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LIMITS OF APPLICABILITY OF THE BARBER AND HILL T-Matrix CODE

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RESEARCH DIRECTORATE

June 1992

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Documentation accompanying the Barber and Hill T-matrix codes for scattering by axisymmetric particles is incomplete regarding the size of particles to which it is applicable. We have calculated the convergence parameters for many spheroidal particles with a variety of sizes, aspect ratios, and refractive indices to indicate the range of particles that are amenable to Barber and Hill's code. To expedite the calculations, the T-matrix program was modified so that it determines convergence parameters without the normal trial and error interaction with the user.

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PREFACE

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OF THE BARBER AND HILL T-Matrix CODE

1. INTRODUCTION

A recent book by Barber and Hill* provides a thorough and rigorous mathematical treatment of light scattering from four different types of systems: plane slabs, infinite circular cylinders illuminated at normal incidence; axisymmetric particles; and spheres. Computer diskettes, formatted for DOS machines and packaged with the book, contain extensively documented FORTRAN source codes for the numerous programs expounded within the text. In the field of light scattering, it is rare to find computer programs in such a polished and final state; that combined with the author's reputations for meticulous work should insure that these programs remain standard tools for some time to come.

This report is concerned only with the chapter on axisymmetric particles, which deals with light scattering from spheroidal bodies, both prolate and oblate. The Barber and Hill programs will not work for spheroids that are too large or too eccentric; but, the text does not adequately indicate where the limits lie. The object of this report is to show the range of particles that can be handled by the Barber and Hill routines, and to suggest suitable values for the convergence parameters discussed below, which the user must supply to the computer program for each particle.

2. OPERATION OF T1.FOR

Nine programs, named T1.FOR through T9.FOR, are supplied for calculating scattering properties of spheroids. Of those, T1.FOR is the primary program in that it calculates a particle's T-matrix, the mathematical structure that contains information about the particles's scattering behavior. The T-matrix is a function of a particle's size, aspect ratio, and refractive index, but does not depend on the particle's orientation. It is used by the other programs to calculate angular scattering intensities and absorption, extinction, and scattering cross sections under various circumstances. (Program T1.FOR can also be used to calculate a related A-matrix, required for internal field calculations performed by program T8.FOR.)

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*Barber, P.W., and Hill, S.C., Light Scattering by Particles: Computational Methods, World Scientific Publishing Co. PTE. LTD, Singapore, 1990.

The operator must run T1 at least three separate times to determine the convergence parameters nrank, ntheta, and nm, and to produce the T-matrix. Nrank, which may not exceed 25 in this implementation, is the order of the highest term to be kept in the vector spherical harmonic expansion of the electric fields. Nm is the number of azimuthal modes required in the expansion, and ntheta is the number of sample points required for accurate numerical integration. As T1 is presently written, ntheta may not exceed 100. Each time T1 is run, the user is asked to enter particle properties (size parameter, axial ratio, and complex refractive index), values for nrank and ntheta, and a case indicator (ic) that "tells" the program what test to carry out.

In the first phase, the operator guesses a value for nrank (with some guidance from Figure 3.4 of Barber and Hill's text), puts ntheta equal to twice nrank, and sets ic=1 to instruct the program to check for convergence over nrank. If the solution converges, a message to that effect is displayed on the screen. Otherwise, the process must be repeated with another value for nrank until convergence does occur. Once a satisfactory value for nrank is established, a second phase is begun (with ic=0) to check for convergence over ntheta. Usually, an ntheta value twice the size of nrank suffices to elicit the "solution has converged" message; but, if not, larger values for ntheta must be attempted. Finally, in a third session, the operator enters the successful values of nrank and ntheta and sets ic=2. The program will determine a satisfactory value for nm and then go on to calculate and write (to disk) the T-matrix.

Convergence, and a valid T-matrix, is by no means guaranteed for every set of particle properties one might submit. The program may fail either because the particle requires a value of nrank >25 (or ntheta >100) or because a run-time math error is generated, which shuts the processing down. To get a useful gauge of when the Barber and Hill codes are applicable, we need to consider a goodly number of particles representing a wide range of sizes, axial ratios, and refractive indices, and ask in each case whether convergence can be attained and with what parameters. This could involve a daunting amount of labor with T1 as it is presently implemented; so, we shall first modify the program to make it run more automatically.

3. MODIFICATIONS TO T1.FOR

We wish to modify the structure of T1.FOR, taking care not to alter its actual calculations, so that it can be called as a subroutine by a new main program we shall write to take the place of the human operator. As it comes from the box, T1.FOR comprises a main program (T1) and seven subroutines, most of which are called to evaluate special math functions. One subroutine, ADDPRC, is of special interest here because the testing for convergence takes place in it.

ADDPRC is called twice when ic=1 (nrank), from lines 388 and 441, and twice when ic=0 (ntheta), both times from line 388. (Line numbers throughout this report refer to positions in the unaltered program T1.FOR.) In both cases, ADDPRC writes "solution has converged" to the screen and terminates the program at line 1133 - within ADDPRC - during its second invocation if convergence has been attained; otherwise, program control returns to T1, and the program ultimately terminates at line 453. The screen message is the only indication that convergence occurred. When ic=2 (nm), ADDPRC may be called numerous times from line 388, once for each increment of nm, until either convergence over nm is attained or the program crashes from a run-time math error. Here, after writing the convergence message but before terminating at line 1133, ADDPRC closes the file 't' (which was written by T1 and contains the now final T-matrix) and also writes a file called 'case' that contains the particle's physical properties for later identification by other programs.

To tell from within the program whether convergence has occurred, we defined a new logical variable CNOK, making it common to both T1 and ADDPRC. Just above each line in T1 where ADDPRC is called, we entered a new line setting CNOK false, and at line 1119 in ADDPRC (reached if and only if convergence has been attained) another new line setting CNOK to true. We also removed the 'stop' line, number 1133 in ADDPRC, and put a 'return' at the end of ADDPRC so that control returns to T1 whether or not there is convergence. In T1, following each 'call addprc' line, we now test CNOK. If it is false, the program keeps running as before; if it is true, the program jumps to the 'stop' line of T1 (line 453) instead of being stopped from within ADDPRC.

We note here an apparent bug which, although not relating directly to the task at hand, had to be fixed before proceeding. Lines 1121 through 1126 write to the disk a file called 'case' when the final convergence has been achieved, i.e., convergence over nm with ic=2. If a file 'case' already exists on the disk - which would be the normal condition, unless T1.FOR is being run for the very first time - then the computer halts operation with a 'Stack Overflow' error message instead of overwriting the old file with a new 'case' as intended. The FORTRAN lines in question appear to be correct, and indeed no tinkering with them improved matters. Finally, we relocated the offending lines to a new environment, in T1 after the line (388) that calls ADDPRC, since we can now check there for convergence via CNOK. The values written into the file 'case' were already known within T1 through common blocks with ADDPRC. With this fix, the program wrote 'case' with no further difficulties. The reason for the problem was never understood but is probably a quirk with the particular compiler used, Microsoft Fortran Version 5.0.

The entire program T1.FOR was then easily transformed to a subroutine, called T1S, with a few more simple changes. The 'stop' line 453 (which is now always reached at termination) was removed, and a 'return' line substituted just before the 'end' line, 471. The read and write lines for interfacing with the operator were removed (lines 76-83 and 105-107), and the whole thing was named SUBROUTINE T1S (XZ, AOVRBZ, CMRZ, CMIZ, NRANKZ, NTHETZ, ICZ, MTXSAX, CNOKZ, FIRZ). The arguments were formed by adding or substituting 'Z' for the last letter of the program variables - new names were required because the variable names already appear in several common blocks. A new set of statements were written near the beginning of T1 to relate these argument variables to their originally defined corresponding program variables.

One last problem had to be solved to make this program work as a subroutine. A logical variable inside ADDPRC, called FIRST, is used to tell whether ADDPRC is being called for the first time in any execution of T1.FOR. Remember that ADDPRC is called twice when ic=1 or ic=0, and numerous times when ic=2, and the program branches differently, depending upon which invocation of ADDPRC is being processed. FIRST is initially set equal to true at a data statement (line 926) each time T1.FOR is called up from the command line. But when T1.FOR is made a subroutine to a main program, FIRST will be properly initialized only once, when the whole program is first started. All subsequent calls to the subroutine will find FIRST=false, since data statements are executed only once, when the program is read in. To overcome this problem, we put FIRST into a common block shared by T1 and ADDPRC, and included a logical variable FIRZ in the argument list of T1S so that FIRST could be properly set from the main program each time T1S is called. A line in T1 sets FIRST=FIRZ.

To control the overall operation, a simple main program called T1A.FOR (Figure) was written. The operator running it is asked to enter only a particle's axial ratio and real and imaginary refractive indices. The program sets the size parameter to 1.0 and begins to check for convergence over nrank, starting with nrank=2 and incrementing nrank (called nrankz in T1A) until either convergence is achieved (indicated by CNOKZ being true) or until nrank is incremented above 25, in which case the program halts. If that convergence is successful, then convergence over ntheta is checked in a similar fashion, starting with ntheta equal to twice nrank and increasing, if necessary, in steps of four. The program halts if ntheta goes above 100. Finally, convergence over nm is checked, and if there is no problem, the size parameter, nrank, ntheta, and nm are written to a file on disk called 'answer'. Then, the size parameter is incremented by 1.0, and the process begins again. This repeats over and over until a size parameter is reached for which convergence fails for one of four possible reasons: nrank exceeding 25; ntheta exceeding 100; floating point math error M6101, invalid operation; or floating point math error M6104, overflow. These reasons for termination are coded as R, T, I, and O, respectively in the tables of results that follow. Once a size parameter is reached at which convergence fails, convergence will fail at all larger size parameters (for the same axial ratio and refractive index), though not necessarily for the same reason.

A listing of the program T1A.FOR follows:

```
      PROGRAM T1A
      LOGICAL CNOKZ,FIRZ
      COMMON /CMVCOM/ NM,KMV,CMV,TVM,PRODM
      WRITE(6,*) 'ENTER aovrb, mr, mi'
      READ(5,*) AOVRBZ,CMRZ,CMIZ
      OPEN(UNIT=8, FILE='ANSWER')
      WRITE(8,*)
      WRITE(8,'(4X,3A8)') 'AOVRB','MR','MI'
      WRITE(8,'(4X,3F8.3)') AOVRBZ,CMRZ,CMIZ
      WRITE(8,'(3X,4A6)') 'X','NRANK','NTHET','NT'
      DO 28 JRB=1,20
      XZ=JRB
C ...............MTXSAZ =1 FOR T=MATRIX, =2 FOR A-MATRIX .........
      MTXSAZ = 1
C ............. CHECK CONVERGENCE OVER NRANK .....................
      ICZ=1
      NRANKZ=1
    3 NRANKZ = NRANKZ+1
        IF(NRANKZ.gt.25) THEN
        WRITE(6,*) 'no convergence for nrank < 25'
        GOTO 21
        END IF
      NTHETZ=2*NRANKZ
      FIRZ = .TRUE.
      CALL T1S(XZ,AOVRBZ,CMRZ,CMIZ,NRANKZ,NTHETZ,ICZ,MTXSAZ,CNOKZ,FIRZ)
      IF(.NOT.CNOKZ) GOTO 3
C ............... CHECK CONVERGENCE OVER NTHETA ............ .....
      ICZ=0
      NTHETZ=NTHETZ-4
    6 NTHETZ=NTHETZ+4
        IF(NTHETZ.GT.100) THEN
        WRITE(6,*) 'no convergence for ntheta < 100'
        GOTO 21
        END IF
      FIRZ = .TRUE.
      CALL T1S(XZ,AOVRBZ,CMRZ,CMIZ,NRANKZ,NTHETZ,ICZ,MTXSAZ,CNOKZ,FIRZ)
      IF(.NOT.CNOKZ) GOTO 6
C .............. CHECK CONVERGENCE OVER NM ......................
      ICZ=2
      FIRZ = .TRUE.
      CALL T1S(XZ,AOVRBZ,CMRZ,CMIZ,NRANKZ,NTHETZ,ICZ,MTXSAZ,CNOKZ,FIRZ)
C ........... IF MADE IT TO HERE, GOOD CONV, T-MATRIX WRITTEN ....
      WRITE(8,27) JRB,NRANKZ, NTHETZ,KMV+1
   27 FORMAT (3X,4I6)
   28 CONTINUE
   21 STOP
      END
```

Figure. T1A.FOR PROGRAM

4. RESULTS

The program T1A was run to find convergence parameters for spheroidal particles with refractive indices typical of nonconducting materials at visible wavelengths in air or water. Tables 1 through 6 show the results for real n equal to 1.1, 1.4, and 1.8, with the imaginary part in each case equal either to 0.0 or to 0.1, which is characteristic of a highly absorbing material. The triplet of numbers in the clear boxes are the convergence values of nrank, ntheta, nm (also called nt) for particles of the boxes' corresponding size parameter and axial ratio. Shaded boxes designate particles that cannot be handled by the Barber/Hill programs, and the letter in the first shaded box of each column indicates the reason why convergence failed, as described at the end of the last section.

Whether a solution converges for a given size and shape of particle and the convergence parameters required to effect that solution are seen to depend only weakly on the particle's refractive index. As either the real or imaginary part of the refractive index increases, the tendency is to lower the size of the largest particle, which can be solved for a given axial ratio, and to increase the value of nrank required to obtain convergence for a given particle. For low values of n, the solvable particle with the largest size parameter is a prolate with an axial ratio of about 1.5. With increasing n, the center of the distribution tends toward the sphere, axial ratio 1.0.

The value of nrank given in the tables is the smallest number that produces convergence. Larger estimates may be used to run T1.FOR; but, they make the T-matrix unnecessarily large. In fact, too large a value of nrank may thwart convergence. The program T1A sets ntheta equal to twice nrank as an initial guess, and in almost all cases, that is adequate. Often a smaller ntheta would have sufficed; but (unlike nrank), there is no penalty for using a larger number. Values of ntheta even greater than twice nrank are required for some prolate particles with very high aspect ratios.

The program was also run with refractive indices characteristic of conductors, n=0.8 + i1.5, and n=1.3 + i7.1, which represent gold and aluminum, respectively, at a visible wavelength. The results, in Tables 7 and 8, are consistent with the trends discussed above. In the case of gold, we observed no apparent problem with specifying a real refractive index less than 1.0.

One does not have an intuitive feeling for the size of spheroidal particles from their size parameters, and such an appreciation would be helpful for judging whether the Barber/Hill programs are useful in a given situation. Now, the size parameter of a spheroid is defined to be the product ka, where

$k=2\pi/\lambda$, and a is the rotational semi-axis. If b is the equatorial semi-axis, then the volume of the spheroid, oblate or prolate, is given by $V=(4/3)\pi ab^2$. Then, it is easily shown that the diameter of a sphere containing the same amount of material as a spheroid with size parameter ka is given by

$$D = \frac{\lambda}{\pi} \frac{ka}{(a/b)^{2/3}}$$

Using a typical wavelength for visible light, $\lambda = 0.5$ microns, we can calculate the diameter, in microns, of spheres equal in volume to the tabulated spheroids.

Table 9 presents the equivalent sphere data. For reference, a heavy line marks the boundary of solvable particles with refractive index $n=1.4 + i0.0$. The size of spheroidal particles whose visible light scattering can be calculated with the Barber/Hill suite of programs is surprisingly small. At best, spheroids equivalent to spheres of about 2 µm in diameter can be treated, and for axial ratios much greater than 2 (or less than 1/2), only particles equivalent to submicron spheres can be handled.

5. CONCLUSIONS

The information generated and displayed in Tables 1 through 8 should help users of Barber and Hill's T-matrix codes. The tables indicate the size and shape range of those spheroidal particles whose light scattering can be calculated with the codes, and specify trial values for convergence parameters so that T1.FOR can be run quickly and efficiently.

To save time running the many cases needed to produce the tabulated data, we wrote a modified version of T1.FOR which can accept particle properties as input and write the particle's T-matrix as output with no further intervention by a human operator, as required with the original T1.FOR. That suggests another variation on the Barber and Hill codes. If an automated T-matrix writing routine can be integrated with a basic light scattering calculation code such as T3.FOR, then it should be possible to produce an independent subroutine which, when given particle properties and orientation, returns a fundamental scattering property. A small supply of such subroutines would allow a casual programmer to write easily customized programs to supplement the applications already provided by Barber and Hill. We plan to explore this possibility in the near future.

TABLE 1. Convergence values of (nrank, ntheta, nt) for n=1.1 + ı0.0

SPHEROIDAL SIZE PARAMETER	*oblate*						AXIAL RATIO							*prolate*		
	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
1	I	9,18,8	7,14,6	6,12,5	5,10,5	4,8,4	4,8,4	4,8,4	4,8,4	4,8,4	4,12,4	4,12,3	5,14,3	5,18,3	I	I
2		I	11,22,10	9,18,7	0,16,6	7,14,6	5,10,5	5,10,5	5,10,5	5,10,4	5,10,4	6,16,4	5,14,4	I		
3			I	11,22,9	9,18,8	7,14,7	6,12,6	7,14,6	7,14,5	7,14,5	8,16,4	T	I			
4				13,26,11	10,20,9	9,18,8	7,14,7	8,16,6	8,16,6	8,16,5	8,16,5					
5				16,32,13	13,26,11	11,22,9	9,18,8	9,18,7	9,18,7	9,18,6	T					
6				18,36,14	14,28,12	12,24,10	10,20,9	10,20,8	10,20,7	10,20,6						
7				O	15,30,13	13,26,12	11,22,10	11,22,9	11,22,8	12,24,6						
8					17,34,14	15,30,12	12,24,10	12,24,9	12,24,8	12,24,7						
9					O	16,32,14	13,26,11	14,28,9	13,26,9	14,28,8						
10						17,34,14	14,28,12	15,30,10	14,28,9	15,30,7						
11						O	15,30,13	16,32,11	16,32,9	15,30,8						
12							17,34,13	17,34,11	17,34,10	16,32,8						
13							18,36,15	18,36,12	18,36,10	17,34,9						
14							O	19,38,12	19,38,11	R						
15								20,40,14	20,40,12							
16								20,40,14	21,42,12							
17								21,42,14	22,44,12							
18								O	23,46,13							
19									O							
20																

TABLE 2. Convergence values of (nrank, ntheta, nt) for n=1.1 + i0.1

SPHEROIDAL SIZE PARAMETER	AXIAL RATIO: oblate .167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	prolate 20.0
1	I	8,16,8	7,14,6	6,12,5	5,10,5	4,8,4	4,8,4	4,8,4	4,8,4	4,8,4	4,8,4	4,12,3	5,14,3	5,38,3	5,18,3	T
2		I	11,22,10	8,16,8	7,14,6	6,12,6	5,10,5	6,12,5	6,12,5	5,10,4	5,10,4	5,14,4	5,14,4			
3			I	11,22,9	9,18,8	8,16,7	6,12,6	7,14,6	7,14,5	7,14,5	8,16,4	T				
4				14,28,11	10,20,9	9,18,8	8,16,7	8,16,7	9,18,6	8,16,5	9,18,5					
5				16,32,13	13,26,11	12,24,9	9,18,8	9,18,7	10,20,7	10,20,6	T					
6				18,36,14	15,30,12	12,24,10	10,20,9	11,22,8	11,22,7	11,22,6						
7				R	16,32,14	14,28,12	11,22,10	12,24,9	11,22,8	12,24,7						
8					17,34,14	16,32,13	12,24,11	13,26,9	13,26,9	13,26,7						
9					Q	17,34,14	13,26,12	14,28,10	14,28,9	14,28,8						
10						17,34,15	15,30,12	15,30,11	14,28,9	15,30,8						
11						O	16,32,14	16,32,12	16,32,9	18,52,8						
12							17,34,14	18,36,12	17,34,11	18,52,9						
13							18,36,14	18,36,13	18,36,11	R						
14							O	19,38,13	19,38,11							
15								20,40,14	20,40,12							
16								21,42,14	22,44,13							
17								O	23,46,13							
18									O							
19																
20																

TABLE 3. Convergence values of (nrank, ntheta, nt) for n=1.4 + i0.0

SPHEROIDAL SIZE PARAMETER	oblate						AXIAL RATIO						prolate			
	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
1	I	9,18,7	7,14,6	6,12,5	5,10,5	4,8,4	4,8,4	4,8,4	4,8,4	4,8,4	4,8,4	4,12,3	5,14,3	5,18,3	5,38,3	T
2		I	11,22,9	8,16,7	7,14,6	6,12,5	5,10,5	5,10,5	5,10,4	5,10,4	5,10,4	6,12,4			I	
3			I	11,22,9	9,18,7	7,14,7	6,12,6	6,12,5	6,12,5	7,14,5	7,14,4	7,14,4				
4				14,28,11	10,20,9	9,18,8	7,14,7	7,14,6	7,14,6	7,14,5	8,16,4	8,16,4				
5				17,34,13	13,26,10	11,22,9	8,16,8	8,16,7	8,16,6	9,18,6	10,20,5	I				
6				O	14,28,12	13,26,10	9,18,8	10,20,8	10,20,7	10,20,6	10,20,5					
7					16,32,13	14,28,11	10,20,10	11,22,8	11,22,7	11,22,6	11,22,5					
8					18,36,15	15,30,13	11,22,10	12,24,9	12,24,8	13,26,7	12,24,5					
9					O	16,32,14	13,26,12	13,26,10	14,28,9	14,28,7	I					
10						18,36,15	14,28,13	15,30,11	15,30,9	15,30,8						
11						O	15,30,14	15,30,11	17,34,10	16,32,8						
12							16,32,15	17,34,12	18,36,11	18,40,8						
13							O	18,36,13	19,38,11	19,38,9						
14								19,38,14	20,40,12	21,50,9						
15								O	22,44,13	R						
16									24,48,13							
17									O							
18																
19																
20																

TABLE 4. Convergence values of (nrank, ntheta, nt) for n=1.4 + i0.1

SPHEROIDAL SIZE PARAMETER	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
	oblate						AXIAL RATIO							*prolate*		
1	I	9,18,7	7,14,6	6,12,5	5,10,5	5,10,4	4,8,4	4,8,4	4,8,4	4,8,4	4,8,4	4,12,3	5,14,3	5,18,3	5,38,3	T
2		I	12,24,9	8,16,7	7,14,6	6,12,6	5,10,5	5,10,5	5,10,5	5,10,4	5,10,4	6,12,4				
3			I	11,22,9	9,18,8	8,16,7	6,12,6	7,14,5	6,12,5	7,14,5	8,16,4	T				
4				15,30,11	11,22,9	9,18,8	7,14,7	8,16,6	7,14,6	8,16,5	8,16,5					
5				17,34,13	13,26,11	11,22,9	9,18,8	9,18,7	8,16,7	9,18,6	10,20,5					
6				O	16,32,12	13,26,11	10,20,9	10,20,8	10,20,7	10,20,6	11,22,5					
7					17,34,13	14,28,12	11,22,10	11,22,8	11,22,8	12,24,7	I					
8					O	16,32,13	12,24,11	12,24,9	12,24,9	13,26,7						
9						17,34,14	13,26,12	13,26,10	14,28,9	14,28,8						
10						O	15,30,13	15,30,11	16,32,9	16,32,8						
11							15,30,14	17,34,12	17,34,10	18,36,9						
12							17,34,15	17,34,12	18,36,11	T						
13							18,36,15	19,38,13	20,40,11							
14							O	20,40,14	21,42,12							
15								O	22,44,13							
16									24,48,13							
17									O							
18																
19																
20																

TABLE 5. Convergence values of (nrank, ntheta, nt) for n=1.8 + i0.0

SPHEROIDAL SIZE PARAMETER	oblate						AXIAL RATIO						prolate			
	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
1	I	10,20,7	8,16,6	7,14,5	5,10,5	5,10,4	3,6,3	4,8,4	4,8,4	4,8,4	5,10,3	5,10,3	5,10,3	4,16,3	5,38,3	T
2		I	I	9,18,7	8,16,6	6,12,5	5,10,5	5,10,5	5,10,4	6,12,4	7,14,4	7,14,4	T	I	I	
3				13,26,9	10,20,8	8,16,7	5,10,5	6,12,5	6,12,5	7,14,4	8,16,4	8,16,4				
4				16,32,12	12,24,9	10,20,8	7,14,7	7,14,6	8,16,6	8,16,5	9,18,4	9,38,4				
5				R	13,26,11	10,20,10	8,16,8	10,20,7	10,20,6	10,20,5	10,20,5	I				
6					17,34,12	14,28,11	10,20,9	11,22,8	11,22,7	12,24,6	I					
7					19,38,13	15,30,12	11,22,11	13,26,9	12,24,8	14,28,7						
8					O	17,34,13	12,24,12	12,24,10	15,30,9	16,32,7						
9						19,38,14	13,26,12	16,32,11	17,34,10	18,36,8						
10						O	15,30,14	17,34,12	18,36,10	19,38,8						
11							16,32,14	18,36,12	19,38,11	T						
12							15,30,15	20,40,14	22,44,11							
13							O	21,42,13	25,54,12							
14								O	R							
15																
16																
17																
18																
19																
20																

TABLE 6. Convergence values of (nrank, ntheta, nt) for n=1.8 + i0.1

SPHEROIDAL SIZE PARAMETER	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
	oblate						AXIAL RATIO						*prolate*			
1	I	10,20,8	8,16,6	7,14,5	5,10,5	5,10,4	3,6,3	4,8,4	4,8,4	4,8,4	5,10,3	5,10,3	5,10,3	4,16,3	5,38,3	I
2		I	I	9,18,7	8,16,6	6,12,5	5,10,5	5,10,5	5,10,5	6,12,4	7,14,4	7,14,4	I	I	I	
3				13,26,9	10,20,8	7,14,7	6,12,6	6,12,6	7,14,5	8,16,4	8,16,4	8,16,4				
4				17,34,12	12,24,9	10,20,8	7,14,7	8,16,6	8,16,6	9,18,5	9,18,4	I				
5				20,52,13	15,30,11	12,24,10	9,18,8	10,20,8	10,20,7	11,22,6	11,22,5					
6				R	18,36,13	14,28,11	10,20,9	11,22,8	11,22,8	12,24,6	I					
7					19,38,14	16,32,12	11,22,11	13,26,9	14,28,8	15,30,7						
8					O	17,34,13	12,24,12	14,28,10	15,30,9	16,32,7						
9						19,38,14	14,28,12	16,32,11	17,34,9	18,36,8						
10						O	15,30,13	17,34,12	18,[illegible]	I						
11							16,32,14	18,36,12	21,42,11							
12							17,34,14	21,42,12	23,46,12							
13							O	21,42,13	23,46,12							
14								O	R							
15																
16																
17																
18																
19																
20																

TABLE 7. Convergence values of (nrank, ntheta, nt) for n=0.8 + i1.5

SPHEROIDAL SIZE PARAMETER	oblate					AXIAL RATIO								prolate		
	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
1	I	I	10,20,7	6,12,6	5,10,5	4,8,4	3,6,3	4,8,4	5,10,4	5,10,4	5,10,3	5,10,3	5,10,3	5,18,3	I	I
2			I	10,20,8	7,14,6	6,12,6	5,10,5	6,12,5	7,14,5	7,14,4	8,16,4	9,50,4	I	I		
3				14,28,10	10,20,8	8,16,7	6,12,6	8,16,6	9,18,5	10,20,5	T	I				
4				19,38,11	12,24,9	10,20,9	8,16,7	9,18,7	10,20,6	12,24,5						
5				R	15,30,11	11,22,10	9,18,8	11,22,7	12,24,7	15,38,6						
6					18,36,12	13,26,11	10,20,9	12,24,8	14,28,7	I						
7					O	14,28,12	11,22,10	14,28,9	16,32,8							
8						16,32,13	13,26,11	14,28,10	24,48,9							
9						17,34,14	13,26,12	17,34,10	R							
10						O	15,30,12	18,36,10								
11							16,32,13	19,38,11								
12							O	21,42,12								
13								22,44,12								
14								O								
15																
16																
17																
18																
19																
20																

TABLE 8. Convergence values of (nrank, ntheta, nt) for n=1.3 + i7.1

SPHEROIDAL SIZE PARAMETER	oblate					AXIAL RATIO							prolate			
	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
1	R	R	R	13,26,5	8,16,5	5,10,4	3,6,3	5,10,4	6,12,4	8,16,3	9,18,3	I	I	I	I	I
2				R	12,24,6	8,16,6	5,10,5	8,16,5	11,22,5	15,50,4	I					
3					R	11,22,7	6,12,6	10,20,5	R	R						
4						13,26,8	7,14,7	13,26,6								
5						R	8,16,8	R								
6							10,20,9									
7							O									
8																
9																
10																
11																
12																
13																
14																
15																
16																
17																
18																
19																
20																

TABLE 9. Diameter of equal volume spheres, in microns, assuming wavelength=0.5 microns.

	oblate					AXIAL RATIO							prolate			
SPHEROIDAL SIZE PARAMETER	.167	.250	.333	.500	.667	.800	1.00	1.25	1.50	2.00	3.00	4.00	6.00	10.0	15.0	20.0
1	0.5	0.4	0.3	0.3	0.2	0.2	0.2	0:1	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0
2	1.0	0.8	0.7	0.5	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.0
3	1.6	1.2	1.0	0.8	0.6	0.6	0.5	0.4	0.4	0.3	0.2	0.2	0.1	0.1	0.1	0.1
4	2.1	1.6	1.3	1.0	0.8	0.7	0.6	0.5	0.5	0.4	0.3	0.3	0.2	0.1	0.1	0.1
5	2.6	2.0	1.7	1.3	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.2	0.1	0.1
6	3.1	2.4	2.0	1.5	1.3	1.1	1.0	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.2	0.1
7	3.7	2.8	2.3	1.8	1.5	1.3	1.1	1.0	0.9	0.7	0.5	0.4	0.3	0.2	0.2	0.2
8	4.2	3.2	2.7	2.0	1.7	1.5	1.3	1.1	1.0	0.8	0.6	0.5	0.4	0.3	0.2	0.2
9	4.7	3.6	3.0	2.3	1.9	1.7	1.4	1.2	1.1	0.9	0.7	0.6	0.4	0.3	0.2	0.2
10	5.2	4.0	3.3	2.5	2.1	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.5	0.3	0.3	0.2
11	5.8	4.4	3.6	2.8	2.3	2.0	1.8	1.5	1.3	1.1	0.8	0.7	0.5	0.4	0.3	0.2
12	6.3	4.8	4.0	3.0	2.5	2.2	1.9	1.6	1.5	1.2	0.9	0.8	0.6	0.4	0.3	0.3
13	6.8	5.2	4.3	3.3	2.7	2.4	2.1	1.8	1.6	1.3	1.0	0.8	0.6	0.4	0.3	0.3
14	7.3	5.6	4.6	3.5	2.9	2.6	2.2	1.9	1.7	1.4	1.1	0.9	0.7	0.5	0.4	0.3
15	7.9	6.0	5.0	3.8	3.1	2.8	2.4	2.1	1.8	1.5	1.1	0.9	0.7	0.5	0.4	0.3
16	8.4	6.4	5.3	4.0	3.3	3.0	2.5	2.2	1.9	1.6	1.2	1.0	0.8	0.5	0.4	0.3
17	8.9	6.8	5.6	4.3	3.5	3.1	2.7	2.3	2.1	1.7	1.3	1.1	0.8	0.6	0.4	0.4
18	9.4	7.2	6.0	4.5	3.8	3.3	2.9	2.5	2.2	1.8	1.4	1.1	0.9	0.6	0.5	0.4
19	10.0	7.6	6.3	4.8	4.0	3.5	3.0	2.6	2.3	1.9	1.5	1.2	0.9	0.7	0.5	0.4
20	10.5	8.0	6.6	5.1	4.2	3.7	3.2	2.7	2.4	2.0	1.5	1.3	1.0	0.7	0.5	0.4