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## Abstract

The problems of pressure shock and longitudinal impact on a semi-infinite circular tube are studied. The considerations are performed within the framework of the linear theory of elasticity. The problems were reduced to the system of Bessel differential equations by using double integral transforms. The solutions are obtained in the terms of Fourier and Laplace integral which could be evaluated by numerical methods.

## Introduction

Problems of wave propagation in waveguides in view of their importance in numerous applications have received considerable attention in the literature devoted to elastodynamics. The bulk of the study of elastic waves is concerned with rods and plates. The cylindrical rod of circular cross section is a classical example of a medium involving the complexities of dispersive propagation. Pochhammer [1] was the first who derived frequency equations for the cases of compressional, flexural and torsional harmonic waves traveling in an infinite elastic rod. The frequency equations are connected with a stress free lateral surface of the rod and they constitute a basis for investigations of both steady vibration and transient wave propagation.

This report deals with two dynamic problems of a semi-infinite circular elastic tube. The first problem is related to mixed pressure conditions, the second one is devoted to the longitudinal impact. The analysis is performed within the framework of the linear theory of elasticity. The considered problems belong to the class involving mixed edge conditions.

The method of solution of the first problem is based on the double integral transforms (Fourier and Laplace transform ) and it leads to solutions given in the terms of Fourier and Laplace integrals , which can be calculated by using analytical approximations or numerical analysis.

A similar mixed-pressure problems of infinite circular rods were solved by Folk et al [2] and is also presented in the Achenbach book [3]. By using integral transforms and employing the analysis of Pochhammer's frequency equations to the procedure of an inversion of Laplace transforms, the solution of the wave problem was expressed in the terms of Fourier integrals. The Fourier integrals could not be evaluated rigorously by analytical means, however, a satisfactory approximation was obtained by the method of stationary phase.

The second problem considered in this report deals with the impact of a semi-infinite circular elastic tube. The solution is given under assumptions that the end of tube has no tangential stresses but is suddenly set into motion with a constant velocity and the lateral surfaces are free of pressures. The analogous problem for a semi-infinite bar was solved by Skalak [11]. His method was to split the problem into two parts. In the first part the collision of two half-spaces was considered. In the second part he investigated an infinite bar with an equal and opposite stresses travelling along the lateral surface. Using the principle of superposition a stress free lateral surface was ensured. In Section 2 Skalak's results are adopted to the problem of circular tube. The solution of the impact problem is presented in the form of Fourier exponential and Laplace integrals, which can be calculated by using analytical approximations or numerical analysis.

Transient wave problems in elastic waveguides were considered by many authors. In the survey by Miklowitz [4] one can find a coverage of general literature up to 1964. Solutions of many wave problems are also presented in Miklowitz book [5]. Longitudinal harmonic motions of a cylindrical tube on the basis of the three-dimensional theory of elasticity were considered by Gosh [6], where the frequency equation are derived. The discussion of the characteristic equations for this problem and the phase velocity (for the case of Poisson's ratio $\nu = 0.3$) are presented in [7]. Some numerical results for the longitudinal impact problem of a circular tube obtained by using the numerical integration along the bicharacteristics, are given in [8].

Understanding of dispersion behaviour in bars and tubes are of great importance in all experimental techniques used to test mechanical characteristics of materials at high loading or strain rates, for example [12-14]. One prominent example is the Split Hopkinson Bar technique (Kolsky apparatus), where a thin wafer specimen is inserted between two instrumented Hopkinson bars [12]. A more exact analysis of wave dispersion in the SHB enables for a more exact interpretation of oscillograms, and consequently, for a more exact determination of a specimen response at different loading conditions [15,16].

Since it is expected that the lateral inertia causing the Pochhammer-Chree vibrations are substantially reduced in tubes, application of tubes instead of bars as measuring devices in wave transmission seems to be reasonable. So far, some applications of tubes for material testing at high strain rates were already proposed, [17,18]. However, in those applications a simple one-dimensional wave approximation was commonly used. The main target of this study is to make analysis of the wave-guide tubes more precise, specially for short pulses.

## 1.The mixed problem of pressure shock

### 1.1 *Formulation of the problem*

A semi-infinite cylindrical elastic tube of inner radius a and outer radius b , which are referred to cylindrical coordinates $(r,\theta,z)$ where the z-axis coinciding with the axis of the tube, Fig.1, is considered. Let $\lambda$, $\mu$ be Lame constants and $\rho$ be the mass density of the tube material. The problem is restricted to the axisymmetric case of motion, thus the displacement vector **u** is independent on the variable $\theta$ and

$$\mathbf{u}(r,z,t) = ( u_r(r,z,t);\ 0;\ u_z(r,z,t) ) \tag{1.1}$$

where t denotes the time.

The equations of motion of the linear elasticity take the form

$$(\lambda + 2\mu) \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r\, \partial z} \right) +$$

$$\mu \left( \frac{\partial^2 u_r}{\partial z^2} - \frac{\partial^2 u_z}{\partial r\, \partial z} \right) = \rho \frac{\partial^2 u_r}{\partial t^2} \quad ,$$

$$(\lambda + 2\mu)\left(\frac{\partial^2 u_r}{\partial r\,\partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) -$$

$$\mu\,\frac{1}{r}\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) - \mu\left(\frac{\partial^2 u_r}{\partial z\,\partial r} - \frac{\partial^2 u_z}{\partial r^2}\right) = \rho\,\frac{\partial^2 u_z}{\partial t^2}\,. \qquad (1.2)$$

The stress components are expressed by the displacements $u_r$ , $u_z$ as follows

$$\sigma_{zz} = 2\mu\,\frac{\partial u_z}{\partial z} + \lambda\left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z}\right),$$

$$\sigma_{zr} = \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \qquad (1.3)$$

$$\sigma_{rr} = 2\mu\,\frac{\partial u_r}{\partial r} + \lambda\left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z}\right).$$

Consider the boundary conditions

$$\sigma_{zz}(\, r,z=0,t\,) = -\,f(t),$$

$$u_r(\, r,z=0,t\,) = 0\ , \quad \text{for } r \in \langle a,b\rangle\ ,\ t \in (0,\infty) \qquad (1.4)$$

and

$$\sigma_{rr}(r=a,z,t) = \sigma_{rr}(r=b,z,t) = 0\ ,$$

$$\sigma_{rz}(r=a,z,t) = \sigma_{rz}(r=b,z,t) = 0, \quad \text{for } z \in (0,\infty)\ ,\ t \in (0,\infty)\ , \qquad (1.5)$$

where f(.) is a given function, provided that $f(t) \equiv 0$ for $t<0$. A futher condition is that the displacements and the stresses vanish at infinity , $z \longrightarrow \infty$ .Assuming that the semi-infinite circular tube is at rest prior to time instant $t = 0$, equations (1.4) and (1.5) are supplemented by the initial conditions

$$u_r(r,z,t=0) = \frac{\partial u_r}{\partial t}(r,z,t=0) = 0 \ ,$$

$$u_z(r,z,t=0) = \frac{\partial u_z}{\partial t}(r,z,t=0) = 0 \ , \text{for } r \in \langle a,b\rangle \ , z \in (0,\infty) . \tag{1.6}$$

By introducing the displacement potentials $\phi$ ,$\psi$ as follows

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial z} \ ,$$

$$u_z = \frac{\partial \phi}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\psi) \tag{1.7}$$

the equations of motion (1.1) are separated into two following equations

$$\left( \nabla^2 - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2} \right) \phi(r,z,t) = 0 \ ,$$

$$\left( \nabla^2 - \frac{1}{r^2} - \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2} \right) \psi(r,z,t) = 0 \ , \tag{1.8}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{1.9}$$

and

$$c_1^2 = \frac{\lambda + 2\mu}{\rho} \ , \qquad c_2^2 = \frac{\mu}{\rho} \ . \tag{1.10}$$

Substituting equations (1.7) into (1.3) the stress components may be written in the terms of displacement potentials in the form

$$\sigma_{rr} = \lambda \nabla^2\phi + 2\mu \frac{\partial}{\partial r}\left( \frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial z} \right) \ ,$$

$$\sigma_{zz} = \lambda \nabla^2\phi + 2\mu \frac{\partial}{\partial z}\left[ \frac{\partial \phi}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\,\psi) \right] \ , \tag{1.11}$$

$$\sigma_{zr} = \mu \left[ \frac{\partial}{\partial z} \left( \frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial r} \left( \frac{\partial \phi}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \psi) \right) \right] .$$

By using equations (1.8) and (1.11) we have

$$\sigma_{rr} = 2\mu \left( \frac{\partial^2 \phi}{\partial r^2} - \frac{\partial^2 \psi}{\partial r \, \partial z} \right) + \frac{\lambda}{c_1^2} \frac{\partial^2 \phi}{\partial t^2} ,$$

$$\sigma_{zz} = \rho \frac{\partial^2 \phi}{\partial t^2} - 2\mu \left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) + 2\mu \frac{1}{r} \frac{\partial^2}{\partial r \, \partial z} (r \psi) , \tag{1.12}$$

$$\sigma_{zr} = 2\mu \left( \frac{\partial^2 \phi}{\partial r \, \partial z} - \frac{\partial^2 \psi}{\partial z^2} \right) + \rho \frac{\partial^2 \psi}{\partial t^2} .$$

From boundary condition $(1.4)_2$ and equation $(1.7)_1$ it follows that

$$\frac{\partial \phi}{\partial r} = \frac{\partial \psi}{\partial z} , \quad \text{for } z = 0 , \quad r \in \langle a,b \rangle , \quad t \in (0,\infty). \tag{1.13}$$

By using (1.13) , $(1.12)_2$ and boundary condition $(1.4)_1$ one obtains

$$\rho \frac{\partial^2 \phi}{\partial t^2} (r,z=0,t) = - f(t) , \quad \text{for} \quad r \in \langle a,b \rangle, \ t \in (0,\infty) . \tag{1.14}$$

Because the potential $\phi$ is independent on $r$ at $z = 0$ (see Eq.(1.14) ), then $\partial\phi/\partial r = 0$ for $z = 0$ , and from (1.13) it follows that

$$\frac{\partial \psi}{\partial z} (r,z=0,t) = 0 \quad \text{for} \quad r \in \langle a,b \rangle , \ t \in (0,\infty) . \tag{1.15}$$

Equations (1.14) and (1.15) constitute boundary conditions at $z = 0$ for the displacement potentials. The initial conditions (1.6) together with (1.7) imply

$$\phi(r,z,0) = \frac{\partial\phi}{\partial t}(r,z,0) = 0 \ ,$$

$$\psi(r,z,0) = \frac{\partial\psi}{\partial t}(r,z,0) = 0 \ . \qquad (1.16)$$

Moreover, the potentials $\phi$ and $\psi$ has to vanish at infinity , $z \to \infty$ .

## 1.2 *Solution by integral transforms*

Let us denote by ,according to [9]:

$$f^S(r,\xi,t) \equiv \mathcal{F}_S\{f(r,z,t);\ z \to \xi\ \} = \int_0^\infty f(r,z,t)\ \sin(\xi z)\ dz \ ,$$

$$f^C(r,\xi,t) \equiv \mathcal{F}_C\{f(r,z,t);\ z \to \xi\ \} = \int_0^\infty f(r,z,t)\ \cos(\xi z)\ dz \ ,$$

$$\bar{f}(r,z,p) \equiv \mathcal{L}\ \{f(r,z,t);\ t \to p\ \} = \int_0^\infty f(r,z,t)\ e^{-pt}\ dt \ . \qquad (1.17)$$

If $\lim_{z \to \infty} f(r,z,t) = \lim_{z \to \infty} \frac{\partial f}{\partial z}(r,z,t) = 0$ then it follows that

$$\mathcal{F}_S\ \{ \frac{\partial f}{\partial z}\ ;\ z \to \xi\ \} = -\xi\ f^C(r,\xi,t) \ ,$$

$$\mathcal{F}_C\ \{ \frac{\partial f}{\partial z}\ ;\ z \to \xi\ \} = \xi\ f^S(r,\xi,t) - f(r,z{=}0,t) \ , \qquad (1.18)$$

$$\mathcal{F}_S\ \{ \frac{\partial^2 f}{\partial z^2}\ ;\ z \to \xi\ \} = -\xi^2\ f^S(r,\xi,t) + \xi\ f(r,z{=}0,t) \ ,$$

$$\mathcal{F}_C\ \{ \frac{\partial^2 f}{\partial z^2}\ ;\ z \to \xi\ \} = -\xi^2\ f^C(r,\xi,t) - \frac{\partial f}{\partial z}(r,z{=}0,t) \ ,$$

and

$$\mathscr{L}\left\{ \frac{\partial^2 f}{\partial t^2} ; t \to p \right\} = p^2 \bar{f}(r,z,p) - p \lim_{t \to 0^+} f(r,z,t) - \lim_{t \to 0^+} \frac{\partial f}{\partial t}(r,z,t) \qquad (1.19)$$

provided ,that $|f(r,z,t)| < K(r,z) \exp(\gamma_1 t)$ , where $K(r,z)$ is an arbitrary function and $\mathrm{Re}\, p = \gamma > \gamma_1$.

Applying Fourier sine transform with respect to z to equation $(1.8)_1$ it follows that

$$\frac{\partial^2 \phi^S}{\partial r^2} + \frac{1}{r} \frac{\partial \phi^S}{\partial r} - \xi^2 \phi^S + \xi\, \phi(r,z=0,t) = \frac{1}{c_1^2} \frac{\partial^2 \phi^S}{\partial t^2} \qquad (1.20)$$

and using Laplace transform to (1.20) one has

$$\frac{\partial^2 \bar{\phi}^S}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\phi}^S}{\partial r} - \xi^2 \bar{\phi}^S + \xi\, \bar{\phi}(r,z=0,p) = \frac{p^2}{c_1^2} \bar{\phi}^S . \qquad (1.21)$$

The boundary condition (1.14) after Laplace transformation yields

$$\bar{\phi}^S(r,z=0,p) = - \frac{\bar{f}(p)}{\rho p^2} , \qquad (1.22)$$

where

$$\bar{f}(p) = \mathscr{L}\{ f(t) ; t \to p \} . \qquad (1.23)$$

Substituting (1.22) into (1.21) the following Bessel equation is found

$$\frac{\partial^2 \bar{\phi}^S(r,\xi,p)}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\phi}^S(r,\xi,p)}{\partial r} - \alpha^2 \bar{\phi}^S(r,\xi,p) = \frac{\xi \bar{f}(p)}{\rho p^2} , \qquad (1.24)$$

where

$$\alpha^2 = \xi^2 + \frac{p^2}{c_1^2} . \qquad (1.25)$$

Applying Fourier cosine transform with respect to z to equation $(1.8)_2$ one finds

$$\frac{\partial^2 \psi^C}{\partial r^2} + \frac{1}{r}\frac{\partial \psi^C}{\partial r} - \xi^2 \psi^C - \frac{\partial \psi}{\partial z}(r,z=0,t) - \frac{1}{r^2}\psi^C = \frac{1}{c_2^2}\frac{\partial^2 \psi^C}{\partial t^2} . \qquad (1.26)$$

However, by means of boundary condition (1.15) the equation (1.26) is reduced to (after Laplace transform):

$$\frac{\partial^2 \bar{\psi}^C}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\psi}^C}{\partial r} - (\beta^2 + \frac{1}{r^2})\,\bar{\psi}^C = 0 , \qquad (1.27)$$

where

$$\beta^2 = \xi^2 + \frac{p^2}{c_2^2} . \qquad (1.28)$$

The general solutions of equations (1.24) and (1.27) may be written in the form, [5]:[1]

$$\bar{\phi}^S(r,\xi,p) = A_1 I_0(\alpha\ r) + A_2 K_0(\alpha\ r) - \frac{\xi}{\rho\ p^2 \alpha^2}\bar{f}(p) ,$$

$$\bar{\psi}^C(r,\xi,p) = B_1 I_1(\beta\ r) + B_2 K_1(\beta\ r) , \qquad (1.29)$$

where $I_0(.)$ , $I_1(.)$ and $K_0(.)$ , $K_1(.)$ are the modified Bessel functions of the first and the second kinds, respectively. Here $A_1$, $A_2$, $B_1$, $B_2$ are unknown functions of the variables $\xi$, $p$ and they have to be determined by the use of boundary conditions on the lateral surfaces of the tube given in (1.5). In order to do this the equations (1.12) are transfomed , thus

$$\bar{\sigma}^S_{rr} = 2\mu\left(\frac{\partial^2 \bar{\phi}^S}{\partial r^2} + \xi\frac{\partial \bar{\psi}^C}{\partial r}\right) + \frac{\lambda\ p^2}{c_1^2}\bar{\phi}^S , \qquad (1.30)$$

---

[1] It is emphasized that for the analogy problem of a rod, the following conditions should be assumed : $A_2 = B_2 \equiv 0$ to ensure finite potentials, displacements and stresses at r=0.

$$\bar{\sigma}^{C}_{zr} = 2\mu \left[ \xi \frac{\partial \bar{\phi}^{S}}{\partial r} - \frac{\partial \bar{\phi}}{\partial r}(r,z=0,p) + \xi^{2}\bar{\psi}^{C} + \frac{\partial \bar{\psi}}{\partial z}(r,z=0,p) \right]$$
$$+ \rho\, p^{2}\, \bar{\psi}^{C} . \tag{1.31}$$

From (1.15) , (1.13) and (1.31) it is obtained

$$\bar{\sigma}^{C}_{zr} = 2\mu \left( \xi \frac{\partial \bar{\phi}^{S}}{\partial r} + \xi^{2}\bar{\psi}^{C} \right) + \rho\, p^{2}\, \bar{\psi}^{C} . \tag{1.32}$$

Employing the following relations, according to [10]:

$$\frac{\partial I_{o}(\beta x)}{\partial x} = \beta\, I_{1}(\beta x) \quad , \qquad \frac{\partial K_{o}(\beta x)}{\partial x} = -\beta\, K_{1}(\beta x) \quad ,$$
$$\frac{\partial I_{1}(\beta x)}{\partial x} = \beta\, I_{o}(\beta x) - \frac{1}{x} I_{1}(\beta x) \; , \tag{1.33}$$
$$\frac{\partial K_{1}(\beta x)}{\partial x} = -\beta\, K_{o}(\beta x) - \frac{1}{x} K_{1}(\beta x) \; ,$$

and using equation (1.29) it is found

$$\frac{\partial \bar{\phi}^{S}}{\partial r} = A_{1}\,\alpha\, I_{1}(\alpha r) - A_{2}\,\alpha\, K_{1}(\alpha r),$$

$$\frac{\partial^{2} \bar{\phi}^{S}}{\partial r^{2}} = A_{1}\,\alpha\, [\alpha\, I_{o}(\alpha r) - \frac{1}{r} I_{1}(\alpha r)] + A_{2}\,\alpha\, [\alpha\, K_{o}(\alpha r) + \frac{1}{r} K_{1}(\alpha r)]$$

$$\frac{\partial \bar{\psi}^{C}}{\partial r} = B_{1}\, [\beta\, I_{o}(\beta r) - \frac{1}{r} I_{1}(\beta r)] + B_{2}[-\beta\, K_{o}(\beta r) - \frac{1}{r} K_{1}(\beta r)],$$

$$\frac{\partial^{2} \bar{\psi}^{C}}{\partial r^{2}} = B_{1}\, [\, (\beta^{2} + \frac{2}{r^{2}})\, I_{1}(\beta r) - \frac{\beta}{r} I_{o}(\beta r) \,] +$$
$$B_{2}\, [\, (\beta^{2} + \frac{2}{r^{2}})\, K_{1}(\beta r) + \frac{\beta}{r} K_{o}(\beta r) \,] . \tag{1.34}$$

Substituting equations (1.29) and (1.34) into (1.30) and (1.32) one finds

$$\bar{\sigma}^{C}_{zr}(r,\xi,p) = 2\mu \left[ A_1 \xi \alpha I_1(\alpha r) - A_2 \xi \alpha K_1(\alpha r) + B_1 (\xi^2 + \frac{\rho p^2}{2\mu}) I_1(\beta r) + B_2 (\xi^2 + \frac{\rho p^2}{2\mu}) K_1(\beta r) \right] ,$$

$$\bar{\sigma}^{S}_{rr}(r,\xi,p) = A_1 \left[ (2\mu \alpha^2 + \frac{\lambda p^2}{c_1^2}) I_0(\alpha r) - 2\mu \frac{\alpha}{r} I_1(\alpha r) \right] + A_2 \left[ (2\mu \alpha^2 + \frac{\lambda p^2}{c_1^2}) K_0(\alpha r) + 2\mu \frac{\alpha}{r} K_1(\alpha r) \right] + B_1 \, 2\mu \xi \left[ \beta I_0(\beta r) - \frac{1}{r} I_1(\beta r) \right] + B_2 \, 2\mu \xi \left[ -\beta K_0(\beta r) - \frac{1}{r} K_1(\beta r) \right] - \frac{\lambda \xi}{(\lambda + 2\mu)\alpha^2} \bar{f}(p) . \qquad (1.35)$$

Let us introduce the following notations:

$$e_1(r) \equiv \xi \alpha I_1(\alpha r) ,$$

$$e_2(r) \equiv - \xi \alpha K_1(\alpha r) ,$$

$$e_3(r) \equiv (\xi^2 + \frac{\rho p^2}{2\mu}) I_1(\beta r) ,$$

$$e_4(r) \equiv (\xi^2 + \frac{\rho p^2}{2\mu}) K_1(\beta r) , \qquad (1.36)$$

$$d_1(r) \equiv (2\mu \alpha^2 + \frac{\lambda p^2}{c_1^2}) I_0(\alpha r) - 2\mu \frac{\alpha}{r} I_1(\alpha r) ,$$

$$d_2(r) \equiv (2\mu \alpha^2 + \frac{\lambda p^2}{c_1^2}) K_0(\alpha r) + 2\mu \frac{\alpha}{r} K_1(\alpha r) ,$$

$$d_3(r) \equiv 2\mu \xi \left[ \beta I_0(\beta r) - \frac{1}{r} I_1(\beta r) \right] ,$$

$$d_4(r) \equiv -2\mu \xi \left[ \beta K_0(\beta r) + \frac{1}{r} K_1(\beta r) \right] .$$

The boundary conditions (1.5) together with equations (1.35) and (1.36) yield the system of four linear algebraic equations, for unknowns $A_1$, $A_2$, $B_1$, $B_2$, in the form:

$$A_1\ d_1(a) + A_2\ d_2(a) + B_1\ d_3(a) + B_2\ d_4(a) = \frac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\ \alpha^2},$$

$$A_1\ d_1(b) + A_2\ d_2(b) + B_1\ d_3(b) + B_2\ d_4(b) = \frac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\ \alpha^2},$$

$$A_1\ e_1(a) + A_2\ e_2(a) + B_1\ e_3(a) + B_2\ e_4(a) = 0\ , \tag{1.37}$$

$$A_1\ e_1(b) + A_2\ e_2(b) + B_1\ e_3(b) + B_2\ e_4(b) = 0\ .$$

The solution of equations (1.37) can be written in the form

$$A_1 = \frac{W_1}{\Delta}\ , \quad A_2 = \frac{W_2}{\Delta}\ , \quad B_1 = \frac{W_3}{\Delta}\ , \quad B_2 = \frac{W_4}{\Delta}\ , \tag{1.38}$$

where

$$\Delta = \det \begin{bmatrix} d_1(a) & d_2(a) & d_3(a) & d_4(a) \\ d_1(b) & d_2(b) & d_3(b) & d_4(b) \\ e_1(a) & e_2(a) & e_3(a) & e_4(a) \\ e_1(b) & e_2(b) & e_3(b) & e_4(b) \end{bmatrix},$$

$$W_1 = \det \begin{bmatrix} \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} & d_2(a) & d_3(a) & d_4(a) \\ \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} & d_2(b) & d_3(b) & d_4(b) \\ 0 & e_2(a) & e_3(a) & e_4(a) \\ 0 & e_2(b) & e_3(b) & e_4(b) \end{bmatrix},$$

$$W_2 = \det\begin{bmatrix} d_1(a) & \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} & d_3(a) & d_4(a) \\ d_1(b) & \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} & d_3(b) & d_4(b) \\ e_1(a) & 0 & e_3(a) & e_4(a) \\ e_1(b) & 0 & e_3(b) & e_4(b) \end{bmatrix}, \tag{1.39}$$

$$W_3 = \det\begin{bmatrix} d_1(a) & d_2(a) & \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} & d_4(a) \\ d_1(b) & d_2(b) & \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} & d_4(b) \\ e_1(a) & e_2(a) & 0 & e_4(a) \\ e_1(b) & e_2(b) & 0 & e_4(b) \end{bmatrix},$$

$$W_4 = \det\begin{bmatrix} d_1(a) & d_2(a) & d_3(a) & \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} \\ d_1(b) & d_2(b) & d_3(b) & \dfrac{\lambda\ \xi\ \bar{f}(p)}{(\lambda+2\mu)\alpha^2} \\ e_1(a) & e_2(a) & e_3(a) & 0 \\ e_1(b) & e_2(b) & e_3(b) & 0 \end{bmatrix}.$$

Substituting the solution given by (1.38) into (1.29) one has

$$\bar{\phi}^S(r,\xi,p) = \frac{W_1}{\Delta}\ I_0(\alpha r) + \frac{W_2}{\Delta}\ K_0(\alpha r) - \frac{\xi\ \bar{f}(p)}{\rho\ p^2\ \alpha^2},$$

$$\bar{\psi}^C(r,\xi,p) = \frac{W_3}{\Delta}\ I_1(\beta r) + \frac{W_4}{\Delta}\ K_1(\beta r)\ . \tag{1.40}$$

From equations (1.7) and (1.40) it follows that

$$\bar{u}^{S}_{r}(r,\xi,p) = \frac{W_1}{\Delta} \alpha\, I_1(\alpha r) - \frac{W_2}{\Delta} \alpha\, K_1(\alpha r) + \frac{W_3}{\Delta} \xi\, I_1(\beta r) + \frac{W_4}{\Delta} \xi\, K_1(\beta r) \,, \tag{1.41}$$

$$\bar{u}^{C}_{z}(r,\xi,p) = \frac{W_1}{\Delta} \xi\, I_o(\alpha r) + \frac{W_2}{\Delta} \xi\, K_o(\alpha r) + \frac{W_3}{\Delta} \beta\, I_o(\beta r) - \frac{W_4}{\Delta} \beta\, K_o(\beta r) + \frac{\xi\, \bar{f}(p)}{\rho\, p^2} \,.$$

Substituting (1.38) into (1.35) one obtains

$$\bar{\sigma}^{C}_{zr}(r,\xi,p) = \frac{2\mu}{\Delta} \left[ W_1\, \xi\, \alpha\, I_1(\alpha r) - W_2\, \xi\, \alpha\, K_1(\alpha r) + W_3 \left(\xi^2 + \frac{\rho p^2}{2\mu}\right) I_1(\beta r) + W_4 \left(\xi^2 + \frac{\rho p^2}{2\mu}\right) K_1(\beta r) \right] ,$$

$$\bar{\sigma}^{S}_{rr}(r,\xi,p) = \frac{W_1}{\Delta} \left[ \left(2\mu\, \alpha^2 + \frac{\lambda p^2}{c_1^2}\right) I_o(\alpha r) - 2\mu \frac{\alpha}{r} I_1(\alpha r) \right] + \frac{W_2}{\Delta} \left[ \left(2\mu\, \alpha^2 + \frac{\lambda p^2}{c_1^2}\right) K_o(\alpha r) + 2\mu \frac{\alpha}{r} K_1(\alpha r) \right] + \frac{W_3}{\Delta} 2\mu\, \xi \left[ \beta\, I_o(\beta r) - \frac{1}{r} I_1(\beta r) \right] - \frac{W_4}{\Delta} 2\mu\, \xi \left[ \beta\, K_o(\beta r) + \frac{1}{r} K_1(\beta r) \right] - \frac{\lambda\, \xi\, \bar{f}(p)}{(\lambda+2\mu)\alpha^2} \,. \tag{1.42}$$

Using equations $(1.12)_2$ and (1.40) after some manipulations one has

$$\bar{\sigma}^{S}_{zz}(r,\xi,p) = \frac{W_1}{\Delta} \left( \rho p^2 - 2\mu\, \alpha^2 \right) I_o(\alpha r) + \frac{W_2}{\Delta} \left( \rho\, p^2 - 2\mu\, \alpha^2 \right) K_o(\alpha r) - \frac{W_3}{\Delta} 2\mu\, \xi\, \beta\, I_o(\beta r) + \frac{W_4}{\Delta} 2\mu\, \xi\, \beta\, K_o(\beta r) - \frac{\xi}{\alpha^2} \bar{f}(p) \,. \tag{1.43}$$

Equations (1.41), (1.42) and (1.43) represent the solution (called the formal one) of the considered problem expressed in the terms of Fourier sine-Laplace or Fourier cosine-Laplace transforms. It should be emphasized that $\Lambda$, $\alpha$, $\beta$, $W_\iota$, $\iota=1,2,3,4$, (see Eqs. (1.25), (1.28), (1.36) and (1.39)) are functions of transform parameters $\xi$ and p. Using the inversion theorems of the Laplace and Fourier transforms the solution can be written in the form

$$u_r(r,z,t) = \frac{1}{\pi^2 i} \int_0^\infty \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{u}_r^s(r,\xi,p)\, e^{pt} \sin(\xi z)dp\, d\xi \,,$$

$$u_z(r,z,t) = \frac{1}{\pi^2 i} \int_0^\infty \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{u}_z^c(r,\xi,p)\, e^{pt} \cos(\xi z)dp\, d\xi \,,$$

$$\sigma_{zr}(r,z,t) = \frac{1}{\pi^2 i} \int_0^\infty \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{\sigma}_{zr}^c(r,\xi,p)\, e^{pt} \cos(\xi z)dp\, d\xi \,, \tag{1.44}$$

$$\sigma_{rr}(r,z,t) = \frac{1}{\pi^2 i} \int_0^\infty \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{\sigma}_{rr}^s(r,\xi,p)\, e^{pt} \sin(\xi z)dp\, d\xi \,,$$

$$\sigma_{zz}(r,z,t) = \frac{1}{\pi^2 i} \int_0^\infty \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{\sigma}_{zz}^s(r,\xi,p)\, e^{pt} \sin(\xi z)dp\, d\xi \,.$$

The function f(.), if the boundary condition is given as $(1.4)_1$, is yet undefined. The possibility of the following cases of impulses can be mentioned:

<u>Case 1</u> *Infinitely short impulse*

Let function f(.) (see Eq.$(1.4)_1$) be given

$$f(t) = -f_o\, \delta(t) \tag{1.45}$$

where $f_o$, $f_o > 0$, is a constant and $\delta(.)$ is the Dirac function, then

$$\bar{f}(p) = -\, f_o \,. \tag{1.46}$$

Case 2 *Infinitely long impulse*

It is assumed

$$f(t) = - f_o \; H(t) \tag{1.47}$$

where H(.) is the Heaviside step function, then

$$\bar{f}(p) = f_o/p \; . \tag{1.48}$$

Case 3 *Impulse of finite lenght*

It is assumed

$$f(t) = -f_o \; H(t) \; H(\tau - t) \tag{1.49}$$

where $\tau$, $\tau > 0$, is a duration of the impulse. In this case one has

$$\bar{f}(p) = - \frac{f_o}{p} \left( 1 - e^{-p\tau} \right). \tag{1.50}$$

## 2.The problem of longitudinal impact

The problem presented above is related to the mixed conditions of shock pressure at the end of the semi-infinite circular elastic tube. Consider now the problem shown in Fig.2. Two semi-infinite circular tubes, moving in opposite directions with speed v ,are assumed to contact at the instant t=0 and at the plane z=0. The lateral surfaces of tubes are free of loads. The analogical problem for two semi-infinite circular bars was solved by Skalak [11], and such approach and also some results of [11] will be used further on.

The solution of the problem considered is derived by application of the superposition principle for two separate problems. The first of them is limited to two elastic semi-spaces, moving in opposite directions with speed v. They are assumed to contact at the instant t=0 and at the plane z=0. After time t=0 the semi-spaces are assumed to behave as a single, solid space, like the tubes considered above, which are assumed to behave for t > 0 as a single, infinite tube. The solution of this problem is given in [11] in the form:

$$u_r(r,z,t) = 0 \ ,$$

$$u_z(r,z,t) = \begin{cases} \mp \ v \ t & \text{for } \pm z > c \ t_1, \\ - \ v \ z/c_1 & \text{for } |z| < c_1 t \ . \end{cases} \tag{2.1}$$

The displacements assumed above induce the following stresses, when using equations (1.3) and (2.1):

$$\sigma_{zr}(r,z,t) = 0 \ ,$$

$$\sigma_{zz}(r,z,t) = \begin{cases} 0 & \text{for } |z| > c_1 t \ , \\ -(\lambda+2\mu) \ v \ /c_1 & \text{for } |z| < c_1 t \ , \end{cases} \tag{2.2}$$

$$\sigma_{rr}(r,z,t) = \begin{cases} 0 & \text{for } |z| > c_1 t \ , \\ - \ \lambda \ v \ /c_1 & \text{for } |z| < c_1 t \ . \end{cases}$$

It is seen from equations (2.2) that the solution of the first problem cause non-zero radial tractions on the lateral surfaces of the circular tube. Applying the principle of superposition to ensure a stress free lateral surfaces of the infinte tube the second problem is considered , which is defined by equations of motion (1.2), the boundary conditions

$$\sigma_{zr}(r=a,z,t) = \sigma_{zr}(r=b,z,t) = 0 \ ,$$

$$\sigma_{rr}(r=a,z,t) = \sigma_{rr}(r=b,z,t) = \begin{cases} 0 & \text{for } |z| > c_1 t \ , \\ \lambda \ v/c_1 & \text{for } |z| < c_1 t \ , \end{cases} \tag{2.3}$$

for $z \in (-\infty \ ,+\infty)$ , $t \in (0, \ +\infty)$ ,

and the initial conditions given in (1.6) for $z \in (-\infty,+\infty)$. A futher condition is that the displacements and the stresses vanish when $|z| \to \infty$ .

The solution of the problem is found by using displacement potentials $\phi$ and $\psi$ introduced in (1.7) and Fourier exponential transforms

$$f^F(r,\xi,t) \equiv \mathcal{F}\{f(r,z,t);\ z\to\xi\} = \int_{-\infty}^{\infty} f(r,z,t)\ e^{-i\xi z}\ dz \qquad (2.4)$$

as well as Laplace transform defined by $(1.17)_3$ with respect to $t$ . Using (2.4) one has

$$\mathcal{F}\{\frac{\partial f}{\partial z};\ z\to\xi\} = i\ \xi\ f^F(r,\xi,t)\ ,$$
$$\mathcal{F}\{\frac{\partial^2 f}{\partial z^2};z\to\xi\} = -\xi^2\ f^F(r,\xi,t) \qquad (2.5)$$

provided, that $\lim_{|z|\to\infty} f(r,z,t) = \lim_{|z|\to\infty} \frac{\partial f(r,z,t)}{\partial z} = 0$ .

Applying Fourier exponential transform and Laplace transform to equations (1.8) one obtains

$$\frac{\partial^2 \bar{\phi}^F(r,\xi,p)}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\phi}^F(r,\xi,p)}{\partial r} - \alpha^2\ \bar{\phi}^F(r,\xi,p) = 0,$$

$$\frac{\partial^2 \bar{\psi}^F(r,\xi,p)}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{\psi}^F(r,\xi,p)}{\partial r} -(\beta^2+\frac{1}{r^2})\ \bar{\psi}^F(r,\xi,p) = 0 \qquad (2.6)$$

where $\alpha^2$ is given in (1.25) and $\beta^2$ is determined by (1.28) .

The general solution of equations (2.6) can be written in the form:[2]

$$\bar{\phi}^F(r,\xi,p) = C_1\ I_0(\alpha r) + C_2\ K_0(\alpha r)\ ,$$

$$\bar{\psi}^F(r,\xi,p) = D_1\ I_1(\alpha r) + D_2\ K_1(\alpha r)\ , \qquad (2.7)$$

where $C_1$, $C_2$, $D_1$, $D_2$ are unknown functions of the variables $\xi$, $p$ and they have to be determined by using boundary conditions (2.3). Applying Fourier and Laplace transforms to equations

[2] It has emphasized that for the analogic problem of a rod the conditions $C_2=D_2\equiv 0$ sholud be assumed to ensure finite potentials, displacements and stresses at r=0 .

(1.12) one has (by using (2.5) and (1.19) ):

$$\bar{\sigma}^F_{rr}(r,\xi,p) = 2\mu \left( \frac{\partial^2 \bar{\phi}^F}{\partial r^2} - i\,\xi \frac{\partial \bar{\psi}^F}{\partial r} \right) + \frac{\lambda p^2}{c_1^2} \bar{\phi}^F ,$$

$$\bar{\sigma}^F_{zr}(r,\xi,p) = 2\mu \left[ i\,\xi \frac{\partial \bar{\phi}^F}{\partial r} + \left(\xi^2 + \frac{\rho p^2}{2\mu}\right) \bar{\psi}^F \right] . \qquad (2.8)$$

Substituting (2.7) into (2.8) and employing (1.33) it follows that

$$\bar{\sigma}^F_{rr}(r,\xi,p) = C_1 \left[ \left(2\mu\,\alpha^2 + \frac{\lambda p^2}{c_1^2}\right) I_o(\alpha r) - 2\mu \frac{\alpha}{r} I_1(\alpha r) \right] +$$

$$C_2 \left[ \left(2\mu\,\alpha^2 + \frac{\lambda p^2}{c_1^2}\right) K_o(\alpha r) + 2\mu \frac{\alpha}{r} K_1(\alpha r) \right] -$$

$$D_1 2\mu\, i\,\xi \left[ \beta\, I_o(\beta r) - \frac{1}{r} I_1(\beta r) \right] + D_2 2\mu\, i\,\xi \left[ \beta\, K_o(\beta r) + \right.$$

$$\left. \frac{1}{r} K_1(\beta r) \right] , \qquad (2.9)$$

$$\bar{\sigma}^F_{zr}(r,\xi,p) = 2\mu \left[ -C_1 i\,\xi\,\alpha\, I_1(\alpha r) + C_2 i\,\xi\,\alpha\, K_1(\alpha r) + \right.$$

$$\left. D_1\left(\xi^2 + \frac{\rho p^2}{2\mu}\right) I_1(\beta r) + D_2\left(\xi^2 + \frac{\rho p^2}{2\mu}\right) K_1(\beta r) \right].$$

Using the notations introduced in (1.36), the boundary conditions (2.3) together with (2.9) the following system of linear algebraic equations for unknowns $C_1$, $C_2$, $D_1$, $D_2$ is found

$$C_1 d_1(a) + C_2 d_2(a) - D_1 i\, d_3(a) - D_2 i\, d_4(a) = \bar{R}^F(\xi,p) ,$$

$$C_1 d_1(b) + C_2 d_2(b) - D_1 i\, d_3(b) - D_2 i\, d_4(b) = \bar{R}^F(\xi,p) ,$$

$$-C_1 i\, e_1(a) - C_2 i\, e_2(a) + D_1\, e_3(a) + D_2\, e_4(a) = 0 , \qquad (2.10)$$

$$-C_1 i\, e_1(b) - C_2 i\, e_2(b) + D_1\, e_3(b) + D_2\, e_4(b) = 0 ,$$

where

$$\bar{R}^F(\xi,p) = \int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{\lambda v}{c_1} H(c_1 t - |z|)\, e^{-i\xi z}\, e^{-pt}\, dz\, dt = \frac{2\lambda v}{p^2 + \xi^2 c_1^2}. \qquad (2.11)$$

The solution of equations (2.10) can be written in the form

$$C_1 = \frac{T_1}{T}, \quad C_2 = \frac{T_2}{T}, \quad D_1 = \frac{T_3}{T}, \quad D_2 = \frac{T_4}{T}, \qquad (2.12)$$

where

$$T = \det\begin{bmatrix} d_1(a) & d_2(a) & -id_3(a) & -id_4(a) \\ d_1(b) & d_2(b) & -id_3(b) & -id_4(b) \\ -ie_1(a) & -ie_2(a) & e_3(a) & e_4(a) \\ -ie_1(b) & -ie_2(b) & e_3(b) & e_4(b) \end{bmatrix},$$

$$T_1 = \det\begin{bmatrix} \bar{R}^F(\xi,p) & d_2(a) & -id_3(a) & -id_4(a) \\ \bar{R}^F(\xi,p) & d_2(b) & -id_3(b) & -id_4(b) \\ 0 & -ie_2(a) & e_3(a) & e_4(a) \\ 0 & -ie_2(b) & e_3(b) & e_4(b) \end{bmatrix}, \qquad (2.13)$$

$$T_2 = \det\begin{bmatrix} d_1(a) & \bar{R}^F(\xi,p) & -id_3(a) & -id_4(a) \\ d_1(b) & \bar{R}^F(\xi,p) & -id_3(b) & -id_4(b) \\ -ie_1(a) & 0 & e_3(a) & e_4(a) \\ -ie_1(b) & 0 & e_3(b) & e_4(b) \end{bmatrix},$$

$$T_3 = \det\begin{bmatrix} d_1(a) & d_2(a) & \bar{R}^F(\xi,p) & -id_4(a) \\ d_1(b) & d_2(b) & \bar{R}^F(\xi,p) & -id_4(b) \\ -ie_1(a) & -ie_2(a) & 0 & e_4(a) \\ -ie_1(b) & -ie_2(b) & 0 & e_4(b) \end{bmatrix},$$

$$T_4 = \det \begin{bmatrix} d_1(a) & d_2(a) & -id_3(a) & \bar{R}^F(\xi,p) \\ d_1(b) & d_2(b) & -id_3(b) & \bar{R}^F(\xi,p) \\ -ie_1(a) & -ie_2(a) & e_3(a) & 0 \\ -ie_1(b) & -ie_2(b) & e_3(b) & 0 \end{bmatrix} ,$$

and $d_i(.)$ , $e_i(.)$ , $i=1$ ,2 ,3 ,4 , are defined by equation (1.36).

Substituting (2.12) into (2.7) and using (1.7) and (1.12) after some manipulations one obtains

$$\bar{u}_r^F(r,\xi,p) = \frac{1}{T} [\ T_1 \alpha\ I_1(\alpha r) - T_2\ \alpha\ K_1(\alpha r) - T_3 i\ \xi\ I_1(\beta r) -$$

$$T_4\ i\ \xi\ K_1(\beta r)\ ] , \tag{2.14}$$

$$\bar{u}_z^F(r,\xi,p) = \frac{1}{T} [\ T_1\ i\xi\ I_0(\alpha r) + T_2\ i\xi\ K_0(\alpha r) +$$

$$T_3\ \beta\ I_0(\beta r) - T_4\ \beta\ K_0(\beta r)\ ] ,$$

and

$$\bar{\sigma}_{zr}^F(r,\xi,p) = \frac{2\mu}{T} [\ -T_1\ i\ \xi\ \alpha\ I_1(\alpha r) + T_2\ i\ \xi\ \alpha\ K_1(\alpha r) +$$

$$T_3\ (\xi^2 + \frac{\rho p^2}{2\mu})\ I_1(\beta r) + T_4\ (\xi^2 + \frac{\rho p^2}{2\mu})\ K_1(\beta r)\ ] ,$$

$$\bar{\sigma}^F_{rr}(r,\xi,p) = \frac{T_1}{T} \left[ \left(2\mu\,\alpha^2 + \frac{\lambda p^2}{c_1^2}\right) I_0(\alpha r) - 2\mu \frac{\alpha}{r} I_1(\alpha r) \right] +$$

$$\frac{T_2}{T} \left[ \left(2\mu\,\alpha^2 + \frac{\lambda p^2}{c_1^2}\right) K_0(\alpha r) + 2\mu \frac{\alpha}{r} K_1(\alpha r) \right] -$$

$$\frac{T_3}{T}\, 2\mu\, i\, \xi \left[ \beta\, I_0(\beta r) - \frac{1}{r} I_1(\beta r) \right] +$$

$$\frac{T_4}{T}\, 2\mu\, i\, \xi \left[ \beta\, K_0(\beta r) + \frac{1}{r} K_1(\beta r) \right] , \qquad (2.15)$$

$$\bar{\sigma}^F_{zz}(r,\xi,p) = \frac{1}{T} \left[ T_1 \left(\rho p^2 - 2\mu\,\alpha^2\right) I_0(\alpha r) + T_2 \left(\rho p^2 - \right.\right.$$

$$\left.\left. 2\mu\,\alpha^2\right) K_0(\alpha r) + T_3\, 2\mu\, \xi\, i\, \beta\, I_0(\beta r) - T_4\, 2\mu\, \xi\, i\, \beta\, K_0(\beta r)\right] .$$

Equations (2.14) and (2.15) represent the solution ,called the formal one ,of the considered impact problem expressed in terms of Fourier and Laplace transforms. The inversion of these transforms would be difficult to carry out because T, $T_\iota$, $\iota$ =1, 2, 3, 4, are the functions of transform parameters $\xi$ and p, equations (2.13), (1.36), (1.28). Using the inversion theorems of the Laplace and Fourier exponential transforms the solution can be written in the form

$$u_r(r,z,t) = \frac{1}{4\,\pi^2 i} \int_{-\infty}^{\infty} \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{u}_r^F(r,\xi,p)\, e^{i\xi z}\, e^{pt}\, dp\, d\xi ,$$

$$u_z(r,z,t) = \frac{1}{4\,\pi^2 i} \int_{-\infty}^{\infty} \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{u}_z^F(r,\xi,p)\, e^{i\xi z}\, e^{pt}\, dp\, d\xi ,$$

$$\sigma_{zz}(r,z,t) = \frac{1}{4\,\pi^2 i} \int_{-\infty}^{\infty} \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{\sigma}^F_{zz}(r,\xi,p)\, e^{i\xi z}\, e^{pt}\, dp\, d\xi ,$$

$$\sigma_{zr}(r,z,t) = \frac{1}{4\,\pi^2 i} \int_{-\infty}^{\infty} \int_{\gamma-i\infty}^{\gamma+i\infty} \bar{\sigma}^F_{zr}(r,\xi,p)\, e^{i\xi z}\, e^{pt}\, dp\, d\xi ,$$

$$\sigma_{rr}(r,z,t) = \frac{1}{4\pi^2 i} \int_{-\infty}^{\infty} \int_{\gamma - i\infty}^{\gamma + i\infty} \bar{\sigma}^{F}_{rr}(r,\xi,p)\, e^{i\xi z}\, e^{pt}\, dp\, d\xi \ .$$

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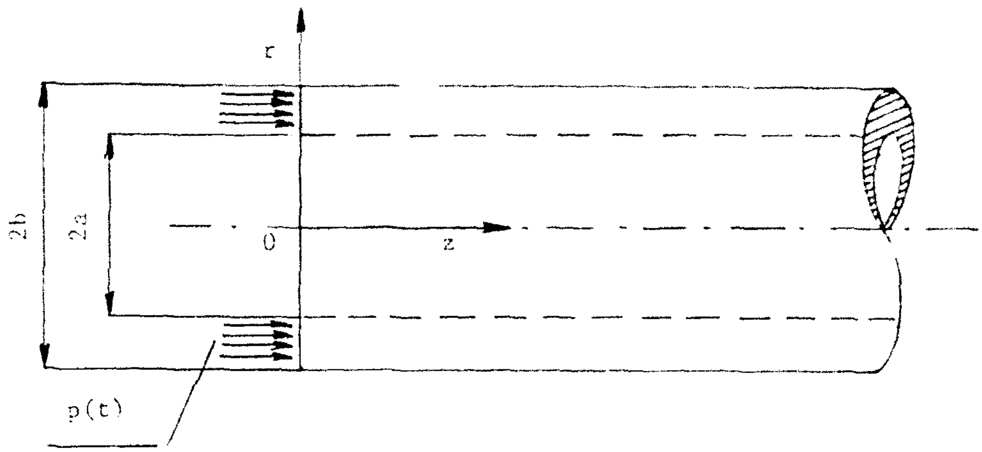


Fig. 1

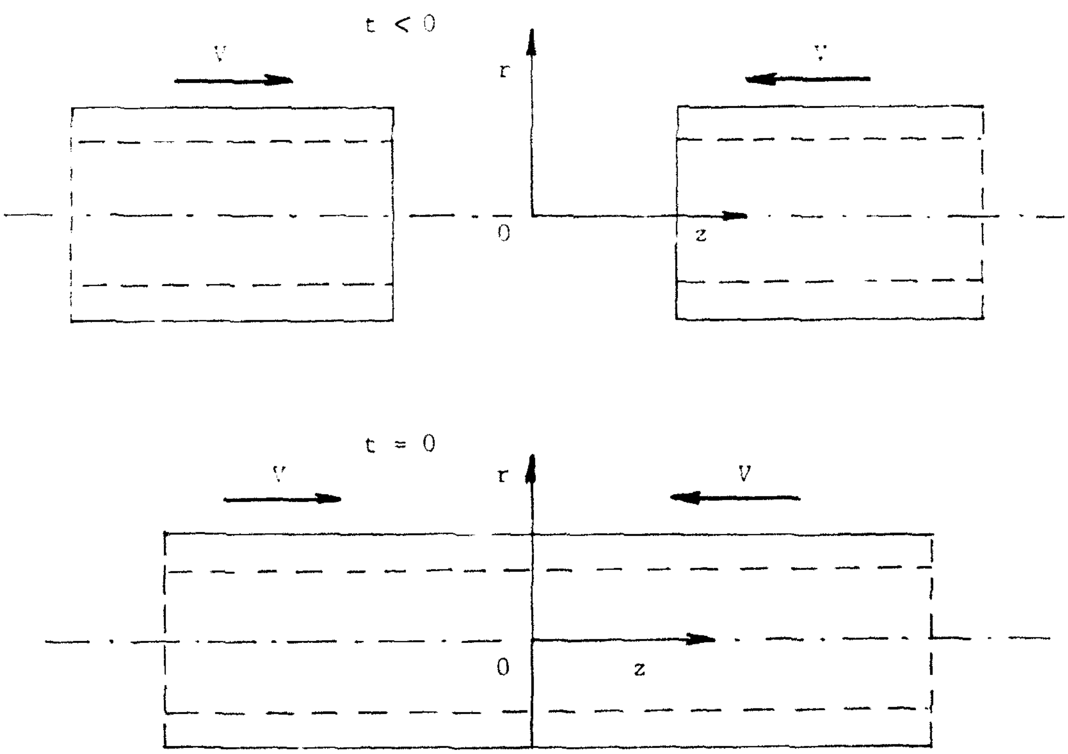


Fig. 2