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Techniques have been demonstrated for time dependent problems that allow rare activation of finest grids; new multiscale approaches in statistical mechanics and many particle problems; fast integral transforms and solvers for integro-differential equations; fast Dirac solvers, and multigrid algorithms on decomposed domains.

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FINAL
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AFOSR 91-0156

Multilevel Techniques in Large Scale Computation

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October, 1992

A. Narrative

Brandt has sorted out the reasons for past failures to obtain high multigrid efficiency in indefinite (wave) and non-elliptic (small viscosity) problems, and is systematically developing algorithms that overcome these difficulties (1), (5), (10), (12), (16), (19) as well as other difficulties associated with boundaries (7), (13), (14), (15), (20). He has demonstrated techniques for time dependent problems that allow rare activation of finest grids (8), (21); new multiscale approaches in statistical mechanics (2), (17), (22) and in many particle problems (3), (6); fast integral transforms (3) and solvers for integro-differential equations; fast Dirac solvers (4), (11), (17); and multigrid algorithms on decomposed domains (18).

Ruge developed efficient multigrid solvers for turbulent flows in meteorology (9) and effective multilevel adaptive methods for the shallow water equatiions (42).

Quinlan (23), (24) developed two C++ class software libraries that provide a software development environment where only single level serial application are specified, but effective parallel adaptive mesh refinement code is automatically obtained.

McCormick (33), (39) developed a new methodology designed to unify a major component of the multigrid discipline and provide a simple and reliable framework for algorithm development. This methodology was applied to parameter estimation (25), (36), optimal control (26) and eigenproblems (35), (40). Multilevel solvers and adaptive methods were developed for various fluid flow problems in transition (27), (28), (29), (37). A theoretical foundation was established (32), (34), (38) for finite volume discretization techniques. Multigrid methods were further developed for elliptic equations with discontinuous coefficients (30), the Eagle grid generation code (31), and parallel processing (41).

B. Publications

1. **A. Brandt and I. Yavneh**
   On multigrid solution of high-Reynolds incompressible entering flows, *J. Comp. Phys.* 101 (1992) 151--164.

2. **A. Brandt, M. Galun** and D. Ron
   Optimal multigrid algorithms for calculating thermodynamic limits, *J. Stat. Phys.*, in press.

3. **A. Brandt**
   Multilevel computations of integral transforms and particle interactions with oscillatory kernels, *Computer Physics Communications* 65 (1991) 24–38.

4. **R. Ben-Av, A. Brandt, M. Harmatz, E. Katznelson, P.G. Lauwers, S. Solomon and K. Wolowesky**
   Fermion simulation using parallel transported multigrid, *Phys. Lett. B.* 253 (1991) 185.

5. **A. Brandt and I. Yavneh**
Accelerated multigrid convergence for high-Reynolds recirculating flows, *J. Comp. Phys.*, in press.

6. **D.S. Balsara and A. Brandt**
Multilevel methods for fast solution of N-body and hybrid systems, *Multigrid Methods III* (W. Hackbusch and U. Trottenberg, eds.), Birkhauser Verlag, Basel, 1991.

7. **V. Mikulinsky and A. Brandt**
Multigrid treatment of free boundary conditions, *Multigrid Methods: Special Topics and Applications II* (W. Hackbusch and U. Trottenberg, eds), GMD-Studien Nr. 189, 1991.

8. **A. Brandt and J. Greenwald**
Parabolic multigrid revisited, *Multigrid Methods III* (W. Hackbusch and U. Trottenberg, eds.), Birkhauser Verlag, Basel, 1991, pp. 143--154.

9. **J. Ruge, A. Brandt, J. McWilliam and R. Milliff**
Multigrid methods applied to turbulent flow problems, *Multigrid Methods III* (W. Hackbusch and T. Trottenberg, eds.), Birkhauser Verlag, Basel, 1991.

10. **D. Sidilkover and A. Brandt**
Multigrid solution to steady-state 2D conservation laws, *SIAM J. Num: Anal.*, in press.

11. **M. Harmatz and P.G. Lauwers with R. Ben-Av, A. Brandt, E. Katznelson, S. Solomon and K. Wolowesky**
Parallel-transported multigrid and its application to the Sci. [illegible]
B (Proc. Suppl.) 20 (1991).

12. **A. Brandt and I. Yavneh**
Improved coarse-grid correction for high-Reynolds flows, *Proc. 5th Copper Mountain Conf. on Multigrid Methods.* Copper Mountain, Colorado, April, 1991.

13. **A. Brandt**
Rigorous quantitative analysis of multigrid, report, April, 1991. (Extension of (52).

14. **A. Brandt and V. Mikulinsky**
Recombining iterants in multigrid algorithms and problems with small islands, report, August, 1991.

15. **A. Brandt and V. Mikulinsky**
Multigrid treatment of problems with highly oscillating boundary and boundary conditions, report, September, 1991.

16. **A. Brandt**
Multiscale computational methods: research activities, *Proc. 1991 Hang Zhou International Conf. on Scientific Computation* (T. Chan and Z.-C. Shi, eds.), World Scientific Publishing Co., Singapore, 1992.

17. **A. Brandt**
Multigrid methods in lattice field computations, *Nucl. Phys. B* (Proc. Suppl.) 26 (1992) 137--180.

18. **A. Brandt**
Multigrid algorithms on decomposed domains, *Proc. of the Sixth International Conference on Domain Decomposition Methods in Science and Engineering* (A. Quarteroni, ed.), to appear.

19. **A. Brandt, C.I. Venner and I. Yavneh**
Efficient multilevel solution of the advection problem with closed characteristics using upstream discretization and downstream relaxation, draft report.

20. **V. Mikulinsky**
Multigrid treatment of boundary and free-boundary conditions, *Ph.D. Thesis* (supervised by A. Brandt), Weizmann Institute of Science, 1992.

21. **J. Greenwald**
Multigrid techniques for parabolic problems, *Ph.D. Thesis* (supervised by A. Brandt), Weizmann Institute of Science, 1992.

22. **M. Galun**
Optimal multigrid algorithms for model problems in statistical mechanics, *M.Sc. Thesis* (supervised by A. Brandt), Weizmann Institute of Science, 1992.

23. **M. Lemke and D. Quinlan**
Fast adaptive composite grid methods on Distributed parallel architectures, *Comm. Appl. Num. Meths.*, to appear.

24. **D. Bulsara, M. Lemke, and D. Quinlan**
AMR++, a C++ object oriented class library for parallel adaptive mesh refinement fluid dynamics applications, *Proc Symp. on Adaptive, Multilevel, and Hierarchical Computational Strategies*, ASME Winter Annual Meeting, Anaheim, California, Nov., 1992.

25. **S. McCormick and G. Wade**
Multilevel paramenter estimation, *Proc. 5th Copper Mountain Conference on Multigrid Methods*, April, 1991.

26. **K. Ito, S. McCormick, and L. Tiejun**
Multilevel Riccati solvers, *Proc. 5th Copper Mountain Conference on Multigrid Methods*, April, 1991.

27. **C. Liu, Z. Liu, and S. McCormick**
High order finite volume and multigrid methods for flow transition in a planner channel, *Proc. 4th Int'l Symp. on CFD*, UC Davis, California, 1991

28. **C. Liu, Z. Liu, and S. McCormick**
Multigrid methods for flow transition in a planar channel, *Computer Physics Comm.* 65 (1991), 188-200.

29. **C. Liu, Z. Liu, and S. McCormick**
Multilevel adaptive methods for incompressible flow in grooved channels, J. *Comp. Appl. Math* 38 (1991), 283-295.

30. **C. Liu, Z. Liu, and S. McCormick**
An efficient multigrid scheme for elliptic equations with discontinuous coefficients, *Comms. Appl. Num. Meths.*, in press.

31. **C. Liu, Z. Liu, and S. McCormick**
Multilevel adaptive methods for Eagle code, *Numerical Grid Generation in C.F.D. and Related Fields* (A.S. Arcilla, J. Hauser, P.R. Eiseman, and J.F. Thompson, eds.), Elsevier, North Holland, 1991, 687-701.

32. **Z. Cai, J. Mandel, and Steve McCormick**
The finite volume element method for diffusion equations on general triangulations, *SIAM J. Num. Anal.* 28 (1991), 392-402.

33. **S. McCormick**
Multilevel Projection Methods for Partial Differential Equations, CBMS-NSF Series, SIAM, Philadelphia, 1992, 115 pps.

34. **S. McCormick and U. Ruede**
A finite volume convergence theory for the fast adaptive composite grid methods, manuscript.

35. **S. McCormick**
Multilevel adaptive methods for elliptic eigenproblems: two-level convergence theory, manuscript.

36. **S. McCormick and G. Wade**
Multigrid solution of linearized, regularized least squares problems in electrical impedance tomography, submitted.

37. **C. Liu, Z. Liu, and S. McCormick**
Fourth order finite difference and multigrid methods for modeling instabilities in flat plate boundary layers, submitted.

38. **S. McCormick**
Finite volume element and multilevel adaptive methods, *Proc. Comp. Mech. Water Resources Conf.*, Denver, Colorado, June 9-12, 1992.

39. **S. McCormick**
Multilevel projection methods for partial differential equations, *Proc. NATO Advanced Res. Workshop on Asymptotic-Induced Numerical Methods for PDE's, Critical Parameters, and Domain Decomposition*, Beaune, France, NATO ASI Series, Kluner, 1992.

40. **Z. Cai, S. McCormick and J. Mandel**
Multigrid methods for nearly singular equations and eigenvalue problems, manuscript.

41. **S. McCormick**
Idealized analysis of asynchronous multilevel methods, *Proc. Symp. on Adaptive, Multilevel, and Hierarchical Computational Strategies*, ASME Winter Annual Meeting, Anaheim, California, Nov., 1992.

42. **S. McCormick, J. Ruge, and S. Yee**
Adaptive grid refinement in a multigrid solver for semi-implicit solution of shallow water equations on a sphere, *Monthly Weather Reviews*, to be submitted.

C. Lectures

Achi Brandt:

1. "Multigrid methods in lattice field theory," in "LATTICE91," the International Conference on Lattice Field Theory, Tsukuba, Japan, 5-9 November 1991.

2. "Multilevel domain decomposition for elliptic and nonelliptic problems," in the Sixth International Conference on Domain Decomposition Methods in Science and Engineering, 15-19 June, 1992.

3. "New directions in multiscale computations," in SIAM 40th Anniversary Meeting, 20-24 July 1992.

Steve McCormick:

1. June 18-23, 1991, Principle Lecturer (10 Lectures), NSF-CBMS Regional Research Conference on "Multigrid Methods," George Washington University, D. C.

2. Oct 29, 1991, Invited Lecture, Physics Computing 1991, "Multigrid Methods in Physics Simulation," San Jose, California.

3. Sept. 9-12, 1992 , Invited Lecturer, 4th Int'l Symp. on CFD, "Multilevel Methods in CFD," Davis, California.

4. May 17-28, Invited Lecturer, US Army Research Office Workshop on Adaptive Methods for PDE's, "Multilevel Adaptive Methods," RPI, New York.

5. May 25-28, Invited Lecturer, NATO Advanced Research Workshop, "Multilevel Methods for Composite Problems," Beaune, France.

6. June 9-12, 1992, Invited Lecturer, 9th Int'l Conf. on Computational Methods in Water Resources, "Finite Volume Element and Multilevel Adaptive Methods," Denver, Colorado.

7. June 23-July 2, Keynote Speaker 3rd Int'l Conf. on Numerical Analysis, "Multilevel Projection Methods," Moscow, Russia.

8. Feb. 6, Colloquium Lecturer, Naval Postgraduate School, "Multigrid Methods," Monterey, California.

9. Jan 14-17, Lecturer Workshop on Parallel Computing for 3D Plasma Simulation, "Multigrid Adaptive Methods," Kirtland AFB, New Mexico.

10. July 19, Lecturer, SIAM Summer Meeting, "MG Tutorial," Los Angeles, California.

11. Aug. 24, Colloquium Lecturer, Texas A&M, "Multilevel Projection Methods," College Station, Texas.

12. Sept. 18, Colloquium Lecturer, Univ. of Colorado at Boulder, "Multilevel Projection Methods," Boulder, Colorado.

D. Graduate Students Supported:

1. Jim Otto, Full Time, Ph.D. received May, 1992,
"Multilevel Methods for Advection-Dominated Elliptic Problems"

2. Dan Quinlan, Summers, Ph.D. expected December, 1992

3. Klaus Ressel, Full Time, Ph.D. expected May, 1994

4. Tiejun Li, Full Time, Ph.D. expected May, 1994

5. David Sholl, Full Time, Ph.D. expected May, 1995

E. Lab Interaction

Jan 23-26: Visited V.P.I. to discuss the feasibility of developing multigrid methods for the blunt body optimization problems at AEDC.

Several contacts (at Hanscomb, in Colorado, and on the phone): John Ruge and Steve McCormick interacted with Sam Yee for guidance and technical expertise in the development of multigrid solvers and multilevel adaptive methods for shallow water equations. The project is being widened to include a graduate student at CU Boulder, Meteorolgist Ray Bates at NASA Goddard, and post doc Irad Yavneh at NCAR, and to take a more fundamental approach to shallow water equations, with later effort directed toward a global climate model.