e behind the primary image, it will form or project a virtual image at optical infinity. This may be seen by inspecting equation 5, for, when $F_e = U_e$, the equation becomes $1/F_e - 1/V_e = 1/F_e$, i.e., $-1/V_e = 0$, from which V_e is optical infinity. Since the displayed virtual image is at optical infinity, the angular subtense, at the eye, of any part of the display image does not change with eye movement along the optical axis or perpendicular to the axis, i.e., back and forth and sideways eye movement does not make the image appear to move or change the angular size of the images of displayed objects. However, moving back from the exit pupil can cut off peripheral parts of the total display field, making field size shrink.

When the displayed image is located at optical infinity, the system is said to be collimated in that all light rays from any one point on the viewed object emerge from the eyepiece parallel to all other rays from that point, but not parallel to rays from other points on the object. It is sometimes said that all the rays from a distant object are parallel when they exit the eyepiece. This is clearly true only when the object is essentially a point object, such as a distant star. The word distant is used here to refer to stars other than our own star, the sun, which subtends, at the earth, an angle of about half of a degree. The word collimate means to render parallel.

Figure 11 illustrates relative magnification by collimating eyepieces. Because of the collimation, at the viewer's eye, as shown in the figure, the angular subtense (Angle B) of the virtual image is the same as the angular subtense at the eyepiece of the primary image. Angle B, is assumed to be a small angle, so that, when measured in radians, where one radian is $180\text{deg}/\text{Pi}$, it is closely approximated by the tangent of the angle. Thus,

$$\begin{aligned} (\text{Angle B}) &\approx \text{Tan B} \\ &= H_o/V_e \\ &= H_1/U_e \\ &= \text{the angular subtense of the displayed virtual image.} \end{aligned}$$

Since, as noted for collimating eyepiece, the object for the eyepiece is the primary image at one eyepiece focal length in front of the eyepiece, i.e., $U_e = F_e$, $\text{Tan B} = H_1/U_e = H_1/F_e$. If the unaided or naked eye were to view the primary image from the standard viewing distance of 250mm or 10 inches, then

(Angle A) $\approx \text{Tan A} = H_1/250$. Eyepiece relative magnification M_e is, by definition, virtual image subtense divided by the subtense of the examined object (here, the primary image) at the unaided eye from a viewing distance of 250mm. By this definition,

$$\begin{aligned} M_e &= (\text{Angle B})/(\text{Angle A}) \\ &\approx \text{Tan B}/\text{Tan A} \\ &= (H_1/F_e)/[H_1/250] = \end{aligned}$$

Eqn. 6. $M_e = 250/F_e$ Eyepiece relative magnification, display image at optical infinity, F_e in mm.

Some observers want the displayed virtual image to be at an optical distance V_e that is closer than optical infinity. Some even want the image to be as close as it can be while still being

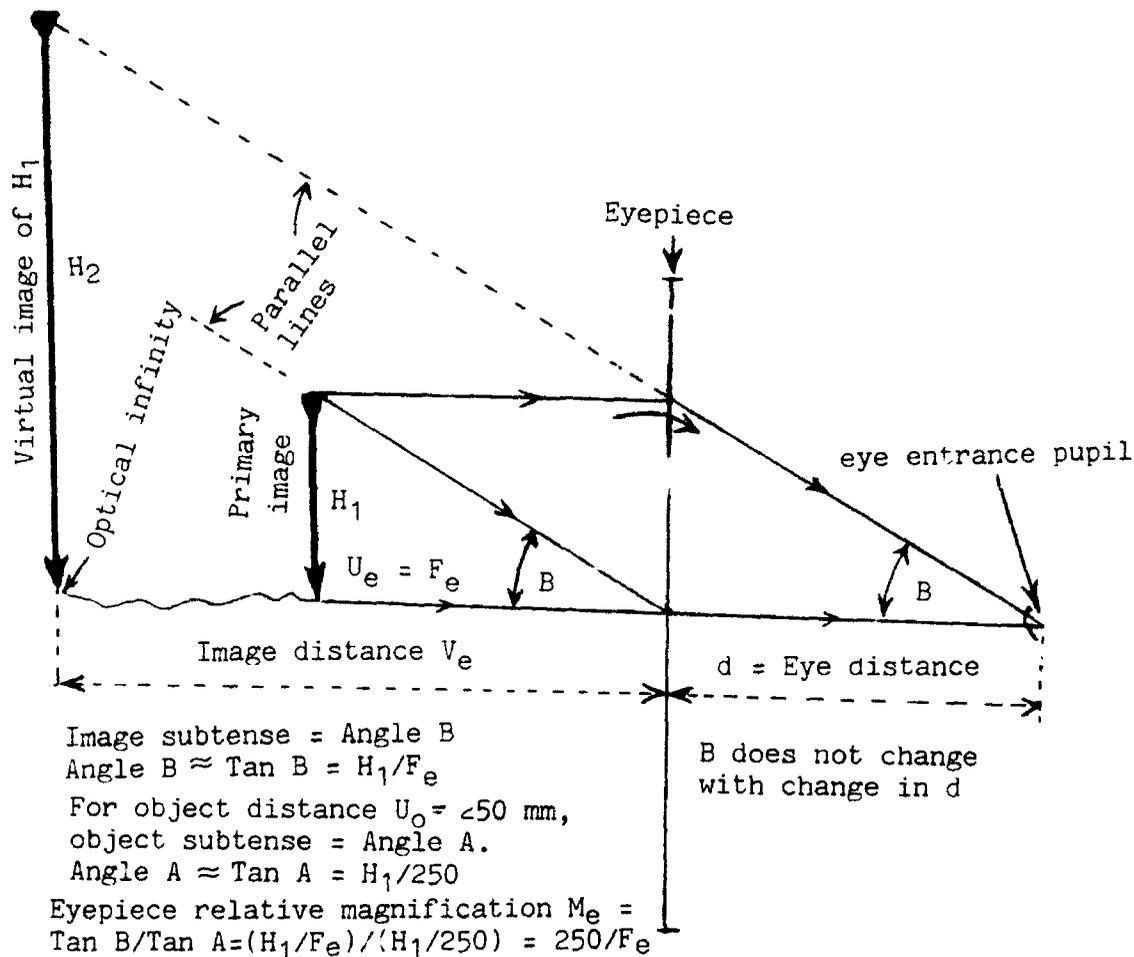


Fig. 11. Relative magnification of a collimating eyepiece.

distant enough to be focussed clearly by the eye. To derive the relative magnification when the image is not at optical infinity, as before, the primary image height is H_1 . When viewed with the unaided eye at the standard 250mm viewing distance, it would subtend an (Angle A) $\approx \tan A = H_1/250$. The virtual image of height H_2 projected by the eyepiece is now a distance V_e from the eyepiece, and subtends an (Angle B) $\approx \tan B = H_2/V_e$. This is shown in part B of figure 10. Since corresponding parts of similar triangles are proportional, $H_2/H_1 = V_e/U_e$, from which $H_2 = H_1 V_e / U_e$. From equation 5, $1/U_e - 1/V_e = 1/F_e$, $1/U_e = 1/V_e + 1/F_e$. Substituting this for $1/U_e$, $H_2 = H_1 V_e (1/V_e + 1/F_e)$. Substituting this for H_2 , $\tan B = H_2/V_e = H_1 V_e (1/V_e + 1/F_e) / V_e = H_1 (1/V_e + 1/F_e)$.

Eyepiece angular magnification is, by definition,
 $M_e = (\text{Angle B}) / (\text{Angle A})$
 $\approx \text{Tan B} / \text{Tan A}$
 $= H_1(1/V_e + 1/F_e) / (H_1/250) =$

Eqn. 7. $M_e = 250/V_e + 250/F_e$ Eyepiece relative magnification,
 image at V_e , F_e in mm.

For an image distance $V_e = 250\text{mm}$, the commonly accepted
 closest average distance for distinct vision, and the image
 distance that yields the highest relative magnification achievable
 with distinct vision and viewing comfort, by equation 7,

$$\begin{aligned}
 M_e &= 250/V_e + 250/F_e \\
 &= 250/250 + 250/F_e =
 \end{aligned}$$

Eqn. 8. $M_e = 250/F_e + 1$ Eyepiece relative magnification for
 an image distance of 250mm, F_e in mm.

From equations 7 and 8, it is clear that focusing an eyepiece
 to bring the virtual display image closer to the viewer's eye than
 optical infinity yields an increase in magnification. However,
 collimation is lost and, for most observers, prolonged viewing is
 less comfortable. The amount of increase in magnification
 obtained by focusing the eyepiece to bring the image to only 250mm
 may be examined by perusing a numerical example. For the majority
 of optical instruments, an eyepiece with a focal length of 60mm is
 one with a rather long focal length, while an eyepiece with a focal
 length of 10mm is a rather short focal length eyepiece. By
 equation 8, for the 60mm eyepiece,

$$M_e = 250/F_e + 1 = 250/60 + 1 = 5.17 \text{ times,}$$

while, for a collimated image, equation 6 yields $250/60 = 4.17$
 times. The gain in magnification by bringing the image to a
 distance of 250mm is $100(5.17 - 4.17)/4.17 = 24\%$. For the 10mm
 eyepiece, $M_e = 250/10 = 25$ times for the infinity image, and
 $250/10 + 1 = 26$ times for the close-up image. Here, the
 magnification gain is $100(26 - 25)/25 = 4\%$. From this example
 with a rather long focal length and a rather short focal length
 eyepiece, it can be seen that appreciable magnification gain from
 bringing the display image to a distance of only 250mm occurs only
 with eyepieces of rather long focal length. An attempt to gain
 even more magnification by bringing the image even closer than
 250mm would make it difficult or impossible for most people to
 clearly focus the image, and, for observers with normal or near
 normal vision, would result in eyestrain and viewing discomfort.

Now that equations have been derived for the magnifications of
 objectives and eyepieces, the magnification of the combination of
 eyepiece and objective can be derived. The overall magnification
 of a microscope is the product of the linear magnification of the
 objective and the relative magnification of the eyepiece i.e.,
 $M = (M.L._o)(M_e)$. From equation 3, $M.L._o = -(V_o - F_o)/F_o$. From
 equation 6, $M_e = 250/F_e$. Multiplying these two magnifications
 together yields microscope magnification:

Eqn. 9. $M = - 250(V_o - F_o)/F_o F_e$ Microscope magnification,
 with V_o , F_o , and F_e in mm.

Microscope objectives can provide their best image quality at only one image projection distance V_o . Long ago, this distance was standardized so that the primary image from the objective would be focused at a distance $V_o = F_o + T$, where T is the standard microscope tube length of most American and British microscopes. Using this value of V_o in equation 7, microscope magnification becomes $M = 250(F_o + T - F_o)/F_o F_e = -250T/F_o F_e$. For a standard microscope tube length T of 180mm,

Eqn. 10. $M = -180 \times 250 / F_o F_e$ Microscope magnification with focal lengths in mm, standard 180mm tube length.

For convenience in use, both the objectives and the eyepieces of microscopes are marked with their magnifications. Most microscope eyepieces have magnifications in the range of 8X to 20X, with 10X being the most common. Most microscope objectives have magnifications in the range of 5X to 100X. As an example, a 40X objective with a 10X eyepiece will provide an overall or microscope magnification of $M = 40 \times 10 = 400X$, i.e., 400 times or 400 diameters. From equation 6, for this example, eyepiece relative magnification $M_e = 250/F_e$, from which $F_e = 250/10 = 25\text{mm}$. Note that equation 10 may be written as $M = - (180/F_o)(250/F_e)$, so that objective magnification $M_o = 180/F_o = 40$, from which $F_o = 180/40 = 4.5\text{mm}$. Alternatively, just after equation 9, tube length was defined by $V_o = F_o + T$. For $T = 180\text{mm}$, $V_o = F_o + 180$. Using this value in equation 3,

$$\begin{aligned} \text{L.M.}_o &= - (V_o - F_o) / F_o \\ &= - (F_o + 180 - F_o) / F_o \\ &= - 180 / F_o = 40, \text{ from which } F_o = 180/40 = 4.5\text{mm, as above.} \end{aligned}$$

The microscope in this example, then, has an objective with a focal length of 4.5mm, an eyepiece with a focal length of 25mm, and a system or overall magnification of 400X.

A second method for deriving equation 10, the standard equation for the microscope, is to regard the compound microscope as a simple magnifier composed of an objective and an eyepiece that are separated by a distance d . In optics textbooks, two lenses separated by a distance d , when the first has a focal length F_1 and the second has a focal length F_2 , have a combined focal length $F_c = F_1 F_2 / (F_1 + F_2 - d)$. This useful equation for the focal length of two combined lenses is derived in Appendix 2. Substituting F_o for F_1 and F_e for F_2 ,

Eqn. 11. $F_c = F_o F_e / (F_o + F_e - d)$ Combined focal length of the objective and eyepiece, lens separation $d > F_o$.

The primary image formed by the objective is a distance V_o behind the objective, and the eyepiece is a distance F_e behind the primary image so that it will provide a virtual display image at optical infinity. The separation of the eyepiece and objective is then $d = V_o + F_e$. Using this value of d in equation 11, the combined focal length of the eyepiece and the objective is

$$\begin{aligned}
 F_c &= F_o F_e / (F_o + F_e - d) \\
 &= F_o F_e / (F_o + f_e - V_o - F_e) \\
 &= F_o F_e / (F_o - V_o). \text{ Used as a simple magnifier, by equation 6,}
 \end{aligned}$$

microscope magnification

$$\begin{aligned}
 M &= -250/F_c \\
 &= -250 / [F_o F_e / (F_o - V_o)] \\
 &= -250(V_o - F_o) / F_o F_e, \text{ as given earlier by equation 9.}
 \end{aligned}$$

Substituting $180 + F_o$ for V_o , where 180mm is tube length, magnification, as given by equation 9, is

$$\begin{aligned}
 M &= -250(V_o - F_o) / F_o F_e \\
 &= -250(180 + F_o - F_o) / F_o F_e \\
 &= -250 \times 180 / F_o F_e, \text{ which is the standard equation for microscope} \\
 &\text{ magnification given by equation 10.}
 \end{aligned}$$

D. The Telescope

A telescope, like a microscope, is a compound magnifier, with an objective of focal length F_o and an eyepiece of focal length F_e . As noted earlier, a microscope is constructed, through the spacing between the objective and the eyepiece, to examine objects close to the objective. The primary image projected by the microscope is larger, in its linear dimensions, than the corresponding linear dimensions of the examined object. In contrast to this, telescopes, through the spacing between their objectives and eyepieces, are constructed to view distant objects, i.e., objects at a distance of several focal lengths of the objective in front of the objective. Telescopes thus have primary images that are smaller to much smaller in linear dimensions than the corresponding linear dimensions of the viewed object.

The telescope objective, receiving light from an object of height H_o at a distance U_o in front of the objective, projects a real inverted primary image of height H_1 at a distance V_o behind the objective. Relative to the object, the primary image is reversed as well as inverted. The geometry of the optical situation is shown in part A of Fig. 10. When this primary image is viewed by the unaided eye from the standard viewing distance of 250mm, if it is projected by an objective whose focal length is such that image distance V_o is 250mm, the relative magnification of the objective would be $V_o/250$. By convention, since the image is inverted, the relative magnification of the objective, when projecting the primary image, becomes

Eqn. 12. $M_o = -V_o/250$ Relative magnification of a telescope objective, for V_o in mm.

Equation 12 may be more formally derived as follows. The image height H_1 of the primary image at a standard viewing distance of 250mm will subtend an angle B . This will be a small angle for small objects that are relatively far away. For small angles, the angle measured in radians is closely approximated by the tangent of the angle. One radian is $180/\pi$, or about 57.296 deg. Using this approximation, (Angle B) $\approx \tan B = H_1/250$.

The object height H_o at a distance U_o from the objective, subtends an angle A approximately equal to $\tan A = H_o/U_o$. Also, from part A of Fig. 10, $\tan A = H_1/V_o$. The relative magnification of the objective is then

$$\begin{aligned} M_o &= (\text{Angle B})/(\text{Angle A}) \\ &\approx \tan B/\tan A \\ &= (H_1/250)/(H_1/V_o) \\ &= V_o/250. \end{aligned}$$

Since the primary image is inverted, M_o is negative, and the relative magnification of the objective is $M_o = -V_o/250$, as given by equation 12.

The eyepiece magnifies the primary image supplied by the objective by $M_e = 250/F_e$, as given by equation 6. The overall magnification, or telescope magnification, is the product of the magnifications of the objective and eyepiece, i.e.,

$$M = M_o M_e = (-V_o/250)(250/F_e) =$$

Eqn. 13. $M = -V_o/F_e$ Telescope angular magnification.

Primary image projection distance V_o may not be known, whereas object distance U_o may be readily obtained. From the lens equation for the objective, given by equation 2 as

$$1/U_o + 1/V_o = 1/F_o, \quad V_o = f_o U_o / (U_o - F_o).$$

Substituting this for V_o ,

$$\begin{aligned} M &= -V_o/F_e \\ &= [-F_o U_o / (U_o - F_o)] / F_e \\ &= -(F_o/F_e) U_o / (U_o - F_o). \end{aligned}$$

Dividing both the numerator and the denominator by U_o ,

Eqn. 14. $M = -(F_o/F_e)/(1 - F_o/U_o)$ Telescope angular magnification.

When object distance U_o is infinity, $M = -(F_o/F_e)/(1 - 0) = -F_o/F_e$. This is the equation used to calculate telescope magnification when changing eyepieces in astronomical telescopes.

Note, from the equation, that, as object distance U_o decreases, F_o/U_o increases, so that $(1 - F_o/U_o)$ decreases, so that telescope magnification increases. This increase in telescope magnification as object distance decreases is quite small until objects are rather close to the telescope, i.e., when at a distance of only a few focal lengths of the objective.

To examine the increase in telescope magnification with decrease in object distance, suppose that an object is n focal lengths of the objective in front of the objective, i.e., is at a distance $U_o = nF_o$. In equation 14, the factor $1/(1 - F_o/U_o)$, with U_o replaced by nF_o , becomes $1/(1 - F_o/nF_o) = 1/(1 - 1/n)$. Thus, the telescope magnification, which is F_o/F_e for infinitely distant objects, increases for objects closer than infinity. The magnification that would be obtained for very distant objects must be multiplied by $1/(1 - 1/n)$ for objects that are at a distance of n focal lengths of the objective. For example, telescope magnification for an object 10 focal lengths away is magnification for very distant objects times $1/(1 - 1/10) =$

$1/0.9 = 1.111$ times. At a distance of $n = 2$ focal lengths, $1/(1 - 1/n) = 2$, i.e., magnification is twice that at infinity.

A different method for deriving telescope magnification is based on the definition of telescope angular magnification as the ratio of the angular subtense of objects, as seen with the telescope, to the angular subtense of the same objects viewed with the unaided eye. Assume, for deriving the magnification equation, that the eye and the objective are both essentially at the same distance U_o from the object. This assumption obviously neglects the length of the telescope plus the exit pupil distance. The optical situation is shown in part A of Fig. 10. At the unaided eye, the object of height H_o at a distance U_o will subtend an (Angle A) $\approx \tan A = H_o/U_o$. The virtual display image of height H_2 at an image distance V_e subtends an (Angle B) $\approx \tan B = H_2/V_e$, as may be noted from part B of Fig. 10. Also, from part B, by corresponding parts of similar triangles being proportional, $H_1/U_e = H_2/V_e$, from which $H_2 = H_1 V_e/U_e$. From part A of the figure, by corresponding parts of similar triangles being proportional, $H_o/U_o = H_1/V_o$, from which $H_1 = H_o V_o/U_o$. Substituting this for H_1 ,

$$\begin{aligned} H_2 &= H_1 (V_e/U_e) \\ &= [H_o V_o/U_o] (V_e/U_e) \\ &= H_o V_o V_e/U_o U_e. \end{aligned}$$

Substituting this for H_2 in the above equation for $\tan B$,

$$\begin{aligned} \tan B &= H_2/V_e \\ &= [H_o V_o V_e/U_o U_e]/V_e \\ &= H_o V_o/U_o U_e. \end{aligned}$$

Telescope angular magnification M is

$$\begin{aligned} M &= (\text{angular subtense with the telescope})/(\text{angular subtense with the unaided eye}) \\ &= (\text{Angle B})/(\text{Angle A}) \\ &\approx \tan B/\tan A \\ &= [H_o V_o/U_o U_e]/(H_o/U_o) \\ &= V_o/U_e. \end{aligned}$$

Since the image is inverted, magnification is, by convention, negative, so that

$$M = -V_o/U_e, \text{ as derived earlier.}$$

Now, for a collimated eyepiece, the primary image is one focal length of the objective in front of the eyepiece, i.e., $U_e = F_e$. Substituting F_e for U_e , $M = -V_o/U_e = -V_o/F_e$, as given by equation 13. From equation 2, $1/U_o + 1/V_o = 1/F_o$,

$$V_o = F_o U_o / (U_o - F_o).$$

Substituting this for V_o ,

$$\begin{aligned} M &= -V_o/F_e \\ &= -F_o U_o / (U_o - F_o) F_e \\ &= - (F_o/F_e) U_o / (U_o - F_o) \\ &= - (F_o/F_e) / (1 - F_o/U_o), \end{aligned}$$

as given by equation 14.

The telescope may be regarded as a simple magnifier with two components, an objective with a focal length F_o , and an eyepiece with focal length F_e , with the two separated by a distance d . The primary image projected by the objective is a distance V_o behind the objective. The eyepiece, to provide a collimated image, is a distance F_e behind the primary image. The eyepiece and the objective are thus separated by a distance $d = V_o + F_e$.

Substituting this for d in equation 11, the equation for the combined focal length of two lenses,

$$\begin{aligned} F_c &= F_o F_e / (F_o + F_e - d) \\ &= F_o F_e / (F_o + F_e - V_o - F_e) \\ &= F_o F_e / (F_o - V_o), \text{ as shown in the discussion of the microscope.} \end{aligned}$$

From equation 2, $1/U_o + 1/V_o = 1/F_o$, the Gauss lens equation for the objective, $V_o = F_o U_o / (U_o - F_o)$. Substituting this value of V_o in the above equation for the combined focal length F_c of the eyepiece and objective,

$$\begin{aligned} F_c &= F_o F_e / (F_o - V_o) \\ &= F_o F_e / [F_o - F_o U_o / (U_o - F_o)]. \text{ Placing the denominator} \\ &\quad \text{over a common denominator } (U_o - F_o), \\ &= F_o F_e / [(F_o U_o - F_o^2 - F_o U_o) / (U_o - F_o)]. \text{ Ignoring the minus,} \\ &= (F_e / F_o) (U_o - F_o). \text{ The object of height } H_o \text{ is at the focus} \\ &\quad \text{of the combined objective and eyepiece, so that a collimated} \\ &\quad \text{display image will be produced, i.e., the image is at a distance} \\ &\quad F_c. \text{ At this distance, the object height } H_o \text{ subtends an (Angle B)} \\ &\quad \approx \tan B = H_o / F_c. \text{ The object height } H_o, \text{ at a distance } U_o \text{ from the} \\ &\quad \text{unaided eye, subtends an (Angle A) } \approx \tan A = H_o / U_o. \text{ By definition,} \\ &\quad \text{the relative magnification of the telescope is} \\ M &= (\text{angular subtense with the telescope}) / (\text{angular subtense with} \\ &\quad \text{no telescope}) \\ &= (\text{Angle B}) / (\text{Angle A}) \\ &\approx \tan B / \tan A \\ &= (H_o / F_c) / (H_o / U_o) \\ &= U_o / F_c. \text{ Replacing } F_c \text{ with its value from above,} \\ &= U_o / F_c \\ &= U_o / [(F_e / F_o) (U_o - F_o)] \\ &= - U_o (F_o / F_e) / (U_o - F_o). \text{ Dividing numerator and denominator by } U_o, \\ &= - (F_o / F_e) / (1 - F_o / U_o), \text{ as derived earlier.} \end{aligned}$$

In deriving the equation for telescope magnification, it was assumed that overall magnification or telescope magnification was the relative magnification of the objective times the relative magnification of the eyepiece. It was also assumed that the angular subtense of the object at the unaided eye was essentially the same as the angular subtense of the object would be if the eye entrance pupil was at the telescope objective instead of at the telescope exit pupil. As noted earlier, this assumption neglects the length of the telescope plus eye distance, both of which move the eye some distance behind the objective. This extra distance, the actual distance of the eye behind the objective, is approximately the primary image distance V_o plus eyepiece focal length F_e plus exit pupil distance V_e plus an allowance for the thickness of the optical elements. The sum of these distances slightly exceeds telescope length. For distant objects, this additional distance is negligible compared to object distance. However, for near objects, it is not trivial. As object distance U_o decreases, due to the length of the telescope, the angular subtense of objects at an unaided eye in the same location as the telescope exit pupil, but with the telescope not present, becomes increasingly less than the object's angular subtense at the objective.

Taking into account the extra distance discussed above, the eye is a distance of approximately $U_o + V_o + F_e + V'_e$ from the object. At this distance, the object of height H_o will subtend, at the unaided eye, an

$$(\text{Angle A}) \approx \tan A = H_o / (U_o + V_o + F_e + V'_e).$$

Earlier, in deriving equation 13 for telescope magnification by a second method, it was shown that, at the eye, the displayed virtual image subtended an

(Angle B) $\approx \tan B = H_o(V_o/U_o)/U_e$. For a collimated eyepiece, as noted earlier, the primary image distance from the eyepiece is

$U_e = F_e$, so that $\tan B = H_o(V_o/U_o)/U_e = H_o V_o / F_e U_o$. The apparent angular magnification M_a of the telescope may be defined, as shown in Fig. 7, as $M_a = (\text{Angle B}) / (\text{Angle A})$, or approximately the ratio of the tangents, i.e., $\tan B / \tan A$. Substituting, from above, $\tan B = H_o V_o / F_e U_o$, and $\tan A = H_o / (U_o + V_o + F_e + V'_e)$,

$$M_a = (H_o V_o / F_e U_o) / [H_o / (U_o + V_o + F_e + V'_e)]$$

$$M_a = (V_o / F_e U_o) (U_o + V_o + F_e + V'_e)$$

$$= V_o / F_e + V_o^2 / F_e U_o + V_o / U_o + (V_o / F_e U_o) V'_e. \text{ From equation 16, derived in Appendix 2 and on page 41, } V'_e = F_e + F_e^2 / V_o.$$

Substituting this value of V'_e in the equation for M_a ,

$$M_a = V_o / F_e + V_o^2 / F_e U_o + V_o / U_o + (V_o / F_e U_o) (F_e + F_e^2 / V_o)$$

$$= V_o / F_e + V_o^2 / F_e U_o + V_o / U_o + V_o / U_o + F_e / U_o.$$

From equation 2, $1/U_o + 1/V_o = 1/F_o$, $V_o = F_o U_o / (U_o - F_o)$.

Substituting this value of V_o in the equation for M_a ,

$$M_a = [F_o U_o / (U_o - F_o)] / F_e + [F_o U_o / (U_o - F_o)]^2 / F_e U_o +$$

$$2 F_o U_o / (U_o - F_o) U_o + F_e / U_o$$

$$= (F_o / F_e) / [(U_o - F_o) / U_o] + F_o^2 U_o / (U_o - F_o)^2 F_e + 2 F_o / (U_o - F_o) + F_e / U_o$$

$$= (F_o / F_e) / (1 - F_o / U_o) + F_o^2 U_o / [(U_o - F_o)^2 / U_o] F_e +$$

$$2 F_o / (U_o - F_o) + F_e / U_o$$

$$= (F_o / F_e) / (1 - F_o / U_o) + F_o^2 / [(U_o - F_o) (U_o - F_o) / U_o] F_e +$$

$$2 F_o / (U_o - F_o) + F_e / U_o$$

$$= (F_o / F_e) / (1 - F_o / U_o) + (F_o^2 / F_e) / (1 - F_o / U_o) (U_o - F_o) +$$

$$2 F_o / (U_o - F_o) + F_e / U_o. \text{ Appending a negative algebraic sign to the value of } M_a \text{ because the virtual display image is on the same side of the eyepiece as the object (here, the primary image), the equation becomes}$$

$$\text{Eqn. 15. } M_a = - [(F_o / F_e) / (1 - F_o / U_o) + (F_o^2 / F_e) / (1 - F_o / U_o) (U_o - F_o) + 2 F_o / (U_o - F_o) + F_e / U_o]$$

Apparent angular magnification.

In this equation, note that the first term, $-(F_o / F_e) / (1 - F_o / U_o)$, is the right side of equation 14. The additional terms are due to the length of the telescope. In deriving equation 15, angle A was smaller, hence $M_a = (\text{Angle B}) / (\text{Angle A})$ was larger. Clearly, apparent angular magnification M_a is increasingly larger than angular magnification M as object distance U_o decreases. For objects at optical infinity, equations 14 and 15 both reduce to $-(F_o / F_e) (1 - F_o / U_o) = -F_o / F_e$, as expected.

A numerical example illustrates how much apparent angular magnification M_a can differ from angular magnification M at small

object distances. Suppose that the objective of a telescope is ten times the focal length F_e of the eyepiece. When used to view an object that is 1.3 focal lengths of the objective in front of the objective. Here, $F_o = 10F_e$, and $U_o = 1.3F_o$. By equation 14, angular magnification $M = - (F_o/F_e)/(1 - F_o/U_o)$. Substituting the F_o and F_e equivalents from above,

$$M = - (F_o/.1F_o)/(1 - F_o/1.3F_o) = - 43.333X \text{ or } -43.333 \text{ times.}$$

By equation 15, apparent angular magnification

$$M_a = [(F_o/F_e)/(1 - F_o/U_o) + (F_o^2/F_e)/(1 - F_o/U_o)(U_o - F_o) + 2F_e/(U_o - F_o) + F_e/U_o]$$

$$= - [(F_o/.1F_o)/(1 - F_o/1.3F_o) + (F_o^2/.1F_o)/(1 - F_o/1.3F_o)(1.3F_o - F_o) + 2(.1F_o)/(1.3F_o - F_o) + .1F_o/1.3F_o]$$

$$= - [10/(1 - 1/1.3) + 10/(1 - 1/1.3)(.3) + 2/.3 + .1/1.3]$$

$$= - [43.333 + 144.444 + .667 + .0769] = - 194.5216 \text{ times.}$$

In this example, apparent angular magnification M_a is larger than angular magnification M by $194.52/43.333 = 4.49$ times. It is also interesting that, for an object at optical infinity, by equation 14, angular magnification

$$M = - (F_o/F_e)/(1 - F_o/U_o) = - (F_o/.1F_o)(1 - 0) = - 1/.1 = -10 \text{ times.}$$

In the example, for an object at a distance of 1.3 focal lengths of the objective, angular magnification M was about 43, while it would be 10 for an object at optical infinity.

E. The Entrance Pupil

The entrance pupil of a compound magnifier, such as a microscope or a telescope, is the virtual image in object space of the optical system aperture stop formed by the optical elements preceding the aperture stop. The entrance pupil diameter D_o is the clear or effective light-gathering aperture of the objective. For telescopes, the entrance pupil is ordinarily both the aperture stop and the entrance pupil. For most objectives, the entrance pupil is at, within, or close to the objective. A diaphragm, or the opening in a lens mount, functions as the aperture stop. The aperture stop is uniformly filled with light from the entrance pupil, light originating at the object or viewed scene.

Most of the light impinging upon the entrance pupil is transmitted to the aperture stop, hence to the primary image projected by the objective. Very little light is lost by reflections from optical surfaces and optics supports and housings. Any unwanted light mixed into the display image reduces display contrast. To avoid scattering unwanted light into the display image from parts of the scene that will not appear in the displayed image, a lens hood and internal stops and baffles are used. Low-reflection black coatings are used on lens edges and on optics. Reflections from optical elements are minimized by coating surfaces with very thin anti-reflection films.

F. The Exit Pupil

The aperture in the aperture stop presents to the eyepiece an evenly-illuminated disc of light. This disc is an object for the eyepiece, which projects or brings to a focus an image of it as a second and smaller real image which is also a uniformly-illuminated disc of light, the exit pupil. The exit pupil is thus the image of the aperture stop. The eyepiece forms or projects two images: the exit pupil, which is a real image of the aperture stop, and a second image, the display image, which is a virtual image formed from the primary image. The observer can see only the display image.

To visualize the formation of the exit pupil of a compound optical system, examine the example illustrated in figure 12. In the figure, a vertical line is drawn down the center of the entrance pupil. Five points on the line are sources of light, points of origin for light rays going to the eyepiece and on to and past the exit pupil. Light from any point on the entrance pupil line goes to a corresponding point on the exit pupil line. The exit pupil is the real inverted and reversed image of the entrance pupil. Imagine that figure 12 is rotated about the optical axis of the system, or that many lines in the entrance pupil with different orientations are used. A three-dimensional solid beam of light is thereby generated that converges to the exit pupil and diverges past the exit pupil, forming a sort of truncated double cone of light that contains all of the light available from the objective. The maximum amount of light enters the eye when the exit pupil and the eye entrance pupil coincide.

When the exit pupil of the viewing instrument and the entrance pupil of the eye coincide and are of the same size, neglecting small light losses from absorption in optical elements and reflection from the surfaces of the optics, retinal image illuminance in the observer's eye is the same as it would be with the unaided eye. That is, what the observer sees appears to have the same luminance with and without the instrument. When the instrument's exit pupil is larger than the entrance pupil of the eye, this is also true. When the exit pupil is smaller than the eye's entrance pupil, retinal illuminance is less with the instrument than without it. For most compound magnifiers, the exit pupil is a circular disc, since entrance pupils for most objectives are circular discs. Since the area of a circular disc with a diameter D is $(\pi/4)D^2$, retinal illuminance varies directly as the square of exit pupil diameter up to that of the entrance pupil, beyond which no further increase in retinal illuminance occurs.

When the entrance pupil of the eye is partially out of the exit pupil, retinal illuminance is decreased relative to when it is entirely within the exit pupil. To insure that no part of the eye pupil ever gets out of the instrument's exit pupil, it is sometimes necessary to have exit pupils that are appreciably larger than the entrance pupil of the eye. For example, the exit pupil of a helmet-mounted display for airborne use has to be much

larger than the eye pupil so that, despite acceleration, bumping, turbulence and rapid head movements, the eye pupil remains wholly within the exit pupil.

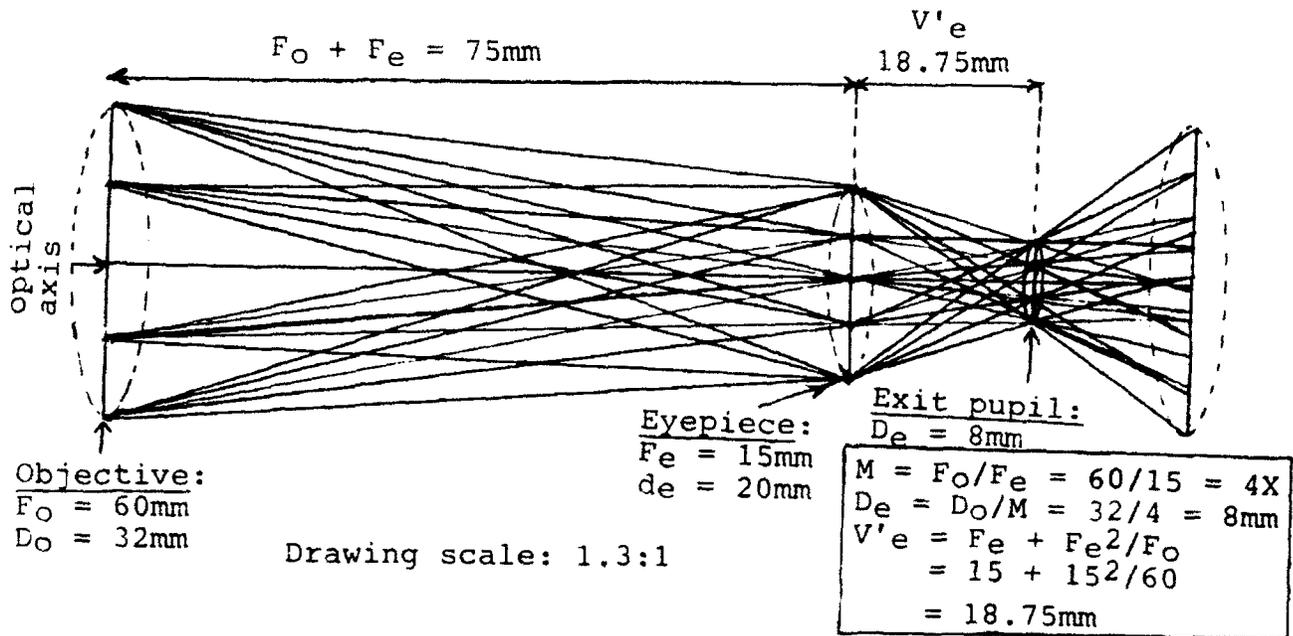


Fig. 12. Exit pupil distance and diameter for a telescope focused on a distant object, illustrating the truncated cone nature of the rays of light forming the exit pupil.

Part of the problem is that scalp motion and slipping of the helmet on the scalp from the conditions mentioned above can move the exit pupil relative to the eye, and conditions of use may not permit repositioning of the helmet on the head: the user's hands may be occupied with critical tasks. Another reason for a large exit pupil is that the center of rotation of the eyeball is about 10 mm behind the entrance pupil of the eye. When the angular field of the eyepiece is large, rotation of the eye in its socket, to view off-center objects, may rotate the eye's entrance pupil partially, or even wholly, out of a small exit pupil.

G. Exit Pupil Distance

The distance from a reference point within an eyepiece to the exit pupil is the exit pupil distance V'_e . This reference point is the second nodal point of the eyepiece. Exit pupil distance V'_e is not the same as eye clearance for two reasons:

- (1) the reference point from which exit pupil distance is measured is within the eyepiece, not at its rearmost surface, and
- (2) the entrance pupil of the eye is not at the very front or vertex of the eye, but a short distance, about 3mm, behind the vertex.

Optically, the distance is about $2 \frac{1}{4}$ mm, due to the

index of refraction of the corneal fluid, rather than the physical distance of 3mm. Thus, part of the exit pupil distance is within the eye and part of it is within the eyepiece. When nothing, such as a beamsplitter or combiner, is between the eye and the eyepiece, exit pupil distance, minus the parts of this distance within the eyepiece and within the eye, is eye clearance. For systems that use a combiner, such as a beamsplitter, there must be enough room between the eyepiece and the eye to provide an adequate distance between the combiner and the eye. For spectacle wearers, there must be adequate clearance between the spectacles and the image combiner.

In many, if not most, applications it is desirable that the virtual display image be located at optical infinity, i.e., that the image be collimated. This avoids apparent image motion with eye motion, which would lead to parallax problems, and it locates the image at a comfortable viewing distance. The display image is at optical infinity when the eyepiece front nodal point is at one eyepiece focal length from the real primary image formed by the objective, i.e., when $U_e = F_e$. As noted earlier, this follows from the lens equation for the eyepiece, given by equation (5) as $1/U_e + 1/V_e = 1/F_e$, since V_e is infinity and $1/V_e = 0$.

For a display image at optical infinity, then, the primary image projected by the objective is a distance V_o behind the second nodal point of the objective, and the eyepiece is a distance F_e behind the primary image. The total distance of the objective from the eyepiece is then $U_e = V_o + F_e$. For most objectives, particularly telescope objectives, the aperture stop of the system is essentially at the objective. For this case, the aperture stop is a distance $F_o + F_e$ from the front nodal point of the eyepiece. The aperture stop is the object for the eyepiece, which focuses or projects it to form the exit pupil, which is a real image. Incidentally, the primary image is also an object for the eyepiece, as noted earlier. The Gauss lens equation of the eyepiece, when it forms a real image, rather than a virtual image, is $1/U'_e + 1/V'_e = 1/F_e$. The algebraic sign of $1/V'_e$ is positive, since the image is a real image located behind the eyepiece. From this equation, $V'_e = F_e U'_e / (U'_e - F_e)$. From above, $U'_e = V_o + F_e$. Substituting for U'_e ,

$$V'_e = \frac{F_e U'_e}{U'_e - F_e}$$

$$= \frac{F_e (V_o + F_e)}{(V_o + F_e) - F_e}$$

$$= \frac{F_e (V_o + F_e)}{V_o} =$$

Eqn. 16. $V'_e = F_e + F_e^2/V_o$ Exit pupil distance.

As primary image distance V_o decreases, exit pupil distance V'_e increases. It also increases when eyepiece focal length F_e increases. The objective lens, when projecting the real primary image, obeys the Gauss lens equation $1/U_o + 1/V_o = 1/F_o$, given by equation 2, from which $1/V_o = 1/F_o - 1/U_o$. Substituting this value of $1/V_o$ for the V_o in equation 16,

Eqn. 17. $V'_e = F_e + F_e^2(1/F_o - 1/U_o)$ Exit pupil distance.

From this equation, as objective focal length F_o decreases, exit pupil distance increases. Also, when object distance U_o decreases, exit pupil distance increases.

From equations 16 and 17, it may be concluded that exit pupil distance increases when:

- (1) the focal length F_o of the objective decreases
- (2) eyepiece focal F_e length increases
- (3) object distance U_o decreases
- (4) primary image distance V_o decreases.

To rephrase the above, to increase exit pupil distance, which increases eye clearance, use an objective with a shorter focal length, or an eyepiece of longer focal length, or view the object or scene from a shorter object distance. However, objectives with shorter focal lengths result in reduced system magnification, as do eyepieces of longer focal length. Using longer focal length objectives and eyepieces, of course, could increase exit pupil distance without changing exit pupil distance, but at a cost of a larger system weight and volume. Decreasing the distance to the object may not be feasible.

H. Exit Pupil Diameter

Since most exit pupils are round discs, a quoted exit pupil size usually refers to the diameter of the exit pupil. For example, a helmet-mounted display with a 15mm exit pupil has an exit pupil that is 15mm in diameter. Since the exit pupil of an instrument is the image of the entrance pupil projected by the eyepiece, exit pupil diameter is directly proportional to the diameter of the entrance pupil. Objectives with larger effective apertures provide larger exit pupils. However, an objective with a larger diameter is bigger and heavier. Also, if diameter is increased without increasing focal length, the relative aperture or f-number, focal length divided by diameter, F_o/D_o , is numerically smaller: the objective is "faster," possibly requiring a more complex structure, such as more optical elements or one or more aspheric surfaces on optical elements, for adequate image quality.

A collimated eyepiece, as noted earlier, is an eyepiece located at its own focal length F_e behind the primary image projected by the objective. The primary image is a distance V_o behind the objective's second nodal point. The objective is thus a distance $V_o + F_e$ from the eyepiece. This is shown in Figure 13. In the figure, D_e is exit pupil diameter, D_o is aperture stop diameter, V'_e is exit pupil distance, and U'_e is the distance of the aperture stop from the eyepiece. From the figure, from the proportionality of corresponding parts of similar triangles, $(D_e/2)/(D_o/2) = D_e/D_o = V'_e/U'_e$. From above, $U'_e = V_o + F_e$. Substituting this for U'_e , $D_e/D_o = V'_e/U'_e = V'_e/(V_o + F_e)$. From equation 16, $V'_e = F_e + F_e^2/V_o$, which may be written as $V'_e = F_e(1 + F_e/V_o)$

$$\begin{aligned}
&= F_e(V_o + F_e)/V_o \\
&= (F_e/V_o)(V_o + F_e). \text{ Substituting this value for } V'_e, \text{ from} \\
&\text{above,} \\
D_e/D_o &= V'_e/(V_o + F_e) \\
&= (F_e/V_o)(V_o + F_e)/(V_o + F_e) \\
&= F_e/V_o. \text{ From equation 13, angular magnification} \\
&M = -V_o/F_e, \text{ from which } F_e/V_o = -1/M. \text{ Substituting this} \\
&\text{value of } F_e/V_o, \\
D_e/D_o &= F_e/V_o = -1/M. \text{ From this equation, ignoring the minus sign,}
\end{aligned}$$

Eqn. 18. $D_e = D_o/M$ Exit pupil diameter, display image at optical infinity.

See Appendix 2 for an alternative derivation of equations 16 and 18.

For some objectives, such as most telescope objectives, the aperture stop is at the objective, so that the entrance pupil and the aperture stop of the system are essentially of the same size and are at the same location. Equation 18 is based on the assumption that the aperture stop is at the entrance pupil. A larger exit pupil requires a larger entrance pupil, i.e., a larger objective. A larger objective is also heavier, and, if of the same focal length, also "faster."

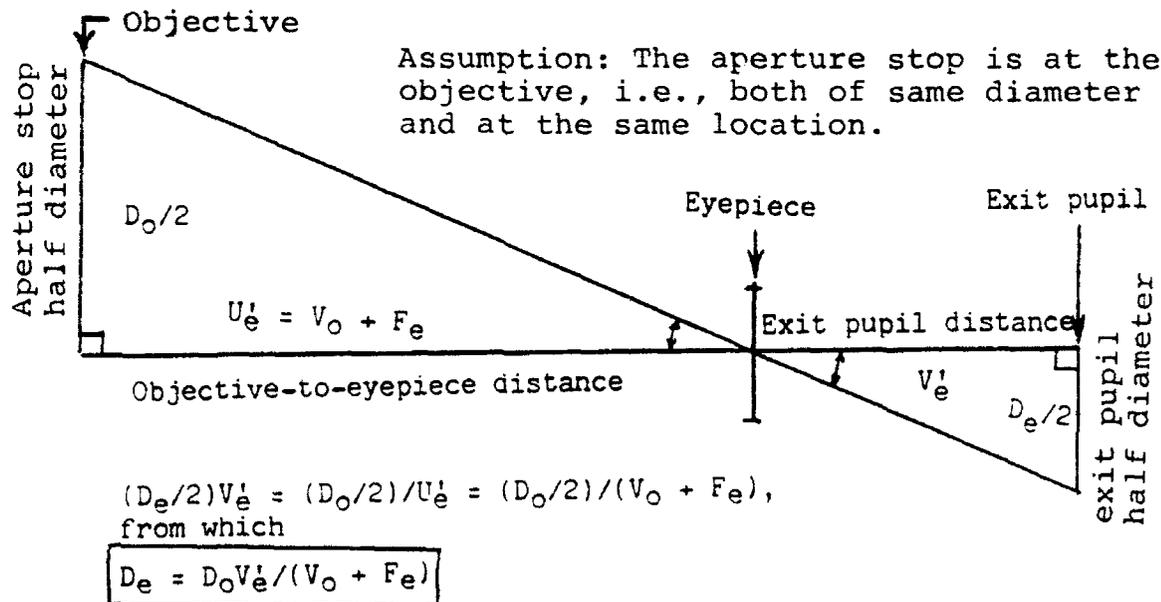


Fig. 13. Optical geometry for deriving an equation for the diameter of the exit pupil.

By equation 14, $M = -(F_o/F_e)/(1 - F_o/U_o)$. Ignoring the minus sign, and substituting in equation 18,

$$\begin{aligned}
D_e &= D_o/M \\
&= D_o / [(F_o/F_e)/(1 - F_o/U_o)] \\
&= D_o (1 - F_o/U_o) / (F_o/F_e) \\
&= F_e (D_o/F_o) (1 - F_o/U_o). \text{ Dividing by } F_o,
\end{aligned}$$

Eqn. 19. $D_e = D_o F_e (1/F_o - 1/U_o)$ Exit pupil diameter, display image at optical infinity.

From the lens equation for the objective, $1/U_o + 1/V_o = 1/F_o$, $1/F_o - 1/U_o = 1/V_o$. Using this in equation 19, the equation becomes $D_e = D_o F_e / V_o$. From this equation, and from equation 19, exit pupil diameter increases when:

- (1) entrance pupil diameter is larger,
- (2) eyepiece focal length is longer,
- (3) objective focal length is shorter,
- (4) primary image distance V_o is less, and
- (5) object distance U_o is greater.

However, equation 14, $M = - (F_o/F_e)(1 - F_o/U_o)$, indicates that any change in the focal length of the eyepiece or of the objective, or in object distance, changes angular magnification.

Equation 19 may be written as

$$\begin{aligned} D_e &= D_o F_e (1/F_o - 1/U_o) \\ &= F_e (D_o/F_o) - D_o F_e / U_o \\ &= F_e / (F_o/D_o) - D_o F_e / U_o. \end{aligned}$$

By definition, relative aperture or f-number = (Focal length)/(Aperture) = F_o/D_o , so that, substituting f-number for F_o/D_o ,

Eqn.20. $D_e = F_e / (\text{f-number}) - D_o F_e / U_o$. Exit pupil diameter, image at optical infinity, f-number = F_o/D_o .

From equation 20, as object distance decreases, D_e decreases. This is due to increased magnification. At great distances, $D_o F_e / U_o$ is essentially zero, and equation 20 becomes

Eqn. 21. $D_e = F_e / (\text{f-number})$. Exit pupil diameter, distant objects, display image at optical infinity, f-number = F_o/D_o .

From equation 21 it is clear that all telescopes used for viewing distant objects, provided that they all have the same relative aperture F_o/D_o or f-number, will provide the same exit pupil diameters when used with the same focal length eyepieces. However, when their objective focal lengths differ, magnifications and fields-of-view will differ.

Earlier, Figure 12 was used to illustrate the truncated double cone nature of the light rays coming out of the eyepiece to form the exit pupil. In that figure, the eyepiece was shown at a distance F_e from the primary image, to provide a display image at optical infinity. Also, in the figure, primary image distance was shown as $V_o = F_o$, since the object was very distant.

As an example of using the above equations, suppose that a telescope used to view distant objects has an objective with a clear or effective aperture $D_o = 50\text{mm}$, and a focal length $F_o = 200\text{mm}$. Let the eyepiece focal length be $F_e = 30\text{mm}$. For this example, equation 14 gives telescope magnification as $M = (F_o/F_e)/(1 - F_o/U_o)$. For distant objects, this equation becomes $M = F_o/F_e = 200/30 = 6.67\text{X}$ or 6.67 times. By equation 18, exit pupil diameter $D_e = D_o/M = 50/6.67 = 7.50\text{mm}$. By equation 2, the

Gauss lens equation, $1/U_o + 1/V_o = 1/F_o$, for very large or infinite object distance, $V_o = F_o$, and $V_o = F_o = 200\text{mm}$. By equation 16, exit pupil distance

$$V'_e = F_e + F_e^2/V_o$$

$$= 30 + 30^2/200 = 34.5\text{mm}.$$

For this example, then, the telescope magnifies 6.67 times, has an exit pupil diameter of 7.5mm, and an exit pupil distance of 34.5mm. This exit pupil distance is distance from the eyepiece second nodal point, which is usually within the eyepiece. It is not eye clearance.

I. Field of View

In the section on magnifiers, magnification was defined as angle B subtended by the eye looking at the displayed image divided by angle A subtended by the object at the eye. When the displayed image covers the diameter of the field of view, the image subtense is the apparent field of view, and the corresponding object subtense is the objective field of view, i.e., the external or instrument field of view. When magnification increases, the apparent field of view, i.e., eyepiece field of view, will include less of the external scene, i.e., the angular subtense of the external view, which is the instrument field of view, decreases. Here, (Apparent field of view)/(Instrument field of view) = B/A, from which

Eqn. 22. $A = B/M$ Instrument or objective field of view. B is apparent field of view.

Note, from equation 14, $M = - (F_o/F_e)/(1 - F_o/U_o)$, that, as object distance U_o decreases, F_o/U_o increases, so that $(1 - F_o/U_o)$ increases, i.e., as object distance U_o decreases, magnification increases. Since, from equation 22, objective or instrument field of view is $A = B/M$, as object distance decreases, the angular field of view of the instrument also decreases.

J. Apparent Field of View

In the following discussion of apparent field of view, keep in mind that, when apparent field of view increases, the overall or system field of view also increases. To simplify visualization and drawing without changing the equations that will be derived, assume that the eyepiece is very thin from front to back. With this stipulation, Figure 14 shows the apparent field of view as determined by the eyepiece clear or effective aperture and the eye distance. From the figure, $\tan B_e = (d_e/2)/V'_e = d_e/2V'_e$. Taking the inverse or arc tangent of both sides of this equation,

Eqn. 23. $B_e = 2 \text{ ArcTan } (d_e/2V'_e)$ Apparent angular field of view.

From this equation, field of view is increased when the clear or effective aperture d_e of the eyepiece is increased and when eye distance V'_e is decreased. A larger eyepiece aperture, or an aperture closer to the eye, subtends a larger angle at the eye. To determine how eye distance V'_e may be decreased to increase the apparent field of view, examine Equation 16, $V'_e = F_e + F_e^2/V_o$. From this equation, V'_e decreases when eyepiece focal length F_e

decreases and when the primary image distance V_0 increases. An eyepiece of the same effective aperture closer to the eye subtends a larger angle: the apparent field of view is larger. Increased primary image distance V_0 increases field of view because, with a larger V_0 , the distance $V_0 + F_e$ of the objective from the eyepiece is greater, so that the eyepiece projects the entrance pupil of the objective a shorter distance behind the eyepiece. This projected image is the exit pupil and is where the eye entrance pupil should be.

To determine how primary image distance V_0 may be increased to increase the field of view, examine Equation (2), $1/U_0 + 1/V_0 = 1/F_0$. This equation may be rearranged to yield $V_0 = 1/(1/F_0 - 1/U_0)$. Clearly, from this equation, V_0 may be increased to increase field of view by increasing the focal length F_0 of the objective, or by decreasing object distance U_0 . Either of these changes also will cause overall or system magnification to change, along with the size of the display exit pupil. As noted elsewhere in the present report, changing object distance has little effect upon V_0 when distance to objects is large, but can be very effective when the viewed object is only a few focal lengths of the objective away. The effect upon field of view of objective focal length and object distance may also be determined from inspection of Equation 17 which includes the same substitution for V_0 .

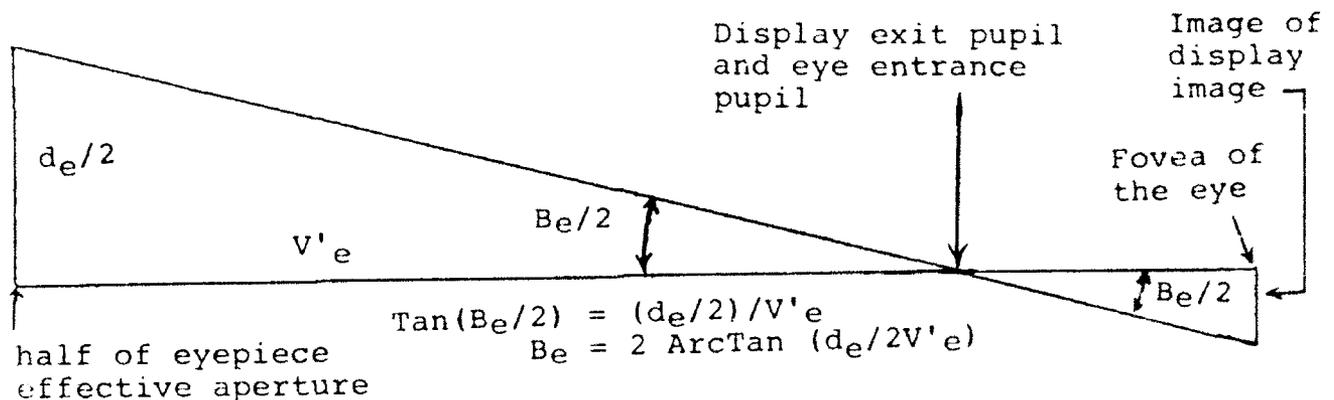


Fig. 14. The relationship of effective eyepiece aperture to apparent field of view and exit pupil distance.

Some virtual image optical systems, such as microscopes and helmet-mounted displays, are used with the viewed object (a microscope slide or a miniature CRT) physically close to the objective. With such optical systems, the total or system angular magnification is the product of the linear magnification of the objective and the angular magnification of the eyepiece. By proper choice of the object distance U_0 , the objective can supply a large amount of linear magnification. The eyepiece, then, has to supply little angular magnification, and can thus have a focal length long enough to provide the required eye distance. With microscopes, a long eye distance is more comfortable to use, and

may be required by spectacle users so that they can view the entire field of view without hitting their glasses on the eyepiece. For helmet-mounted displays, a long eye distance is required for safety reasons and to permit locating the eyepiece out of the field of view of the user by means of a beamsplitter. However, to obtain a given apparent field of view requires a corresponding clear or effective eyepiece aperture and focal length. This is why helmet-mounted displays that have very wide fields of view have huge eyepieces, giving them a "bug eye" appearance.

To summarize the above points made about apparent field of view, the apparent field of view is increased by using an eyepiece with a larger clear or effective aperture, by using an eyepiece with a shorter focal length and large effective aperture, by using an objective with a longer focal length, and by any change in the optical system that reduces eye distance. However, some changes will alter eye distance, system magnification, and exit pupil diameter. Optical system design involves compromises between many characteristics to attain an acceptable balance of characteristics.

K. Eyepiece Diameter

Eyepieces of different optical devices vary greatly in physical size. Eyepieces that have short focal lengths and small apparent fields of view are physically small. Conversely, eyepieces with large fields of view and long focal lengths are large. Eyepieces for use by one eye are sometimes appreciably less than an inch in both length and diameter and are very light, while some eyepieces are over three inches in both length and diameter and weigh several pounds. An eyepiece large enough to be used simultaneously by both eyes will, obviously, be quite large.

The apparent field of view of an optical instrument is the angle subtended at the eye by the field of view. When the field of view is circular, as almost all are, the apparent field of view is the angle subtended by the diameter of the displayed field. It is also the angular subtense at the eye of the diameter d_e of the effective aperture of the eyepiece. The wider the field of view and the greater the distance of the eye from the eyepiece, the greater the required eyepiece diameter. Figure 14 illustrates the relationship of apparent field of view B_e , exit pupil diameter D_e , and effective eyepiece aperture d_e . Keep in mind that, for reasons discussed elsewhere in the present paper, exit pupil distance V'_e is not the same as eye clearance.

From Equation 23, $B_e = 2 \text{ ArcTan}(d_e/2V'_e)$, solving for d_e ,
 Eqn. 24. $d_e = 2V'_e \text{ Tan}(B_e/2)$ Effective eyepiece optical diameter
 (effective eyepiece aperture).

When V'_e in Equation 24 is replaced by V'_e given by equation 17 as $V'_e = F_e + F_e^2(1/F_o - 1/U_o) \text{ Tan}(B_e/2)$, Equation 24 becomes

Eqn. 25 $d_e = 2[F_e + F_e^2(1/F_o - 1/U_o)] \text{ Tan}(B_e/2)$ Eyepiece
 effective aperture

From examination of the relationships of the variables of Equations 24 and 25, it is apparent that the required effective or clear aperture or diameter of the eyepiece of a virtual image optical display instrument increases with increase in:

1. eyepiece field of view, B_e ,
2. eyepiece focal length, F_e ,
3. exit pupil distance, V'_e ,
4. objective focal length, F_o .

The above equations also show that required eyepiece diameter decreases with increase in object distance U_o . However, changing any one of these variables changes other variables, or may require changes in other variables to obtain a desired magnification, etc.

For some compound magnifier optical systems, such as helmet-mounted displays, the size and weight of the system is critical: saving only a few ounces of weight or a few cubic inches of volume can make an unacceptable system acceptable. However, minimizing the weight of a system that contains a compound optical system is difficult. For example, for airborne use of a helmet-mounted display by an aircraft pilot, the exit pupil diameter D_e must be large, e.g., 12-20mm, so that the eye entrance pupil always stays in the exit pupil of the system despite high accelerations. The exit pupil distance V'_e must be large enough to accommodate both a beamsplitter, usually called a combiner, and spectacles, so that there is adequate clearance between the eye and the device or the user's spectacles and the device. This is required for safety during bumping or other impacts. In addition, the field of view must be large to facilitate awareness of the external environment, commonly called situational awareness. A large exit pupil requires a large objective, while a large exit pupil distance and a wide field-of-view both require a large eyepiece. Objectives and eyepieces with these characteristics, if composed of glass elements, would weigh more than the allowable combined weight of both the helmet and the optics plus other viewing system components, such as the electronics, optics supports and mounts, the combiner and the CRT.

To save weight and volume, glass elements are sometimes replaced in part, or even entirely, by a system of concave aspheric mirrors. When a display with only one hue is permissible, and the light used in the optical system has a narrow bandwidth, as is the case for some CRT phosphors, some of the refractive (glass or plastic) elements may be replaced with holographic optical elements. Holographic optical elements are extremely thin layers, and work by diffraction rather than by refraction, as do ordinary glass or plastic lenses. Because they are very thin and can be supported on a light-weight substrate, holographic optical elements are very light.

TABLE 1

SUMMARY OF NUMBERED EQUATIONS

1. $L.M._o = H_1/H_o$ Linear magnification by the objective.
2. $1/U_o + 1/V_o = 1/F_o$ Lens equation for the objective.
3. $L.M._o = - (V_o - F_o)/F_o$ Linear magnification by the objective.
4. $L.M._o = - F_o/(U_o - F_o)$ Linear magnification by the objective.
5. $1/U_e - 1/V_e = 1/F_e$ Eyepiece lens equation when projecting a virtual image.
6. $M_e = 250/F_e$ Eyepiece relative magnification, image at optical infinity.
7. $M_e = 250/V_e + 250/F_e$ Eyepiece relative magnification, image distance V_e .
8. $M_e = 1 + 250/F_e$ Eyepiece relative magnification, image distance 250mm.
9. $M = - 250(V_o - F_o)/F_oF_e$ Microscope magnification, image at optical infinity.
10. $M = - 180 \times 250/F_oF_e$ Microscope magnification, optical tube length 180mm, image at optical infinity.
11. $F_c = F_oF_e/(F_o + F_e - d)$ Focal length of two lenses, F_o the first lens, F_e the second, lens separation distance d .
12. $M_o = - V_o/250$ Relative magnification of a telescope objective for distant objects.
13. $M = - V_o/F_e$ Telescope angular magnification.
14. $M = - (F_o/F_e)/(1 - F_o/U_o)$ Telescope angular magnification, image at optical infinity.
15. $M_a = - [(F_o/F_e)/(1 - F_o/U_o) + F_o^2/F_e(1 - F_o/U_o)(U_o - F_o) + 2F_o/(U_o - F_o) + F_e/U_o]$ Telescope apparent angular magnification
16. $V_e = F_e + F_e^2/V_e$ Telescope exit pupil distance, image at optical infinity.

17. $V'_e = F_e + F_e^2(1/F_o + 1/U_o)$ Telescope exit pupil distance, image at optical infinity.
18. $D_e = D_o/M$ Exit pupil diameter.
19. $D_e = D_o F_e (1/F_o - 1/U_o)$ Exit pupil diameter, image at optical infinity.
20. $D_e = (F_e/f\text{-number}) - D_o F_e / U_o$ Exit pupil diameter, image at optical infinity. $f\text{-number} = F_o/D_o$.
21. $D_e = F_e/(f\text{-number})$ Exit pupil diameter, object and image at optical infinity. $f\text{-number}$ is F_o/D_o .
22. $A = B/M$ Instrument or external or objective field of view.
23. $B_e = 2 \text{ ArcTan}(d_e/2V'_e)$ Apparent angular field of view
24. $d_e = 2V'_e \text{ Tan}(B_e/2)$ Eyepiece effective aperture.
25. $d_e = 2F_e + 2F_e^2(1/F_o - 1/U_o) \text{ Tan}(B_e/2)$ Effective eyepiece aperture.

TABLE 2

SUMMARY OF SYMBOLS USED IN NUMBERED EQUATIONS

- A The instrument or objective or external field of view.
- B The image angular subtense at the eye.
- B_e The apparent angular field of view
- d The separation or distance between two lenses.
- d_e The effective aperture of an eyepiece: the effective optical diameter.
- D_e The exit pupil diameter.
- D_o The entrance pupil diameter of an objective, the effective clear diameter.
- F Focal length.
- F_c The combined focal length of two lenses.
- F_e Eyepiece focal length.
- F-number The relative aperture of an objective, F_o/D_o .
- F_o Focal length of an objective.
- H_1 The linear height of the primary image projected by the objective.
- H_o The linear height above the optical axis of a small object.
- H_2 The linear height above the optical axis of the displayed image.
- $L.M._e$ The linear magnification of the eyepiece.
- $L.M._o$ The linear magnification of the objective.
- M relative magnification, also called relative angular magnification.
- M_a Apparent angular magnification.
- M_e Eyepiece relative magnification or relative angular magnification.
- M_o Relative magnification or relative angular magnification by the objective.

- U_e The distance in front of an eyepiece of the primary image projected by the objective. This image is the object for the eyepiece.
- U'_e The optical distance from the eyepiece to the objective.
- U_o Object distance. Distance of the object or scene from the objective.
- V_e Display image distance. Distance of the displayed image from the rear nodal point of the eyepiece.
- V'_e Exit pupil distance. Distance from the eyepiece rear nodal point to the exit pupil of the instrument.
- V_o Primary image distance. The distance from the rear nodal point of the objective to the primary image projected by the objective.

TABLE 3

EQUATIONS FOR COMPOUND MAGNIFIERS

(a) Primary image distance behind the objective

$$V_o = F_o U_o / (U_o - F_o)$$

(b) Magnification by the objective

1. Linear Magnification

$$L.M._o = - V_o / U_o$$

$$L.M._o = - F_o / (U_o - F_o)$$

$$L.M._o = - (V_o - F_o) / F_o$$

2. Relative Magnification

$$M_o = - V_o / 250 \quad V_o \text{ in millimeters}$$

$$M_o = - V_o / 10 \quad V_o \text{ in inches}$$

(c) Relative magnification by the eyepiece

1. Collimated image at optical infinity

$$M_e = - 250 / F_e \quad F_e \text{ in millimeters}$$

$$M_e = - 10 / F_e \quad F_e \text{ in inches}$$

2. Virtual display image at V_e

$$M_e = - 250 (1/F_e + 1/V_e) \text{ millimeter values}$$

$$M_e = - 10 (1/F_e + 1/V_e) \text{ inch values}$$

3. Virtual display image at 250mm or 10 inches

$$M_e = - (250/F_e + 1) \quad F_e \text{ in millimeters}$$

$$M_e = - (10/F_e + 1) \quad F_e \text{ in inches}$$

(d) Telescope magnification with a collimated display image

$$M = (\text{Image angular subtense with telescope}) / (\text{object angular subtense without telescope})$$

$$M = (\text{Apparent field of view}) / (\text{Instrument or external field of view}) = B/A$$

$$M = \text{Tan (Apparent field of view)} / \text{Tan (Instrument or external field of view)} = \text{Tan } B / \text{Tan } A$$

$$M = M_o M_e$$

$$M = D_o / d_e$$

$$M = - (F_o / F_e) / (1 - F_o / U_o)$$

$$M = F_o / F_e, \text{ Object at optical infinity}$$

$$M_a = - \left[\frac{(F_o / F_e) / (1 - F_o / U_o) + F_o^2 / (1 - F_o / U_o)}{(U_o - F_o) + 2F_o / (U_o - F_o) + F_e / U_o} \right]$$

(e) Exit pupil distance

$$V'_e = F_e + F_e^2 / V_o$$

$$V'_e = F_e + F_e^2 / (1/F_o - 1/U_o)$$

(f) Exit pupil diameter

$$D_e = D_o / M$$

$$D_e = F_e / (\text{f-number of objective})$$

$$D_e = F_e D_o / F_o$$

$$D_e = D_o (F_e / F_o) (1 - F_o / U_o)$$

(g) Instrument or external field of view

$$A = B/M \quad \text{Angle } A = (\text{Angle } B)/(\text{Magnification})$$

L Supplementary References

Recommended reading for this section for those with little or no knowledge of optics is the easy-reading classic introductory textbook on optics and optical instruments by Johnson (1960). It requires no college mathematics, and it covers basic optics from refraction and lenses to microscopes and telescopes. A second useful reference, also at an introductory level, is the textbook "Optical Man 3 and 2" prepared for a U.S. Navy training course (anonymous author, 1966). A textbook at a more advanced level covering both optical and mechanical aspects of optical instruments is the classic textbook by Jacobs (1932) on the fundamentals of optical engineering. A third classic optics textbook by Hardy and Perrin covers many aspects of optics. Its chapters on microscopes and telescopes are recommended reading. For those with some knowledge of optics, two widely-used excellent college-level textbooks are Hecht (1987) and O'Shea (1985).

9.0 USING A MAGNIFIER TO VARY OPTICAL DISTANCE

It is sometimes desirable to vary the optical distance to an object, an image or a display. Optical distance is the distance for which the eye, a camera, or a telescope must be focused to obtain a sharp image. When no optical device intervenes between the eye and an object or a display, optical distance is the physical distance between the eye and the display. When optics are not used, optical distance is varied by varying physical distance. When space is available, this may be an acceptable way to vary optical distance. However, for long distances, an object or a display must be very large to subtend an appreciable angle at the observer's eye. Thus, when either long distances to a display or an angularly-large display at a distance is required, varying display distance by varying physical distance may be unacceptable.

When varying physical distance is unacceptable, optical distance can be varied by optical means. When either an object or a real image provided by an objective is viewed through a magnifying glass, a magnifier or an eyepiece, the observer is presented with a virtual image display. The optical distance of the image from the observer's eye varies as the distance between the object and the magnifier varies. Any optical distance, from optical infinity to a few centimeters away, can be selected by varying this distance. Focussing a magnifier or an eyepiece is done by adjusting the distance between the magnifier and the object (or a primary image serving as an object) until the desired optical distance to the displayed virtual image is obtained.

How display image distance varies with the distance between a magnifier and an object is described by the Gauss lens equation, which relates object distance U , image distance V , and magnifier focal length F . The equation for virtual images, is $1/U - 1/V = 1/F$. The $1/V$ term, by convention, is negative, because the viewed object and the displayed virtual image are both on the same side of the magnifier. Solving this equation for image distance, $V = FU/(U - F)$. Dividing both the numerator and denominator of this fraction by U , $V = F/(F/U - 1)$. When $U = F$, $(F/U - 1) = 0$, and $V = F/0$, i.e., image distance is optical infinity. As object distance decreases from one lens focal length, F/U increases, so that $(F/U - 1)$ also increases, and $V = F/(F/U - 1)$ decreases. Thus, decreasing the object distance from one focal length, where the image is at optical infinity, moves the image closer to the lens.

As noted above, when $U = F$, i.e., when the object is one focal length away, the image is at optical infinity, and the image is said to be collimated. With a collimated image, movement of the eye laterally (side-to-side) is not accompanied by image motion, nor does the angular subtense of the image vary with eye movement along the lens axis, i.e., movement toward or away from the magnifier. However, at less than optical infinity distances to the image, lateral eye movement makes the image appear to

move, and axial movement changes the image subtense at the eye, as well as the optical distance between the eye and the image.

To examine variation of image distance with variation in object distance, and for generality, object distance U may be specified as n focal lengths of the magnifier, i.e., $U = nF$. Substituting nF for U in the above equation for image distance, $V = FU/(F - U) = F(nF)/(F - nF) = nF/(1 - n)$. Dividing both numerator and denominator by n , $V = F/(1/n - 1)$. For $n = 1$, image distance V is $1/0$, which is infinity, while V is less than optical infinity for n less than 1. For n less than 1, V is negative, indicating that the image is on the same side of the lens as the viewed object, i.e., the image is virtual. The equation $V = F/(1/n - 1)$ is plotted in Fig. 15 for object distances from .5 to 1 focal length and image distances V from 0 to 50 lens focal lengths, ignoring the minus sign.

Note, from the graph, that, since V is infinity for $n = 1$, i.e., when the viewed object is one focal length from the magnifier, the curve is asymptotic to the vertical line at $n = 1$. Because of this, to change image distance V from one large value to another large value involves only minute changes in object distance U . When image distance is many focal lengths, the curve is almost vertical and image distance changes by large amounts with small changes in object distance. Here, the viewed object is close to 1 lens focal length in front of the magnifier.

When object distance is one focal length ($n = 1$), image distance is optical infinity, as noted earlier. As an example of variation in image distance V with changes in object distance U , suppose that the object is moved from one focal length away ($U = F$) from the magnifier to $U = .9F$, i.e. object distance is decreased by 10% of magnifier focal length F . By the equation derived above, image distance from the lens is $V = F/(1/n - 1) = F/(1/.9 - 1) = 9F$. Note that the graph also yields $9F$. Moving the magnifier closer to the object by 10% of magnifier focal length from an initial distance of one focal length from the object brings the displayed image from optical infinity to only 9 focal lengths away from the magnifier. When n is down to .5, the image is one focal length in front of the magnifier.

A magnifier may be placed in front of a display, such as a CRT, to provide a desired optical distance to the image. In such a situation, it may be desirable to use both eyes with a large diameter magnifier with long focal length. In this case, a large CRT can be viewed. With a magnifier of long focal length, the eye may not be essentially at the magnifier, but at a distance D behind the magnifier. When D is not essentially zero, it must be kept in mind that the image distance V , given by the Gauss lens equation, is the optical distance of the displayed virtual image from the magnifier, not the distance of the image from the eye: The eye-to-image distance is $V + D$. When V is small, neglecting D in computing image distance from the eye can result in large errors. With short distances from eye-to-image,

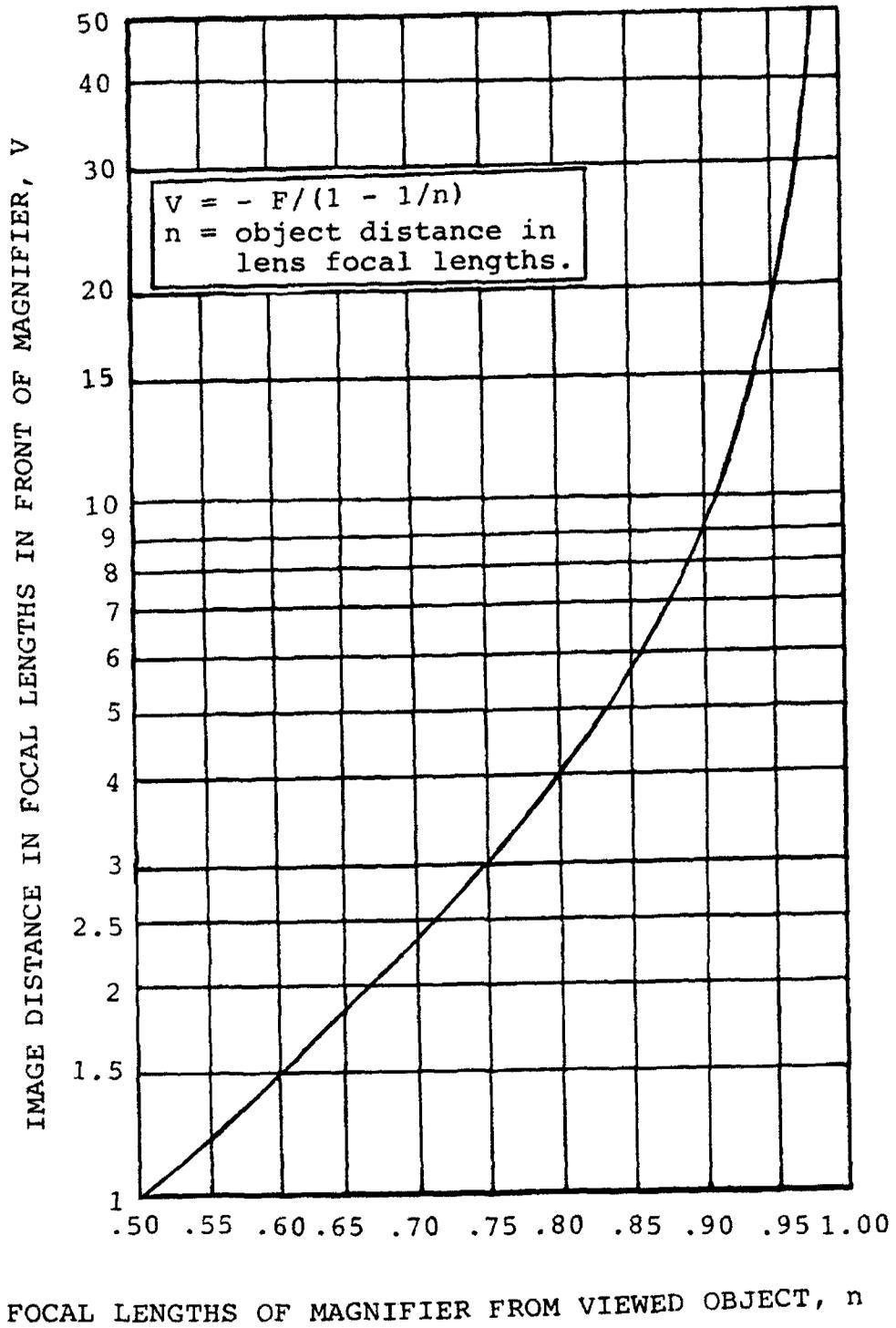


Fig. 15. Image distance with a magnifier as a function of the distance of the viewed object from the magnifier. Image distances are in magnifier focal lengths.

maintaining accurate eye-to-image distance ($D + V$) may require fixing or limiting eye distance from the magnifier by using a forehead rest, a bite board, a clear pane or glass, or some other method that prevents head motion.

When adjusting the distance U between an object (or a primary image serving as an object) and a magnifier or eyepiece to obtain a given image distance V to the virtual image on display, it is sometimes convenient to first adjust object distance U to yield an image at optical infinity, then move the lens or magnifier inward toward the object, or move the object closer to the lens. Suppose that the distance that the lens must be moved to move the virtual image in from infinity to the desired image distance V is X . The proper value of X is determined by calculation. This procedure is sometimes used because it may be easy, by using a collimator, to adjust magnifier position to obtain an image at optical infinity, then to change U by a calculated amount, while available equipment may be inadequate for accurate adjustment of image distances other than infinity.

As noted earlier, the magnifier is first positioned at one focal length F from the object to provide an image at optical infinity. After moving the lens a distance X toward the object, object distance U is then $U = F - X$. Since the object is closer than one focal length from the lens, the image will be a virtual image for which the Gauss lens equation is $1/U - 1/V = 1/F$. Solving the equation for object distance, $U = FV/(V + F)$. Replacing U with $U = F - X$, $F - X = FV/(V + F)$, from which $X = F - FV/(V + F) = (FV + F^2 - FV)/(V + F) = F^2/(V + F)$. Thus, to obtain a desired image distance V from the magnifier (not from the eye, for eye distance must be added in), the magnifier, initially at a position $U = F$, where it provides an image at optical infinity is moved toward the object by a distance $X = F^2/(V + F)$. For some purposes, it may be desirable to specify image distance in terms of magnifier focal lengths P , then $V = PF$. In this case, $X = F^2/(V + F) = F^2/(PF + F) = F/(P + 1)$.

For an example of using the $X = F^2/(V + F)$ equation, suppose that, in a laboratory study on the effect of display distance on performance, a large magnifier is used to view a small television display with both eyes. For this example, the magnifier is a lens with a focal length of 200mm and the eyes of the observer are to be a distance $D = 180$ mm behind the lens. The desired image distance from the observer's eyes of the displayed TV image is 500 meters = 500,000mm. Image distance V for the lens is thus $V = 500,000 - D = 500,000 - 180 = 499,820$ mm. Distance X of the lens from the infinity image position is then $X = F^2/(V + F) = 200^2/(499,820 + 200) = .0800$ mm = .00315 inch. This is a small distance.

In the above numerical example, the calculations used $F = 200$ mm. However, manufacturing tolerances, even for quality magnifiers, are unlikely to be better than plus or minus 2%. For a large single-element lens used as a magnifier, actual focal length may differ by 5% or more from marked or catalog values, which are often called nominal values. For example, suppose that the 200mm nominal focal

length lens in the above example was actually 2% longer in focal length than 200mm. Then, $F = 1.02 \times 200 = 204\text{mm}$. If this 204mm lens is moved the distance $X = .0800\text{mm}$ calculated above, then object distance $U = 204 - .0800 = 203.920\text{mm}$. Image distance V is then, from the Gauss lens equation $1/U - 1/V = 1/F$, $V = FU/(F - U) = (204)(203.920)/(204.00 - 203.920) = 519,996\text{mm} = 519.996$ meters. Optical distance from the eye to the virtual image is $V + D = 519,996 + 180 = 520,176\text{mm} = 520.176$ meters. Error from the desired 500 meters is $(500-520.176)(100)/500 = 4.04\%$. When focal length F is 2% low, $F = (.98)(200) = 196\text{mm}$, and $U = 196 - .0800 = 195.9200\text{mm}$, and $V = FU/(F - U) = (196)(195.9200)/(196 - 195.9200) = 480,0040\text{mm} = 480.00$ meters. Here, $V + D = 480,004 + 180 = 480,184\text{mm} = 480.1840$ meters. Here, error in image distance from the eye is $(500 - 480.004)(100)/500 = 3.96\%$.

As another example, one where optical distance $V + D$ is not large relative to D , suppose that the desired display distance is 700mm, or 27.55 inches, and that the eye is 300mm behind a magnifier whose focal length is 360mm. Here, $V + D = V + 300 = 700$, so that $V = 400\text{m}$. Then, from above, $X = F^2/(V + F) = 360^2/(400 + 360) = 170.5263\text{mm}$. Had the 300mm distance to the eye from the magnifier not been accounted for, so that optical distance from the magnifier of the image was 700mm, then actual eye-to-image distance $D + V = 300 + 700 = 1,000\text{mm}$, not 700mm. Here, the error would be $(1000-700)(100)/700 = 42.9\%$, a very large error.

In the above discussion and examples, the object, a CRT, was a real object. Actually, the object was the real image on the CRT face. This is not a projected image, so that, when viewed through a magnifier, no exit pupil is present. Thus, the eye can be at different distances from the magnifier without changing the luminance of the retinal image.

When, however, a simple magnifier or an eyepiece is used to examine a real projected image, there is an exit pupil, so that eye movement perpendicular to the optical axis can take the eye pupil out of or partly out of the exit pupil, reducing retinal illuminance for the eye. Also, axial movement can change retinal illuminance. However, whether the object is a real object or a projected image, the $1/U - 1/V = 1/F$ equation for the magnifier is valid, as are the other equations in this section that are based on this equation.

In the discussion above, it must be kept in mind that distances are distances from reference points, such as nodal points. For example, to say that the eye is $X\text{mm}$ behind the second nodal point of the eyepiece means that the entrance pupil of the eye is $X\text{mm}$ behind the second nodal point of the eyepiece. The exit pupil of the eyepiece is a short distance within the eyepiece, and the entrance pupil of the eye is a short distance within the eye, as noted elsewhere in this tutorial.

In the distance calculations in this section, the eyepiece

lenses were assumed to be thin, so that the distance S between the first and second nodal points could be assumed to be quite small and be neglected in computations. Thus, display image distance was given as $V + D$. For lenses that are thick from front to back, a more accurate image distance would include the distance S between the front or first nodal points and the rear or second nodal point. The more accurate image distance would be $V + S + D$.

10. OPTICAL AND OPTICS-RELATED CONSIDERATIONS FOR HELMET-MOUNTED DISPLAYS

A. GENERAL DISCUSSION

Helmet mounted displays (HMDs) provide a virtual display image while simultaneously permitting a direct view of external objects. They are a type of head-up display. The display image appears to be located in the external world: it is projected upon the external scene. The display may present symbols, instructions, instrument readings, maps, and pictures from sensors or from a computer. The sensor image may be from a radar system, a closed-circuit television, a forward-looking infrared system, etc.

Before discussing the parts, construction and performance of an HMD, it is instructive to visualize the general appearance of an HMD. Figure 16 is a rough sketch of an HMD. It is not drawn to scale. Note, in the figure, that the display parts consist of a CRT, an objective, a folding mirror, an eyepiece, and a combiner or beamsplitter. Many systems do not contain the folding mirror. In the figure, the image source or original image is a cathode ray tube (CRT), but other image sources, such as a liquid crystal display (LCD) matrix, a matrix of solid-state light-emitting diodes (LEDs), a laser line scanner, etc. may be used. In the figure, the CRT image is projected by an objective, sometimes referred to as a relay or as relay optics, as an enlarged real image of the real CRT image. The real primary image projected by the objective is viewed through an eyepiece which magnifies it and presents a virtual display image to the observer. A beamsplitter located between the eyepiece and the observer's eye permits both the objective and the CRT to be located in positions that are out of the line of sight of the observer. The beamsplitter reflects the display image to the observer. Simultaneously, the observer obtains a direct view of the outside scene by looking through the beamsplitter. The beamsplitter thus superimposes the display image upon the external scene: both display image and the scene appear to be outside in the real world. Because the beamsplitter enables combining the display image with the direct view of the scene, the beamsplitter is referred to as a combiner.

The beamsplitter, then, transmits the outside scene and reflects the display image. In the figure, the beamsplitter is a flat plate inclined to the line of sight of the observer. However, not all beamsplitters are flat. Some helmet-mounted displays do not have a beamsplitter and a direct or see-through view of the outside scene. With these types of HMDs, at least part of the outside scene is obscured by the display. It is the see-through type of HMD that is addressed in the present report.

In some HMD systems, the helmet user can aim or direct the sensor by moving his head so as to position a displayed reticle on an object or area of interest seen by direct vision looking through the beamsplitter, or seen on the display image from a sensor. Sensor aiming in such systems is accomplished by an

electro-optical, an ultrasonic, or a magnetic system that senses helmet orientation within the cockpit, and, through computer control, slews the sensor to point along the line of sight indicated by the reticle. Such HMD systems are said to be helmet-aimed or helmet-slewed. Another type of sensor-aiming systems uses an electro-optical system viewing one of the observer's eyes to determine, through computer image processing, the eye's line of sight, i.e., where the observer is looking. Actually, what is determined by computer image processing is the orientation, relative to the aircraft, of the visual axis of the eye. The line of sight information is used by the computer to control electric motors that slew a gimbal-mounted sensor. Thus, there are two general types of sensor-aiming systems, referred to as helmet-slaved and eye-slaved, respectively, and both are visually-coupled systems. They can automatically direct sensors, cameras, slewable guns, or missiles to areas or objects of interest without the vehicle pilot having to aim the aircraft or having to manipulate special controls for aiming. The HMD user's hands are free for flight control and other tasks. This can be very beneficial in some situations, for example, in aerial combat.

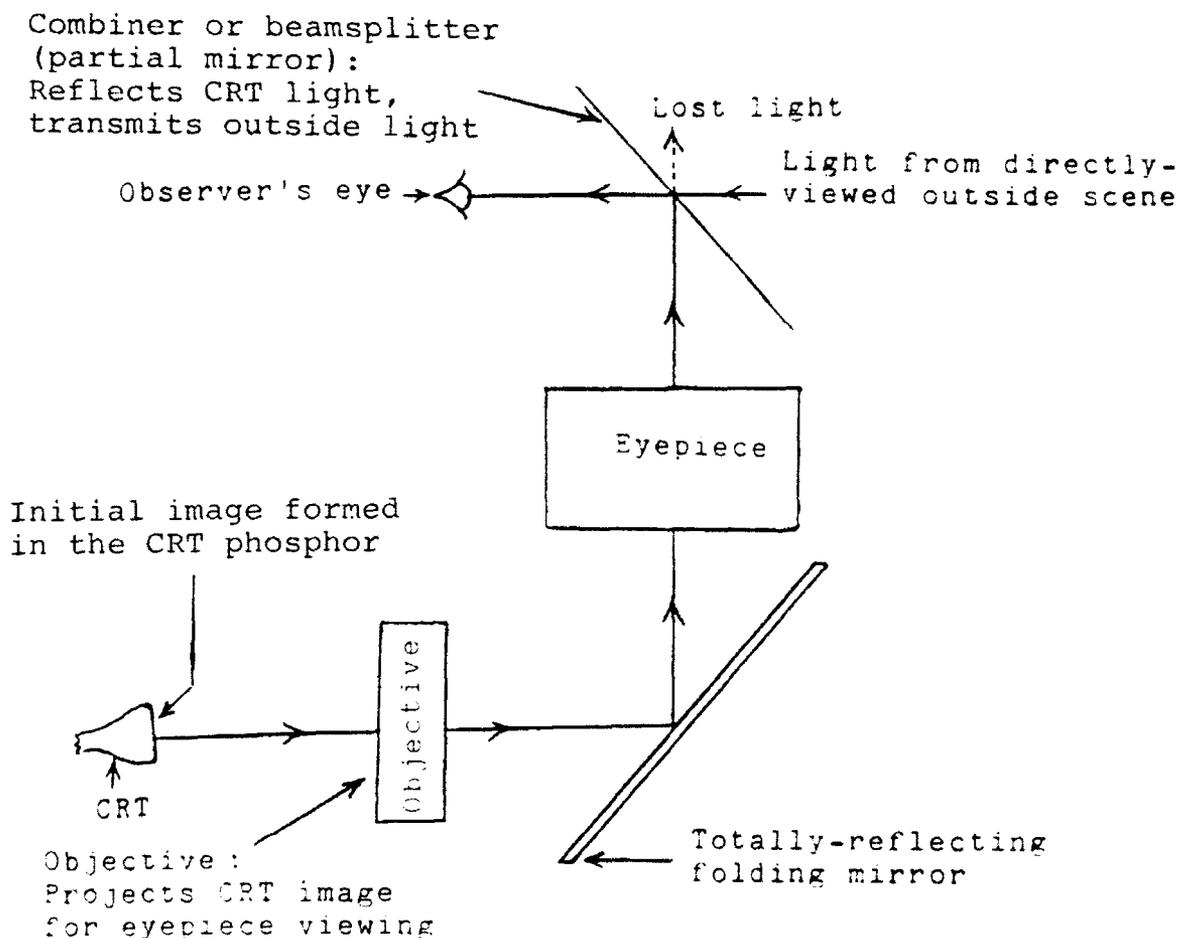


Fig. 16. Illustrative sketch of a head-up display such as might be used as a helmet-mounted display. Not to scale.

Since flight and weapons data are optically superimposed upon the outside scene at either optical infinity or at the distance of objects in the scene, attention shifting from the scene to display information or vice versa does not require refocusing (accommodation) of the eyes, or adaptation to a luminance difference between scene and display. Also, and of major importance, with a head-up display system, both display and outside scenes are always in the field of view: neither is lost while viewing the other. A suitable sensor location on the vehicle permits the HMD user to view parts of the outside world obscured from direct vision by opaque parts of the vehicle.

Helmet-mounted displays, particularly visually-coupled systems, because of the advantages mentioned above, permit enhanced system performance when compared to systems that use panel-mounted displays or other forms of fixed-position displays. HMD systems, whether or not they are visually coupled, are increasingly used by crew members of aircraft, tanks and remotely-operated (teleoperator) vehicle. They are also useful in operating remote-control manipulators and for maintenance, repair and inspection work. They would make valuable teaching aids.

Ideally, an HMD presents an image that:

- (1) is at an optimum optical distance from the eye;
- (2) is of sufficient angular subtense to satisfy field of view requirements;
- (3) contains image details of adequate angular subtense, luminance, contrast, resolution, dynamic range, and freedom from jitter and noise to permit the observer to perform required tasks;
- (4) is adequate in contrast in ambients ranging from bright sunlight to totally dark for day and night systems and;
- (5) does not prevent look-down at instruments and controls in the cockpit.

In addition, the HMD system must also:

- (1) provide adequate clearance for spectacles;
- (2) have an adequate exit pupil diameter;
- (3) if binocular, permit interpupillary distance (IPD) adjustment;
- (4) permit see-through;
- (5) have a system weight, volume, and center of gravity that is acceptable to users;
- (6) be comfortable, or at least tolerable, for hours-long use, and;
- (7) protect the wearer from high noise levels, nuclear bomb light flashes, laser weapons, and chemical agents, while providing impact and crash protection.

Items listed in (6) and (7) are provided, in part, by accessories that must be integrated with the display system.

HMD system weight and center of gravity are primary factors in system acceptance by HMD users, even with the lightest and best-balanced one yet (1991) produced. Since system weight must be minimized, weights can't be added to balance the system. Also, since the system center of gravity is not at the center of the wearer's head, the head is forced away from a desired comfortable position. In high-acceleration conditions, the weight and

imbalance of the HMD system cause large forces to be exerted on the neck muscles of the helmet wearer.

To minimize weight and volume, the optical, electro-optical, and mechanical parts must all be very small. Typically, the CRT or other primary image source in available HMD systems has an image with a diameter of less than one inch. Such small images require considerable magnification to provide a satisfactory angular field of view at the eye and to display image details adequate in angular size. This magnification, often 5X-10X, must be achieved by an optical system whose eyepiece is of long enough focal length to provide an adequate clearance between the display optics and the user's eyes or spectacles. A simple magnifier with a large field of view, high magnification, and adequate eye clearance is not feasible when the source image is small, so that an objective or relay is required to supply part of the total magnification. Although many HMD systems are monocular, displaying an image to only one eye, some are binocular, which obviously adds appreciable weight and volume due to both duplicated parts and to supporting fixtures that permit interpupillary (IPD) adjustment and binocular alignment, and that have adequate strength and rigidity to maintain adjustments.

A see-through capability, to be discussed later, can add weight and imbalance, as well as optics and vision problems. The primary image source, such as a CRT, being opaque, must not be in the field of view, nor can optical and support parts of objectives and eyepieces be located between the eye and the external scene. Thus, the beamsplitter prevents such objects from interfering with viewing the outside scene by allowing them to be out of the field of view.

As noted earlier, the diameter of the exit pupil must be adequate to retain all of the pupil of the user's eye (really, the eye's entrance pupil) during relative motion between the helmet and the head due to motion of the scalp on the head or to slipping of the helmet on the scalp. Such movements are maximum during bumping, heavy vibration, and the high accelerations that occur during turbulence and combat maneuvers. When any part of the eye pupil is outside of the HMD exit pupil, the retinal image receives less light. Another reason for a large exit pupil is that the center of rotation of the eyeball is not at the eye's entrance pupil, but at a point located about 13mm behind the vertex (extreme front) of the cornea of the eye (Southall 1937/1961). Rotation of the eyeball away from a straight-ahead exit-pupil-centered position thus moves the eye entrance pupil relative to the HMD exit pupil: exit and entrance pupils no longer coincide. The problem can be serious with wide-angle displays, which require large exit pupils to prevent image dimming when rotating the eye through large angles to examine off-axis portions of the display. Military HMDs for pilots require exit pupils in excess of 10-15mm to assure retention of the eye pupil in the exit pupil when the helmet moves on the head and during viewing at large off-axis angles.

A large exit pupil requires a large display system entrance pupil, since the exit pupil is the image of the entrance pupil, and exit pupil diameter is numerically equally to entrance pupil diameter

divided by magnification. For example, an HMD with a 5 times magnification of a CRT image and a 10mm exit pupil requires an entrance pupil 50mm in diameter. The objective, here, must have a clear or effective aperture of 50mm. However, to keep the system short requires a short focal length objective so that the CRT-to-objective distance, plus the distance of the projected image from the objective, is small. Now, a short focal length objective with a large diameter is a "fast" objective, i.e., one with a low relative aperture or "F-number" (focal length/aperture). This configuration may require complex optics in the objective at a cost of added optical elements and more weight.

A large entrance pupil for the HMD objective requires large diameter optical elements, and large glass elements are heavy. In some applications, part, or even all, of the optical elements may be constructed of clear optical quality plastics, which is appreciably less dense, hence lighter, than glass. Transmissive (or refractive) optical elements may be partially or entirely replaced with reflective (or curved mirror) elements. When the display contains only a narrow bandwidth of light, holographic lenses may be used. These lenses work by diffraction, rather than by refraction or reflection, and are contained in thinner-than-paper films. Gard (1982) discusses holographic head-up displays.

B. THE COMBINEP IN HELMET-MOUNTED DISPLAYS.

Users of helmet-mounted displays, especially vehicle operators, often require direct or unaided vision of the external world, while simultaneously viewing the displayed image. The displayed image is optically superimposed upon the outside scene. The ability to look through the HMD optics as if they were not present, as noted earlier, is referred to as a "see-through" capability. Incorporating see-through into an HMD poses several problems.

The image source, such as a CRT, being opaque, has to be in a location that is outside of the field of view. As discussed earlier, this is made possible by using a beamsplitter or partial reflector. A partial reflector can be seen through and at the same time it reflects light to the eye from the CRT or other image source. The CRT or other image source may be at the top, bottom, or side of the helmet. Beamsplitters are inclined at an angle to the eye's line of sight. In its simplest form, a beamsplitter is a thin flat plate or disc of glass or plastic with a thin surface coating that transmits some light and reflects some. Most beamsplitters are elliptical in shape so that, when inclined to the observer's line of sight, they present to the observer a round or circular shape. Because the beamsplitter combines the display image and the external scene, the beamsplitter is usually referred to as a combiner. Because the beamsplitter is close to the eye, the eye can not bring the beamsplitter into focus. The optics of the display system do not form an image on the beamsplitter. It is thus incorrect to talk about images being combined on it: images are combined with the assistance of the beamsplitter, but not on it. To the observer, the CRT image appears to be superimposed on the part of the scene that is within the display field of view.

Also visible to the user is the scene that surrounds the HMD field of view.

To obtain, for the eye, more light from the CRT, most of the light impinging on the combiner from the objective must be reflected to the eye. An optically neutral combiner, if it reflected most of the CRT light, would also reflect back to the outside most of the light from outside, i.e., would transmit little light from the outside scene. The outside scene included in the combiner would be quite dim, and the see-through scene outside of the area covered by the beamsplitter could be very much lighter than the part of the scene inside of the beamsplitter area. Replaceable beamsplitters with neutral (nonselective) transmission, one with low transmission of outside light for use in bright sunlit scenes and one with high transmission for dim or dark scenes, are not without problems, including possible damage to optics coatings during handling and storage, loss of alignment and boresight when changing combiners, and a sturdier heavier mounting to allow changing combiners without losing optical alignment. What is needed is a combiner that transmits to the eye almost all of the light from the outside environment, while reflecting to the eye almost all of the light from the CRT.

A widely-used solution to the problem is to use a CRT phosphor that emits light in a narrow spectral band, typically a narrow band of green light. A phosphor that emits green light is usually used because of its high phosphor efficiency and luminance relative to other available CRT phosphors. The beamsplitter coating for use with the green phosphor reflects to the eye almost all of the green light incident upon it, and transmits to the outside nearly all of the remaining visual spectrum light from the phosphor. The coating also transmits most of the light from the external scene, except for a narrow spectral band in the green. The result of this reflection and transmission behavior is little loss of either CRT or outside light. Looking at the combiner from outside of the HMD, the combiner appears to be green. Looking through the combiner, it has a purple or magenta tint.

Because the eye becomes less sensitive (adapts) to green when exposed to green light, when the HMD is removed after wearing it for awhile, color vision is disturbed. The environment appears to have less green than usual. Visually, the world appears to have a pink or purple cast, sometimes called "pink eye." No harm is done, and normal color vision returns within a few minutes to a few hours. To fabricate a display of similar nature that used red, blue, and green colors would be a technically difficult problem: one would require a phosphor that emitted light in narrow red, blue and green bands, and a combiner coating whose reflectivity was high in these three narrow bands, while transmission was high in the remainder of the visible spectrum. A CRT and a beamsplitter with these characteristics would have to be developed. To obtain enough light, phosphor efficiency in converting electron beam energy to visible light would have to be high in each of the three narrow spectral bands. A nonselective or neutral combiner coating might be required for a full-color display, provided very high luminance could be

obtained with a color CRT. A neutral coating would lose too much CRT light by transmission through the beamsplitter to the outside, and would lose too much outside light by reflection back to the outside.

The HMD combiner must also provide adequate eye clearance, i.e., eye relief. Since an appreciable percentage of users will wear spectacles, adequate clearance behind the beamsplitter must be available to prevent contact of spectacles and beamsplitter during buffeting, high "G," when the helmet is bumped, and during aircraft crashes. The combiner, if not part of the visor, must fit inside of the visor.

C. DISPLAY RESOLUTION AND FIELD OF VIEW

Briefly mentioned earlier was the need for the displayed sensor imagery to cover a wide field of view, while also providing enough angular resolution of the outside world. For some tasks, such as navigating, piloting at very low altitude, or searching for other aircraft, a wide field of view is highly desirable, even at the cost of reduced resolution of small details of objects. For other tasks, such as identifying objects, or for aiming guns, missiles, or cameras with very narrow fields of view, high angular resolution may be desirable, or even essential. However, the number of available resolution elements across the field of view of the sensor or of the display is either fixed or limited. Spreading available resolution elements over a wider field of view results in a larger angle per resolution element, i.e., a reduced (poorer) angular resolution. Conversely, higher angular resolution must be at the expense of a narrower field of view. At present (1991), wide fields of view are available, as is high angular resolution, but not both simultaneously in HMD systems with acceptable weight and balance. The FOV-resolution problem is one of determining the optimum balance or compromise between the two.

Since required or desired FOV and resolution are both task-dependent and mission-dependent, there are no available simple rules for achieving an optimum balance. The use of zoom optics to vary the FOV of the sensor, hence the field of view of the displayed scene, may be required for some tasks, but zooming results in loss of one-to-one image-object size correspondence. This loss may or may not be important, depending upon the situation.

When discussing FOV/resolution trade-off and eye versus display resolution, it is essential to keep in mind that eye resolution and the resolution of scan-type displays, such as that provided by a CRT, are defined and measured differently, even when both are expressed in the same units, such as angular resolution in minutes of arc. When eye resolution is measured with a standard letter-type of eye test chart, such as the Snellen chart, each letter fills a square with the lines making up the letter having line widths that are one-fifth of letter width. At a standard distance of 20 feet, individual letters on the normal acuity (or 20/20) line of letters subtend 5 arc minutes in height and 5 arc minutes in width. Individual lines of which letters are composed subtend, in width, one arc minute. Letters on the test chart have a high contrast with

their background, and chart illumination is high. These conditions favor high visual acuity or resolution by the human eye.

Many other resolution test patterns or targets, besides letters of the alphabet, have been used to measure visual acuity. Examples are two points of light, discs of equal size separated by one disc diameter, checkerboards, and gratings composed of straight uniformly-luminous alternate light and dark bars of equal width called Ronchi gratings or square-wave patterns. Farrell and Booth (1984) present eye resolution data obtained with various test patterns. With Ronchi gratings, a resolution element is the distance between the centers of two dark bars separated by a light bar, or vice versa. A grating resolution element, or cycle, is thus the width of two bars. How well the eye can resolve (discern) bar separation varies with a host of factors, including test instructions, the number of cycles in a test pattern, the ratio of bar length to bar width, bar contrast or modulation, bar orientation (horizontal, vertical, etc.), chart illuminance, and observer confidence. For several years, sine-wave test patterns covering a range of spatial frequencies have been used as a more advanced type of test pattern. Instead of having uniform luminance across a bar, luminance varies as the amplitude of a sine wave. Such sine wave patterns are used because they provide a more comprehensive measure of eye response than is available from limiting resolution measurements obtained with letters and Ronchi gratings. Sine wave test patterns are used in modulation transfer function testing. The result of such testing is plotted as the ratio of output modulation to input modulation as a function of spatial frequency, such as cycle per degree.

With Ronchi gratings, one cycle, or one line pair, is one light bar and one dark bar. With sine wave patterns, the resolution element is one cycle. However, the resolution element used for testing television displays is individual TV lines, not line pairs. Because of this difference in how resolution is defined, there is a 2:1 ratio between optical resolution and TV resolution. In addition, TV scan line intensity across a line is approximately Gaussian, the normal probability distribution, not square wave, as are the luminances or lightnesses of the bars or lines of a Ronchi grating.

Since adjacent TV lines slightly overlap each other, each line, across its width, being an approximation to a Gaussian distribution, and, in addition, the lines are not perfectly spaced, there can be as many as 2.85 television scan lines to one optical line pair. Keep in mind that, in terms of what an observer can see, the number of scan lines across a target is not the number of TV resolution elements across the target image. The atmosphere, aircraft motion and vibration, the optics of the sensor, the system electronics, and the CRT and the HUD optics can cause the actual displayed resolution of a target or pattern image to be very low. The number of scan lines across an image must not be confused with resolution.

For many tasks, missions or activities, observers using an HMD will prefer, or even need, a display picture of the outside scene

with image details as fine as, or almost as fine as, they could discern with the unaided eye. This resolution need is particularly likely when the only information about the outside scene is that provided by the display, such as occurs on dark nights, or when flying a remotely-piloted vehicle. Observing with the unaided eye in ideal laboratory conditions, where there is no vibration or buffeting and no observer motion, and the viewed object or image has both high contrast and luminance, visual acuity of one arc minute is regarded as normal. In an airborne environment, visual acuity is no better than about 2-3 arc minutes. Since there are 60 arc minutes in one degree, the 3-arc minutes case equates to $60/3 = 20$ visual or optical resolution elements per degree. If the Kell factor is included, to account for partial overlap of scan lines on a CRT, 2.83 TV lines are equivalent to one optical line pair. Thus, in this case, 20 optical resolution elements/degree is equivalent to $2.83 \times 20 = 56.6$ TV lines. A 40-degree field of view with 56.6 TV lines/degree requires $40 \times 56.6 = 2,264$ lines, while a 60 degree FOV requires $60 \times 56.6 = 3,396$ lines. Such resolutions across the field of view are beyond today's technology for small high-luminosity CRTs. Eventually, wide fields of view will be possible for displays that provide high luminance, adequate image contrast, and acceptable angular resolution.

D. INTERFERENCE OF SEE-THROUGH AND SENSOR IMAGES

An HMD problem that has had little research applied to it is that of the visual interference between the displayed image from a sensor and the image supplied by the directly-viewed outside scene. The images from the environment and from the CRT are simultaneously present on the same area of the retina. When the display contains only a few computer-generated lines and symbols, they do not significantly interfere with scene imagery, nor does scene imagery interfere to any significant degree with symbol legibility. However, two different pictures that are superimposed, one from see-through and one from an imaging sensor, represent a quite different situation. The sensor, such as forward-looking infrared (FLIR), may have a spectral sensitivity quite different from that of the eye. Thus, for example, a part of an object may appear light to direct vision and dark to an infrared system, or vice versa. Even with an optimal balancing of average scene and display luminances, overlapping images cause some image details to be difficult to observe, even with perfect registration or matching of the images. A small amount of picture misregistration may create bas-relief effects in addition to the loss of contrast due to superimposition on the retina of two images whose individual image details differ in luminance and resolution of fine details. Some degree of misregistration is unavoidable due to quick eye movements and quick head movements, including head tilting, for which the sensor-directing system can not respond rapidly enough. Also, movement of the helmet on the head, for those systems that monitor helmet orientation, rather than observer line of sight, will cause misregistration. In addition, it is necessary for the sensor-supplied image to compete, in some cases, with an outside sky, terrain or water scene that may have a very high luminance. Both weight and volume constraints, as noted, require a small CRT

or other image source. Thus, considerable magnification of the source image is required, with the result that slight helmet motion on the head can cause considerable misregistration.

A highly-luminous display image that has been considerably magnified requires both a large objective (relay lens) and a highly-luminous (very bright) image source. If the source image is supplied by a CRT, an electron beam with both high electron beam current and high electron gun voltage is required. These conditions degrade both CRT line width, hence CRT resolution, and operational life of the CRT. In addition to high voltage on the helmet, the hard driving of the CRT, to obtain high phosphor luminosity, produces heat which must be dissipated.

E. FIELD OVERLAP

An observer's nose blocks out part of the left field of view of the right eye and part of the right field of view of the left eye. There is, however, a large common or overlap area seen by both eyes. By moving the head, an object in either one of the two one-eye-only areas can be brought into the common area. In a binocular helmet-mounted display whose primary image is electronically generated, the horizontal field of view can be made wider by reducing the overlap or common area viewable by both eyes. By widening the horizontal field of view, situational awareness can be increased. On the other hand, human vision is slightly better for objects or images that can be seen by both eyes. In addition, if there is only one sensor of the outside world supplying the image for the HMD, increasing the HMD horizontal angular coverage spreads out the available scan lines and resolution elements. Thus, as noted earlier, the number of resolution elements per degree is less. A wider field of view, unless total number of horizontal resolution elements is proportionally increased, reduces the ability of the system to display fine details of the external world.

In systems designed for some missions or tasks, 100% overlap of right and left eye imagery may be desirable, even with some loss of peripheral vision, while other tasks or missions with less demand for fine detail resolution and more demand for a wider horizontal FOV may require little overlap of the fields of view of the two eyes.

Table 4 lists some optics and vision considerations for helmet-mounted displays. The order of the items in the table is not necessarily the order of importance of the consideration or factors, and the factors are not independent. The tables also contain some repetition.

Table 5 lists some optically-relevant or optically-impacting concerns for HMD systems, and Table 6 on trade-offs in system design presents some of the consequences of changing values of some system parameters to attain desired values of other aspects of system performance. Included in Table 6 are changes to: a) reduce system weight and volume, b) increase eye relief or clearance, c) increase magnification, d) increase field of view, e) increase exit pupil diameter, and f) increase luminance of the image source, for example

phosphor luminance of a CRT. As in Tables 4 and 5, order in the list of Table 6 is not an indication of importance. The rationale for much of the trade-offs in the tables is developed in Section 8, Compound Magnifiers. For example, why a larger exit pupil requires an objective, sometimes referred to as a relay lens, with a larger effective diameter, i.e., a larger entrance pupil. In the tables, as well as in the text, some topics are not discussed or explained, or are only briefly covered, so that further reading in references may be required.

For readers with little or no previous familiarity with helmet-mounted sights and helmet-mounted displays, a recommended source of information is a symposium on development and application of visually-coupled systems edited by Birt and Task (1973). The 25 papers in this symposium discuss systems that aim sensors and weapons by automatic equipment using either helmet position or data obtained by measurement of the observer's line of sight. Included are operational considerations and experience, design problems, application concepts, dual field of view concepts, etc.

TABLE 4

OPTICAL CONSIDERATIONS FOR HELMET-MOUNTED DISPLAYS

1. Field of View and Resolution
FOV trade-offs with angular resolution (object detail), system weight and volume; FOV reduction by facial anatomy, oxygen mask, chemical-biological-nuclear (CBN) and laser weapon protection accessories, and optics supports and baffles; amount of field overlap.
2. Image-Quality
Total resolution, angular resolution, off-axis resolution, MTF, etc.; contrast, luminance, signal-to-noise ratio, gray shades (dynamic range), perspective error, shape distortion, computer image distortion correction, and scan line visibility.
3. Display-Image versus See-Through Scene
Scene illuminance (day, twilight, night) versus display luminance (optics transmission and reflections, phosphor luminance and persistence, optics diameters and coatings, magnification); image registration, color.
4. Exit Pupil Diameter
Helmet movement on scalp, scalp motion (bumping, buffeting, acceleration), eye rotation; exit pupil diameter versus HMD weight and volume.
5. Eye Relief and Clearance
Spectacle use, safety (in impact, crashes); trade-off with field of view.
6. Eye Dominance and Retinal Rivalry
Monocular versus binocular displays; see-through luminance versus display luminance with monocular display; right eye and left eye monocular display models; effect on tasks.
7. Masking and Obscuration
Optics supports and required baffling (for stray light); display symbols, display versus see-through; oxygen mask; cockpit.
8. Image Registration
Registration of see-through scene and display image during tracking, head tilt and rotation, quick head motions; display image matching (size, rotation, luminance) with binocular display.
9. Masking and Obscuration
Optics supports and required baffling (for stray light), displayed symbols, display versus see-through; oxygen mask; cockpit.
10. Stray Light
Unwanted reflections and transmissions (sunspots, ghost images, reflections from clothing and cockpit).

Table 4 continued

11. Display Color versus See-Through Color
Green trees and other vegetation versus green phosphor; most appropriate display color versus most luminous phosphor; lower resolution of multicolor displays.
12. Color Distortion (Pink Eye, Adaptation)
Distorted color vision after hours-long use with a narrow-band phosphor (usually a green one).
13. Monochrome versus Multicolor Display
less resolution with multicolor, also more weight; color coding, attention, user preference.
14. Vignetting
Allowable off-axis dimming versus larger optics.
15. Aiming and Tracking Error
Lag and overshoot; jitter, hysteresis, image stabilization, control dynamics, boresight accuracy, helmet motion on head (turbulence, bumping, acceleration, helmet fit).
16. Symbols, Pictorial Data
Value, quantity and clutter, configuration, type, color, luminance, conspicuity, interference between displayed sensor image and see-through scene, optical distance of display image.
17. Monocular versus Binocular Display
Weight and volume; cost, maintenance of alignment tolerance, safety (redundant display); comparative user comfort, visual resolution and contrast sensitivity; multiuser capability of a given helmet-mounted display; width of FOV (overlap); fitting and adjustment.
18. System Integration
Control of light loss, unwanted reflections, and masking down of see-through FOV by oxygen mask and CBN and laser protection gear. Effect of donning such protective gear on see-through field of view, optical adjustments, tolerances, boresighting, image distortion, special problems with eye trackers. Effect of windscreen optical distortions on aiming and tracking accuracy, and computer correction of distortion for displays. Display luminance control to suit ambient light conditions.

TABLE 5

OPTICALLY-RELEVANT CONCERNS FOR HELMET-MOUNTED DISPLAYS

1. Weight and Volume
HMD optics plus helmet, visor, oxygen mask, CBN and laser accessories: comfort, neck strain, cockpit egress, aiming and tracking under high acceleration.
2. Balance and Center of Gravity (Vertical, Horizontal)
Neck strain, comfort, tracking.
3. Helmet Fitting and Optical Alignment
Ease, speed, accuracy, reliability, comfort. Use of trained helmet fitters and special equipment.
4. Stability
Maintenance of collimation, boresighting, alignment tolerances, (helmet slip on scalp, scalp-on-head slip, buffeting, acceleration), vibration and tracking effect on image jitter, optics mount rigidity.
5. Aiming and Tracking Error
Weapon and sensor scanning, head versus eye tracking, image stability, helmet slip, flexing of helmet and optics mount.
6. Comfort
Weight, balance, center of gravity, helmet fit (tightness, pressure points, hot spots), hours-long use, padding, insulation and helmet heating (hot head, sweating and condensation on optics, required cooling and ventilation, facial access), CBN equipment (heat, perspiration, ventilation, weight on head).
7. Head and Face Anatomy
FOV restriction, non-optimal exit pupil location, CRT location on helmet, personalized equipment for some users, multi-user for some).
8. Optics Delicacy
Coatings (abrasion, scratches, cleaning), storage, transportation, damaged parts replacement.
9. Mechanical Delicacy
Weight versus delicacy; retention of boresight, collimation, and alignment; storage, maintenance, parts replacement and other repairs, transportation and handling.
10. Disconnect and Discard Quick, easy, certain.
11. Aircraft Crash and Bird Impact Protection
12. Cables
High voltage (safety), weight, pull on head, catching on clothes, harness, and controls, in the way, emergency disconnect.

Table 5 Continued

13. Noise Protection and Communication
Ear cups, ear seal, earphones, microphone.
14. Required Back-Up Displays and Controls
15. CRT Luminance and Luminance Loss with Use
16. User Acceptance
Usability, capability, accuracy, reliability, comfort, required training time, pilot wash-out rate.
17. Dollar Cost
Affordability; cost versus performance; software and computers, maintenance, lifetime cost.
18. System Integration
Combining with sensors, software, computers, oxygen mask, CBN and laser protection accessories, and with weapon system, ventilation equipment, power supply, earphones, cockpit egress system, and with back-up displays and controls.

TABLE 6

CHANGING HMD SYSTEM PARAMETERS

APPROACH	CONSEQUENCES
(A) <u>To REDUCE SYSTEM WEIGHT:</u>	
1. Narrower field of view optics.	Less situational awareness: impacting objects; missing hostiles, friendlies; loss of orientation.
2. Smaller CRT	Reduced image quality (luminance, resolution, etc.) More magnification required.
3. Matrix display (Liquid crystal, LED, etc.)	Reduced image quality (resolution, aliasing). Liquid crystals are heat-sensitive. They can be lighter yet bigger, requiring less magnification
4. Laser scan display.	Requires multiple lasers for RGB color. Using solid-state lasers is promising
5. Less complex optics.	Less image quality.

- | | | |
|-----|---|--|
| 6. | Fresnel Lens Optics. | Grooves limit resolution, cause scatter, diffraction, Moire patterns from scan-line-groove interaction. |
| 7. | Holographic Optics
(Diffraction) | Excellent see-through, very light weight, works only with narrow spectral bandwidth. Limited sharp FOV, very angle-sensitive. Sensitive to stray light (baffling required). |
| 8. | Gradient Index Lenses. | Immature technology, future promise. |
| 9. | Removing Parts from Head
via Fiber Optics Bundle | Heavy bundle.
Fiber breaks (image defects). |
| 10. | Visor a Part of Eyepiece
(Parabolic, etc. reflectors) | Complex surface shapes very promising. Immature technology. FOV limited by pupil aberration. |
| 11. | Shorter Focal Length Objective
(Relay lens a misnomer) | Faster, more complex objective, maybe little weight decrease. |
| 12. | Shorter Focal Length Eyepiece | Safety (less eye clearance). |
| 13. | Non-adjustable IPD | Individual (custom) HMD, or larger exit pupils required, possible weight increase, not decrease. |
| 14. | Smaller Diameter Objective | Smaller exit pupil. |
| 15. | Less Eye Clearance | Safety hazard. |
| 16. | Flimsy Construction | Easily damaged. Loss of alignment and calibration |
| 17. | Exotic Materials
(Kevlar, carbon fiber, etc.) | Must use them.
Cost, manufacturability. |
| 18. | Monocular HMD | <p style="margin-left: 40px;">Plus: (A) Much lighter, less bulky, less costly, less delicate, easier cockpit egress. No perspective distortion. Easier to align and boresight.</p> <p style="margin-left: 40px;">Minus: (B) Less visual comfort. Reduced visual resolution and contrast sensitivity. Eye dominance and retinal rivalry problems. No display redundancy (failure insurance)</p> |

(P) TO INCREASE EYE CLEARANCE:

- | | |
|-----------------------------------|--|
| 1. Longer Focal Length Eyepiece | More weight and volume.
More primary magnification required (longer projection distance or shorter focal length objective). |
| 2. Shorter Focal Length Objective | More complex objective, but does move exit pupil back. |
| 3. Less CRT magnification | Bigger CRT. |
| 4. Reduced FOV | Safety (situational awareness). |

(C) TO INCREASE CRT MAGNIFICATION:

- | | |
|--|---|
| 1. Shorter Focal Length Objective | Possibly more complex (faster) objective. |
| 2. More Primary Magnification by the Objective | Bulkier, heavier metal parts for longer projection distance |
| 3. Shorter Focal Length Eyepiece | Safety (reduced eye clearance). |

(D) TO INCREASE EXIT PUPIL DIAMETER:

- | | |
|--|--|
| 1. Less Magnification | Larger CRT required, or loss of unity (display/scene) magnification ratio. |
| 2. Objective with Larger Entrance Pupil (Clear Diameter) | Increased objective size, complexity, weight, volume. |

(E) TO INCREASE FIELD of VIEW:

- | | |
|---------------------------------------|--|
| 1. Wider FOV eyepiece | Note: Decreased angular resolution.
Larger, heavier eyepiece. |
| 2. Reduce Eye Clearance | Safety Hazard. |
| 3. Less Magnification | Loss of unity magnification, or use a larger CRT. |
| 4. Decrease Field Overlap (Binocular) | Less angular resolution (poorer vision). |

(F) TO INCREASE DISPLAY LUMINANCE

- | | |
|-------------------------------|---|
| 1. Higher Voltage and Current | Shorter life for CRT. |
| 2. Larger CRT | Less magnification required.
Bigger and heavier CRT and optics required. |

For a general review of psychological problems with helmet-mounted displays, see Hughes, Chason and Schwank (1973). A wide range of HMD topics, both psychological and equipment-wise, is included in the papers of an HMD symposium edited by Birt and Task (1973). A technical report, "Optical and Human Performance Evaluation of HUD System Design," edited by Martin (1983), is a collection of papers by engineers and psychologists that is of value to anyone with an interest in head-up displays, including helmet-mounted displays. It discusses some of the technical aspects included in the present paper. Recommended reading on binocular rivalry and eye dominance is Herschberger and Guerin (1975). In a series of four laboratory studies they found that retinal rivalry was significantly, and to the greatest extent, influenced by HMD and scene luminance. Also statistically significant, but with less influence, were ambient scene complexity, HMD accommodation distance, HMD field of view, and HMD contrast. No significant effect was found for HMD resolution, percent see-through, eye dominance and scene accommodation distance. Also recommended reading on binocular rivalry and eye dominance are Porac and Cohen (1978) and Peli (1990). Contrast sensitivity and its application to assessing HUD systems is covered in detail by Ginsburg (1983).

For further reading on system analysis, design considerations and trade-offs, see Task, Kocian and Brindle (1980), Kocian (1983), and Buchroeder and Kocian (1989) (limited distribution). HUD system integration is discussed by Coonrod and Ernstoff (1977), and by Schwartz (1983). Diffractive optics and holographic optics are examined by Close (1975), Colburn and Fairchild (1982), Chorley (1974), and Lewis et al (1976). An extensive examination of operational problems associated with head-up displays was conducted by Newman (1980). Although concerned primarily with fixed (cockpit-mounted) head-up displays, some of the operational problems with fixed HUDs also are found with helmet mounted displays. Head-up display users and designers will find useful Newman's 1987 head-up display design guide. An excellent coverage of human factors issues in head-up display design is provided by Weintraub and Ensing (1992).

As noted elsewhere, any change to improve some characteristic will cause changes in other characteristics, often in an undesirable direction. It is often, even usually, necessary to balance competing demands by making compromises. Trade-offs must be made. A common example is compromising between the need for a wider field of view for improved situational awareness, and higher angular resolution to discern finer details of objects. When the total number of resolution elements is fixed, a wider field of view is accompanied by reduced (poorer) angular resolution.

Many of the helmet-mounted display system concerns mentioned in this section have been examined and researched by other investigators, and research continues. The optimum balance between conflicting requirements varies with tasks and missions. Because of this specificity, little useful data for making trade-offs is available. In operational situations, a great many factors or variables are

present and interacting, making it almost impossible to collect usable trade-off data. In addition, the most capable laboratory simulators available are very limited in their ability to simulate the effects of equipment, situations, and tactics on system performance. Because of the difficulty of adequately simulating the real world flight environment in the laboratory, it will probably be necessary to perform some of the necessary research in flight test vehicles.

APPENDICES

APPENDIX 1

DERIVATION OF THE GAUSS LENS EQUATION

The lens equation $1/U + 1/V = 1/F$ relates object distance U , image distance V , and lens focal length F . The proper name for this equation is the Gauss lens equation. It is sometimes referred to as the lens equation or the basic lens equation, although, in other documents, these names are sometimes applied to quite different equations. Because the Gauss lens equation is used extensively in the present paper, it will be derived in the following text.

The Gauss lens equation is based on three characteristics of lenses. First, a point light source on the optical axis of the lens and infinitely distant from the lens will be imaged at the second or rear focal point F_2 behind the lens. Second, a light ray parallel to the optical axis will also pass through the rear focal point F_2 . Third, a light ray passing through the first focal point F_1 in front of the lens will exit the lens parallel to the optical axis. Figure 17 is based on these three lens characteristics. From the figure, note that an object of height H_o lying on the lens axis at an object distance U in front of the lens is projected by the lens to form a real image at an image distance V behind the lens. Due to the light rays crossing the lens axis, the image is inverted relative to the object. In the figure, image height is H_i . Note, from the figure, that, at point B, a light ray traveling parallel to the lens axis and coming from point A of the object will pass through the rear focal point F_2 on its way to image point G. Another light ray from object point A is shown as passing through the first focal point F_1 of the lens, thus emerging from the lens parallel to the lens axis and also going to image point G.

From Figure 17, in right triangles AOF_1 and F_1CD , by corresponding parts of similar triangles being proportional, $H_o/H_i = (U - F)/F$. In similar triangles BCF_2 and F_2EG , $H_o/H_i = F/(V - F)$ by corresponding parts of similar triangles being proportional. Equating the H_o/H_i values from these two equations, $F(V - F) = (U - F)F$. Cross multiplying, $F^2 = (U - F)(V - F)$. This is Newton's form of the lens equation. Expanding the right side of the equation, $F^2 = UV - FV - FU + F^2$. Canceling the F^2 terms on the two sides of the equation and transposing FV and FU , $FV + FU = UV$. Dividing both sides of the equation by FUV , $1/U + 1/V = 1/F$. This is the Gauss lens equation.

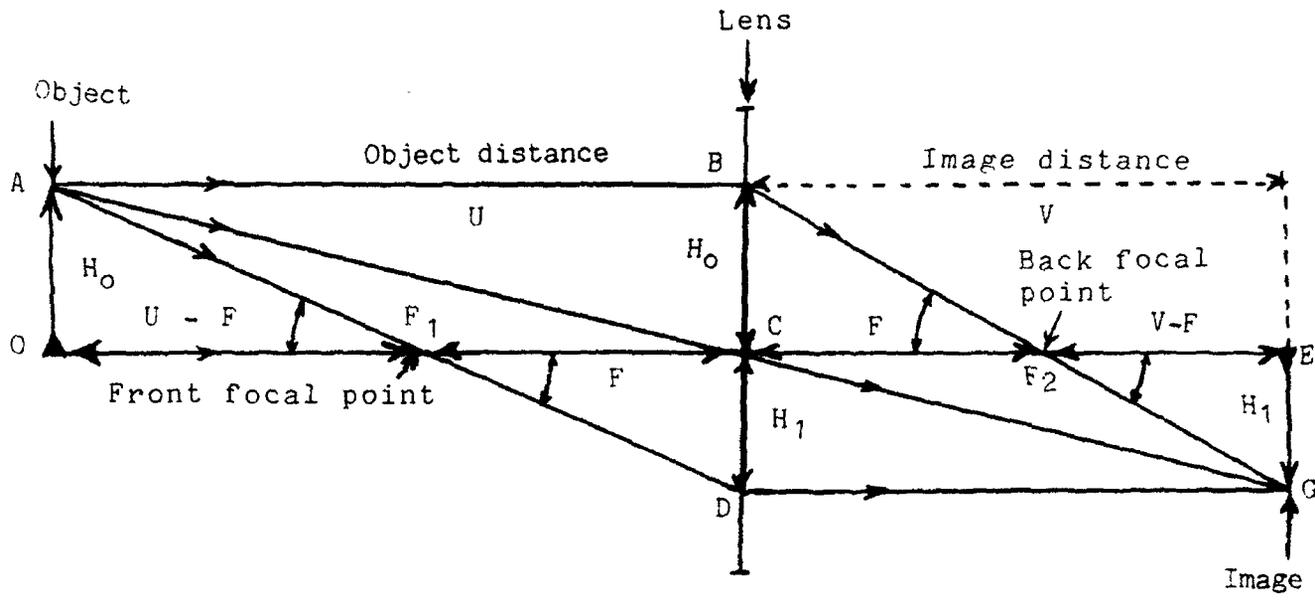


Fig. 17. The relationship of image size and distance to object size and distance and to the focal length of the imaging lens.

APPENDIX 2
EXIT PUPIL DIAMETER AND DISTANCE

It is of some tutorial value to derive equations for the diameter D_e of the exit pupil and its distance v'_e from the second nodal point of the eyepiece without using the Gauss lens equation, as was done in the body of this report. The geometry for the derivation is shown in Fig. 22. The basis of the derivations is the following:

- (1) A light ray from the entrance pupil passing through the first nodal point (point D) will emerge from the eyepiece parallel to the optical axis. This is ray BDHG.
- (2) A ray on the optical axis is undeviated. This is ray ADEF.
- (3) A ray passing through the center of a thin lens (point E) is undeviated. This is ray BEG.

In the figure, in triangles BAE and EFG, by corresponding parts of similar triangles being proportional, $BA/AE = FG/EF$, i.e., $(D_o/2)/(V_o + F_e) = (D_e/2)/v'_e$, from which $D_e = D_o v'_e / (V_o + F_e)$.

In triangles BAD and DEH, by corresponding parts of similar triangles being proportional, $BA/AD = EH/DE$, i.e., $(D_o/2)/V_o = (D_e/2)/F_e$, from which $D_e = (F_e/V_o)D_o$. Earlier in the text, by Eqn.(13), $M = -V_o/F_e$. Replacing F_e/V_o with $1/M$, $D_e = D_o(F_e/V_o) = D_o/M$, which is Eqn.(18).

Equating the two values of exit pupil diameter from above, $D_o v'_e / (V_o + F_e) = (F_e/V_o)D_o$, from which $v'_e = (V_o + F_e)(F_e/V_o) = F_e + F_e^2/V_o$, which is Eqn.(16).

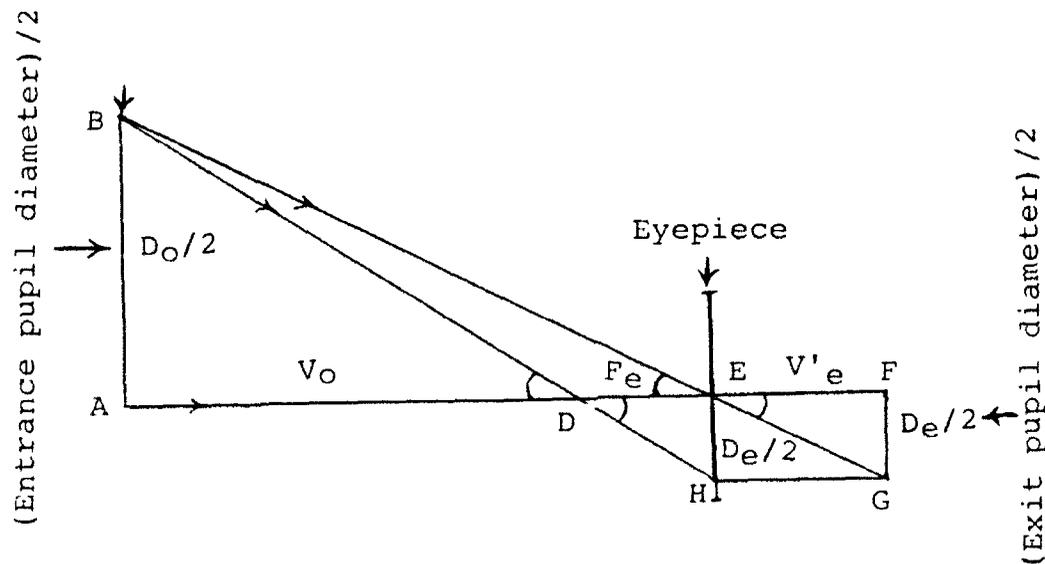


Fig. 18. Geometry for deriving equations for the diameter and distance of the exit pupil.

APPENDIX 3

'THE FOCAL LENGTH OF TWO COMBINED LENSES

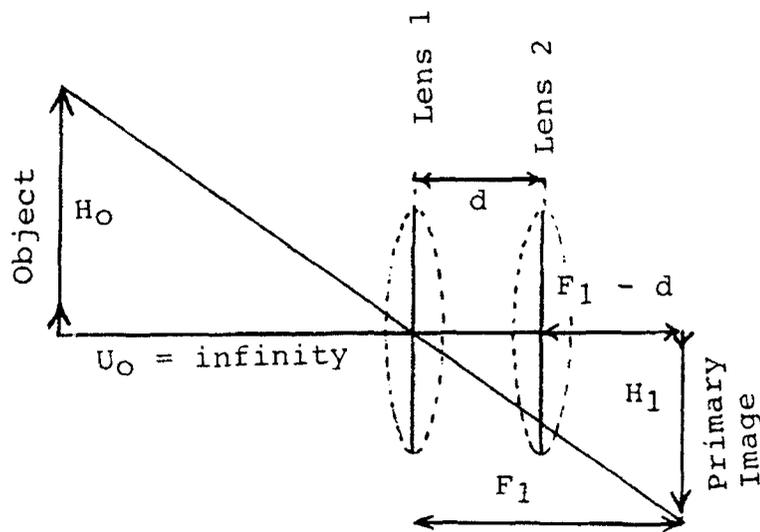
Two lenses separated by a distance d form an optical system that may be regarded as a combined lens with a focal length F_c . Let the focal length of the first lens be F_1 and the focal length of the second lens be F_2 . Assume that both lenses are optically thin lenses, i.e., their front-to-back thicknesses are negligible relative to object and image distances. The first lens projects a real inverted image, the primary image, which serves as the object for the second lens. The second lens projects an image of the primary image as a secondary image which is the output of the combined lenses. Object and image relationships are shown in figure 18.

The focal length of the two combined lenses is the focal length of the first lens multiplied by the linear magnification L.M.₂ of the second lens. By definition, L.M.₂ is the ratio of output image height to input image height. In part (B) of figure 18, it is shown that this linear magnification is the ratio of image distance V_2 to object distance U_2 . Since the image is inverted, the ratio is negative, so that $L.M._2 = -V_2/U_2$. Assume that the first lens is focussed upon an object at optical infinity, so that the primary image of the object is at a distance $V_1 = F_1$ behind the first lens. Assume that F_1 exceeds lens separation d , so that the primary image, the object for lens 2, is a distance $F_1 - d$ behind the second lens. Since this image is behind lens 2, object distance U_2 is negative, so that $U_2 = - (F_1 - d)$, as shown in figure 18.

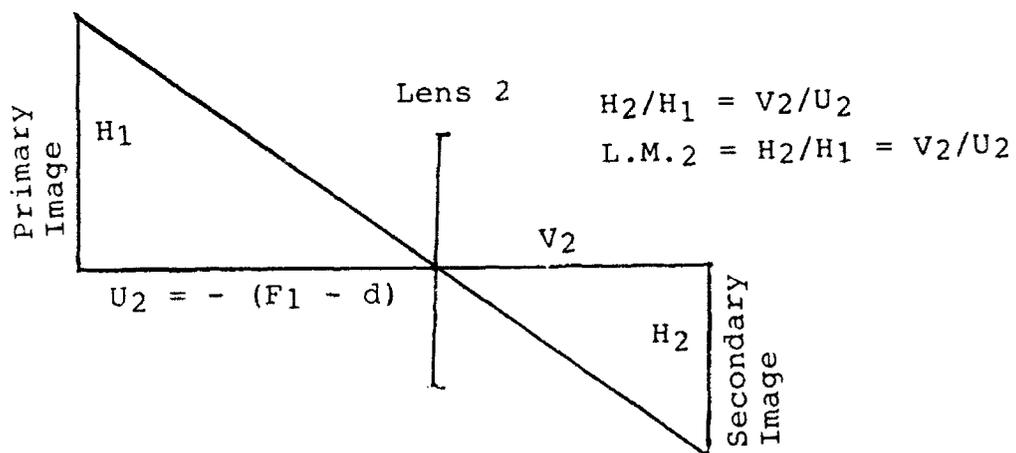
The Gauss lens equation for the second lens, $1/U_2 + 1/V_2 = 1/F_2$, with $-(F_1 - d)$ substituted for U_2 , becomes $-1/(F_1 - d) + 1/V_2 = 1/F_2$, from which $V_2 = F_2(F_1 - d)/(F_1 + F_2 - d)$. The linear magnification of the second lens, as noted above, is $L.M._2 = -V_2/U_2$. Substituting the values of V_2 and U_2 from above, $L.M._2 = - [F_2(F_1 - d)/(F_1 + F_2 - d)] / [- (F_1 - d)] = F_2/(F_1 + F_2 - d)$. The combined focal length of the two lenses, as noted above, is the focal length of the first lens multiplied by the linear magnification of the second lens:

$$\begin{aligned} F_c &= (L.M._2)F_1 \\ &= [F_2/(F_1 + F_2 - d)]F_1 \\ &= F_1F_2/(F_1 + F_2 - d) \end{aligned}$$

Note, from examining the above equation for combined focal length, that as lens separation d approaches zero, F_c approaches $F_1F_2/(F_1 + F_2)$. When $F_1 = F_2$, combined focal length approaches $F_1/2$ as d approaches zero, as might be expected. When lens separation $d = F_1 + F_2$, the primary image is a distance F_2 in front of lens 2, so that lens 2 images the secondary image at optical infinity. The two combined lenses, in this case, have a combined focal length of infinity. This system is the same as that of a telescope focussed on an object at optical infinity,



(A) Basic geometry with the first lens focussed on an object at optical infinity.



(B) Linear magnification by the second lens.

Fig. 18. Object-image relationships for two lenses separated by a distance d .

with the first lens being the objective and the second lens being the eyepiece.

As an example of the use of the equation for the combined focal length of two separate lenses forming a lens system, let $F_1 = 40\text{mm}$, and $F_2 = 20\text{mm}$, with $d = 30\text{mm}$. For this particular configuration, $F_C = F_1 F_2 / (F_1 + F_2 - d) = 40 \times 20 / (40 + 20 - 30) = 26.67\text{mm}$. If d were the sum of the two focal lengths, $F_1 + F_2 = 60\text{mm}$, then F_C is infinity, and the system could be used as a telescope focussed on distant objects, with the first lens being the objective and the second being the eyepiece. As shown earlier in this report, the telescope magnification would be $M = F_O / F_e = F_1 / F_2 = 40 / 20 = 2$ times or diameters. Turning the combination around, with the second lens facing the distant object and being the objective, and the first lens now being the eyepiece, the magnification would be $M = 20 / 40 = .5$ times. Increasing the lens spacing to more than 60mm would permit focussing for objects nearer than optical infinity.

The equation for combined focal length was derived with the assumption that the two lenses were thin lenses. It is also valid for real lenses, i.e., thick lenses. With thick lenses, the lens separation distance d is the distance between the second nodal point of the first lens and the first nodal point of the second lens.

APPENDIX 4

THE RELAY LENS IN AN OPTICAL VIEWING SYSTEM

A relay lens is a lens located between the objective and the eyepiece of an optical viewing system. It is sometimes called an erector or an erector lens. It may contain several optical elements. A relay lens projects an image of the primary image of the object or viewed scene that was projected by the objective, relaying the primary image to the eyepiece. A relay lens may be used to:

- (1) provide a display image that is, relative to the viewed object, erect and unreversed,
- (2) provide a system angular magnification that is different from magnification without the relay lens,
- (3) provide, with a given eyepiece, an exit pupil that is farther behind the eyepiece, thus increasing eye relief or eye clearance behind the eyepiece.
- (4) Permit use of an eyepiece with a longer focal length, hence longer exit pupil distance and eye relief.
- (5) Permit use of an objective of shorter focal length.

Since a relay lens can provide a secondary or relay image that is larger than the primary image provided by the objective, less magnification may be required of the eyepiece, permitting use of an eyepiece with a longer focal length. A longer focal length eyepiece provides more eye relief or eye clearance. However, it weighs more and is physically larger. The magnification provided by a relay lens may also allow using an objective with a shorter focal length.

The optical geometry of an optical viewing system containing an objective, a relay lens, and an eyepiece is shown in Figure 19. In an optical viewing system containing a relay lens, the object of height H_o to be observed is projected by the objective as a real inverted and reversed image of height H_1 . This image, since it is the first image in the system, is called the primary image. The primary image, as noted above, is a real image and serves as the object for the relay lens, which projects a real secondary image or relay image of height H_r of the primary image. The relay image is inverted and reversed with respect to the primary image. Since the primary image is inverted and reversed with respect to the object or scene to be observed, the relay image is erect and unreversed with respect to the object. The relay lens corrects the orientation of the primary image.

The linear magnification $L.M._r$ of the relay lens is, by the definition of linear magnification, the ratio of a linear dimension of the secondary image or relay image to the corresponding linear dimension of the primary image. The linear size of the image presented to the eyepiece for magnification is the linear size of the primary image of the object times the linear magnification $L.M._r$ of the relay lens. The amount of magnification provided by the relay lens is determined by its focal length F_r and its distance U_r from

the primary image. The magnification provided by a relay lens may be unity or may be more or less than unity.

The eyepiece of focal length F_e magnifies the relay image, providing a virtual display or final image. In projecting the display image, the eyepiece does not invert or reverse the correctly-oriented image projected by the relay image, so that the display or eyepiece image is erect and unreversed with respect to the object whose image is displayed. Although both the objective and the relay lens project real images that are behind them, the virtual image projected by the eyepiece is in front of the eyepiece, i.e., on the same side of the eyepiece as the image projected by the relay lens.

In an optical viewer containing only an objective and an eyepiece, the exit pupil is the image projected by the eyepiece of the light-filled clear aperture of the objective. The exit pupil is the image of the entrance pupil. In a system containing a relay lens, the relay lens projects a primary image of the featureless light-filled clear aperture of the objective as a uniform disc of light. This disc of light is an object for the eyepiece, and the eyepiece projects a second real image of it as the exit pupil of the instrument.

Understanding how a viewing system with a relay lens functions in forming images, for those not familiar with such systems, will be facilitated by a step-by-step equation-deriving analysis that progresses from the object to the exit pupil and the final display image. Figure 19 shows the imaging geometry of an optical viewing system that contains a relay lens. In figure 20, which is a dissection of figure 19, image formation by each lens in the system

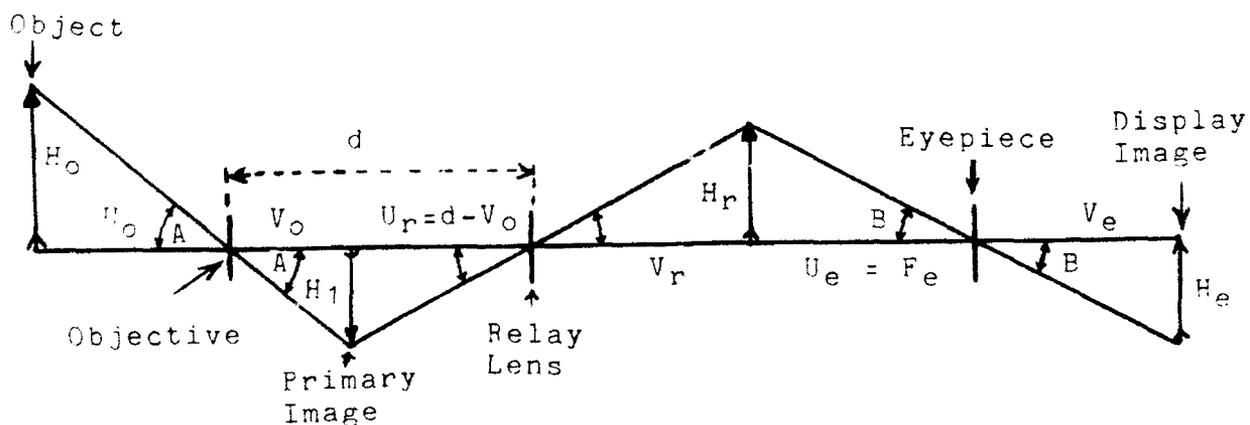
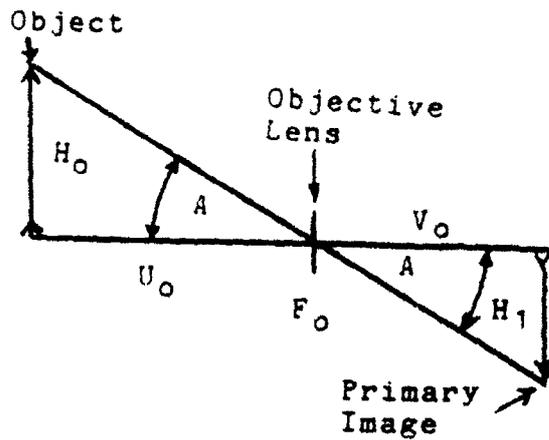
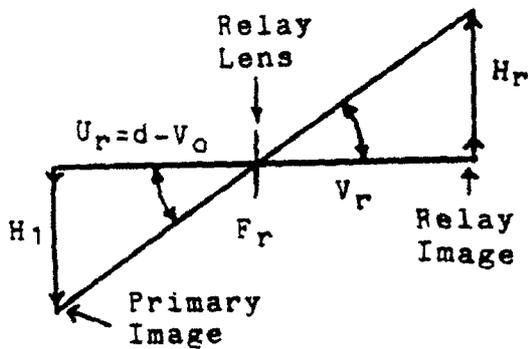


Fig. 19. Image formation by a viewing system with an objective, a relay lens, and an eyepiece.



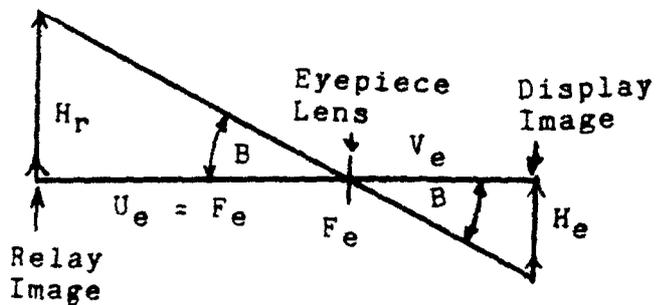
$$\begin{aligned}
 H_1/H_o &= V_o/U_o \\
 H_1 &= H_o V_o/U_o \\
 &= H_o V_o/[F_o V_o/(U_o - F_o)] \\
 &= H_o(U_o - F_o)/F_o
 \end{aligned}$$

A. Image formation by the objective lens.



$$\begin{aligned}
 L.M.r &= H_r/H_1 = V_r/U_r \\
 &= V_r/(d - V_o) \\
 H_r &= H_1 V_r/(d - V_o) \\
 &= H_o (V_o/U_o) V_r/(d - V_o)
 \end{aligned}$$

B. Image formation by the relay lens.



$$\begin{aligned}
 \tan B &= H_e/V_e = H_r/U_e \\
 \text{For a collimated image,} \\
 U_e &= F_e, \text{ so that} \\
 \tan B &= H_r/F_e.
 \end{aligned}$$

C. Image formation by the eyepiece.

Fig. 20. Image formation by the objective, by the relay lens, and by the eyepiece: a system dissection into the individual components.

is explicitly depicted. For clarity in exposition, in both figures, the final image or eyepiece image, which is the display image for the device user, is shown as being behind the eyepiece, although it is actually in front of the eyepiece. This does not invalidate the derivations of image sizes and locations. In the display system example which will be analyzed in the following pages, the distance d separating the objective and the relay lens is greater than the focal length F_o of the objective. Lens separation d is also large enough for the primary image to be farther in front of the relay lens than the focal length F_r of the relay lens.

The Gauss lens equation for the objective, when projecting the primary real image of the object, is $1/U_o + 1/V_o = 1/F_o$, from which

Eqn. (A) $V_o = F_o U_o / (U_o - F_o)$ Image distance behind the objective of the primary image of the object.

From Part A of Figure 20, by corresponding parts of similar triangles being proportional, $H_1/H_o = V_r/U_r$. By the definition of linear magnification, relay lens linear magnification is $L.M._r = -H_1/H_o$, minus because the image is inverted. Thus, $L.M._r = H_1/H_o = -V_r/U_r$, from which

Eqn. (B) $V_r = -L.M._r U_r$ Image distance from the relay lens to the relay lens image or secondary image of the object.

From the Gauss lens equation $1/U_r + 1/V_r = 1/F_r$ for the relay lens when it projects a real image,

Eqn. (C) $V_r = F_r U_r / (U_r - F_r)$ Image distance from the relay lens image to the secondary image of the object.

From Figure 20, $U_r = d - V_o$. Replacing U_r in Eqn. (C),

Eqn. (D) $V_r = F_r (d - V_o) / (d - F_r - V_o)$ Image distance of secondary image of object from relay lens.

From Eqn. (B), $V_r = -L.M._r U_r$, $L.M._r = -V_r/U_r$. Replacing V_r with, from Eqn. (C), $V_r = F_r U_r / (U_r - F_r)$,
 $L.M._r = -V_r/U_r = -[F_r U_r / (U_r - F_r)] / U_r =$

Eqn. (E) $L.M._r = -F_r / (U_r - F_r)$ Relay lens linear magnification.

As shown earlier in Figure 20, $U_r = d - V_o$. Replacing U_r ,
 $L.M._r = -F_r / (U_r - F_r) =$

Eqn. (F) $L.M._r = -F_r / (d - F_r - V_o)$ Relay lens linear magnification.

By dividing the numerator and the denominator of Eqn. (F) by F_r ,

it may be written as $L.M._r = -1/[(d - V_o/F_r - 1)]$. In this equation, $(d - V_o)/F_r$ decreases as F_r increases. In Eqn. (E), $L.M._r = -F_r/(d - F_r - V_o)$, $L.M._r$ increases as d decreases. Also, $L.M._r$ increases, for a fixed d , as V_o increases. Eqn. (A), $V_o = F_o U_o / (U_o - F_o)$, by dividing both the numerator and the denominator by U_o , becomes $V_o = F_o / (1 - F_o/U_o)$, which increases as U_o decreases. In other words, as object distance decreases, image distance increases. From this analysis, relay lens linear magnification $L.M._r$ increases as object distance U_o decreases.

The above results may be summarized by noting that the linear magnification provided by the relay lens increases when:

- (1) the object distance U_o decreases,
- (2) the focal length F_r of the relay lens increases (with a fixed d),
- (3) the distance d between the objective and the relay lens decreases.

By Eqn. (F),

$$L.M._r = -F_r / (d - F_r - V_o). \text{ Replacing } V_o \text{ with, from Eqn. (A),}$$

$$V_o = F_o U_o / (U_o - F_o),$$

$$L.M._r = -F_r / [(d - F_r) - F_o U_o / (U_o - F_o)]$$

$$= -F_r / \{ [(d - F_r)(U_o - F_o) - F_o U_o] / (U_o - F_o) \}$$

$$\text{Eqn. (G) } L.M._r = -F_r(U_o - F_o) / [(d - F_r)(U_o - F_o) - F_o U_o]$$

Relay lens linear magnification.

From Eqn. (C), $V_r = F_r U_r / (U_r - F_r)$, replacing U_r with $(d - V_o)$,

$$V_r = F_r U_r / (U_r - F_r) =$$

$$\text{Eqn. (H) } V_r = F_r(d - V_o) / (d - F_r - V_o) \text{ Secondary or relay}$$

scene image distance behind the relay lens.

Equating the values of V_r given by Equations (B) and (C), neglecting the minus sign, $L.M._r U_r = F_r U_r / (U_r - F_r)$, from which

$$\text{Eqn. (I) } U_r = (F_r / L.M._r) (L.M._r + 1) \text{ Distance of primary scene image}$$

in front of the relay lens.

From Part A of Figure 20, by corresponding parts of similar triangles being proportional, $H_1/H_o = V_o/U_o$, from which

$$\text{Eqn. (J) } H_1 = H_o(V_o/U_o) \text{ Height of primary image of}$$

the object or the viewed scene.

Note that (V_o/U_o) is the linear magnification of the objective. The height of the primary image of the object or scene is the height of the object times the linear magnification of the objective.

From Part B of Figure 20, by the proportionality of corresponding parts of similar triangles, $H_r/H_1 = V_r/U_r = L.M._r$, from which $H_r = H_1 L.M._r$. Replacing H_1 with $H_1 = H_o V_o / U_o$ from Eqn. (J), $H_r = H_1 L.M._r =$

$$\text{Eqn. (K) } H_r = H_o(V_o/U_o)L.M._r \text{ Height of secondary scene image,}$$

i.e. of secondary image of object.

Note, from Eqn. (K), that the height of the secondary image of the object is the height of the object multiplied by the linear magnifications of both the objective and the relay lens.

The angular magnification M of the optical viewing system is defined as

$$M = (\text{Angular subtense of display image of the object}) / (\text{Angular subtense at the objective of the object})$$

$$= (\text{Angle B}) / (\text{Angle A}), \text{ approximated by the ratio of the tangents,}$$

$$\approx \text{Tan B} / \text{Tan A. From Part C of Figure 20,}$$

$$\text{Tan B} = H_e / V_e$$

$$= H_r / U_e$$

$$= H_r / F_e. \text{ From Eqn. (K), } H_r = (H_o V_o / U_o) \text{L.M.}_r.$$

Replacing H_r ,

$$\text{Tan B} = H_r / F_e$$

$$= H_o V_o \text{L.M.}_r / F_e U_o.$$

$$\text{From Part A of Figure 20, Tan A} = H_1 / H_o = H_o / U_o.$$

Replacing Tan B and Tan A with these values,

$$M = [H_o V_o \text{L.M.}_r / (F_e U_o)] / (H_o / U_o) =$$

$$\text{Eqn. (L) } M = (V_o / F_e) \text{L.M.}_r \text{ Angular magnification of the optical viewing system.}$$

Earlier, in Section 8 of this tutorial, by Eqn. (13), $M = - (V_o / F_e)$ for an optical viewing system without a relay lens. From this equation, and from and Eqn. (L), note that the angular magnification of a system with a relay lens is the angular magnification of a system without a relay lens times the linear magnification of the relay lens. In effect, primary image distance V_o is multiplied by the linear magnification L.M._r of the relay lens.

From Eqn. (L), it is clear that increased primary image distance V_o , from decreased distance of the object or viewed scene, or from an objective of longer focal length F_o , increases viewing system angular magnification M. It is also apparent from this equation that decreased eyepiece focal length F_e increases M. From these observations, and the discussion following the derivation of Eqn. (F), it may be concluded that:

System angular magnification increases with:

- (1) A longer focal length objective,
- (2) A shorter focal length eyepiece,
- (3) A shorter object distance,
- (4) A relay lens of longer focal length,
- (5) A reduced space between objective and relay lens,
- (6) Increased relay lens linear magnification.

In this list note that the greater relay lens linear magnification of (6) would be due to (4) or (5), or to a combination of them. It must also be kept in mind that conditions or parameter relationships other than those assumed at the start of the system analysis may lead to different conclusions.

The optical geometry of an optical viewing system with a relay lens when the system is forming the exit pupil of the system is shown in Figure 21. For the relay lens, the light-filled featureless disc that is the entrance pupil of the objective is an

object. The relay lens projects this image as a real image, also a featureless disc, that is the primary image of the entrance pupil of the objective. In doing this, the relay lens obeys the Gauss lens equation $1/U'_R + 1/V'_R = 1/F_R$. The primes are used to avoid confusion with the equation for the relay lens when it projects an image of the scene or object imaged by the objective, rather than an image of the entrance pupil. In this equation, U'_R is the distance of the entrance pupil of the objective from the relay lens. This distance is d . Substituting d for U'_R in the Gauss lens equation for the relay lens when projecting a real image, $1/d + 1/V'_R = 1/F_R$, from which

Eqn. (M) $V'_R = dF_R/(d - F_R)$ Relay lens image distance when projecting the primary image of the entrance pupil of the objective

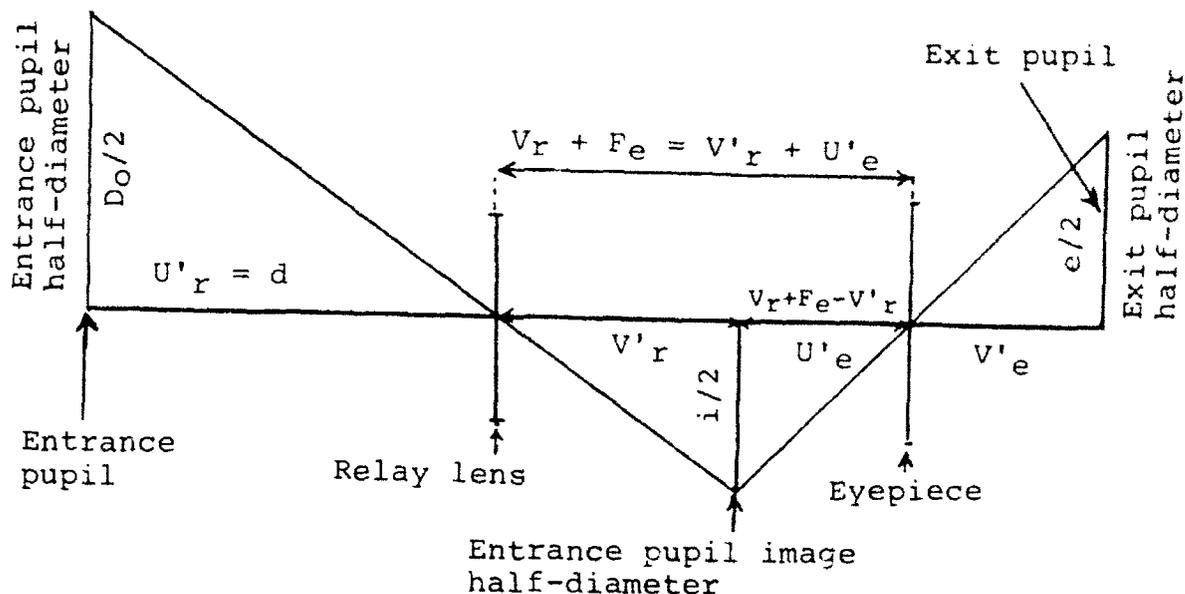


Fig. 21. Optical geometry for determining the exit pupil diameter of a viewing system containing a relay lens.

In Figure 21, the real primary image projected by the relay lens of the entrance pupil of the objective has an image height i . Also from the figure, by the proportionality of corresponding parts of similar triangles, $(i/2)/(D_o/2) = V'_R/U'_R = V'_R/d$, from which $i = (D_o/d)V'_R$. Replacing, from Eqn. (M),

$$i = (D_o/d)V'_R = (D_o/d)[F_R d/(d - F_R)] =$$

Eqn. (N) $i = D_o F_R/(d - F_R)$ Height of primary image of the entrance pupil.

From Figure 21, the distance between the relay lens and the eyepiece is $V_R + F_e$. This distance is also $V'_R + U'_e$. Equating the two distance values, $V_R + F_e = V'_R + U'_e$, from which

Eqn. (O) $U'_e = V_R - V'_R + F_e$ Distance from the eyepiece of the primary image of the entrance pupil.

In Eqn. (O), replacing, from Eqn. (M), $V'_r = dF_r/(d - F_r)$, and replacing, from Eqn. (H), $V_r = F_r(d - V_o)/(d - F_r - V_o)$,

$$\begin{aligned}
 U'_e &= V_r - V'_r + F_e \\
 &= F_r(d - V_o)/(d - F_r - V_o) - dF_r/(d - F_r) + F_e \\
 &= F_r[(d - V_o)/(d - F_r - V_o) - d/(d - F_r)] + F_e \\
 &= F_r[(d - V_o)(d - F_r) - d(d - F_r - V_o)]/(d - F_r)(d - F_r - V_o) \\
 &\quad + F_e \\
 &= F_r[\cancel{d^2} - \cancel{dF_r} - \cancel{dV_o} + F_rV_o - \cancel{d^2} + \cancel{dF_r} + \cancel{dV_o}]/(d - F_r)(d - F_r - V_o) + F_e \\
 &= F_r^2V_o/(d - F_r)(d - F_r - V_o) + F_e \text{ Replacing } V_o \text{ with, from Eqn. (A), } V_o = F_oU_o/(U_o - F_o), \\
 U'_e &= [F_r^2F_oU_o/(U_o - F_o)(d - F_r)]/[(d - F_r) - F_oU_o/(U_o - F_o)] + F_e \\
 &= [F_r^2F_oU_o/(U_o - F_o)(d - F_r)]/[(d - F_r)(U_o - F_o - F_oU_o)/(U_o - F_o)] + F_e \\
 &= [F_r^2F_oU_o/(U_o - F_o)(d - F_r)](U_o - F_o)/[(d - F_r)(U_o - F_o) - F_oU_o] + F_e =
 \end{aligned}$$

Eqn. (P) $U'_e = F_r^2F_oU_o/(d - F_r)[(d - F_r)(U_o - F_o) - F_oU_o] + F_e$
 Image distance from the eyepiece of the primary image of the entrance pupil.

In projecting a real image of this primary image of the entrance pupil of the objective, the eyepiece obeys the Gauss lens equation $1/V'_e + 1/V'_e = 1/F_e$, from which

$V'_e = F_eU'_e/(U'_e - F_e)$. Replacing U'_e with, from Eqn. (O),

$$\begin{aligned}
 U'_e &= V_r + F_e - V'_r \\
 V'_e &= F_e(V_r + F_e - V'_r)/(V_r + F_e - V'_r - F_e) \\
 &= F_e(V_r + F_e - V'_r)/(V_r - V'_r); \text{ Splitting into two parts,} \\
 &= F_e(V_r - V'_r)/(V_r - V'_r) + F_e^2/(V_r - V'_r) \\
 &= F_e + F_e^2/(V_r - V'_r). \text{ Rearranging terms,}
 \end{aligned}$$

Eqn. (Q) $V'_e = F_e^2/(V_r - V'_r) + F_e$ Exit pupil distance.

Replacing, from Eqn. (D), $V_r = F_r(d - V_o)/(d - V_o - F_r)$, and replacing, from Eqn. (M), $V'_r = dF_r/(d - F_r)$,

$$\begin{aligned}
 V'_e &= F_e^2/(V_r - V'_r) + F_e \\
 &= F_e^2/[F_r(d - V_o)/(d - V_o - F_r) - dF_r/(d - F_r)] + F_e. \\
 &\text{Factoring out } F_r, \\
 &= (F_e^2/F_r)/[(d - V_o)/(d - V_o - F_r) - d/(d - F_r)] + F_e. \text{ Putting the denominator terms over a common denominator,} \\
 &= (F_e^2/F_r)/\{[(d - V_o)(d - F_r) - d(d - V_o - F_r)]/(d - F_r)(d - V_o - F_r)\} + F_e. \\
 &= (F_e^2/F_r)(d - F_r)(d - V_o - F_r)/[(d - V_o)(d - F_r) - d(d - V_o - F_r)] + F_e. \text{ Expanding this equation,} \\
 &= (F_e^2/F_r)(d - F_r)(d - V_o - F_r)/[\cancel{d^2} - \cancel{dF_r} - \cancel{dV_o} + F_rV_o - \cancel{d^2} + \cancel{dV_o} + \cancel{dF_r}] + F_e. \text{ Removing terms that cancel each other.} \\
 &= (F_e^2/F_r)(d - F_r)(d - V_o - F_r)/[F_rV_o] + F_e \\
 &= (F_e^2/F_r^2V_o)(d - F_r)(d - V_o - F_r) + F_e \\
 &\text{From Eqn. (A), } V_o = F_oU_o/(U_o - F_o), \text{ from which } 1/V_o = (U_o - F_o)/F_oU_o. \text{ replacing } 1/V_o \text{ and } V_o,
 \end{aligned}$$

$$V'_e = (F_e/F_r^2) [(U_o - F_o)/F_o U_o] (d - F_r) [(d - F_r) - F_o U_o / (U_o - F_o)] + F_e$$

$$= (F_e^2/F_r^2) [(U_o - F_o)/F_o U_o] (d - F_r) [(d - F_r)(U_o - F_o) - F_o U_o] / (U_o - F_o) + F_e$$

Eqn. (R) $V'_e = (F_e^2/F_o F_r^2 U_o) (d - F_r) [(d - F_r)(U_o - F_o) - F_o U_o] + F_e$ Exit pupil distance.

From Figure 21, since corresponding parts of similar triangles are proportional, $(e/2)/(i/2) = V'_e/U'_e$, from which $e = i(V'_e/U'_e)$. Also from Figure 21, $V'_e/U'_e = (e/2)/(i/2)$, which is the linear magnification $L.M._e$ of the eyepiece. Replacing V'_e/U'_e with $L.M._e$, $e = i(V'_e/U'_e) = iL.M._e$. The eyepiece, in projecting the relay image of the entrance pupil as the exit pupil of the instrument, obeys the Gauss lens equation $1/U'_e + 1/V'_e = 1/F_e$, from which $U'_e = F_e V'_e / (V'_e - F_e)$. Replacing U'_e with this,

$$e = i(V'_e/U'_e)$$

$$= iV'_e / [F_e V'_e / (V'_e - F_e)]$$

$$= (i/F_e) (V'_e - F_e)$$

Replacing, from Eqn. (R),

$$V'_e = (F_e^2/F_o F_r^2 U_o) (d - F_r) [(d - F_r)(U_o - F_o) - F_o U_o] + F_e$$

$$e = (i/F_e) \{ (F_e^2/F_o F_r^2 U_o) (d - F_r) [(d - F_r)(U_o - F_o) - F_o U_o] + F_e - F_e \}$$

$$= (iF_e/F_o F_r^2 U_o) (d - F_r) [(d - F_r)(U_o - F_o) - F_o U_o]$$

Replacing, from Eqn. (N), $i = D_o F_r / (d - F_r)$,

$$= [D_o F_r / (d - F_r)] (F_e/F_o F_r^2 U_o) (d - F_r) [(d - F_r)(U_o - F_o) - F_o U_o] =$$

Eqn. (S) $e = (D_o F_e / F_o U_o) [(d - F_r)(U_o - F_o) - F_o U_o]$ Exit pupil diameter.

An alternative procedure for deriving an equation for exit pupil diameter is to use the equation

$$e = D_o / M. \text{ Replacing, from Eqn. (L), } M = (V_o / F_e) L.M._r,$$

$$e = D_o / [(V_o / F_e) L.M._r]$$

$$= D_o F_e (1/V_o) (1/L.M._r) \text{ From Eqn. (A) of Section 8,}$$

$$V_o = F_o U_o / (U_o - F_o), \text{ from which}$$

$$(1/V_o) = (U_o - F_o) / F_o U_o. \text{ From Eqn. (G),}$$

$$L.M._r = F_r (U_o - F_o) / [(d - F_r)(U_o - F_o) - F_o], \text{ from which}$$

$$(1/L.M._r) = [(d - F_r)(U_o - F_o) - F_o U_o] / F_r (U_o - F_o).$$

Replacing $(1/V_o)$ and $(1/L.M._r)$ with these values,

$$e = D_o F_e [(U_o - F_o) / F_o U_o] [(d - F_r)(U_o - F_o) / F_r (U_o - F_o)], \text{ which is Eqn. (S) above.}$$

When the display image is collimated and a point light source on a black background is the object, the light exiting the eyepiece is a parallel beam with a diameter $e = D_o / M$. However, in many actual systems containing a relay lens, the entrance pupil of the relay lens is small and will not pass all of the light from the objective. In this case, the system aperture stop is the entrance pupil of the relay lens, rather than the entrance pupil of the objective. The exit pupil diameter is reduced and the exit pupil is closer to the eyepiece. When the relay lens is the system aperture stop, the system entrance pupil is the image formed by the objective in object space (the space in front of the objective) of the entrance pupil of the relay lens, not the entrance pupil of the objective. In this case, the system entrance pupil may be an

appreciable distance in front of the objective.

When the relay lens is the aperture stop, the Gauss lens equation of the eyepiece in projecting the exit pupil is $1/U''_e + 1/V''_e = 1/F_e$, from which $V''_e = F_e U''_e / (U''_e - F_e)$. Here, the distance of the relay lens from the eyepiece is U''_e , and the exit pupil distance is V''_e . From Figure 21, $U''_e = V_r + F_e$. Replacing U''_e ,

$$V''_e = F_e U''_e / (U''_e - F_e)$$

$$= F_e (V_r + F_e) / (V_r + F_e - F_e) =$$

Eqn. (T) $V''_e = F_e + F_e^2/V_r$ Exit pupil distance when the relay lens is the system aperture stop.

When the relay lens is not the aperture stop, by Eqn. (Q), $V'_e = F_e + F_e^2/(V_r - V'_r)$. Since $(V_r - V'_r)$ is smaller than V_r , $V'_e = F_e + F_e^2/(V_r - V'_r)$ is clearly larger than $V'_e = F_e + F_e^2/V_r$. In other words, exit pupil distance is reduced when the relay lens acts as the system aperture stop.

The eyepiece of an optical viewer determines both the apparent field of view and the instrument or true field of view, whether or not the device contains a relay lens. The apparent field of view is the angle subtended at the eye of the observer by the effective or clear aperture of the eyepiece. The true or instrument field of view is the field of view of the external environment, defined as the angular subtense at the objective of the field of view that is included in the display presented to the observer. In optical display systems with or without a relay lens, the true field of view or instrument field of view A is the apparent field of view B divided by the angular magnification of the instrument.

Eqn. (U) $A = B/M$ Instrument or true field of view.
 B is eyepiece or apparent field of view.

REFERENCES

- Birt, J. A. and Task, H. L. (Editors). Proceedings of a Symposium on Visually-Coupled Systems: Development and Application. AMD-TR-73-1 (Sep. 1973). Aerospace Medical Research Laboratory, Wright-Patterson AFB OH, (NTIS 916 572)
- Buchroeder, R. A. and Kocian D. F. Display System Analysis for the LHX Helicopter Application (U). AAMRL-TR-89-001 (1989). Armstrong Aerospace Medical Research Laboratory, Wright-Patterson AFB OH, (NTIS B 134 738, Limited Distribution)
- Chorley, R. A. Head-Up Display Optics. AGARD Lecture Series No. 11, Electro-Optics, (Sep. 1974) (NTIS AD 787 014)
- Close, D. H. Holographic Optical Elements. Optical Engineering, Vol. 14, No. 5, (Oct. 1975)
- Colburn, W. S. and R. C. Fairchild. Design Study for a Low-Distortion Holographic HUD. Air Force Wright Aeronautical Laboratories Technical Report, AFWAL-TR-81-1263 (Jan 1982), Wright-Patterson AFB OH, (NTIS A 113 982)
- Coonrod, J. F. and Erntoff, M. N. Advanced HUD Technology - the Integrated HUD. NAECON Conference Record, May 1977, Page 871
- Farrell, R. J. and Booth, J. M. Design Handbook for Imagery interpretation Equipment, (1984). Boeing Aerospace Company, Seattle, Washington
- Gard, J. H. Holographic HUD de-mystified. NAECON Proceedings, 1982, pp. 752-759
- Ginsburg, A. P. Direct performance assessment of HUD display systems using contrast sensitivity. In Martin, W. L. (Editor). (See W.L. Martin et al Reference, Dec. 1983)
- Hardy, A. C. and Perrin, F. H. The Principles of Optics, McGraw-Hill Book Company, Inc., New York, 1932.
- Hecht, E. Optics. Addison-Wesley Publishing Co., New York, 1987.
- Herschberger, E. and Guerin, D. F. Binocular rivalry in helmet-mounted display. AMRL-TR-75-48, (June 1975). Aerospace Medical Research Laboratory, Wright-Patterson AFB OH, (NTIS A-013838)
- Hughes, R. L., Chason, L. R., and Schwank, C. H. Psychological problems in the design of helmet-mounted displays and sights: Overview and annotated bibliography. AMRL-TR-73-16, (Aug. 1973). Aerospace Medical Research Laboratory, Wright-Patterson AFB OH, (NTIS A 770-297)
- Jacobs, D. H. Fundamentals of Optical Engineering. McGraw-Hill Book Company, Inc. New York, (1942)

- Johnson, B. K. Optics and Optical Instruments. Dover Publications, Inc., New York, (1960)
- Kocian, D. F. Design considerations for virtual panoramic display (VPD) Helmet Systems. AGARD Symposium on man-machine interface in tactical aircraft design and combat automation, AGARD-CP-425 (July 1983)
- Lewis, W. N., D.H. Close, and R. S. Jacobs. Diffraction optics study Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433-6573 (NTIS A043 632)
- Martin, W. L. (Editor), L. V. Genco, A. P. Ginsburg, H. C. Self, H. L. Task, R. Lee, R. W. Schwartz, and W. Wilson. Optical and human performance evaluation of HUD systems design. AFAMRL-TR-84-095 (December 1983). Air Force Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433-6573 (NTIS A140 601)
- Melzer, J. E, and Kirk Moffitt. Partial binocular overlap in helmet-mounted displays. SPIE Vol. 1117 Display system optics 11 (1988)
- Newman, R. L. Operational problems associated with head-up displays during instrument flight. AFAMRL TR-80-116 (October 1980). Crew Systems consultants TR-79-20 (NTIS A 092 954)
- Newman, R. L. Improvement of head-up display standards. Vol. 1: Head-up display design guide (Appendix). AFWAL TR-87-3055 (1987) (Crew Systems Consultants). Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433-6553 (NTIS ADA 192973)
- O'Shea, D. C. Elements of Modern Optical Design, John Wiley & Sons. Inc., New York, 1985
- Peli. E. Visual issues in the use of a head-mounted monocular display. Optical-Engineering Aug. 1990, Vol 29, No. 8, page 883.
- Porac C. and S. Corey. Sighting dominance and binocular rivalry. Am. J. Physiol. Opt. 55, pp.208-213 (1978)
- Schwartz, R. W. Physical integration of the HUD. In Martin, W. L, (Editor). (see W. L. Martin et al reference, Dec. 1983)
- Self, H. C. Optical tolerances for alignment and image differences for binocular helmet-mounted displays. AAMRL TR-86-019 (May 1968). Harry G. Armstrong Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433-6573 (NTIS A 174 536)
- Southall, J. P. C. Introduction to Physiological Optics. Dover, New York, 1961 (Originally published in 1938).
- Task, H. L., D. F. Kocian and J. H. Brindle. Helmet-mounted displays: Design considerations. Advancement of Visualization Techniques. AGARDograph No. 225 October 1980

Task, H. L. Measurement of HUD Optical Quality. In Martin, W. L. (See W. L. Martin et al reference, Dec. 1983).

Weintraub, D. J., and Ensing, M. State of the Art Report: Human Factors in Head-Up Display Design: The Book of HUD. May, 1992. Armstrong Laboratory. AL/CFH/CSERIAS, Wright-Patterson Air Force Base, Ohio 45433-6573.

BIBLIOGRAPHY

- Buchroeder, R. A., Helmet-mounted Displays, Tutorial T2. SPIE 1987 Technical Symposium 21, May 1987.
- Chisum, G. T. and P. E. Morway, Visual Accommodation responses in a Virtual Image Environment, NADC 79216-60, 1977 (NTIS AD-A074415).
- D. H. Close, A. Au, and A. Graube Holographic Lens for Pilot's Head-up Display, Hughes Aircraft , August 1874 (NTIS AD-787605).
- Dekker, F. E. D., "Head-up Display Symbology," Problems of the Cockpit Environment, AGARD CP-55, November 1968.
- Doucette, A. R., "Design Decisions for a Head-up Display," IEE Spectrum, 13, August 1976, 28-32
- Egan, D. E. and J. E. Goodson, Human Factors Engineering for Head-up Displays: a Review of Military Specifications and Recommendations for Research, NAMRL Monograph 23, April 1978.
- Fischer, E., R. F. Haines, and T. A. Price, Cognitive Issues in Head-up Displays, NAMRL Monograph 23, April 23 1978.
- Fry, Glenn A. Geometrical Optics, Chilton Book Company, New York 1969.
- Gibson, C. P., "Binocular Disparity and Head-up Displays," Human Factors 22, 1980. 435-444.
- Gibson, C. P., Collimation and the Human Visual System--A Review and Literature Search, RAE TM-FS-173, April 1978.
- Ginsburg, A. P., W. L. Martin, and H. C. Self, "Contrast Sensitivity Performance Assessment of HUD Display Systems," Proceedings of the 2nd Symposium on Aviation Psychology, Ohio State University, April 1983, pp.473-480.
- Gold, T. and A. Hyman, Visual requirements Study for Head-up Displays, JANAIR 680712, March 1970.
- Gold, T. and R. F. Perry. Visual Requirements Study for Head-up Displays, JANAIR 700407, March 1972.
- Hull, J. C., R. T. Gill, and S. N. Roscoe, "Locus of the Stimulus to Visual Accommodation: Where in the World, or Where in the Eye?," Human Factors 24, 1982, 311-319.
- Kane, R. S., Study of Requirements for Head-up Displays (HUD), NASA CR-66912, March 1970.
- Kingslake, R. (Editor), Applied Optics and Optical Engineering. Volume 3, Academic Press, New York, 1965.
- Leitner E. F. and R. F. Haines, Magnitude of Visual Accommodation to a Head-up Display. NASA TP-1796, February 1981.

Military Specification: Display, Head-up, General Specification for, MIL-D-81641 (AS), June 1972.

Naish, J. M., Properties and Design of the Head-up Display, Douglas DP-4951, April 1968. McDonnell Douglas 409, February 1970.

Naish, J. M., "Factors Affecting Head-up Display Design," 8th Annual Symposium on Human Factors in Electronics, Palo Alto, May 1967.

Newman, R. L. Operational Problems Associated with Head-up Displays During Instrument Flight, AFAMRL TR-80-116, October 1980; Crew Systems Consultants TR-79-20 (NTIS A 092 954).

Newman, R. L. and T. G. Foxworth, A Review of Head-up Display Specifications, Crew Systems Consultants TR-84-04, April 1984.

Roscoe, S. N., S. G. Hasler, and D. J. Dougherty, "Flight by Telescope: Making Takeoffs and Landings; the Influence of Image Magnification; Practice and Various Conditions of Flight," Human Factors, 3, 1966, pp. 13-40.

Shrager, J. J. Head-up Displays: A Literature Review and Analysis with an Annotated Bibliography, FAA RD-78-31, April 1978.

Strong, J. Concepts of Classical Optics, W. H. Freeman and Company, San Francisco, 1959.

Wallace, F. L., "Head-up Displays... Some Unanswered Questions," 15th Alpha Air Safety Forum, July 1968.

Warren, R., L. V. Genco, and T. R. Connon, Horizontal Diplopia Thresholds for Head-up Displays, AFAMRL TR-84-18, April 1984 (NTIS A 141 965).

GLOSSARY

Aberration. An error or defect in an image. Rays of light from an object do not converge precisely at the corresponding image points, the conjugate points.

Alphanumeric. Letters and numerals.

Aperture. An opening that limits the amount of light passing into or out of an optical element or system.

Apparent Field of View. (1) The field of view as it appears to an observer looking into an eyepiece. (2) The angle subtended at the eye by the display. When the display is not circular, FOV may be defined as the angular diagonal of the display or by the vertical and horizontal angular subtenses at an observer's eye.

Beamsplitter. An optical device that transmits part of the incident light and reflects part of it. It may be wavelength-neutral, or it may be selective, i.e., have different wavelength compositions in the reflected transmitted energies.

Cathode ray tube (CRT). An evacuated (or vacuum) tube in which a thin beam of high-speed electrons scans over a layer of phosphor coated on the back of a transparent faceplate, causing the phosphor to emit light (glow). A type of picture tube, such as found on most current television receiving sets and HMDs.

Combiner. A beamsplitter that permits superimposition upon an external scene of an image from a CRT or other image source.

Compound magnifier. An optical system containing an objective for projecting a real primary image and an eyepiece for magnifying the primary image to present to an observer a virtual display image.

Cornea. The transparent lens-shaped front surface of the eyeball. The first optical surface of the eye.

Dichroic Coating. (Dichroic means two-color). An extremely thin wavelength-selective coating on an optical element. Such a coating that reflects part of the electromagnetic spectrum, e.g., a narrow band of green, and transmits the remainder of the spectrum. Dichroic beamsplitters are used on some helmet-mounted displays to reflect most of the green from a narrow-band green-emitting CRT phosphor, while transmitting most of the light from the outside scene, except for a narrow band of green..

Electromagnetic. Pertaining to that type of radiation which includes X-rays, ultraviolet, visible light, infrared, and radio and television broadcast wavelengths. Called electromagnetic because such radiant energy consists of transverse electric and magnetic oscillating fields propagating together through space.

Entrance Pupil. The virtual image of the system aperture stop in object space formed by the optical elements preceding the stop. For example, the eye's entrance pupil is the virtual image of the real eye pupil (the iris) formed by the cornea and aqueous humor.

Erector. A lens or prism system that turns an upside-down image over, making it erect. It corrects image orientation to make image orientation correspond to object orientation.

Eversion (reversing). Turning an image right-to-left to correct image orientation.

Exit pupil distance. The distance from the second or rear nodal point of the eyepiece to the exit pupil.

Exit Pupil. The image of the aperture stop formed by the optical elements following the stop. It is a uniform disc of light that contains all of the energy available for the display. The eye's entrance pupil should intercept some of the light that is used to form the exit pupil.

Eye Clearance(≠Eye Relief). The distance from the cornea of the eye to the nearest optical or mechanical element of the optical system. Eye clearance is less than eye relief, because the exit pupil should be at the eye entrance pupil which is approximately 2 1/4mm within the eye. The nearest optical element may be a beamsplitter rather than the eyepiece.

Eyepiece. An optical device consisting of one or more optical elements with focussing power that acts as a magnifier. It is used to magnify the primary image from the objective, thus providing a virtual display image for an observer. It is called an eyepiece because it is the part of the device nearest the eye.

Field of View (FOV). The angular subtense (usually measured in degrees or radians) of the display at the objective (true or instrument FOV) or at the eye (apparent FOV or AFOV).

Focal Length. (1). A measure of the light-bending or refractive power of a lens or other image-forming optical device, (2) The distance from the rear nodal point of a lens to the image projected by the lens when imaging an object at optical infinity or very distant. For distant objects, image dimensions are proportional to focal length. (3) The characteristic of a lens or a curved mirror that relates image distance to object distance according to the Gauss lens equation.

f-number. (Also called relative aperture). The ratio of focal length to effective aperture.

Helmet-Mounted Display (HMD). A system, usually electro-optical, mounted on a helmet to present images to an observer.

Holographic (optical element). An optical element that works by diffraction.

Image. An optical representation, copy, or duplication of an object, scene or prior image.

Interpupillary Distance (IPD). The distance between the centers of an observer's eye pupils when the lines of sight of the two eyes are parallel, as when observing a distant object.

Inversion. Turning an image upside-down to provide an image that is right-side-up (erect) relative to an object or scene.

Kell Factor. A dimensionless number that is multiplied times the number of TV-type scan lines when determining resolution to take into account the overlapping of the lines due to the Gaussian distribution of energy across the width of the lines. The value usually quoted for the Kell factor is .7.

Lens. A transparent optical component with one or more curved surfaces that bends light to converge or diverge it to form or modify images, i.e., that has focussing characteristics. A lens may have one to several elements or pieces.

Magnification. A ratio which may be anywhere in a range from zero to infinity that relates image size to object size. Sizes may be linear or angular. If angular and small, tangents may replace angles. Angular size may be subtense at the eye, at the objective, or at a standard viewing distance.

Magnifying Power. (1) The numerical value of magnification, (2) the ratio of image angular subtense to object angular subtense, (3) the ratio of the tangents of the angular subtenses of the image and the object.

Mirror. An optical element that is smooth and highly reflective. A mirror surface may be flat, concave, or convex.

Narrow-band light. Light from a small region of the spectrum, i.e., light containing a short range of wavelengths.

Objective. An optical component, such as a lens or a curved mirror, that forms or focuses a primary image in a compound optical device, such as a telescope or a microscope. Called an objective because it is the part closest to the object or scene to be examined.

Optical. Pertaining to light or to devices using light. Lenses, mirrors, prisms, filters, etc. are examples of optical devices.

Optical element. A lens, mirror, prism, grating, etc. used to reflect, transmit, refract, focus, etc. light. An objective or an eyepiece may contain several elements or pieces.

Optics. (1) The science of light, (2) optical devices or elements.

Phosphor. A substance that emits light (glows) when struck by a high-speed electron, or by high-energy radiation, such as ultraviolet or x-rays. The image on a CRT is formed by light emitted by the phosphor coating on the inside of the faceplate of the tube when the phosphor is struck by the electrons of a scanning beam.

Primary image. A real image of an object or scene projected by the objectives of a compound optical system. An eyepiece projects the primary image as a secondary image, the virtual display image.

Prism. A transparent optical device that has one or more flat surfaces inclined to one or more of its other surfaces to bend the path of the transmitted light. Prisms are used to bend a light path or to change the orientation of an image. Some prisms are used to refract different wavelengths of light by different amounts, thus forming a spectrum or spread of colors.

Real image. An image made of electromagnetic energy, where the energy image is at the location of the optical image. A reflecting screen at the image location would reveal it, and it could be recorded by an unexposed photographic film placed at the real image location.

Relay Lens. (1) A lens or system of lenses that changes the location, size, or orientation of the primary image from the objective, thus providing a secondary image at a more suitable location, or with a more suitable size or orientation for examination with the eyepiece. A relay lens is located between the objective and the eyepiece. Riflescopes, periscopes, and some telescopes use relay lenses. (2) The objective of a helmet-mounted display. Physicists and optical engineers regard this definition as incorrect, although it is widely used.

Resolution or resolving power. (1) The ability to display images containing fine details. (2) The ability to discern fine details of images or of objects. (3) The ability to see, or to image as separate, two or more discs, bars, or other elements of resolution test patterns. (4) Ability to read the letters on an eye resolution test chart or discern the orientation of a test pattern element.

Selective Coating. On an optical surface, a thin layer of material that reflects or transmits more of some wavelengths than of others. An example of a selective coating is a coating that reflects green light, but transmits light of other colors.

Spectrum. (1) The gamut or range of wavelengths that can cause visual sensations. It covers a range of about 350-750 nanometers, and slightly more at very high light intensities. (2) The spread of colors produced by a prism.

Subtend. To include in or to cover by an angle whose vertex is at a reference point, such as an observer's eye or an objective.

Thin Lens. A lens whose front-to-back thickness is negligible relative to object and image distances.

Virtual image. An image whose energy is not in the form of the image at the optical location of the image, such as the image in a mirror, or the image provided by a magnifier or an eyepiece. Virtual images are actual images, not imaginary images, but the label of real image is applied to images where image energy is at the optical location of the image.