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design of an adaptive scheme and in the analysis of its robustness. In our research these three heretofore separate techniques have been merged into a methodology which eliminates their individual shortcomings. In a separate research direction we have initiated a study of systems with practically important nondifferentiable nonlinearities such as dead-zone, backlash and hysteresis. These systems can not be analyzed by existing methods.

During this research period, major advances have been made in our study of nonlinear dynamic systems with parametric uncertainties and in the development of a systematic design methodology for adaptive nonlinear control.

First, we have demonstrated that the phenomenon of controller and/or observer peaking is of fundamental importance for nonlinear feedback design, and that interference of peaking with uncertain nonlinearities can result in a drastic decrease of the stability region. Geometric-asymptotic conditions under which this type of interference can be avoided have been developed. Further advance in this area has been made by other researchers who were motivated by our results. A coherent theory of semiglobal stabilization is emerging from this collective effort.

Second, we have shown that adaptive control methods can reduce the effects of parametric uncertainties without introducing high-gain loops, thus avoiding the danger of peaking. We have solved the adaptive tracking problem with full-state feedback. Our solution is in the form of a systematic recursive procedure called backstepping. Using backstepping and a new concept of tuning functions we have designed adaptive controllers which avoid overparametrizations.

Third, we have formulated and partially solved a class of nonlinear output-feedback problems by developing a design toolkit applicable to a wide range of systems. Among the tools developed so far are our nonlinear damping terms which compensate for the effects of the state estimation error. We are in the process of applying this toolkit to the design of a new generation of nonlinear adaptive and robust controllers whose applicability and performance surpass all earlier designs.

Fourth, we have initiated the study of adaptive control of systems with unknown nondifferentiable nonlinearities - a new area of adaptive control. Typical examples of such nonlinear characteristics are dead-zone, backlash and hysteresis which are common in control systems and often severely limit system performance. We have developed an adaptive inverse approach for handling these nonlinearities.

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**NONLINEAR SYSTEM DESIGN:
ADAPTIVE FEEDBACK LINEARIZATION
WITH UNMODELED DYNAMICS**

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TECHNICAL REPORT

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Abstract

The main goal of this research has been to develop a unified geometric-asymptotic-adaptive methodology for feedback design of nonlinear control systems. Such a methodology is needed because the existing differential geometric results are restrictive and often violated by small modeling errors. Effects of these errors can be analyzed asymptotically by singular perturbation methods, which, however, are still lacking a clear geometric interpretation. Neither geometric, nor perturbational problem formulations can cope with large parametric uncertainty, for which an adaptive approach seems suitable. Conversely, both geometric and asymptotic techniques can become constructive steps in the design of an adaptive scheme and in the analysis of its robustness. In our research these three heretofore separate techniques have been to be merged into a methodology which eliminates their individual shortcomings. In a separate research direction we have initiated a study of systems with practically important nondifferentiable nonlinearities such as dead-zone, backlash and hysteresis. These systems can not be analyzed by existing methods.

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systematic recursive procedure called **backstepping**. Using backstepping and a new concept of **tuning functions** we have designed adaptive controllers which avoid overparametrizations.

Third, we have formulated and partially solved a class of nonlinear output-feedback problems by developing a design toolkit applicable to a wide range of systems. Among the tools developed so far are our nonlinear damping terms which compensate for the effects of the state estimation error. We are in the process of applying this toolkit to the design of a new generation of nonlinear adaptive and robust controllers whose applicability and performance surpass all earlier designs.

Fourth, we have initiated the study of adaptive control of systems with unknown nondifferentiable nonlinearities - a new area of adaptive control. Typical examples of such nonlinear characteristics are **dead-zone**, **backlash** and **hysteresis** which are common in control systems and often severely limit system performance. We have developed an adaptive inverse approach for handling these nonlinearities.

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1 Introduction

Realistic models of physical systems include incompletely known nonlinearities, unmodeled dynamics, and unknown disturbances. Our adaptive and nonlinear design methods employ two types of uncertainty characterizations.

The first type uncertainty consists of unknown constant parameters multiplying known or preselected nonlinearities. This characterization is convenient for adaptive control. The second characterization, suitable for the design of fixed robust nonlinear controllers, assumes that unknown nonlinearities satisfy some known nonlinear bounds.

To allow for a fuller use of the available modeling information, such uncertainty characterizations are further refined by employing geometric properties of the model. Among the key geometric properties in our approach are the complexity of the unknown nonlinearity, and its distance, in terms of the number integrators from the control input. These properties have been crucial in our recent development of a systematic adaptive nonlinear design with full-state feedback. They have also been extended to more demanding output- and partial-state-feedback designs.

Whenever we develop a design tool for systems without uncertainties, we proceed to extend it for an adaptive or robust design. With our tools the design starts with some basic system structures and then grows through a recursive application of the same tools.

Available nonlinear design tools assume that the system nonlinearities are differentiable, thus excluding some of the most practical nonlinearities such as dead-zone, backlash and hysteresis. Further results of this research have made our design tools applicable to such common nonlinearities.

In this report, our research accomplishments will be grouped in two broad areas:

- Geometric-Asymptotic Methods;

Publications: J1, J3, J5-J7, J10, J12; C8, C14, C15.

- Adaptive Nonlinear Control;

Publications: J2, J4, J8, J9, J11; C1-C7, C9-C13, C16-C20.

2 Geometric-Asymptotic Methods

Our major geometric-asymptotic results deal with approximate feedback linearization designs, peaking and semiglobal stabilization.

2.1 Perturbed Zero-dynamics

Feedback linearization designs were expected to preserve stability under small regular perturbations. However, our results on perturbed zero dynamics [J7] show that in many situations this is not the case. We have shown that, although fundamental and very useful, the concept of zero dynamics is, unfortunately, extremely nonrobust. Simply stated, our result is that the zero dynamics of regularly perturbed systems may be, and often are, singularly-perturbed. Our singular perturbation analysis proves that a relative-degree perturbation of three and higher leads to fast instabilities. A rather far reaching conclusion is that instead of the exact feedback linearization designs for perturbed systems, approximate designs may be more robust. Two such approximate designs are presented in [J6] and [C8].

2.2 Peaking and Semiglobal Stabilization

Another major result of our current research is our analysis of the peaking phenomenon. To analyze the destabilizing effects of peaking, we have addressed the problem of global stabilization for a class of cascade systems [J3]. In this problem, the first part of the cascade is a linear controllable system and the second part is a nonlinear system receiving the inputs from the states of the first part. In linear systems, a peaking phenomenon occurs when high gain feedback is used to produce eigenvalues with very negative real parts. Then some states peak to very large values, before they rapidly decay to zero. Such peaking states act as destabilizing inputs to the nonlinear part and may even cause some of its states to escape to infinity in finite time, as illustrated by simple examples. We have given precise structural conditions for peaking and proceeded to show that the destabilizing effects of peaking can be reduced if the nonlinearities have sufficiently slow growth. Based on our detailed analysis of the peaking phenomenon we have examined the tradeoffs between linear peaking and nonlinear growth conditions. To provide for realistic trade offs between performance and stability, we have introduced the concepts of semiglobal stability and nonlinear overshoot function. Using these concepts we have given a method

for computing robustness bounds. These results have been further extended by other researchers who have obtained sharper semiglobal stability conditions.

For the output feedback problem we have proposed a design in which the effects of observer peaking are counteracted by "nonlinear damping" terms. This design is a part of a more general Design Toolkit which is being developed in our current research.

3 Adaptive Nonlinear Control

Until a few years ago, adaptive linear and geometric nonlinear methods belonged to two separate areas of control theory. They were helpful in the design of controllers for plants containing either unknown parameters or known nonlinearities, but not both. In the last few years, the problem of adaptive nonlinear control was formulated to deal with the control of plants containing both unknown parameters and known nonlinearities. With our backstepping method we have first solved the full-state feedback problem and then proceeded to more challenging output-feedback problems. We have also obtained preliminary results in the adaptive design of systems with dead-zone, backlash and hysteresis nonlinearities.

3.1 Adaptive Backstepping Design

Our major result, most favorably received by the research community, is for pure-feedback systems. This is the broadest class of nonlinear systems for which adaptive controllers can now be systematically designed without imposing any growth constraints on system nonlinearities. The geometric characterization of this class identifies the level of uncertainty and nonlinear complexity as structural obstacles to adaptive feedback linearization. For an unknown parameter, the level of uncertainty is its "distance," in terms of the number of integrators, from the control input. The larger this distance is, the smaller is the number of state variables on which the nonlinearity multiplying this parameter is allowed to depend.

Our adaptive scheme is designed by a systematic backstepping procedure which recursively constructs, at each step, a parameter update law and a new Lyapunov function to be used for a direct proof of stability.

One of the most important stability and robustness properties of every adaptive system is the size of its region of attraction, relative to the size of the region that would have been achieved if

all the parameters were known. The region of attraction for the new adaptive scheme is global if the feedback linearization is global. A subclass of pure-feedback systems for which this global property is easy to establish are strict feedback systems. For these systems, our adaptive scheme achieves both global regulation and global tracking of smooth bounded reference inputs. In contrast to earlier schemes, these global results are obtained without any growth constraints on system nonlinearities.

A complete presentation of our state-feedback results, their proofs, and examples illustrating their properties, is given in [J4]. The sequence of results which culminated in [J4] can be traced through [J2], [C3], [C5] and [C6]. Our recent breakthrough [J9] introduced the concept of tuning functions which avoids overparametrization and significantly improves the results of [J4].

A backstepping procedure has also been developed for nonadaptive robust design of nonlinear systems without matching restrictions on uncertainties [J12].

3.2 Adaptive Output-Feedback Control

By far the most difficult problems in adaptive nonlinear control are those with incomplete state measurement. Adaptive output-feedback designs may follow either a direct model-reference path or an indirect path, via adaptive observers. Our first two results on output-feedback adaptive nonlinear control, [C1] and [C4], followed the indirect path and imposed linear growth constraints similar to those used in earlier state-feedback adaptive schemes. In our current research we formulate and solve truly nonlinear output-feedback and partial-state-feedback adaptive control problems [J8], [C7], [C9], [C10], [C13], [C19].

For systems in the so called output-feedback form we do not require any growth conditions on the output-dependent nonlinearities. For these systems we systematically design adaptive controllers with global stability properties which guarantee that the tracking error converges to zero. At the present time, this is the broadest class of nonlinear systems that can be adaptively controlled by output feedback.

The above class of nonlinear systems encompasses all minimum phase linear systems. When applied to linear systems, our approach results in a whole new generation of adaptive controllers with remarkable features. Not only is the design of these controllers systematic and with the simplest stability proof, but also is their performance superior to the performance achievable by other adaptive schemes.

In conventional adaptive control, the applicability of the passivity approach has been limited to systems with relative degree less than two. Until recently, this obstacle seemed insurmountable because of the feedback invariance of the relative degree. The relative degree limitation has now been removed by our backstepping procedure [J3], [J4], [J5], [J9]. The idea of backstepping, illustrated in Fig.1, is to design a sequence of “virtual” systems S_i of relative degree one, finishing with the actual system as the last member of the sequence. For each virtual system S_i the relative degree is reduced to one by selecting an available signal as a virtual input and then achieving passivity with respect to a virtual output. The choice of the virtual input-output pairs is flexible and different designs are possible with essentially the same procedure. For the adaptive design, shown in Fig.1, the last virtual output τ_p is used to close the adaptive feedback loop via the passive parameter update law $\dot{\tilde{\theta}} = \Gamma \tau_p$, where $\Gamma > 0$.

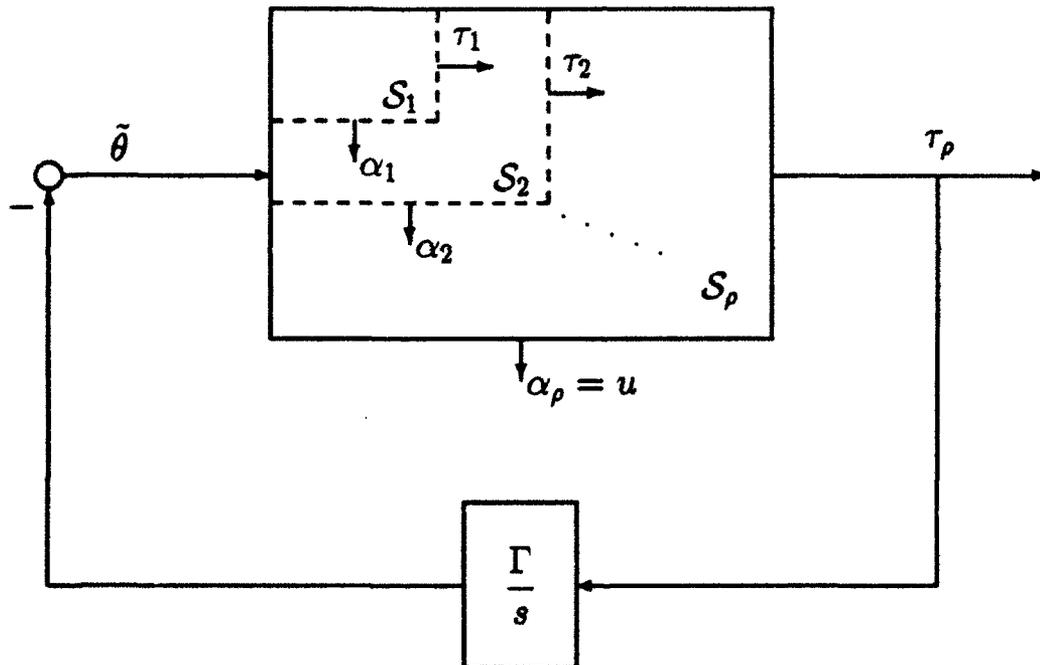


Figure 1: The schematic representation of the backstepping design procedure. At each step, a virtual system S_i is designed, with a strict passivity property from the virtual input $\tilde{\theta}$ to the virtual output, the tuning function τ_i .

3.3 Adaptive Control of Systems with Nondifferentiable Nonlinearities

Many physical components of control systems have nonsmooth nonlinear characteristics such as dead-zones, backlash and hysteresis. These practical nonlinearities often severely limit system performance, giving rise to inaccuracy and oscillations, or even leading to instability. Yet, these nonlinearities have not been treated in modern nonlinear feedback control theory.

In [J11], [C11, C12, C17, C18], we have developed a new adaptive inverse control approach for systems consisting of a linear part preceded by an actuator with an unknown dead-zone, backlash or hysteresis characteristic. Our adaptive inverse controllers employ an adaptive inverse for the unknown nondifferentiable nonlinearity and a linear adaptive controller structure for the unknown linear part. Simulation results show significant improvement of system performance with the use of an adaptive inverse controller.

Adaptive control of systems with nondifferentiable nonlinearities is a new area of adaptive control, in which there are many open theoretical problems of major relevance to applications. We have initiated this research area because of urgent demands from industry, including Rockwell and Ford. Our future research in this direction will include stability, convergence and robustness analysis of the proposed adaptive control schemes, and extension of our adaptive designs to wider classes of systems with nondifferentiable nonlinearities.

4 Principal Investigator's Activities

The Principal Investigator was the co-organizer (with Alan J. Laub) of an NSF-NASA workshop on Nonlinear Control, April 5-7, 1990, at Cliff House, UCSB. Many of the topics covered by this research grant were discussed at the workshop. Another major event organized by P. V. Kokotović, in his capacity as Grainger Professor at the University of Illinois, was the series of fifteen Grainger Lectures on "Foundations of Adaptive Control," September 28-October 1, 1990. A volume (more than 500 pages) of extended texts of these lectures was published by Springer in June 1991. At the World Congress of IFAC in August 1990, P. V. Kokotović received the IFAC's highest award - the Quazza Medal - which has been given triennially since 1981. He delivered the Bode Prize lecture at the 1991 IEEE Conference on Decision and Control.

5 Publications

The details of results supported in part by our AFOSR grant are documented in 12 journal papers listed below as [J1] to [J12] and 20 conference papers [C1] to [C20].

- [J1] H. K. Khalil and P. V. Kokotović, "On stability properties of nonlinear systems with slowly-varying inputs," *IEEE Transactions on Automatic Control*, vol. AC-36, p. 229, Feb. 1991.
- [J2] I. Kanellakopoulos, P. V. Kokotović, and R. Marino, "An extended direct scheme for robust adaptive nonlinear control," *Automatica*, vol. 27, pp. 247-255, March 1991.
- [J3] H. J. Sussmann and P. V. Kokotović, "The peaking phenomenon and the global stabilization of nonlinear systems," *IEEE Transactions on Automatic Control*, vol. AC-36, pp. 424-439, April 1991.
- [J4] I. Kanellakopoulos, P. V. Kokotović, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Transactions on Automatic Control*, vol. AC-36, pp. 1241-1253, Nov. 1991.
- [J5] I. Kanellakopoulos, P. V. Kokotović, and A. S. Morse, "A toolkit for nonlinear feedback design," *Systems & Control Letters*, vol. 18, pp. 83-92, Feb. 1992.
- [J6] J. Hauser, S. S. Sastry, and P. V. Kokotović, "Nonlinear control via approximate input-output linearization: the ball and beam example," *IEEE Transactions on Automatic Control*, vol. AC-37, 1992 (to appear).
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- [J8] I. Kanellakopoulos, P. V. Kokotović, and A. S. Morse, "Adaptive output-feedback control of systems with output nonlinearities," *IEEE Transactions on Automatic Control*, vol. AC-37, pp. 1666-1682, 1992.
- [J9] M. Krstić, I. Kanellakopoulos, P. V. Kokotović, "Adaptive nonlinear control without over-parametrization," *Systems & Control Letters*, vol. 19, pp. 177-185.

- [J10] P. V. Kokotović, "The joy of feedback: nonlinear and adaptive," 1991 Bode Prize Lecture, *Control Systems Magazine*, vol. 12, pp. 7-17, June 1992.
- [J11] G. Tao and P. V. Kokotović, "Adaptive control of systems with backlash," *Automatica*, vol. 25, 1993 (to appear).
- [J12] R. A. Freeman and P. V. Kokotović, "Backstepping design of robust controllers for a class of nonlinear systems," *Automatica* (to appear).
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- [C3] P. V. Kokotović and I. Kanellakopoulos, "Adaptive nonlinear control: a critical appraisal," *Proceedings of the Sixth Yale Workshop on Adaptive and Learning Systems*, pp. 1-6, New Haven, CT, 1990.
- [C4] I. Kanellakopoulos, P. V. Kokotović, and R. H. Middleton, "Indirect adaptive output-feedback control of a class of nonlinear systems," *Proceedings of the 29th IEEE Conference on Decision and Control*, pp. 2714-2719, Honolulu, HI, 1990.
- [C5] I. Kanellakopoulos, P. V. Kokotović, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *Proceedings of the 1991 American Control Conference*, pp. 649-654, Boston, MA.
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- [C14] R. A. Freeman and P. V. Kokotović, "Backstepping design of robust controllers for a class of nonlinear systems," *Proceedings of the IFAC Symposium on Nonlinear Control Systems Design*, pp. 307-312, Bordeaux, France, 1992.
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- [C16] P. V. Kokotović, I. Kanellakopoulos, M. Krstić, "On letting adaptive control be what it is: nonlinear feedback," *Plenary Talk, ACASP, IFAC Symposium on Adaptive Systems in Control and Signal Processing*, Grenoble, France, July 1992.

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