

AD-A260 394



COPY **KEEP THIS COPY FOR REPRODUCTION PURPOSES**

DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

2

on is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, letting and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this loading this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE Nov 92	3. REPORT TYPE AND DATES COVERED Final 15 Oct 89-14 Oct 92	
4. TITLE AND SUBTITLE Large-Amplitude Forced Response of Dynamic Systems		5. FUNDING NUMBERS DAAL03-89-K-0180	
6. AUTHOR(S) Ali H. Nayfeh			
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Virginia Polytechnic Institute and State Univ Blacksburg, VA 24061		8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709-2211		10. SPONSORING/MONITORING AGENCY REPORT NUMBER ARO 25630.32-EG	
11. SUPPLEMENTARY NOTES The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.			
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.		12b. DISTRIBUTION CODE S FEB 19 1993 J	
13. ABSTRACT (Maximum 200 words) The objective of this work was to study analytically, numerically, and experimentally the nonlinear dynamic behavior of metallic and composite structural elements to parametric and external excitations. The study was focused on resonance conditions that may produce large and possibly damaging motions. Special attention was given to modal coupling and consequent energy exchanges.			
14. SUBJECT TERMS Dynamic Systems, Dynamic Behavior, Metallics		15. NUMBER OF PAGES 24	
		16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL

93-03356



25P8

FINAL REPORT

1. ARO PROPOSAL NUMBER: 25630-EG
3. TITLE OF PROPOSAL: Large-Amplitude Forced Response of Dynamic Systems
4. CONTRACT OR GRANT NUMBER: DAAL03-89-K-0180
5. NAME OF INSTITUTION: Virginia Polytechnic Institute and State University
6. AUTHORS OF REPORT: Ali H. Nayfeh

ABSTRACT

The objective of this work was to study analytically, numerically, and experimentally the nonlinear dynamic behavior of metallic and composite structural elements to parametric and external excitations. The study was focused on resonance conditions that may produce large and possibly damaging motions. Special attention was given to modal coupling and consequent energy exchanges.

The following is a brief summary of the accomplishments under this grant.

A. PUBLICATIONS

1. A. H. Nayfeh and B. Balachandran, "Motion Near a Hopf Bifurcation of a Three-Dimensional System," *Mechanics Research Communications*, Vol. 17, 1990, pp. 191-198.

In this article, we study the three-dimensional autonomous system. The method of multiple scales is used to obtain an asymptotic expansion for the amplitude and frequency of the limit cycle near the Hopf bifurcation of the system. The resulting analytical approximation is valid only in a small range near the Hopf bifurcation point. This range is ascertained using numerical simulations and stability analysis. Floquet theory is used to determine the stability of the periodic solutions. Also, Rand's analysis is discussed in light of the present study. The perturbation analysis

presented here can be easily applied to higher-dimensional systems. The discussion on the range of validity of the analytical approximation and the stability analysis are pertinent to higher-dimensional systems as well.

2. P. F. Pai and A. H. Nayfeh, "Three-Dimensional Nonlinear Vibrations of Composite Beams--I. Equations of Motion," *Nonlinear Dynamics*, Vol. 1, 1990, pp. 477-502.

Newton's second law is used to develop the nonlinear equations describing the extensional-flexural-flexural-torsional vibrations of slewing or rotating metallic and composite beams. Three consecutive Euler angles are used to relate the deformed and undeformed states. Because the twisting-related Euler angle ϕ is not an independent Lagrangian coordinate, twisting curvature is used to define the twist angle, and the resulting equations of motion are symmetric and independent of the rotation sequence of the Euler angles. The equations of motion are valid for extensional, inextensional, uniform and nonuniform, metallic and composite beams. The equations contain structural coupling terms and quadratic and cubic nonlinearities due to curvature and inertia. Some comparisons with other derivations are made, and the characteristics of the modeling are addressed. The second part of the paper will present a nonlinear analysis of a symmetric angle-ply graphite-epoxy beam exhibiting bending-twisting coupling and a two-to-one internal resonance.

3. B. Balachandran and A. H. Nayfeh, "Nonlinear Motions of a Beam-Mass Structure," *Nonlinear Dynamics*, Vol. 1, 1990, pp. 39-61.

We study the planar dynamic response of a flexible L-shaped beam-mass structure with a two-to-one internal resonance to a primary resonance. The structure is subjected to low excitation (mili g) levels and the resulting nonlinear motions are examined. The Lagrangian for weakly nonlinear motions of the undamped structure is formulated and time averaged over the period of the primary oscillation, leading to an autonomous system of equations governing the amplitudes and phases of the modes involved in the internal resonance. Later, modal damping is

assumed and modal-damping coefficients, determined from experiments, are included in the analytical model. The locations of the saddle-node and Hopf bifurcations predicted by the analysis are in good agreement, respectively, with the jumps and transitions from periodic to quasi-periodic motions observed in the experiments. The current study is relevant to the dynamics and modeling of other structural systems as well.

4. L. D. Zavodney, A. H. Nayfeh, and N. E. Sanchez, "Bifurcations and Chaos in Parametrically Excited Single-Degree-of-Freedom Systems," *Nonlinear Dynamics*, Vol. 1, 1990, pp. 1-21.

The behavior of single-degree-of-freedom systems possessing quadratic and cubic nonlinearities subject to parametric excitation is investigated. Both fundamental and principal parametric resonances are considered. A global bifurcation diagram in the excitation amplitude and excitation frequency domain is presented showing different possible stable steady-state solutions (attractors). Fractal basin maps for fundamental and principal parametric resonances when three attractors coexist are presented in color. An enlargement of one region of the map for principal parametric resonance reveals a Cantor-like set of fractal boundaries. For some cases, both periodic and chaotic attractors coexist.

5. B. D. Heydon, A. H. Nayfeh, and W. T. Baumann, "An Adaptive Quenching Algorithm for a Nonlinear Single-Degree-of-Freedom System," *Nonlinear Dynamics*, Vol. 1, 1990, pp. 193-208.

An indirect adaptive quenching algorithm for a nonlinear single-degree-of-freedom system with unknown constant system parameters is presented. The system is subject to external or parametric sinusoidal disturbances and the resulting control signal is also sinusoidal. The quenching algorithm provides a reduction in the control effort required compared to direct disturbance cancellation. The disturbance sinusoid and the unknown parameters are incorporated into the system model and an extended Kalman filter (EKF) with modified update equations is used to estimate the system state and parameters. The estimates are then used to

form the quenching signal. The adaptive quenching algorithm is found to work well inside a quenching region defined by the separatrices and suggests the use of a hybrid control law. The algorithm was verified by implementing it on an analog computer.

6. B. Balachandran and A. H. Nayfeh, "Nonlinear Vibrations of a Composite Structure," *Proceedings of the SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics*, Atlanta, GA, March 22-23, 1990.

We conducted an experimental study of the influence of modal interactions on the nonlinear response of a composite-beam structure to harmonic excitations. Energy exchange between the excited and nonexcited modes of vibration leads to nonlinear periodic, periodically modulated, and chaotically modulated responses. Results obtained in the experiments with the composite structure are discussed and compared with the results obtained in earlier studies of a steel-beam structure of a similar form.

7. T. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "A Theoretical-Experimental Investigation of a Three-Degree-of-Freedom Structure," *Proceedings of the SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics*, Atlanta, GA, March 22-23, 1990.

A theoretical-experimental investigation is conducted into the response of a three-degree-of-freedom structure having an autoparametric combination resonance of the additive type to a primary resonance of the higher mode. The structure consists of two light-weight beams and three concentrated masses arranged in a T-shape. The theoretically predicted natural frequencies are in good agreement with those obtained experimentally. The method of multiple scales is used to determine six first-order differential equations describing the modulation of the amplitudes and phases of the three modes with nonlinearity, damping, and resonances. Some of the predicted nonlinear phenomena include the saturation phenomenon, coexistence of stable solutions, jumps, and periodic, two-period quasiperiodic, and phase-locked or synchronized

motions. The experiment confirms these predictions and shows other phenomena that are not yet explained by the theory.

8. H. L. Neal and A. H. Nayfeh, "Nonstationary Vibrations of Parametrically Excited Systems," Proceedings of the SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics, Atlanta, GA, March 22-23, 1990.

A parametrically-excited system with excitation amplitude or frequency that is a linear function of time is considered. The method of multiple scales equations and the governing equations are integrated on a digital computer. Deviations from the stationary solution, including penetration, jump, overshoot, convergence to the stationary solution, and lingering are considered. The effects of sweep rate on the maximum amplitude and unboundedness are discussed.

9. P. F. Pai and A. H. Nayfeh, "Nonlinear Flexural-Flexural-Torsional Dynamics of Metallic and Composite Beams," Proceedings of the SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics, Atlanta, GA, March 22-23, 1990.

Newton's second law is used to develop the nonlinear equations describing the flexural-flexural-torsional vibration of metallic and composite beams. Three consecutive Euler angles are used to relate the deformed and undeformed states. Because the twisting-related Euler angle ϕ is not an independent Lagrangian coordinate, the torsional equation is used to define the real twist angle. The resulting equations contain structural coupling terms and cubic nonlinearities due to curvature and inertia. These equations are used to analyze the nonplanar responses of a cantilever composite beam to a lateral harmonic base excitation. A powerful technique based on the state space concept is used to solve for the linear mode shapes. A combination of the Galerkin procedure and the method of multiple scales is then used to construct a first-order uniform expansion for the case of a one-to-one internal resonance and a primary resonance. Quantitative results are obtained for nonplanar motions and

their dynamic behavior is investigated. For different range of parameters, the nonplanar motions can be steady whirling motions, whirling motions of the beating type, or chaotic motions.

10. T. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "A Theoretical-Experimental Investigation of a Three-Degree-of-Freedom Structure," AIAA Paper No. 90-1081, 1990.

In this paper we present the results of a theoretical-experimental investigation of the response of a three-degree-of-freedom structure with quadratic nonlinearities to a harmonic excitation. The structure consists of three light-weight steel beams and three concentrated masses, as shown in Figure 1. The positions of the masses on the vertical beams are adjusted to make $\omega_3 \approx \omega_2 + \omega_1$ and $\omega_2 \approx 2\omega_1$, where the ω_n are the linear undamped natural frequencies. These conditions are related to combination and two-to-one internal (sometimes called autoparametric) resonances, respectively. Under these conditions the response to a simple harmonic excitation can exhibit complicated behavior, such as quasi-periodic and chaotically modulated motions. Here we call the response linear if it can be described accurately by linear equations of motion; we call the response nonlinear when nonlinear equations of motion are necessary

11. B. Balachandran and A. H. Nayfeh, "Modal Interactions in Composite Structures," AIAA Paper No. 90-1212, 1990.

We present a collection of experimental results on the influence of modal interactions (i.e., internal or autoparametric resonances) on the nonlinear response of flexible composite-beam structures to harmonic excitations. During primary resonant excitations at low excitation (mili g) levels, coupling between the directly excited and indirectly excited modes of vibration of the structure led to nonlinear planar and nonplanar periodic, nonplanar periodically modulated, and nonplanar chaotically modulated motions. Interaction between flexural and torsional modes led to the nonplanar responses. The transition to chaotically modulated motions

occurred via quasi-periodic motions. During combination resonant excitations, even at high excitation (2.0 g) levels, we could not excite the nonlinear responses of the structure. Theoretical analyses for internally-resonant systems are used to predict and explain the structure's responses. In addition, the experimental results are compared with those obtained in earlier experiments with a metallic structure of a similar form.

12. P. F. Pai and A. H. Nayfeh, "Nonlinear Flexural-Flexural-Torsional Dynamics of Metallic and Composite Beams," AIAA Paper No. 90-1229, 1990.

Three nonlinear integro-differential equations of motion are used to investigate the forced nonlinear vibration of a symmetric laminated graphite-epoxy composite beam. The analysis focuses on the case of primary resonance of the first flexural mode when its frequency is approximately two times the frequency of the first out-of-plane flexural-torsional mode. A combination of the fundamental-matrix method, the Galerkin procedure, and the method of multiple scales is used to derive four first-order ordinary-differential equations describing the modulation of the amplitudes and phases of the interacting modes with damping, nonlinearity, and resonances. The eigenvalues of the Jacobian matrix of the modulation equations are used to determine the stability of constant solutions, and Floquet theory is used to determine the stability and bifurcations of limit-cycle solutions. Hopf bifurcations, symmetry-breaking bifurcations, amplitude- and phase-modulated motions, period-multiplying sequences, and chaotic motions are studied. The results show that the motion can be planar and/or nonplanar although the input force is planar. Nonplanar responses may be periodic motions, amplitude- and phase-modulated motions, or chaotically modulated motions.

13. P. F. Pai and A. H. Nayfeh, "Three-Dimensional Nonlinear Vibrations of Composite Beams--II. Flapwise Excitations," *Nonlinear Dynamics*, Vol. 2, 1991, pp. 1-34.

The nonlinear equations of motion derived in Part I are used to investigate the response of an inextensional, symmetric angle-ply graphite-epoxy beam to a harmonic base-excitation along the flapwise direction. The equations contain bending-twisting couplings and quadratic and cubic nonlinearities due to curvature and inertia. The analysis focuses on the case of primary resonance of the first flexural-torsional (flapwise-torsional) mode when its frequency is approximately one-half the frequency of the first out-of-plane flexural (chordwise) mode. A combination of the fundamental-matrix method and the method of multiple scales is used to derive four first-order ordinary-differential equations to describe the time variation of the amplitudes and phases of the interacting modes with damping, nonlinearity, and resonances. The eigenvalues of the Jacobian matrix of the modulation equations are used to determine the stability and bifurcations of their constant solutions, and Floquet theory is used to determine the stability and bifurcations of their limit-cycle solutions. Hopf bifurcations, symmetry-breaking bifurcations, period-multiplying sequences, and chaotic solutions of the modulation equations are studied. Chaotic solutions are identified from their frequency spectra, Poincare sections, and Lyapunov's exponents. The results show that the beam motion may be nonplanar although the input force is planar. Nonplanar responses may be periodic, periodically modulated, or chaotically modulated motions.

14. P. F. Pai and A. H. Nayfeh, "Three-Dimensional Nonlinear Vibrations of Composite Beams--III. Chordwise Excitations," *Nonlinear Dynamics*, Vol. 2, 1991, pp. 137-156.

Three nonlinear integro-differential equations of motion derived in Part I are used to investigate the forced nonlinear vibration of a symmetrically laminated graphite-epoxy composite beam. The analysis focuses on the case of primary resonance of the first in-plane flexural (chordwise) mode when its frequency is approximately twice the frequency of the first out-of-plane flexural-torsional (flapwise-torsional) mode. A combination of the fundamental-matrix method and the method of multiple scales is used

to derive four first-order ordinary-differential equations describing the modulation of the amplitudes and phases of the interacting modes with damping, nonlinearity, and resonances. The eigenvalues of the Jacobian matrix of the modulation equations are used to determine the stability of their constant solutions, and Floquet theory is used to determine the stability and bifurcations of their limit-cycle solutions. Hopf bifurcations, symmetry-breaking bifurcations, period-multiplying sequences, and chaotic motions of the modulation equations are studied. The results show that the motion can be nonplanar although the input force is planar. Nonplanar responses may be periodic, periodically modulated, or chaotically modulated motions.

15. B. Balachandran and A. H. Nayfeh, "Observations of Modal Interactions in Resonantly Forced Beam-Mass Structures," *Nonlinear Dynamics*, Vol. 2, 1991, pp. 77-117.

We present a collection of experimental results on the influence of modal interactions (i.e., internal or autoparametric resonances) on the nonlinear response of flexible metallic and composite structures subjected to a range of resonant excitations. The experimental results are provided in the form of frequency spectra, Poincaré sections, pseudo-phase planes, dimension calculations, and response curves. Experimental observations of transitions from periodic to chaotically modulated motions are also presented. We also discuss relevant analytical results. The current study is also relevant to other internally-resonant structural systems.

16. A. H. Nayfeh, J. F. Nayfeh, and D. T. Mook, "On Methods for Continuous Systems with Quadratic and Cubic Nonlinearities," *Nonlinear Dynamics*, Vol. 3, 1991, pp. 145-162.

Methods for determining the response of continuous systems with quadratic and cubic nonlinearities are discussed. We show by means of a simple example that perturbation and computational methods based on first discretizing the systems may lead to erroneous results whereas

perturbation methods that attack directly the nonlinear partial-differential equations and boundary conditions avoid the pitfalls associated with the analysis of the discretized systems. We describe a perturbation technique that applies either the method of multiple scales or the method of averaging to the Lagrangian of the system rather than the partial-differential equations and boundary conditions.

17. B. Balachandran and A. H. Nayfeh, "Cyclic Motions Near a Hopf Bifurcation of a Four-Dimensional System," *Nonlinear Dynamics*, Vol. 3, 1992, pp. 19-39.

We study motion near a Hopf bifurcation of a representative four-dimensional autonomous system. Special cases of the four-dimensional system represent the envelope equations that govern the amplitudes and phases of the modes of an internally resonant structure subjected to resonant excitations. Using the method of multiple scales, we derive asymptotic expansions for the frequency and amplitude of a limit cycle near a bifurcation point and determine the nature of the bifurcation (i.e., subcritical or supercritical). The range of validity of the analytical approximation is ascertained using numerical simulations. The perturbation analysis and discussions are pertinent to other autonomous systems as well.

18. P. F. Pai and A. H. Nayfeh "A Nonlinear Composite Plate Theory," *Nonlinear Dynamics*, Vol. 2, 1991, pp. 445-477.

A general nonlinear theory for the dynamics of elastic anisotropic plates undergoing moderate-rotation vibrations is presented. The theory fully accounts for geometric nonlinearities (moderate rotations and displacements) by using local stress and strain measures and an exact coordinate transformation, which result in nonlinear curvatures and strain-displacement expressions that contain the von Karman strains as a special case. The theory accounts for transverse shear deformations by using a third-order theory and for extensionality and changes in the configuration due to in-plane and transverse deformations. Five

third-order nonlinear partial-differential equations of motion describing the extension-extension-bending-shear-shear vibrations of plates are obtained by an asymptotic analysis, which reveals that laminated plates display linear elastic and nonlinear geometric couplings among all motions.

19. P. F. Pai and A. H. Nayfeh "A Nonlinear Composite Beam Theory," *Nonlinear Dynamics*, Vol. 3, No. 4, 1992, pp. 273-303.

Presented here is a general theory for the three-dimensional nonlinear dynamics of elastic anisotropic initially straight beams undergoing moderate displacements and rotations. The theory fully accounts for geometric nonlinearities (large rotations and displacements) by using local stress and strain measures and an exact coordinate transformation, which result in nonlinear curvature and strain-displacement expressions that contain the von Karman strains as a special case. Extensionality is included in the formulation, and transverse shear deformations are accounted for by using a third-order theory. Six third-order nonlinear partial-differential equations are derived for describing one extension, two bending, one torsion, and two shearing vibrations of composite beams. They show that laminated beams display linear elastic and nonlinear geometric couplings among all motions. The theory contains, as special cases, the Euler-Bernoulli theory, Timoshenko's beam theory, the third-order shear theory, and the von Karman type nonlinear theory.

20. P. F. Pai and A. H. Nayfeh, "A Nonlinear Composite Shell Theory," *Nonlinear Dynamics*, Vol. 3, No. 6, 1992, pp. 431-463.

A general nonlinear theory for the dynamics of elastic anisotropic circular cylindrical shells undergoing small strains and moderate-rotation vibrations is presented. The theory fully accounts for extensionality and geometric nonlinearities by using local stress and strain measures and an exact coordinate transformation, which result in nonlinear curvatures and strain-displacement expressions that contain the von Karman strains as a special case. Moreover, the linear part of the theory contains, as special cases, most of the classical linear theories when appropriate stress

resultants and couples are defined. Parabolic distributions of the transverse shear strains are accounted for by using a third-order theory and hence shear correction factors are not required. Five third-order nonlinear partial differential equations describing the extension, bending, and shear vibrations of shells are obtained using the principle of virtual work and an asymptotic analysis. These equations show that laminated shells display linear elastic and nonlinear geometric couplings among all motions.

21. T. A. Nayfeh, W. Asrar, and A. H. Nayfeh, "Three-Mode Interactions in Harmonically Excited Systems with Quadratic Nonlinearities," *Nonlinear Dynamics*, Vol. 3, No. 5, 1992, pp. 385-410.

In this paper the response of a three-degree-of-freedom structure with quadratic nonlinearities subject to harmonic forcing of its third mode is investigated. The method of multiple scales is used to obtain the equations that govern its amplitudes and phases. The fixed points of these equations are obtained and their stability is determined. The fixed points are found to undergo Hopf bifurcations and hence the response undergoes amplitude- and phase-modulated motions. Regions where the amplitudes and phases undergo periodic, quasiperiodic, and chaotic motions and hence regions where the overall system response is periodically, quasiperiodically, and chaotically modulated are determined.

22. P. F. Pai and A. H. Nayfeh "A Fully Nonlinear Model of Cables," accepted for publication, *AIAA Journal Technical Note*.

Because engineering cables are usually lengthy and flexible, their vibrations are essentially nonlinear and dominated by geometric nonlinearities. Large-amplitude vibrations and nonlinear phenomena of cables have been extensively studied analytically and experimentally. There are many nonlinear cable models due to the many ad hoc assumptions adopted by different researchers during different steps in the derivation. Because extensional forces are the only elastic loads acting on cables, compressibility and Poisson's effect can be significant. For

example, for rubber-like materials the Poisson's effect results in a nonlinear relationship¹² between the tension force and the axial strain. In the literature, most of cable models do not account for the change in the cross-section area due to Poisson's effect or assume that the material is incompressible, and hence the Poisson's ratio ν is absent from the equations of motion; such theories are valid only for materials with $\nu \approx 0$ or $\nu \approx 0.5$. But, the Poisson's ratios of most engineering materials are between 0.25 and 0.35 except those of rubber and paraffin, for which $\nu \approx 0.5$. To account for geometric nonlinearities, some researchers account only for cubic nonlinearities, and some researchers account only for quadratic nonlinearities. However, cubic nonlinearities can greatly change the dynamic responses^{1,11} due to quadratic nonlinearities. Moreover, some theories do not account for initial sags, static loads, and/or extensionality, but these effects always exist in real cable structures.

23. J. Hadian, A. H. Nayfeh, and J. F. Nayfeh, "Modal Interaction in the Response of Composite Rectangular Plates," submitted for publication, Composites Engineering.

A higher-order shear-deformation theory is used to analyze the interaction of two modes in the response of thick laminated rectangular plates to transverse harmonic loads. The case of a two-to-one autoparametric resonance is considered. Four first-order ordinary differential equations describing the modulation of the amplitudes and phases of the internally resonant modes are derived using the averaged Lagrangian when the higher mode is excited by a primary resonance. The fixed-point solutions are determined and their stability is analyzed. It is shown that besides the single-mode solution, two-mode solutions exist for a certain range of parameters. It is further shown that, in the multi-mode case, the lower mode, which is indirectly excited through the internal resonance may dominate the response. For a certain range of parameters, the fixed points lose stability via a Hopf bifurcation, thereby giving rise to limit-cycle solutions. It is shown that these limit cycles undergo a series of period-doubling bifurcations, culminating in chaos.

24. P. F. Pai and A. H. Nayfeh, "Dynamic Characteristics of Slewing Metallic and Composite Beams," submitted for publication, *Journal of Guidance, Control, and Dynamics*.

The exact natural frequencies and mode shapes of a slewing isotropic beam are determined, and the effects of the distributed rotary inertia and the hub inertia are investigated. The results show that the influence of the rotary inertia is not negligible for a beam vibrating in the principal direction of strong flexural rigidity and it decreases the natural frequencies and increases the modal rotation angles at the hub. Because the modal rotation angles at the hub and the natural frequencies are large when the hub inertia is small, a small hub inertia is desirable for controlling a slewing beam. For a composite beam made of three prismatic segments and exhibiting bending-twisting coupling, a powerful technique based on the state-space concept and the fundamental-matrix solution is used to obtain the exact frequencies and mode shapes. The influence of the hub inertia on the modal frequencies and mode shapes is similar to that for an isotropic beam. But, the bending-twisting coupling results in a serious control problem because it creates a large twisting angle at the tip of the beam, especially for higher modes. The results obtained by the fundamental-matrix method are compared with those obtained by a finite-element method.

25. J. Hadian and A. H. Nayfeh, "Free Vibration and Buckling of Shear-Deformable Cross-Ply Laminated Plates using the State-Space Concept," accepted for publication, *Computers & Structures*.

Generalized Levy-type solutions are obtained for the stability of cross-ply laminated plates. The governing equations, which are derived by using the principle of virtual displacement and a third-order shear-deformation plate theory, are transformed into a set of first-order linear ordinary-differential equations with constant coefficients. The general solution of these equations can be obtained by using the state-space concept. Then, application of the boundary conditions yields equations for the buckling loads. Unfortunately, a straightforward

application of the state-space concept yields numerically ill-conditioned problems as the plate thickness is reduced. A combination of an initial-value method and the modified Gram-Schmidt orthonormalization procedure is used to overcome this problem. It is shown that this method not only yields results that are in excellent agreement with the results in the literature, but also it converges fast and gives all the buckling loads regardless of the plate thickness.

26. B. Balachandran, A. H. Nayfeh, S. W. Smith, and R. S. Pappa, "On Identification of Structures with Internal Resonances," submitted for publication, AIAA Journal.

Structural identification is of paramount importance for the prediction of the response of a given structure to various loading conditions. Although the responses of many structures used in practice can be determined by modeling them with linear equations, there are many physical phenomena that cannot be modeled by linear equations. These phenomena include subharmonic, superharmonic, and combination resonances, jumps, flutter limit cycles, saturation, multi-mode responses, and aperiodic responses. In several structures, some of the natural frequencies are commensurate or nearly commensurate. In the presence of appropriate nonlinearities, these frequency relationships can lead to modal interactions. In this event, these frequency relationships are called internal resonances. Modal interactions can occur during both free and forced oscillations of a structure and can lead to the aforementioned nonlinear phenomena. With the advent of complex space and aircraft structures, there is a growing need for the identification of nonlinear systems. Conditions for modal interactions do exist in most such structures. However, the identification of systems which exhibit modal interactions is just beginning to receive attention.

27. P. F. Pai, A. H. Nayfeh, K. Oh, and D. T. Mook, "A Refined Nonlinear Model of Composite Plates with Integrated Piezoelectric Actuators and Sensors," accepted for publication, International Journal of Solids and Structures.

A fully nonlinear theory for the dynamics and active control of elastic laminated plates with integrated piezoelectric actuators and sensors undergoing large-rotation and small-strain vibrations is presented. The theory fully accounts for geometric nonlinearities (large rotations and displacements) by using local stress and strain measures and an exact coordinate transformation. Moreover, the model accounts for continuity of interlaminar shear stresses, extensionality, orthotropic properties of piezoelectric actuators, dependence of piezoelectric strain constants on induced strains, and arbitrary orientations of the integrated actuators and sensors. Extension and shearing forces and bending and twisting moments are introduced onto the plate along the boundaries of the piezoelectric actuators. Five nonlinear partial-differential equations describing the extension-extension-bending-shear-shear vibrations of laminated plates are obtained, which display linear elastic and nonlinear geometric couplings among all motions. Also, the piezoelectric actuator-induced warping is addressed, and comparisons with other simplified models and nonlinear theories are made.

28. P. F. Pai and A. H. Nayfeh, "A Fully Nonlinear Theory of Curved and Twisted Composite Rotor Blades Accounting for Warping and 3-D Stress Effects," submitted for publication, *International Journal of Solids and Structures*.

A geometrically exact nonlinear beam model is developed for naturally curved and twisted solid composite rotor blades undergoing large vibrations in three-dimensional space. The theory accounts for in-plane warpings due to bending and extensional loadings, out-of-plane warpings due to shearing and torsional loadings, elastic couplings among warpings, and three-dimensional stress effects by using the results of a two-dimensional, static, sectional, finite-element analysis. Also, the theory fully accounts for extensionality, initial curvatures, and geometric nonlinearities by using local stress and strain measures and an exact coordinate transformation, which result in fully nonlinear curvature and strain-displacement expressions. Six fully nonlinear equations of motion

describing one extension, two bending, one torsion, and two shearing vibrations of composite beams are obtained by using a combination of the extended Hamilton principle and the concept of virtual local rotations. The equations display linear elastic couplings due to structural anisotropy and initial curvatures and nonlinear geometric couplings. The theory contains most of beam theories as special cases. Moreover, the formulation is based on an energy approach, but the derivation is fully correlated with the Newtonian approach and the final equations of motion are put in compact matrix form.

29. P. F. Pai and A. H. Nayfeh, "Sectional Analysis of Warpings and Elastic Constants for Curved and Twisted Anisotropic Beams," submitted for publication, International Journal of Solids and Structures.

A nonlinear theory of curved and twisted beams can be used to model helicopter rotor blades, aviation propeller blades, turbine blades, arm-type positioning mechanisms of magnetic disk drives, robot manipulators, helical springs, etc. Moreover, beam-type elements are usually used in material characterization as well as basic structural elements. Recently, the rapid developments in aerospace exploration stimulated extensive research into the dynamics and control of flexible structures because space structures must be extremely light weight and hence very flexible in order to be cost effective. In this paper, we develop a general nonlinear theory for curved and twisted beams.

30. T. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "A Theoretical-Experimental Investigation of a Three-Degree-of-Freedom Structure," submitted for publication, Nonlinear Dynamics.

The response of a structure to a simple-harmonic excitation is investigated theoretically and experimentally. The structure consists of two light-weight beams arranged in a T-shape turned on its side. Relatively heavy and concentrated weights are placed at the upper and lower free ends and at the point where the two beams are joined. The base of the "T" is clamped to the head of a shaker. Because the masses of the

concentrated weights are much larger than the masses of the beams, the first three natural frequencies are far below the fourth; consequently, for relatively low frequencies of the excitation, the structure has, for all practical purposes, only three degrees of freedom. The lengths and weights are chosen so that the third natural frequency is approximately equal to the sum of the two lower natural frequencies, an arrangement that produces an autoparametric (also called an internal) resonance. A linear analysis is performed to predict the natural frequencies and to aid in the design of the experiment; the predictions and observations are in close agreement. Then a nonlinear analysis of the response to a prescribed transverse motion at the base of the "T" is performed. The method of multiple scales is used to obtain six first-order differential equations describing the modulations of the amplitudes and phases of the three interacting modes when the frequency of the excitation is near the third natural frequency. Some of the predicted phenomena include periodic, two-period quasiperiodic, and phase-locked (also called synchronized) motions; coexistence of multiple stable motions and the attendant jumps; and saturation. All the predictions are confirmed in the experiments, and some phenomena that are not yet explained by theory are observed.

B. PRESENTATIONS

1. A. H. Nayfeh and P. F. Pai, "Nonlinear Vibrations of a Cantilever Composite Beam Under Planar Harmonic Base Excitation," Third Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Duke University, Durham, NC, March 12-14, 1990.
2. B. Balachandran and A. H. Nayfeh, "Nonlinear Vibrations of a Composite Structure," SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics, Atlanta, GA, March 22-23, 1990.
3. T. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "A Theoretical-Experimental Investigation of a Three-Degree-of-Freedom Structure," SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics, Atlanta, GA, March 22-23, 1990.

4. H. L. Neal and A. H. Nayfeh, "Nonstationary Vibrations of Parametrically Excited Systems," SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics, Atlanta, GA, March 22-23, 1990.
5. P. F. Pai and A. H. Nayfeh, "Nonlinear Flexural-Flexural-Torsional Dynamics of Metallic and Composite Beams," SECTAM XV, 1990 Southeastern Conference on Theoretical and Applied Mechanics, Atlanta, GA, March 22-23, 1990.
6. T. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "A Theoretical-Experimental Investigation of a Three-Degree-of-Freedom Structure," AIAA/ASME/ASCE/AHS/ASC 31st Structures, Structural Dynamics and Materials Conference, Long Beach, CA, April 2-4, 1990.
7. B. Balachandran and A. H. Nayfeh, "Modal Interactions in Composite Structures," AIAA Dynamics Specialists Conference, Long Beach, CA, April 5-6, 1990.
8. P. F. Pai and A. H. Nayfeh, "Nonlinear Flexural-Flexural-Torsional Dynamics of Metallic and Composite Beams," AIAA Dynamics Specialists Conference, Long Beach, CA, April 5-6, 1990.
9. H. L. Neal and A. H. Nayfeh, "Nonstationary Vibrations of Parametrically Excited Systems," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.
10. M. U. Biddash, E. M. Cliff, and A. H. Nayfeh, "Nonlinear Behavior of Time-Optimal Soft-Constrained Control of Linear Systems," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.
11. A. Abou-Rayan, A. H. Nayfeh, D. T. Mook, and M. A. Nayfeh, "Nonlinear Analysis of a Parametrically Excited Buckled Column," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and

Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.

12. P. F. Pai and A. H. Nayfeh, "Nonlinear Flexural-Flexural-Torsional Dynamics of Composite Beams," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.
13. B. Balachandran and A. H. Nayfeh, "Nonlinear Motions of a Beam-Mass Structure," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.
14. J. Hadian and A. H. Nayfeh, "Modal Interaction in Circular Plates," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.
15. T. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "Modal Interactions in a Three-Degree-of-Freedom Structure: Theory and Experiment," Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, Virginia Polytechnic Institute and State University, Blacksburg, VA, June 25-27, 1990.
16. B. Balachandran and A. H. Nayfeh, "Nonlinear Oscillations of Resonantly Forced Composite Structures," CCMS/CASS Review, VPI&SU, Blacksburg, VA, September 23-26, 1990.
17. P. F. Pai and A. H. Nayfeh, "Nonlinear Flexural-Flexural-Torsional Dynamics of Composite Beams," CCMS/CASS Review, VPI&SU, Blacksburg, VA, September 23-26, 1990.
18. M. Bikdash, E. M. Cliff, and A. H. Nayfeh, "Feedback Soft-Constrained Time-Optimal Control of a Flexible Beam," Eighth VPI&SU Symposium on Dynamics and Control of Large Structures, Blacksburg, VA, May 6-8, 1991.

19. B. Balachandran and A. H. Nayfeh, "Observations of Modal Interactions in Resonantly Forced Beam-Mass Structures," ASME Applied Mechanics Division Summer Annual Meeting, Columbus, OH, June 16-19, 1991.
20. Nayfeh, J. F., Nayfeh, A. H., and Mook, D. T., "Nonlinear Response of Thick Laminated Composite Plates," the 13th Biennial Conference on Mechanical Vibration and Noise, Miami, FL, September 22-25, 1991.
21. A. H. Nayfeh, "Nonlinear Structural Dynamics," Contractors Meeting on Mechanics of Materials, Sponsored by Air Force Office of Scientific Research, Dayton, OH, October 7-8, 1991.
22. P. F. Pai and A. H. Nayfeh, "Three-Dimensional Nonlinear Vibrations of Beams," 62nd Shock and Vibration Symposium, Springfield, VA, October 29-31, 1991.
23. A. Abou-Rayan, A. H. Nayfeh, D. T. Mook, and M. A. Nayfeh, "Theoretical and Numerical Analysis of a Parametrically Excited Buckled Beam," 62nd Shock and Vibration Symposium, Springfield, VA, October 29-31, 1991.
24. B. Balachandran, T. J. Anderson, and A. H. Nayfeh, "Detection of Nonlinear Interactions in Structures," 62nd Shock and Vibration Symposium, Springfield, VA, October 29-31, 1991.
25. T. J. Anderson, B. Balachandran, and A. H. Nayfeh, "Investigation of Multi-Mode Interactions in a Continuous Structure," 62nd Shock and Vibration Symposium, Springfield, VA, October 29-31, 1991.
26. T. A. Nayfeh, W. Asrar, and A. H. Nayfeh, "Modal Interaction in the Response of Systems with Quadratic Nonlinearities to a Harmonic Excitation," 62nd Shock and Vibration Symposium, Springfield, VA, October 29-31, 1991.
27. P. F. Pai and A. H. Nayfeh, "A Nonlinear Theory of Curved and Twisted Composite Beams," Fourth Workshop on Dynamics and Aeroelastic

Stability Modeling of Rotorcraft Systems, College Park, MD, November 19-21, 1991.

28. B. Balachandran, A. H. Nayfeh, S. W. Smith, and R. S. Pappa, "On Identification of Structures with Internal Resonances," AIAA 30th Aerospace Sciences Meeting, Reno, NV, January 6-9, 1992.
29. P. F. Pai and A. H. Nayfeh, "A Nonlinear Composite Shell Theory," AIAA 30th Aerospace Sciences Meeting, Reno, NV, January 6-9, 1992.
30. A. H. Nayfeh, "Modal Interactions in the Nonlinear Response of Dynamical Systems-Theory and Experiment," seminar given at the University of California at Santa Barbara, Santa Barbara, CA, January 10, 1992.
31. K. Oh, P. F. Pai, A. H. Nayfeh, and M. L. Fripp, "Response of an Adaptive Structure to a Broadband Excitation" CCMS/CASS Review, VPI&SU, Blacksburg, VA, March 22-24, 1992.
32. P. F. Pai, A. H. Nayfeh, and K. Oh, "A Nonlinear Theory of Laminated Piezoelectric Plates," AIAA Paper No. 92-2407, AIAA Structures, Structural Dynamics, and Materials Conference, Dallas, TX, April 12-16, 1992.
33. T. J. Anderson, B. Balachandran, and A. H. Nayfeh, "Observations of Nonlinear Interactions in a Flexible Cantilever Beam," AIAA Paper No. 92-2332, AIAA Structures, Structural Dynamics, and Materials Conference, Dallas, TX, April 12-16, 1992.
34. K. Oh, A. H. Nayfeh, and P. F. Pai, "Control of the Resonant Response of a Cantilever Plate Using PVDF Films as the Sensor/Actuator System: An Experimental Study," Recent Advances in Adaptive and Sensory Materials and their Applications, VPI&SU, Blacksburg, VA, April 27-29, 1992.
35. P. F. Pai, A. H. Nayfeh, and K. Oh, "A Refined Nonlinear Model of Piezoelectric Plates," Recent Advances in Adaptive and Sensory Materials and their Applications, VPI&SU, Blacksburg, VA, April 27-29, 1992.

36. A. H. Nayfeh, "Modal Interactions in the Nonlinear Response of Dynamic and Structural Systems," Workshop on Bifurcation and Chaos in Mechanical Systems, L'Aquila, Italy, May 19-20, 1992.
37. P. F. Pai and A. H. Nayfeh, "A Geometrically-Exact Beam Theory Accounting for Warpings and 3-D Stress Effects," Fourth Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, VPI&SU, Blacksburg, VA, June 7-11, 1992.
38. S. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, "Modal Interactions in Parametrically and Externally Excited Strings," Fourth Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, VPI&SU, Blacksburg, VA, June 7-11, 1992.

C. SEMINARS

1. "Energy Transfer from High-to-Low-Frequency Modes in Flexible Structures," Istanbul Technical University, Istanbul, Turkey, May 5, 1992.
2. "Modal Interactions in the Nonlinear Response of Structural Elements-Theory and Experiment," Istanbul Bosph. University, Istanbul, Turkey, May 7, 1992.
3. "Modal Interactions in the Nonlinear Response of Structural Elements-Theory and Experiment," M.E.T. University, Ankara, Turkey, May 11, 1992.
4. "A Refined Nonlinear Model of Piezoelectric Plates," Bilkent University, Ankara, Turkey, May 12, 1992.

D. STUDENTS

1. B. Balachandran, Ph.D., "A Theoretical and Experimental Study of Modal Interactions in Structures," Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1990.

2. J. F. Nayfeh, Ph.D., "Nonlinear Dynamics of Composite Plates and Other Physical Systems," Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1990.
3. P. F. Pai, Ph.D., "Nonlinear Flexural-Flexural-Torsional Dynamics of Metallic and Composite Beams" Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1990.
4. T. A. Nayfeh, M.S., "Dynamics of Three-Degree-of-Freedom Systems with Quadratic Nonlinearities," Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1991.
5. J. Hadian, Ph.D., "Modal Interactions in the Dynamic Response of Isotropic and Composite Plates," Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1991.
6. A. Abou-Rayan, Ph.D., "Deterministic and Stochastic Responses of Nonlinear Systems," Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1991.

DTIC QUALITY INSPECTED 3

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
Availability Codes	
01st	02nd
A-1	