E 201 387

AD-A259 981

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 2, 1992	Reprint

4. TITLE AND SUBTITLE Similitude Modeling of Internal Gravity Wave Spectra	5. FUNDING NUMBERS PE 61102F PR 2310 TA G2 WU 01
6. AUTHOR(S) Edmond M. Dewan	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Phillips Lab/GPOS Hanscom AFB Massachusetts 01731-5000	8. PERFORMING ORGANIZATION REPORT NUMBER PL-TR-92-2321
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

Reprinted from Geophysical Research Letters, Vol. 18, No. 8, pages 1473-1476, August 1991

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Approved for public release; Distribution unlimited	

13. ABSTRACT (Maximum 200 words)

Abstract. Internal gravity waves are present in both the ocean and atmosphere. The oceanic gravity wave power spectra have been modeled semi-empirically by Garrett and Munk (1972, 1975) and shown to have "universal" properties. Their model seems to be useful also for atmospheric waves. Recently a similitude type of "saturation" model for vertical wave number horizontal velocity fluctuation spectra has met with empirical support in the atmosphere. The present paper employs a further similitude argument related to "cascade" properties which would apply to the frequency and horizontal wave number spectra. Testable consequences are exhibited. If validated the model would provide useful insight into the physics of these waves.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Gravity waves, Internal waves, Wave spectra, Middle atmosphere	4
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

GEOPHYSICAL RESEARCH LETTERS, VOL. 18, NO. 8, PAGES 1473-1476, AUGUST 1991

SIMILITUDE MODELING OF INTERNAL GRAVITY WAVE SPECTRA

Edmond M. Dewan

PL/(OPS), Hanscom AFB

Abstract. Internal gravity waves are present in both the ocean and atmosphere. The oceanic gravity wave power spectra have been modeled semi-empirically by Garrett and Munk (1972, 1975) and shown to have "universal" properties. Their model seems to be useful also for atmospheric waves. Recently a similitude type of "saturation" model for vertical wave number horizontal velocity fluctuation spectra has met with empirical support in the atmosphere. The present paper employs a further similitude argument related to "cascade" properties which would apply to the frequency and horizontal wave number spectra. Testable consequences are exhibited. If validated the model would provide useful insight into the physics of these waves.

Introduction

The power spectral densities, or PSD's, of oceanic internal waves were shown by Garrett and Munk (1972, 1975) to have certain universal characteristics. They also created a semi-empirical model (called here GM 72, 75) for these PSD's. Internal waves have also been measured in the atmosphere, and Dewan (1979) suggested that GM 72, 75 may be relevant to this case as well. Van Zandt (1982) has made a convincing case that this is indeed correct. Subsequently, a striking example of universality was found by Dewan et al (1984) for the vertical wave number, k_z, PSD for horizontal stratospheric wind fluctuations, u (Figure 1) and this demanded physical explanation. One based upon "saturation" due to turbulent instability was proposed by Dewan and Good (1984), (1986), extended by Smith et al (1987) and tested by numerous authors by means of radar and lidar. Tsuda (1989) summarized the good experimental support of this theory. Up to that time, the model described the data very well. Most recently Hines (1991) has proposed a new physical approach which promises to be very useful. It contradicts the previous picture of "turbulent saturation", but it does not seem to contradict the picture to be presented below.

The purpose of the present paper is to extend the similitude approach to the remainder of the spectra of GM 72,75. It presumes that, unlike the k_z PSD's, the frequency, ω, and horizontal wave number, k_x, PSD's are determined by local wave interactions which give rise to a "cascade" of energy through time and space scales. In a way, this is like the well known turbulence cascades, but certain wave properties are retained. The final picture emerging is "wave-like turbulence" of "turbulence-like waves".

Prior Indications of a Wave Cascade

When Crooks et al (1968) presented their horizontal wave number PSD's of stratospheric velocity fluctuations, the general interpretation given at that time was that it was due to inertial range turbulence. This was because their spectra were of the form approximating $k_x^{-5/3}$ and it is well known that PSD's of velocity fluctuations of the inertial range obey the relation

$$\psi(k_x) = c_1 \varepsilon^{2/3} k_x^{-5/3} \qquad (2.1)$$

where ε is the dissipation rate and c_1 is a universal dimensionless constant. In Dewan (1979) the inertial range turbulence interpretation of that data was shown to be ruled out on the basis of scale size. An alternative interpretation was therefore suggested that assumed that the fluctuations were due to gravity waves and that these waves interacted in a cascade fashion which gave rise to Eq. (2.1) in a manner analogous to turbulence.

A second indication of evidence for a wave cascade comes from the empirically based GM model itself where for most frequencies (sufficiently larger than ω_i) the frequency dependence of the kinetic energy density is given by (using GM symbols, approximately - see next section).

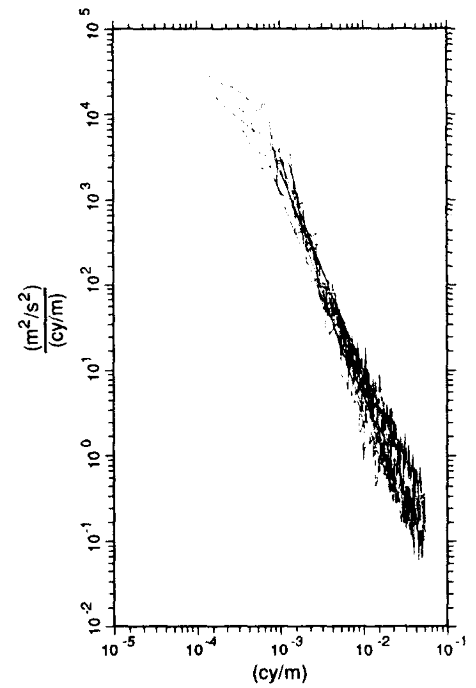

Fig. 1. The Vertical Wave Number, k_z, PSD of the Vector Horizontal Velocity, u, in the Stratosphere, $\psi_u(k_z) = \psi_{u\ zonal}(k_z) + \psi_{u\ meridional}(k_z)$, based on data in Dewan et al (1984). A cubic trend was removed. This particular figure has not been previously published. Note: These spectra are not "normalized". It is their "universality" that causes superposition.

Paper number 91GL01522

$$E'B(\omega) \approx \omega^{-2} \tag{2.2}$$

Tennekes and Lumley (1972) have shown that, for the case of inertial range turbulence

$$\psi_E(\omega) = \frac{c'\,\varepsilon}{\omega^2} \tag{2.3}$$

where ψ_E is the PSD for kinetic energy. Eq. (2.3) is obtained by dimensional analysis (c.f. Bridgeman (1963)).

If the ε dependence in Eq. (2.3) and (2.1) were to be verified for the case of internal gravity waves it would constitute further evidence for the postulated cascade. (Meanwhile, the scalings are suggestive). Unfortunately there are, at present, no measurements of ω or k_X spectra simultaneously with ε measurements.

It is well known that for turbulence the cascade goes from small to large k and ω, the mechanism is vortex stretching, (Tennekes and Lumley, 1972), and interactions are local in k and ω. In the strongly interacting waves presently under discussion the description of Phillips (1966) p. 178 seems appropriate. For steep enough internal waves he describes interactions that can be, to use his word, "promiscuous", and hence possibly local. At this time there is no proof that local cascades in k and ω are ruled out. The phenomenological nature of the model to be proposed below obviates the need to describe the physical interaction mechanisms or even the directions of the cascades thus leaving these topics for future discussion.

The GM (72, 75) Formalism

A proper review of this formalism is not possible here within the space permitted; therefore, the reader may need to consult Garrett and Munk (1972, 1975). The heart of their formalism is E (ω, k), an energy density spectrum assumed separable. [Fritts and Chou (1987) gave evidence of incomplete separability, but since the frequency spectral slopes did not have a wave number dependence, the new predictions of the present approach would not be critically affected. In any case, this matter is beyond the scope of this paper.] GM 72,75 uses

$$E(\omega, k) \equiv \frac{E'B(\omega)\, A(\beta)}{k_*} \tag{3.1}$$

where k_* is the dominant wave number, $\beta \equiv k/k_*$, and $A(\beta) \equiv (t-1)(1+\beta)^{-t}$, where $E'B(\omega)$ and t are determined by experiment. The total range integrals of $A(\beta)$ and $B(\omega)$ are defined as unity. Below, both t and $EB(\omega)$, will be derived from dimensional analysis. $EB(\omega)$ is a dimensional energy spectrum per unit volume as compared to the dimensionless form $E'B(\omega)$ in GM 72, 75.

A crucial part of their formalism is the ratio of vertical or horizontal velocity fluctuations, v or u respectively, to total energy density $E(\omega,k)$ given by the so-called "wave functions" (derivable from the polarization relations), defined respectively by

$$\tilde{U}^2(\omega)^2 \equiv \frac{\left(\omega^2+\omega_1^2\right)\cdot n}{\omega^2} \quad \text{and} \quad \tilde{V}^2(\omega) \equiv \frac{\left(\omega^2-\omega_1^2\right)\cdot n}{N^2} \tag{3.2}$$

where n is the ratio of buoyancy frequency at the depth in question to the extrapolated surface value. A one dimensional PSD with ω or k dependence is obtained by integration over the entire range of wave values of the alternate variable as will be illustrated below. The atmospheric model uses n set to unity.

The New PSD Model

Using the dimensional argument of Dewan and Good (1984, 1986) and the extension of Smith et al (1987) we see that t = 3 in A (β_z), thus

$$A(\beta_z) = 2\left(1+\beta_z\right)^{-3} \tag{4.1}$$

in the GM formalism. This leads to the assumption (c.f. the last two references).

$$\psi_u(k_z) = \frac{\alpha N^2}{k_{z_*}^3}\left(1+\beta_z\right)^{-3} \tag{4.2}$$

where α is a universal constant of order unity which should be about 1/6 as Smith et al (1987) showed. In a similar way the conclusions of Dewan (1979) lead to the assumption

$$\psi_u(k_x) = \frac{\alpha'\,\varepsilon^{2/3}\left(1+\beta_x\right)^{-5/3}}{k_{x_*}^{5/3}}. \tag{4.3}$$

The frequency spectra are based on E B (ω) which, in analogy to Eq. (2.3), is given by

$$E\,B(\omega) = \frac{\alpha'\,\varepsilon}{\omega^2}. \tag{4.4}$$

This is obtained from a similitude argument based only on the assumption that the waves interact so as to exhibit a cascade in ω space. In other words, one assumes, as in the case of inertial range turbulence, that there is a cascade. Then $EB(\omega)$ must depend solely on ε and ω. Eq. (4.4) results immediately from forcing both sides to have the same dimensions. On the basis of the GM 72, 75 formalism,

$$\psi_v(\omega) = E\,B(\omega)\,\tilde{V}^2(\omega)\int_0^{\beta_{max}} A(\beta)\,d\beta \tag{4.5}$$

hence

$$\psi_v(\omega) = \frac{\alpha'\,\varepsilon}{\omega^2}\,\frac{\left(\omega^2-\omega_1^2\right)}{N^2}\,n \tag{4.6}$$

and in analogous fashion

$$\psi_u(\omega) = \frac{\alpha'\,\varepsilon}{\omega^2}\,\frac{\left(\omega^2+\omega_1^2\right)}{\omega^2}\,n. \tag{4.7}$$

The ε dependences in Eq.(4.7), (4.6) and (4.3) are new predictions for the gravity waves. Table 1 contains a summary of predictions of this approach including the values of k_{x_*} and k_{z_*}. The derivation of the latter can be found in Dewan (1990).

Table 1. Proposed Model.

$$\Psi_u(k_z) = \frac{\alpha N^2}{k_{z_*}^3}\left(1+\frac{k_z}{k_{z_*}}\right)^{-3}, \quad \Psi_u(k_x) = \frac{\alpha'' \varepsilon^{2/3}}{k_{x_*}^{5/3}}\left(1+\frac{k_x}{k_{x_*}}\right)^{-5/3}$$

$$\left(m^2/s^2\right)/(\mathrm{rad/m}) \qquad \left(m^2/s^2\right)/(\mathrm{rad/m})$$

$$\Psi_v(k) = \frac{3}{4}\left(\frac{\omega_i}{N}\right)\Psi_u(k) \qquad \alpha = \left[\ln\left(\frac{k_{max}}{k_{z_*}}\right) - \frac{3}{2}\right]^{-1}$$

$$\Psi_u(\omega) = \frac{\alpha' \varepsilon}{\omega^2}\left[\frac{\left(\omega^2+\omega_i^2\right)}{\omega^2}\right], \quad \Psi_v(\omega) = \frac{\alpha' \varepsilon}{\omega^2}\left[\frac{\left(\omega^2-\omega_i^2\right)}{N^2}\right]$$

$$\left(m^2/s^2\right)/(\mathrm{rad/s}) \qquad \left(m^2/s^2\right)/(\mathrm{rad/s})$$

$$k_{z_*} = S_z \sqrt{\left(N^2\omega_i\right)/\varepsilon} \qquad k_{x_*} = S_x \sqrt{\left(\omega_i^3\right)/\varepsilon}$$

S_z and S_x are universal constants of order unity and

$$\alpha'' \equiv \left(\alpha\, S_x^{2/3}/\left(3\, S_z^2\right)\right), \ \alpha' \equiv (3/8)\, \alpha/\left(S_z^2\right)$$

Details are in Dewan (1990)

In order to test the model (Table 1) simultaneous measurements of ε, $\psi_{u,v}(\omega)$, $\psi_{u,v}(k_x)$, $\psi_{u,v}(k_z)$, and from them k_{x_*} and k_{z_*} are needed. In the ocean sonar would be used whereas radar and lidar would be the instruments of choice for the atmosphere.

Concerning Quantitative "Reasonableness" of the Model

Due to the fact that, as of this writing, there does not exist a single simultaneous measurement of $\psi_{u,v}(k_x)$ or $\psi_{u,v}(\omega)$ together with suitably averaged ε, it is not possible to subject the model to immediate test. The only question that can be addressed right now is whether or not the predictions are within reasonable bounds of known measurements. This section will study this question. Note the citations used for "reasonable" parametric values.

Atmosphere

The parametric values used here will be

$\varepsilon = 5 \times 10^{-4}\ m^2/s^3$	Barat (1975)
$N = 10^{-2}$ rad/s	Chosen to fit the relevant
$\omega_i = 10^{-4}$ rad/s	altitude and latitude
$\alpha = 0.24$	- using above in Table 1
$S_z = 0.2$	- fit to experiment

Ocean

$\varepsilon = 10^{-8}\ (m^2/s^3)$	Woods (1974), GM (1972, p. 251)
$N = 3.5 \times 10^{-3}$ (rad/s)	GM (1972), 2 (cy./hr.)
$\omega_i = 7 \times 10^{-5}$ (rad/s)	GM (1972), 0.04 (cy./hr.)
$\alpha = 0.256$	using above in Table 1
$S_z = 0.2$	(from previous section)

Table 2 presents a comparison between theory and cited experimental values.

Concerning the choice of the value of ε for the oceanic case above, it is the value cited for the upper thermocline and is therefore appropriate for the case n = 1 in the GM model used for comparison in Table 2.

Table 2.

Quantity (units)	Theory Atmos.	Exper. Atmos.	Theory Oceanic	Exper. Oceanic
$(2\pi/k_z)_{min}$ (m)	22	40(1)	0.48	1.0(4)
$(2\pi/k_{z_*})$ (km)	7	5(2)	107	170(5)
$\psi_u(\omega_i)$ $[(m^2/s^2)/(Hz)]$	1.4×10^6	10^5 to 2×10^6(3)	62	56(6)
$\psi_u(N)$ $[(m^2/s^2)/(Hz)]$	69	63 to 3.2×10^3(3)	1.2×10^{-2}	1.7×10^{-2}(6)

(1) Dewan and Good (1986), (2) Smith et al (1987), (3) Van Zandt (1982), Figure 1, (4) Woods (1974), (5) Holloway (1981) p. 58, (6) GM (1972), Figure 4, and graphed value times $(N/(2\pi))$.

Vertical Coordinate Dependence

The average value of ε as measured in the atmosphere depends on altitude. For example Barat (1975) found stratospheric values of ε to be in the range 10^{-5} to $5 \times 10^{-4}\ m^2 s^{-3}$. We shall use the upper limit in what follows. At mesospheric heights (86-88 km) Hocking (1988) found average values of ε to be around $0.1\ m^2 s^{-3}$. From the model derived above, $\psi_u(\omega)$ is directly proportional to ε and hence one would expect that it would be of order 200 times larger at 86 km than at 11 km. The question raised therefore is whether or not such differences in $\psi_u(\omega)$ are in fact measured in the atmosphere. According to Balsley and Carter (1982) "In the $\omega^{-5/3}$ portions of the spectra, the spectral energy density/unit mass is about 250 times smaller in the troposphere (8 km) than in the mesosphere (86 km)".

From this one can conclude that the new prediction that $\psi_u(\omega)$ will be proportional to ε as the latter changes with altitude is reasonable.

Regarding the oceanic case, it can be shown that the present model implies that ε should scale as $e^{-y/b}$ where y is depth and b ~ 1.3 km where use is made of GM 72, 75 plus WKB scaling which implies that total energy per unit volume scales as N (see Van Zandt and Fritts (1989) for example). I do not know if this, in fact, has ever been observed. It is a prediction.

Conclusions

In this paper a similitude model has been suggested which, if experimentally validated, would phenomenologically extend the GM 72, 75 model. At the same time, it would increase somewhat our understanding of the underlying physics of the processes in question. This model is based on a turbulent-like wave field picture. Wave-like properties (e.g. polarization relations, etc.) are imagined as coexisting with strong mode-interactions giving rise to cascade-like transfers in the k_x and ω scales and saturation-like processes in k_z scales.

A process known as "buoyancy range turbulence" has been described in the literature, sometimes in connection with GM 72, 75. See Lumley (1964), and Weinstock (1985). It is possible that the model offered here will turn out to be an improved description of such "buoyancy range turbulence", but since wave-like properties are assumed, the word turbulence would have to be modified. "Wavelike turbulence" might be appropriate (compare Gage (1979)).

Finally it should be mentioned that phenomenological models are helpful, not only to applied scientists and engineers, but also to theoreticians. It is therefore hoped that this work will be of assistance in the development of a "first principles" physical theory of GM 72, 75.

Acknowledgements

This work was sponsored by AFOSR under project MAPSTAR. I am also grateful to Drs. David Fritts, Colin Hines, and Thomas Van Zandt for many important conversations and suggestions. In addition I acknowledge many helpful criticisms from T. Van Zandt regarding the present manuscript.

References

Balsley, B.B. and Carter, D.A. (1982) The spectrum of atmospheric velocity fluctuations at 8 km and 86 km, *Geophys. Res. Lett., 9*, 465-468.

Bridgman, P.W. (1963) *Dimensional Analysis*, Yale University Press, New Haven.

Barat, J. (1975) Une methode de measure directe du taux de dissipation d'energie turbulente dans la stratosphere, *C.R. Acad. Sc. Paris, 281-B*:53-56.

Crooks, W., et al (1968), *Project HICAT, Rep. AD 847-497*, (NTIS).

Dewan, E.M., and Good, R.E. (1986) Saturation and the "universal" spectrum for vertical profiles of horizontal scalar winds in the atmosphere, *J. Geophys. Res 91*, 2742-2748.

Dewan, E.M. and Good, R.E. (1984) Universal gravity wave spectra in the atmosphere and turbulent saturation, *EOS, 65*, p. 1030.

Dewan, E.M., Grossbard, N., Quesada, A.F., and Good, R.E. (1984) Spectral analysis of 10m scalar velocity profiles in the stratosphere, *Geophys. Res. Lett. 11*, 80-83.

Dewan, E.M. (1979) Stratospheric wave spectra resembling turbulence, *Science 204*, 832-835.

Dewan, E.M. (1990) Power spectra of internal gravity waves, *GL-TR-90-0233, Special Reports #265*.

Fritts, D.C., and H.-G. Chou (1987) An investigation of the vertical wave number and frequency spectra of gravity wave motions in the lower stratosphere, *J. Atmos. Sci., 44*:3610-3624.

Gage, K.S. (1979) Evidence for a $k^{-5/3}$ law inertial range in mesoscale two-dimensional turbulence, *J. Atmos. Sci., 36*, 1950-1954.

Garrett, C. and Munk, W. (1972) Space-time scales of internal waves, *Geophys. Fluid Dyn. 2*, 225-264.

Garrett, C. and Munk, W. (1975) Space-time scales of internal waves: a progress report, *J. Geophys. Res. 80*, 291-297.

Hines, C.O. (1991) The saturation of atmospheric gravity waves in the Middle Atmosphere (4 papers) *J. Atmos. Sci.* (in press).

Hocking, W.K. (1988) Two years of continuous measurements of turbulence parameters in the upper mesosphere and lower thermosphere made with a 2-MHZ radar, *J. Geophys. Res. 93*, 2475.

Holloway, G. (1981) Theoretical approaches to interactions among internal waves, turbulence and finestructure, in *Non-linear Properties of Internal Waves*, Am. Inst. Phys., B. West (ed).

Lumley, J. (1964) The spectrum of nearly inertial turbulence in a stably stratified fluid, *J. Atmos. Sci., 21*, 99-102.

Munk, W. (1981) Internal waves and small scale processes, Ch. 9, *Evolution of Physical Oceanography*, ed. B.A. Warren and C. Wunsch, MIT Press.

Monin, A. and Yaglom, A. (1971) *Statistical Fluid Mechanics, 1*, MIT Press, Cambridge, MA.

Phillips, O.M. (1966) *The Dynamics of the Upper Ocean*, (1st Edition), Cambridge Univ. Press.

Phillips, O.M. (1980) *The Dynamics of the Upper Ocean*, (2nd Edition), Cambridge Univ. Press.

Smith, S.A., Fritts, D.C. and VanZandt, T.E. (1987) Evidence for a saturated spectrum of atmospheric gravity waves, *J. Atmos. Sci. 44*, 1404-1410.

Tennekes, H. and Lumley, J.L. (1972) *A First Course in Turbulence*, MIT Press, Cambridge, MA.

Tsuda, T., Inoue, T., Fritts, D.C., Van Zandt, T.E., and Fukao, S. (1989) MST radar observations of a saturated gravity wave spectrum, *J. Atmos. Sci. 46*, 2440-2447.

U.S. Standard Atmosphere (1976) *NOAA, NASA, USAF, NOAA-S/T 76-1562*, U.S. Gov. Printing Office, Washington, D.C.

Van Zandt, T.E. (1982) A universal spectrum of buoyancy waves in the atmosphere, *Geophys. Res. Lett., 9*, 575-578.

Van Zandt, T.E. and Fritts, D.C. (1989) A theory of enhanced saturation of the gravity wave spectrum due to increases in atmospheric stability, *PAGEOPH, 130*, 399-420.

Weinstock, J. (1985) Theoretical gravity wave spectrum in the atmosphere: strong and weak wave interactions, *Radio Sci. 20*, 1295-1300.

Woods, J.D. (1974) Diffusion due to fronts in the rotation sub-range of turbulence in the seasonal thermocline, *La Houille Blanche*, 589-598.

(Received February 12, 1991;
revised April 12, 1991;
accepted May 30, 1991.)