(2)

ARMY RESEARCH LABORATORY

Least-Squares Averaging of Burst Points

Aivars Celmiņš

ARL-TR-21

December 1992

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION IS UNLIMITED.

NOTICES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1992	Final, Apr 91 - Mar 92

4. TITLE AND SUBTITLE
Least-Squares Averaging of Burst Points

5. FUNDING NUMBERS
PR: 1L162618AH80

6. AUTHOR(S)
Aivars Celmiņš

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRL-CI-CA
Aberdeen Proving Ground, MD 21005-5067

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRL-OP-CI-B (Tech Lib)
Aberdeen Proving Ground, MD 21005-5066

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARL-TR-21

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The average burst point of artillery shells is computed from individual burst-point observations. We review such calculations within the framework of general least-squares averaging of observations in an n-dimensional space and propose an iterative solution for cases where the event scatter is unknown and neither it nor the observational scatters can be neglected. The solution produces the average burst point and an estimate of the event scatter, that is estimates of the expected burst point location and the dispersion of the artillery fire.

14. SUBJECT TERMS
least squares method; artillery fire; least squares; vector averaging; model variance; iterative solution; burst-point coordinates; dispersion of artillery fire

15. NUMBER OF PAGES
19

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

INTENTIONALLY LEFT BLANK.

TABLE OF CONTENTS

INTENTIONALLY LEFT BLANK.

LIST OF FIGURES

INTENTIONALLY LEFT BLANK.

1. INTRODUCTION

To determine the accuracy of artillery fire one measures the coordinates of the shell's burst point in repeated firings and calculates an average burst point and its scatter from these measurements. The task amounts to the computation of an average vector in $\mathbb{R}^3$. The accuracy of each observed vector is known from an analysis of the actual measurements and depends mainly on the geometry of the setup and properties of the measuring instruments (theodolites in general). We characterize this accuracy by an estimated variance-covariance matrix of the vector components. If the cannon would fire every time exactly alike (i.e., if the event scatter would be zero) then the burst-point coordinates could be obtained from these data by a weighted averaging where the weights are the inverses of the variance-covariance matrices. However, in real life the event scatter can be of the same order or even larger than the measurement scatters. Also, in general the principal directions of the event distribution are different from the principal directions of the measurement-error distributions. Therefore, a weighted averaging in $\mathbb{R}^3$ can produce unacceptable results. On the other hand an unweighted averaging would not take into account the estimates of measurement errors that can be quite different for different observations. In this paper we present a new algorithm for the computation of an average vector that does not have the disadvantages of unweighted or observation-weighted averages. The algorithm provides in addition to the average vector, also an estimate of the event variance that is consistent with the observations and their estimated variances.

In Section 2 we define a problem of vector averaging in $\mathbb{R}^n$ that corresponds to the outlined artillery problem and propose a solution. Section 3 contains some examples and Section 4 is a summary.

2. ESTIMATION OF AN AVERAGE VECTOR

Let the observed vectors be $x_i \in \mathbb{R}^n$, $i = 1,\ldots,s$ and let the estimated variance-covariance $n \times n$ matrices of the observations be Q_i, $i = 1,\ldots,s$. Let the unknown average vector be $a \in \mathbb{R}^n$ and the variance-covariance matrix of the event be P. The model equation of the event is

$$f(x,a) = x - a = 0 \quad . \tag{1}$$

We define the least-squares value of a as the solution of the following constrained optimization problem.

Minimize $$W = \sum_{i=1}^{s} \left(c_i^T Q_i^{-1} c_i + b_i^T P^{-1} b_i \right) \tag{2}$$

subject to
$$f(x_i + c_i,\, a + b_i) = x_i + c_i - a - b_i = 0\,, \quad i = 1,\ldots,s\,, \tag{3}$$

where c_i is the correction of the i-th observation and b_i is the deviation of the i-th event from the average a.

To solve the minimization problem we introduce a modified objective function $\widetilde{W}$ using Lagrange multiplier vectors k_i:

$$\widetilde{W} = \frac{1}{2} \sum_{i=1}^{s} \left(c_i^T Q_i^{-1} c_i + b_i^T P^{-1} b_i \right) - \sum_{i=1}^{s} k_i^T \left(x_i + c_i - a - b_i \right)\ . \tag{4}$$

We obtain a system of normal equations by setting equal to zero the partial derivatives of $\widetilde{W}$ with respect to c_i, b_i, a and k_i. The result is

$$\left.\begin{array}{lll} Q_i^{-1} c_i - k_i & = 0\,, & i = 1,\ldots,s\ , \\ P^{-1} b_i + k_i & = 0\,, & i = 1,\ldots,s\ , \\ \sum_{i=1}^{s} k_i & = 0\,, & \\ x_i + c_i - a - b_i & = 0\,, & i = 1,\ldots,s\ . \end{array}\right\} \tag{5}$$

Eliminating the k_i we obtain the following simpler equation system

$$\left.\begin{array}{lll} a & = \left[\sum_{i=1}^{s} (Q_i + P)^{-1} \right]^{-1} \sum_{i=1}^{s} (Q_i + P)^{-1} x_i\ , & \\ b_i & = \quad P\,(\,Q_i + P\,)^{-1}\,(x_i - a)\ , & i = 1,\ldots,s\ , \\ c_i & = -\,Q_i\,(\,Q_i + P\,)^{-1}\,(x_i - a)\ , & i = 1,\ldots,s\ . \end{array}\right\} \tag{6}$$

We also obtain

$$W_b = \sum_{i=1}^{s} b_i^T P^{-1} b_i = \sum_{i=1}^{s} (x_i - a)^T (\,Q_i + P\,)^{-1}\, P\,(\,Q_i + P\,)^{-1}\,(x_i - a)\ , \tag{7}$$

$$W_c = \sum_{i=1}^{s} c_i^T Q_i^{-1} c_i = \sum_{i=1}^{s} (x_i - a)^T (\,Q_i + P\,)^{-1}\, Q_i\,(\,Q_i + P\,)^{-1}\,(x_i - a)\ , \tag{8}$$

$$W = W_b + W_c = \sum_{i=1}^{s} (x_i - a)^T\,(Q_i + P)^{-1}\,(x_i - a) \tag{9}$$

and the variance of weight one

$$v_o = \frac{1}{n\,(s-1)}\; W \;. \tag{10}$$

Let the total variance-covariance matrix including both the measurement scatter and the event scatter of the observed x_i be R_{xi}. Then the variance-covariance matrix of a is

$$R_a = \Big[\sum_{i=1}^{s} (Q_i + P\,)^{-1}\Big]^{-1} \sum_{i=1}^{s} \Big[\,(Q_i+P)^{-1} R_{xi}\,(Q_i+P)^{-1}\,\Big]\,\Big[\sum_{i=1}^{s} (Q_i + P\,)^{-1}\Big]^{-1}. \tag{11}$$

If we estimate as usual

$$R_{xi} = v_o\,(Q_i + P)\;, \qquad i = 1,\ldots,s \tag{12}$$

then it follows from eq. (11)

$$R_a = v_o\,\Big[\sum_{i=1}^{s} (Q_i + P\,)^{-1}\Big]^{-1} = \Big[\sum_{i=1}^{s} R_{xi}^{-1}\Big]^{-1} \;. \tag{13}$$

The formulas (6) through (13) provide the general least-squares solution of the averaging problem if the Q_i and P are known. In practice such a situation is an exception, because in general P is not known. Therefore, commonly used are two special cases of the solution that are based on the assumption that either the Q_i or P can be neglected. We now outline these special cases.

In the first special case we assume that $P = 0$, that is, we assume that either the event scatter is negligible or that the estimated Q_i already contain the matrix P. With this assumption we obtain from eqs. (6) and (13) the usual observation-weighted least-squares averaging formulas:

$$a = \Big[\sum_{i=1}^{s} Q_i^{-1}\Big]^{-1} \sum_{i=1}^{s} Q_i^{-1} x_i \;, \tag{14}$$

$$\left.\begin{aligned} b_i &= 0\;, & i &= 1,\ldots,s\;,\\ c_i &= a - x_i\;, & i &= 1,\ldots,s\;, \end{aligned}\right\} \tag{15}$$

$$R_a = v_o\,\Big[\,\sum_{i=1}^{s} Q_i^{-1}\,\Big]^{-1} \;. \tag{16}$$

Usually the Q_i are positive definite matrices but in some applications they may be only semi-definite. Also, the sum of their inverses in eqs. (14) and (16) is not necessarily positive definite. The formulas are, however, generally valid if Moore-Penrose generalized inverses are used in both formul .

In the second case we assume that the measurement errors are negligible, that is, $Q_i = 0$ for all $i = 1,\ldots,s$ (or that all Q_i are equal and included in P). In that case eqs. (6) and (13) provide the formulas

$$a = \frac{1}{s} \sum_{i=1}^{s} x_i \ , \tag{17}$$

$$\left.\begin{aligned} b_i &= x_i - a \ , \qquad i = 1, \ldots, s \ , \\ c_i &= 0 \ , \qquad\quad\ \ i = 1, \ldots, s \ , \end{aligned}\right\} \tag{18}$$

$$R_a = v_o \frac{1}{s} P \ . \tag{19}$$

Equations (17) through (19) are the formulas for simple unweighted averaging. To complete the calculation we also need an estimate for P. The usual estimate is

$$P = \alpha \widetilde{P} = \alpha \sum_{i=1}^{n} b_i \, b_i^T \ , \tag{20}$$

where the factor α is determined such that v_o equals unity. To that end we compute

$$\widetilde{U} = \sum_{i=1}^{s} (x_i - a)^T \, \widetilde{P}^{-1} \, (x_i - a) \ , \tag{21}$$

$$\alpha = \frac{1}{n\,(s-1)} \, \widetilde{U} \tag{22}$$

and obtain for the variance-covariance matrix of the average

$$R_a = v_o \frac{\alpha}{s} \widetilde{P} = \frac{\widetilde{U}}{n\,s\,(s-1)} \widetilde{P} \ . \tag{23}$$

As in the first case, we use the Moore-Penrose generalized inverse in eq. (21) if the matrix $\widetilde{P}$ is not positive definite.

In real life, estimates of the Q_i usually are available but P is not known so that the general formulas cannot be used. If also neither of the two special cases apply one needs a method that provides an estimate of P concurrently with the average vector a. We propose for this purpose the following iterative algorithm. It produces an estimate of P and solves the general problem, eqs. (2) and (3), using this estimate. Because the solution takes into account the distribution of the observed vectors x_i we call the resulting a the *distribution-weighted* average.

We initialize the iteration with an unweighted averaging

$$a_o = \frac{1}{s} \sum_{i-1}^{s} x_i \ , \tag{24}$$

$$b_{o,i} = x_i - a \ , \qquad i = 1, \ldots, s \ , \tag{25}$$

and obtain an initial estimate of P

$$\widetilde{P}_o = \sum_{i=1}^{s} b_{o,i}\, b_{o,i}^T \ , \tag{26}$$

$$\widetilde{U}_o = \sum_{i=1}^{s} b_{o,i}^T\, \widetilde{P}_o^{-1}\, b_{o,i} \ , \tag{27}$$

$$P_1 = \frac{\widetilde{U}_o}{n\,(s-1)}\, \widetilde{P}_o \ . \tag{28}$$

Next, we update the initial estimates of a and b_i and obtain an initial estimate for the scaling factor α.

$$a_1 = \Big[\sum_{i=1}^{s} (Q_i + P_1)^{-1} \Big]^{-1} \sum_{i=1}^{s} (Q_i + P_1)^{-1}\, x_i \ , \tag{29}$$

$$b_{1,i} = P_1\,(Q_i + P_1)^{-1}\,(x_i - a_1)\,, \qquad i = 1,\ldots,s \ , \tag{30}$$

$$\widetilde{P}_1 = \sum_{i=1}^{s} b_{1,i}\, b_{1,i}^T \ , \tag{31}$$

$$\widetilde{U}_1 = \sum_{i=1}^{s} b_{1,i}^T\, \widetilde{P}_1^{-1}\, b_{1,i} \ , \tag{32}$$

$$\alpha = \frac{\widetilde{U}_1}{n\,(s-1)} \ . \tag{33}$$

In subsequent iteration steps we do update P but keep the value of α unchanged. The iteration formulas for $k = 1,2,\ldots$ are

$$P_{k+1} = \alpha\, \widetilde{P}_k \ , \tag{34}$$

$$a_{k+1} = \Big[\sum_{i=1}^{s} (Q_i + P_{k+1})^{-1} \Big]^{-1} \sum_{i=1}^{s} (Q_i + P_{k+1})^{-1}\, x_i \ , \tag{35}$$

$$b_{k+1,i} = P_{k+1}\,(Q_i + P_{k+1})^{-1}\,(x_i - a_{k+1}) \ , \qquad i = 1,\ldots,s \ , \tag{36}$$

$$\widetilde{P}_{k+1} = \sum_{i=1}^{s} b_{k+1,i}\, b_{k+1,i}^T \ . \tag{37}$$

The variance-covariance estimate R_a of the average is computed with eqs. (9), (10) and (13). Iteration end conditions can be expressed, for instance, in terms of changes of the elements of a and R_a. Experience shows that the average vector a becomes stationary after a few iteration steps whereas the elements of R_a need more steps to meet such convergence criteria. Convergence enhancement techniques were found unnecessary in numerical experiments with this algorithm.

The result of the iterations depends on the scaling factor α that was initially estimated by eq. (33). We want to determine its value such that the variance of weight one v_o, defined by eqs. (9) and (10), equals unity. We achieve this by embedding the iteration eqs. (34) through (37) in a regula falsi algorithm for the solution of the equation $v_o(\alpha) = 1$. In general, a solution with positive α exists if $v_o(0) > 1$, because v_o decreases with increasing α. If $v_o(0) \leq 1$ then the estimated observational errors (the matrices Q_i) are so large that a solution with $\alpha = 0$, i.e., with neglected P suffices to explain the data.

The final result of the calculations is a solution of the general minimization problem, eqs. (2) and (3), whereby the event variance matrix satisfies eq. (20) and $v_o = 1$.

3. EXAMPLES

We present two examples. The first example is chosen to illustrate the main characteristics of the three types of averaging. In the second example we use actual data.

In the first example we compute the average of three points on a straight line in a plane. The coordinates of the points are (0.5, 2.0), (1.5, 2.1) and (8.5, 2.8). We assume that the observational errors are equal for all three points and given by the following estimate of their variance-covariance matrix

$$Q = \begin{pmatrix} 2.0 & 2.0 \\ 2.0 & 2.0 \end{pmatrix} .$$

The matrix Q is not positive definite which means that the observational errors are distributed in a subspace of $\mathbb{R}^2$, that is, along a straight line. In other words, the observational-error ellipses are degenerated into error bars. Figure 1 shows the data and the result of a weighted average. The coordinates of the average are (3.5, 2.3) and the variance-covariance matrix of the average is

$$R_a = \begin{pmatrix} 0.95792 & 0.95792 \\ 0.95792 & 0.95792 \end{pmatrix} .$$

The corresponding standard-deviation ellipse is again degenerated and shown in Figure 1 as a dashed line. The location of the average point is reasonable but its estimated variance is not because the structure of the variance-covariance matrix that is computed with eq. (16) is independent of the observations and does not reflect the event scatter.

Next we use the same data and compute their unweighted average by eqs. (17) through (23). The average vector is the same as in the previous calculation but its variance-covariance matrix is

$$R_s = \begin{pmatrix} 4.75 & 0.475 \\ 0.475 & 0.0475 \end{pmatrix} .$$

The result is shown in Figure 2. The image of the one-standard-error ellipse of the

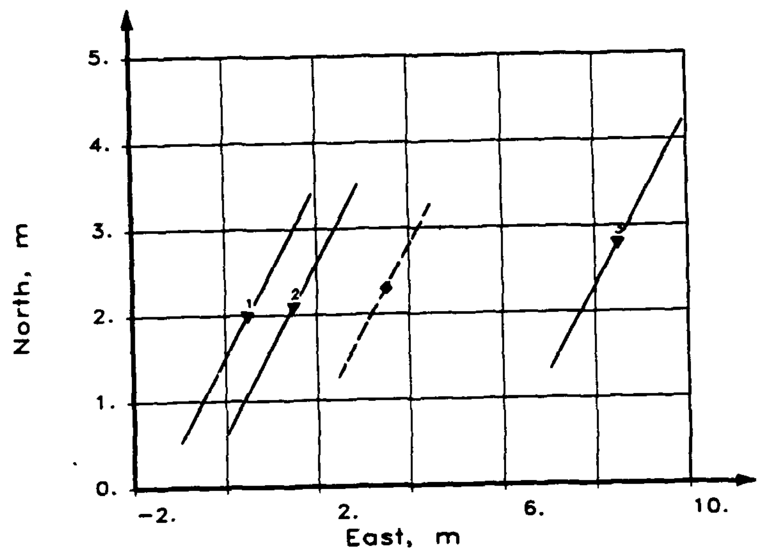

Figure 1. Observation-weighted average.

average is an error bar in the direction of the scatter of the observations, because in this case R_a is independent of the observational-error variances.

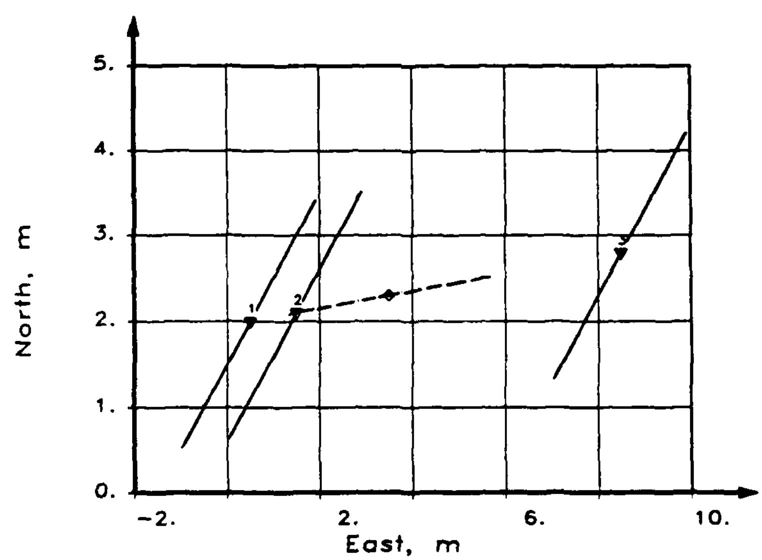

Figure 2. Unweighted average.

Finally, we compute the distribution-weighted average by iteration, eqs. (24) through (37). The average vector again is the same as before. Its variance-covariance matrix is

$$R_s = \begin{pmatrix} 4.12682 & 1.13567 \\ 1.13567 & 0.73027 \end{pmatrix} .$$

Figure 3 shows the corresponding one-standard-error ellipse. The figure also contains the correction vectors b_i plotted as rays from the average point. The end points of the b_i are indicated by dots. In this example, all vectors b_i are parallel so that their end points are located on a straight line and the matrix P, eq. (34), is only positive semi-definite. The image of the ellipse representing P is a segment of a straight line in the direction of the b_i. The differences between the dots and the corresponding observations (inverted triangles) are the corrections c_i. We observe that all corrections c_i and b_i are in the direction of the corresponding error bars, as they should be. In this example the iteration with eqs. (34) through (37) became stationary after two steps. The initial scaling factor and the variance of weight one were, respectively, $\alpha = 0.250$ and $v_o = 1.008$. After three regula falsi steps, we had the values $\alpha = 0.252006$ and $v_o = 1.00006$.

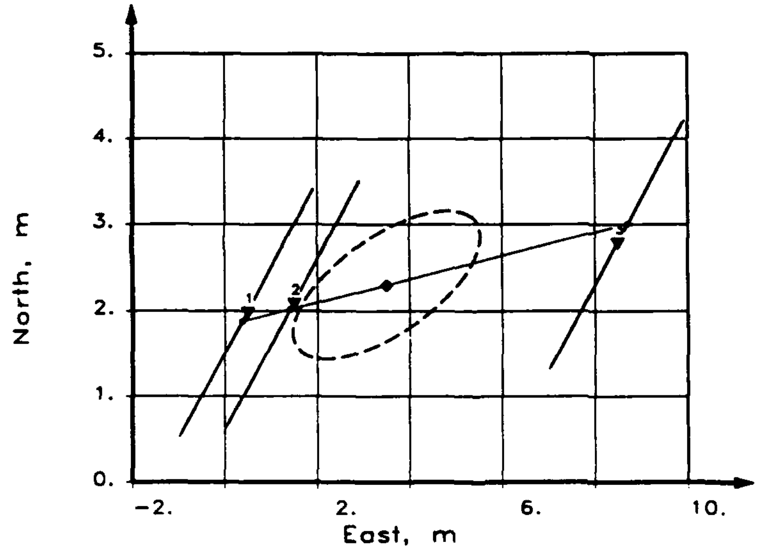

Figure 3. Distribution-weighted average.

In our second example we use actual observations of artillery burst-point coordinates. The observations are three-dimensional vectors consisting of range, deflection and height of the burst. The coordinates of each vector were obtained from simultaneous measurements of directional angles of the burst points from four observation towers. An analysis of these measurements provided the components of each burst location vector and estimates of the accuracies of the burst points in form of the variance-covariance matrices Q_i. The estimated accuracies of the observations vary widely and are in this example smaller than the scatter between the burst points, but are not negligible. The observation set in our example consists of eight observed burst points from the same howitzer. Figures 4, 5 and 6 show the observed points as inverted triangles and their distribution-weighted (iteratively determined) average as a diamond. The figures also contain the projections of the corresponding one-standard-error ellipsoids. The standard-error ellipsoid of the average, defined by R_s, is plotted with a dashed line. The standard-deviation ellipsoid of a single shot is represented by the matrix P and plotted with a dotted line. We note that contrary to the appearance in the plots P is not proportional to R_s: the relation between P and R_s is given by eq. (13). The correction vectors b_i are plotted as rays from the average point, as in Figure 3. We observe that these corrections are in general not in the direction towards the observations, but towards other points such that the corrections b_i and c_i are in directions of large error estimates thus minimizing W, eq. (2). The initial value of the scaling factor was $\alpha = 0.143$ for a variance of weight one of $v_0 = 1.137$. After four regula falsi steps, the results were $\alpha = 0.176998$ and $v_0 = 1.00004$. The iteration for a and b_i, eqs. (34) through (37), required eight steps at the beginning and three steps at the end of the calculations.

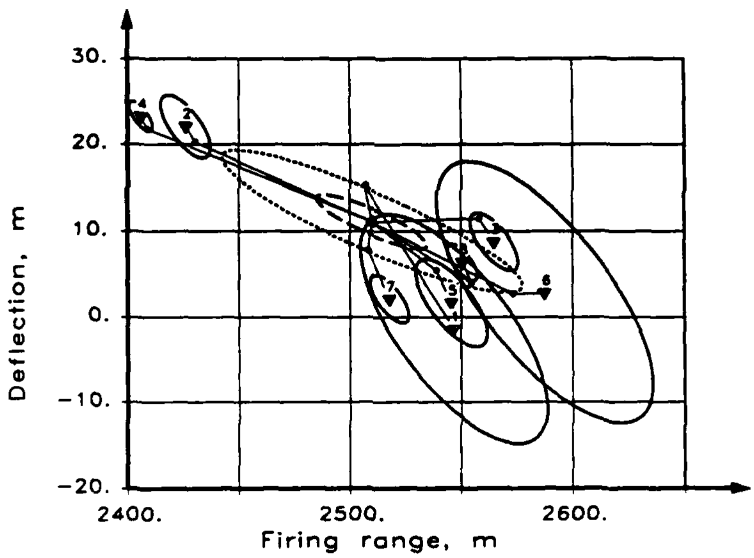

Figure 4. Burst-point range and deflection.

To illustrate the advantage of the distribution-weighted average we show in Figures 7, 8 and 9 the usual observation-weighted average [eqs. (14) through (16)] of the same observations. We notice that in this example the result is completely unrealistic because the observation-weighted average burst point is shifted far outside the cloud of observations. From an inspection of the figures, we conclude that this shift is caused by the high sensitivity of the location of the average to the estimated principal directions of observational errors. The variance of weight one was in this case $v_0 = 5996$ indicating

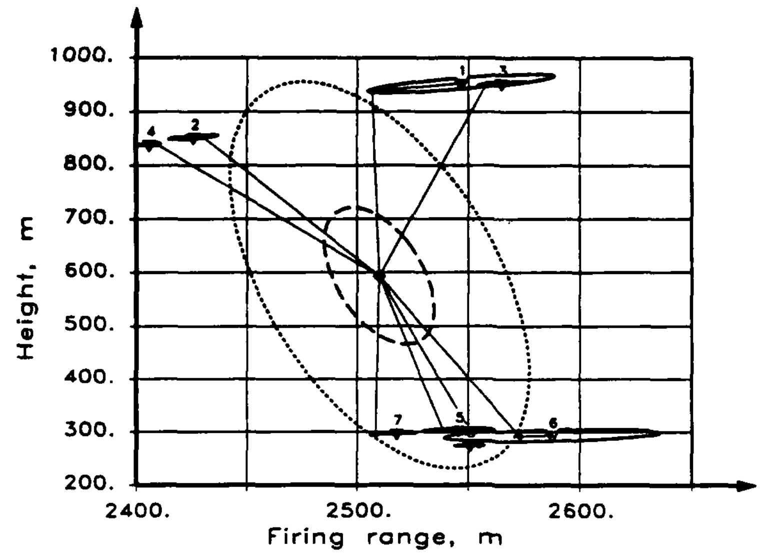

Figure 5. Burst-point range and height.

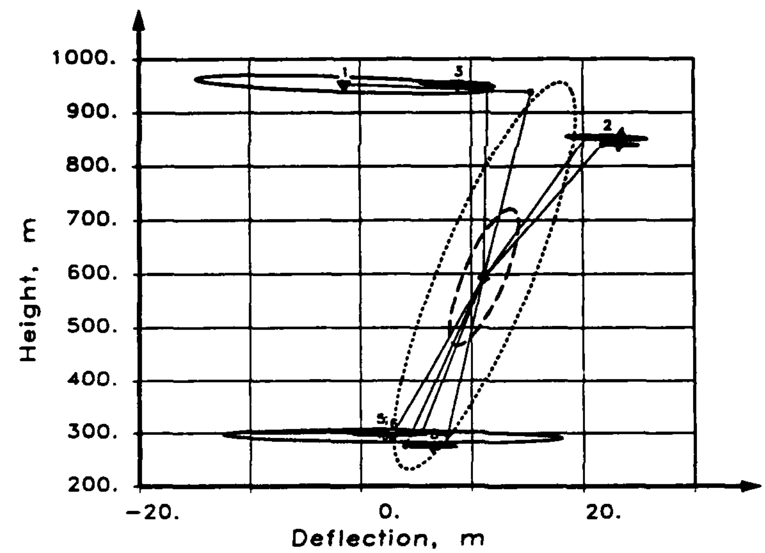

Figure 6. Burst-point deflection and height.

that measurement errors alone are not sufficient to explain the data scatter.

4. SUMMARY

We have considered least-squares computations of vector averages. We assume that the observations in an n-dimensional space contain inaccuracies from two sources: observational errors and variations of the observable itself, that is, event scatter.

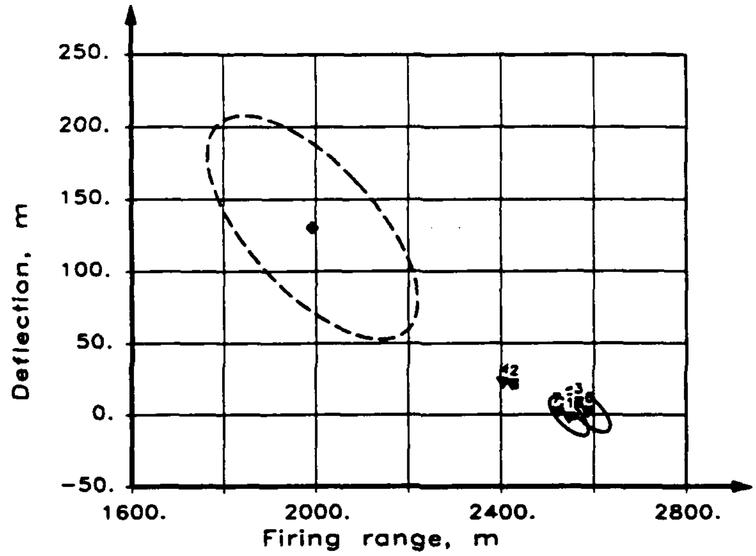

Figure 7. Observation-weighted range and deflection.

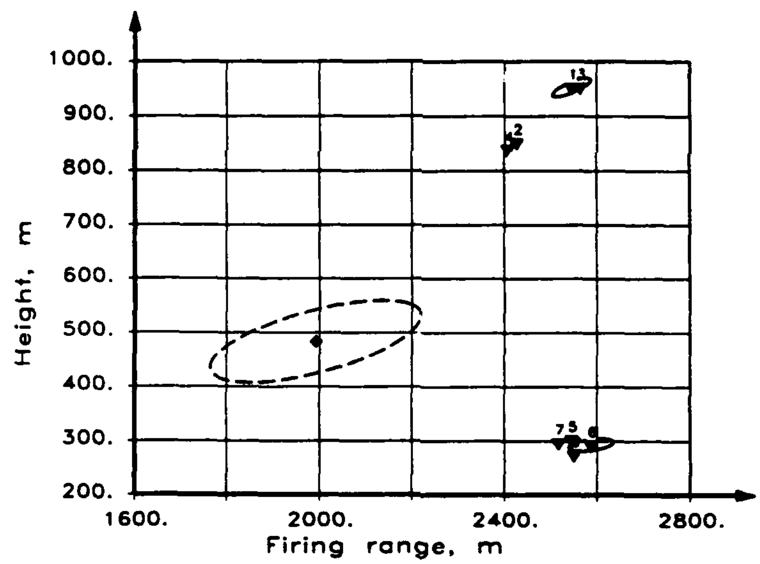

Figure 8. Observation-weighted range and height.

Usually one of these error sources is neglected. If a simple unweighted average is computed then one assumes implicitly that the observational errors are negligible. If a weighted average is computed then the implicit assumption is that the event scatter can be neglected. Most often the event variation is not known and one has no grounds for using one of these special algorithms. If event scatter is known to exist we propose to use the distribution-weighted average that is computed by an iterative algorithm. The

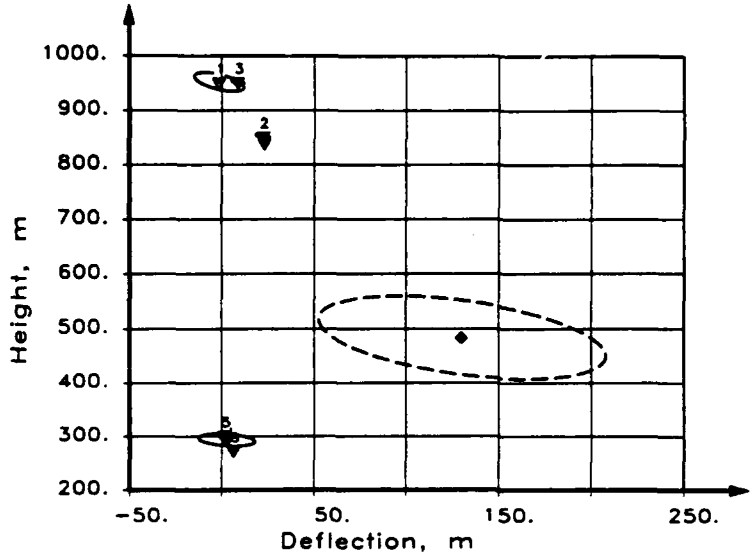

Figure 9. Observation-weighted deflection and height.

algorithm provides in addition to the average of the observed vectors with its variance, also an estimate of the variance-covariance matrix of the event.

No. of Copies	Organization
2	Administrator Defense Technical Info Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22304-6145
1	Commander U.S. Army Materiel Command ATTN: AMCAM 5001 Eisenhower Ave. Alexandria, VA 22333-0001
1	Director U.S. Army Research Laboratory ATTN: AMSRL-D 2800 Powder Mill Rd. Adelphi, MD 20783-1145
1	Director U.S. Army Research Laboratory ATTN: AMSRL-OP-CI-AD, Tech Publishing 2800 Powder Mill Rd. Adelphi, MD 20783-1145
2	Commander U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-IMI-I Picatinny Arsenal, NJ 07806-5000
2	Commander U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-TDC Picatinny Arsenal, NJ 07806-5000
1	Director Benet Weapons Laboratory U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-CCB-TL Watervliet, NY 12189-4050
(Unclass. only) 1	Commander U.S. Army Rock Island Arsenal ATTN: SMCRI-IMC-RT/Technical Library Rock Island, IL 61299-5000
1	Director U.S. Army Aviation Research and Technology Activity ATTN: SAVRT-R (Library) M/S 219-3 Ames Research Center Moffett Field, CA 94035-1000

No. of Copies	Organization
1	Commander U.S. Army Missile Command ATTN: AMSMI-RD-CS-R (DOC) Redstone Arsenal, AL 35898-5010
1	Commander U.S. Army Tank-Automotive Command ATTN: ASQNC-TAC-DIT (Technical Information Center) Warren, MI 48397-5000
1	Director U.S. Army TRADOC Analysis Command ATTN: ATRC-WSR White Sands Missile Range, NM 88002-5502
1	Commandant U.S. Army Field Artillery School ATTN: ATSF-CSI Ft. Sill, OK 73503-5000
(Class. only) 1	Commandant U.S. Army Infantry School ATTN: ATSH-CD (Security Mgr.) Fort Benning, GA 31905-5660
(Unclass. only) 1	Commandant U.S. Army Infantry School ATTN: ATSH-CD-CSO-OR Fort Benning, GA 31905-5660
1	WL/MNOI Eglin AFB, FL 32542-5000
	<u>Aberdeen Proving Ground</u>
2	Dir, USAMSAA ATTN: AMXSY-D AMXSY-MP, H. Cohen
1	Cdr, USATECOM ATTN: AMSTE-TC
1	Dir, ERDEC ATTN: SCBRD-RT
1	Cdr, CBDA ATTN: AMSCB-CI
1	Dir, USARL ATTN: AMSRL-SL-I
10	Dir, USARL ATTN: AMSRL-OP-CI-B (Tech Lib)

No. of Copies	Organization
1	Commander TRADOC ATTN: ATAN-AP Fort Monroe, Va 23651-5143
1	Director TRAC-FLVN ATTN: ATRC Fort Leavenworth, KS 66027-5200
2	Department of the Army Office of the Product Manager 155-mm Howitzer, M109A6, Paladin ATTN: SFAE-AR-HIP-IP, Mr. R. De Kleine Mr. D. Griggs Picatinny Arsenal, NJ 07806-5000
1	OPM Nuclear ATTN: AMCPM-NUC Picatinny Arsenal, NJ 07806-5000
4	Commander U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-AET, Mr. F. Scerbo Mr. J. Bera SMCAR-AET-A, Mr. R. Kline Mr. H. Hudgins Picatinny Arsenal, NJ 07806-5000
3	Commander U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-FSA, Mr. F. Brody Mr. R. Kantenwein SMCAR-FSS, Mr. J. Brooks Picatinny Arsenal, NJ 07806-5000
2	Commandant U.S. Army Field Artillery School ATTN: ATSF-CCM ATSF-GD Fort Sill, OK 73503

No. of Copies	Organization
1	Director U.S. Army Field Artillery Board ATTN: ATZR-BDW Fort Sill, OK 73503
1	Commander U.S. Army Dugway Proving Ground ATTN: STEDP-MT, Mr. G. C. Travers Dugway, UT 84022
1	Commander U.S. Army Yuma Proving Ground ATTN: STEYP-MTW Yuma, AZ 85365-9103
1	Director Sandia National Laboratories ATTN: Mr. A. Hodapp Division 1631 Albuquerque, NM 87185
1	Commander Naval Surface Warfare Center Aerodynamics Branch ATTN: Dr. W. Yanta K-24, Building 402-12 White Oak Laboratory Silver Spring, MD 20910
1	Director Lawrence Livermore National Laboratory ATTN: Mr. T. Morgan, Mail Code L-35 P.O. Box 808 Livermore, CA 94550
1	Commander U.S. Army Research Office ATTN: CRD-AA-EH P.O. Box 12211 Research Triangle Park, NC 27709

No. of Copies	Organization
	Aberdeen Proving Ground
4	Dir, USAMSAA ATTN: AMXSY-RV, Mr. E. Hilkemeyer AMXSY-RA, Mr. R. Scungio AMXSY-GS, Mr. B. King AMXSY-CC, Mr. R. Sandmeyer
1	Cdr, USATECOM ATTN: AMSTE-TE-F, Mr. W. Vomocil
1	PM-SMOKE, Bldg. 324 ATTN: AMCPM-SMK-M, Mr. J. Callahan
2	Cdr, USACSTA ATTN: STECS-AS-H STECS-EN-B

INTENTIONALLY LEFT BLANK.

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. ARL Report Number ARL-TR-21 Date of Report December 1992

2. Date Report Received ______

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) ______

4. Specifically, how is the report being used? (Information source, design data, procedure, source of ideas, etc.) ______

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided, or efficiencies achieved, etc? If so, please elaborate. ______

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.) ______

CURRENT ADDRESS

Organization

Name

Street or P.O. Box No.

City, State, Zip Code

7. If indicating a Change of Address or Address Correction, please provide the Current or Correct address above and the Old or Incorrect address below.

OLD ADDRESS

Organization

Name

Street or P.O. Box No.

City, State, Zip Code

(Remove this sheet, fold as indicated, staple or tape closed, and mail.)

DEPARTMENT OF THE ARMY

OFFICIAL BUSINESS

BUSINESS REPLY MAIL
FIRST CLASS PERMIT No 0001, APG, MD

Postage will be paid by addressee.

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES

Director
U.S. Army Research Laboratory
ATTN: AMSRL-OP-CI-B (Tech Lib)
Aberdeen Proving Ground, MD 21005-5066