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\ PSYCHOMETRICALLY SOUND COGNITIVE DIAGNOSTIC MODEL: :FFECT OF REMEDIATION AS EMPIRICAL VALIDITY

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It was also found that the designing of instructional units for remediation can be effectively guided by the rule-space model, because the determination of all possible ideal item-score patterns, given an incidence matrix, is based on a tree structure of cognitive attributes, knowledge states, and items.

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A Psychometrically Sound Cognitive Diagnostic Model:
Effect of Remediation as Empirical Validity

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Abstract

The purpose of this study was to validate the results of cognitive diagnoses using the rule-space model and to demonstrate the usefulness of cognitive diagnoses for instruction. The results of the study strongly indicated that the rule-space model can effectively diagnose students' knowledge states and can point out ways for remediating their errors quickly with minimum effort.

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Introduction

The diagnosing of cognitive errors committed by students taking a test is a matter of interest not only to teachers, but also to cognitive psychologists and scientists who investigate the cognitive processes that underlie problem solving and reasoning (Greeno & Simon, 1984). To carry out such diagnoses adequately, it is necessary first to do a task analysis of the test by constructing what is called an incidence matrix, which associates with each item an attribute vector. The latter is a binary vector with 1s and 0s as elements, representing the presence or absence, respectively, of various basic-skill attributes in each item. That is, if the jth basic skill on a list is required for correctly answering a given item, then the jth element of the associated attribute vector is 1; otherwise it is 0.

The determination of classification groups must be done prior to formulating a classification space, rule space which is defined in the next section. Tatsuoka (1991) and Varadi and Tatsuoka (1989) successfully introduced a Boolean algebra algorithm by which all possible knowledge states can be derived from the information embedded in the test items --more precisely, from an incidence matrix. The number of knowledge states can often be as large as one thousand.

However, knowledge and capability states are impossible to observe directly unless we use computers for testing and prepare special computer programs such as intelligent tutoring systems. However, developing such intelligent systems is very expensive because they are essentially domain-specific and require extensive programming efforts.

Since only item scores are observable in practical testing situations, one must develop a probabilistic method by which unobservable performances on

unobservable cognitive tasks can be inferred from observable item scores.

One of the assumptions used in this study is that only item scores are observable and the underlying cognitive tasks are not expected to be observable.

The purposes of this study are: 1) To validate empirically whether or not the fraction diagnostic test based on the rule-space model is effectively classifying each examinee into his/her true state, and 2) to test whether diagnostic information is useful and efficient for remediation.

A brief summary of rule-space model, a psychometrically sound cognitive diagnostic model, will be given in the next section, and our procedure for the empirical validation of the model will be introduced. Finally the results from our study will be discussed.

<u>Classification Space: Rule Space</u>

A convenient tool for facilitating error diagnosis is rule space, which was developed by Tatsuoka and her associates (Tatsuoka, 1983, 1985; Tatsuoka & Tatsuoka, 1987; Tatsuoka & Tatsuoka, 1989). One of the upshots of utilizing the rule-space model is that various "misconception groups" are representable by ellipses centered on what might be called "pure rule points"--i.e., points onto which are mapped the response patterns resulting from the consistent use of specific erroneous rules of operation throughout the entire test (Tatsuoka, 1986).

The formulation of rule space is done by utilizing Item Response Theory (IRT) in order to maintain continuity with current psychometric theories. Item response functions are used to derive an index ζ (defined in the next section, Eq.7) that is sensitive to the unusualness of item response patterns (Tatsuoka, 1984; Tatsuoka & Linn, 1983). A Cartesian product space of IRT

ability θ and the variable ζ is used to formulate a classification space.

We first define a function $f(\mathbf{x})$ that is proportional to the coefficient of regression of $\mathbf{P}(\theta)-\mathbf{x}$ on $\mathbf{P}(\theta)-\mathbf{T}(\theta)$, so that when its value is close to zero, it means that the two probability vectors are almost collinear. Since $P_j(\theta)$ is the expectation of item score x_j given θ, the conditional expectation of $f(\mathbf{x})$ equals zero. Therefore, when the elements of an observed vector $\mathbf{x}$ are close to the average performances on the test items, the absolute value of θ values will be nearly zero.

Students' misconceptions diagnosed by the rule-space model can be related with the IRT ability, θ. The unusualness of diagnosed cognitive errors can be judged by examining ζ-values because the expectation of ζ is zero. ζ-values close to zero, that is, close to the θ-axis, indicate that such item response patterns are frequently observed, which in turn means the corresponding knowledge states are observed for many students. If the ζ-value associated with a knowledge state is large, then such a state is unusual. The larger a ζ-value is, the more unusual is the state associated with this value. A similar argument holds for negative ζ values.

By locating the position of a knowledge state of interest in rule space, one can make two inferences: (1) What IRT-ability level is likely to produce this particular state, and (2) what percentage of students in a sample will be classified into this state. If some sources of error (or the lack of specific knowledge) are shared by many students, then the states involving such errors will be found closer to θ-axis (Tatsuoka, 1990).

Bug distribution The performance on the test items is not always perfectly consistent with the erroneous rule or "bug" (denoted by **R**) used most frequently by a student, and the responses that deviate from the modal rule

are called slips or random errors. The item response patterns deviating by various numbers of slips from a pure rule pattern R will form a cluster around the pure rule pattern. We assume that occurrences of slips on items are independent across the items. Tatsuoka and Tatsuoka (1987) showed that the distribution of the number of slips is a compound binomial distribution if the slippage probabilities of the items are different, and a binomial distribution if the slippage probabilities are the same across items.

Let us map all the "fuzzy" response patterns in the cluster around R into rule space, by computing their θ and ζ values. Then the images of the "fuzzy" response patterns form a subset that swarms around the point (θ_R, ζ_R) which corresponds uniquely to R. The swarm of mapped "fuzzy" points in the rule space follows approximately a multivariate normal distribution with centroid (θ_R, ζ_R) (Tatsuoka, 1990), which is called the bug distribution or state distribution associated with R.

When two sets of "fuzzy" response patterns are mapped into the rule space, one can apply Bayes' decision rule for minimum errors to classify a student's point (θ_x, ζ_x) into one of the states.

In summary, rule-space model is a probabilistic model for cognitive diagnosis and the model is applicable to any domain of interest at very low cost. It is a general and cost-effective method for cognitive diagnosis when the remediation of errors is our concern.

<u>Error-Diagnostic Adaptive Testing System</u>

Utilizing the rule-space model described above, Tatsuoka, Baillie, Tatsuoka (1986) have developed an adaptive testing system by which a students' sources of misconception (which produce bugs or erroneous rules of operation) can be diagnosed. A computer program that sequentially selects an optimal set

of items to administer to each individual, and then terminates the testing as soon as a specified stopping rule is satisfied, has been written on the PLATO system. The item-sequence selection strategy is an extension of the method commonly used in IRT-based adaptive testing procedures. The new system allows us to attain a specified level of accuracy in estimating θ and ζ most rapidly. The procedure is an application of the theory of convergence in functional space, the space of item response functions in this case.

Before proceeding to a description of our diagnostic testing system, we must explain what θ and ζ are. The first is the standard person parameter in item response theory (IRT), commonly characterized as the latent ability of an individual in the domain for which he/she is being tested. However, it can just as well be construed as the individual's achievement level in that domain at the time of testing; it is this interpretation (or definition) of θ that is more appropriate for our purposes here. Either way, IRT postulates that the probability of an individual's answering a given item correctly is a joint function of θ and one, two or three item parameters. The latter characterize the discrimination power ($\underline{a}$), the difficulty level ($\underline{b}$) and the "guessability" ($\underline{c}$) of the item--i.e. the chances that an examinee with absolutely no ability or prior achievement in the domain will answer the item correctly. The particular functional relation between the probability of correct response and θ, $\underline{a}$, $\underline{b}$, and $\underline{c}$ may in principle be chosen at will by the researcher-- with some reasonable constraints such as its being a monotone increasing function of θ for fixed $\underline{a}$, $\underline{b}$, and $\underline{c}$. In practice, however, only two functional forms are widely used. These are the logistic function and the normal ogive. Here we use the logistic model and confine ourselves to the case of $c=0$, which is appropriate when we are dealing with open-ended (or "constructed-response")

items as against multiple-choice items. The two-parameter logistic model has the following form:

$$P_j(X_j=1|\theta) = 1/[1 + \exp(-1.7a_j(\theta - b_j))] \tag{1}$$

where x_j is the binary score for item j, so that

$$x_j = \begin{cases} 1 \text{ when item j is answered correctly;} \\ 0 \text{ otherwise} \end{cases}$$

For short, the left-hand side of Equation (1) is often written as $P_j\ (\theta)$, as we do below.

We now define the true score $T(\theta)$ as the average of the $P_j(\theta)$ over the n items:

$$T(\theta) = 1/n\sum P_j(\theta) \tag{2}$$

Using this quantity as the repeated element of an n-dimensional vector, we obtain

$$\mathbf{T}(\theta) = [T(\theta), T(\theta), \ldots, T(\theta)]'$$

and hence the residual (or deviation score) vector

$$\mathbf{P}(\theta)-\mathbf{T}(\theta) = [P_1(\theta)-T(\theta),\ P_2(\theta)-T(\theta), \ldots, P_n(\theta)-T(\theta)]'$$

Similarly, we define the residual vector of $\mathbf{P}(\theta)$ from the binary-score vector $\mathbf{x}$:

$$\mathbf{P}(\theta)-\mathbf{x} = [P_1(\theta)-x_1,\ P_2(\theta)-x, \ldots,\ P_n(\theta)-x_n]'$$

We then form the scalar product of these two residual vectors, thus:

$$\begin{aligned} f_\theta(\mathbf{x}) &= [\mathbf{P}(\theta)-\mathbf{T}(\theta)]'[\mathbf{P}(\theta)-\mathbf{x}] \\ &= \sum [P_j(\theta)-T(\theta)][P_j(\theta)-x_j] \end{aligned} \tag{3}$$

To see how it functions, we distribute the multiplication over the second factor to get

$$f_\theta(\mathbf{x}) = [\mathbf{P}(\theta)-\mathbf{T}(\theta)]'\mathbf{P}(\theta)-[\mathbf{P}(\theta)-\mathbf{T}(\theta)]'\mathbf{x} \tag{4}$$

For fixed θ, the first term is a constant. Let us see how $f_\theta(\mathbf{x})$ varies with $\mathbf{x}$

due to the second term. Rewriting this term as

$$- \sum [P_j(\theta)-T(\theta)]x_j$$

helps us make the following observations: Without loss of generality, we may suppose the items to be arranged in descending order of magnitude of $P_j(\theta)$. Then, for some m, the first m terms of the summation will have positive coefficients associated with the x_j, and the remaining n-m terms will have negative coefficients. Thus, to the extent that there is a preponderance of 1s among the scores on the first m items $(x_1,x_2,\ldots,x_m)$ and a preponderance of 0s among the last n-m item scores, the sum (exclusive of the minus sign) will have a large value, and hence $f_\theta(\mathbf{x})$ will be small. Conversely, when there are many 0s among the earlier items, $f_\theta(\mathbf{x})$ will be large. Since the $P_j(\theta)$ values are in descending order of magnitude $[P_1(\theta) > P_2(\theta) > \ldots > P_{n-1}(\theta) > P_n(\theta)]$, a response pattern having many 1s among the earlier items and many 0s among the latter ones may be regarded as a "normal" or "typical" response pattern for the group in which the a_j and b_j values were estimated (i.e., in which the items were calibrated). On the other hand, response patterns having many 0s among the earlier items and many 1s among the later items would be "atypical" ones. Thus typical response patterns are associated with small value $f_\theta(\mathbf{x})$ while atypical (i.e., unusual) response patterns get larger $f_\theta(\mathbf{x})$ values. Hence, $f_\theta(\mathbf{x})$ may be taken as a measure of atypicality of response patterns--the larger $f_\theta(\mathbf{x})$ is, the more atypical the response pattern is.

The function $f_\theta(\mathbf{x})$ described above suffices to serve as a measure of atypicality of response patterns only for the population in which the items were calibrated. To measure the atypicality of response patterns observed for examinees who are not members of the calibration population, we need to standardize $f_\theta(\mathbf{x})$. It was shown by Tatsuoka (1985) that the expectation and

variance of $f_\theta(\mathbf{x})$ for fixed θ are

$$E[f_\theta(\mathbf{x})] = 0 \tag{5}$$

and

$$\mathrm{Var}[f_\theta(\mathbf{x})] = \sum P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2 \tag{6}$$

where $Q_j(\theta) = 1- P_j(\theta)$. Thus the standardized $f_\theta(\mathbf{x})$, denoted ζ, is given by

$$\zeta = f(\theta)/(\mathrm{var}[f_\theta(\mathbf{x})])^{1/2} \tag{7}$$

This constitutes the ordinate of rule space, the abscissa being the estimated value of θ. Thus, each point in rule space, which represents a particular response pattern (or an individual who has that response pattern) is associated with a coordinate pair (θ, ζ). The error-diagnostic adaptive testing procedure may be regarded as a random walk in this space, whose path is determined by the changing values of the vector (θ, ζ) as successive items are selectively administered to each examinee in accordance with a certain selection rule.

The item-selection rule may take several forms, one of which may be described as follows. Suppose that, if an examinee under consideration were to take the entire test consisting of n items, he/she would be diagnosed as having a misconception that places him/her in misconception group G in accordance with the classification rule described in the previous section. Then, it stands to reason that each new item to be administered should be chosen in such a way that the examinee would be hurried toward his/her ultimate destination as rapidly as possible. This purpose will be served--even though the final destination is unknown--if successive items are chosen so that the individual's response pattern is located as far as possible in rule space from its location at the time the previous item was taken. This is somewhat analogous to the method of steepest descent often used in certain

types of optimization problems.

Once an examinee's response-pattern point stabilizes on (or converges to) some point in rule space, we take that to be his/her "true point", and no further items are administered. The next step is to determine the misconception group to which that point most probably belongs. This, of course, is a problem of statistical classification theory as described earlier. Thus, the problem of error diagnosis is translated into one of statistical classification using a suitable model. The model often chosen for mathematical convenience is the normal model--i.e., that θ and ζ jointly follow a bivariate normal distribution with a specifiable centroid and covariance matrix specific to each misconception group.

Since θ and $f(\mathbf{x})$ are uncorrelated, the covariance matrix becomes diagonal with $1/I(\theta)$ and the variance of $f(\mathbf{x})$ as the diagonal elements (Tatsuoka, 1985; 1990). The classification procedure is described in Tatsuoka & Tatsuoka (1987).

Method

A Task Analysis and Cognitive Attributes

A task analysis that was conducted in the domain of fraction addition problems identified 15 basic cognitive tasks for carrying out the addition of two fractions of which there are three types: Addition of two simple fractions, F + F; addition of two mixed numbers, M + M; and addition of F + M or M + F. These tasks are closely associated with types of items in which three positive integers are used to form either a mixed or fraction number. The 15 basic cognitive tasks are listed in Table 1.

Insert Table 1 about here

However, these tasks are used for generating exercise problems in instructional units for remediation and, further, their mastery patterns obtained from diagnostic classification are used for routing an examinee to his/her remediation unit(s). Since these tasks are oriented toward generating various types of items and focused on a finer, micro-level analysis, it is difficult to see a close connection to the cognitive attributes discussed in the previous studies (Tatsuoka, 1984, 1986; Birenbaum & Shaw, 1986). In order to clarify the continuity of these 15 basic tasks with the attributes discussed in our other papers, they are grouped into eight categories: {Task 1 or Task 2 }, {Tasks 1 and 2}, {Task 3 or 4 or 5 or 6}, {Tasks 7 and/or 8}, {Tasks 9 and/or 10}, {Tasks 11 and/or 12 and/or 13}, {Task 14} and {Task 15}. Let us denote these new categories as cognitive attributes A_1, through A_8. The eight attributes characterize 38 items with attribute involvement vectors. Table 2 describes the items by various combinations of attributes. The 38 addition problems

Insert Table 2 about here

are also listed in Appendix I and in Appendix II with their parameter values in the two-parameter logistic model.

Adaptive Testing System for Cognitive Diagnoses in Fraction Addition Problems

An adaptive diagnostic testing system for fraction addition problems was developed on a computerized instructional system at the University of Illinois

(PLATO system). The system is designed to classify examinees' responses into one of the predetermined misconception groups (called states of knowledge and capabilities). For fraction problems, the misconception groups were determined by reviewing the previous studies (Tatsuoka, 1984; Klein, Birenbaum, Standiford & Tatsuoka, 1981; Shaw & Tatsuoka, 1982). Tatsuoka (1984) grouped erroneous rules into several basic categories by examining where each erroneous rule originated. For example, if erroneous rules are due to lack of knowledge for making equivalent fractions, then they are labeled "errors in making equivalent fractions".

The 15 prime cognitive skills were first defined as stated earlier and 33 knowledge states that consist of various combinations of the prime cognitive skills were selected based on the frequency statistics of erroneous rules. These 33 states constituted the "bug bank" in our adaptive cognitive-diagnostic testing. Paper-and-pencil tests that were given in the previous years were analyzed for students' errors and approximately 90 percent of the examinees (N=593) were classified into one of the states (Tatsuoka, 1986).

The 10 most popular knowledge states are:

No.4 Cannot get the common denominator (CD) but can do simple fraction addition problems.

No.6 Cannot get CDs for the problems involving mixed number(s).

No.9 Have problems in simplifying their answers into the simplest form.

No.10 Mastery state: all cognitive attributes are correct.

No.11 Can do only addition of two simple fractions (F) when they have the same denominators.

No.16 Cannot get CDs and cannot add two reducible mixed numbers (M). Also having problems with simplification of the answers.

No.21 Non-mastery state: all attributes are wrong.

No.24 Cannot add a mixed number and a fraction number. Cannot get CDs. Don't reduce fraction parts correctly before getting the common denominators.

No.25 Cannot add the combinations of M and F numbers. Also cannot get CDs.

No.26 Don't realize that adding zero to a non-zero number a is a itself, $a + 0 = a$; Identity Principle.

Table 3 describes the ten most popular knowledge states by nine attribute-mastery patterns. Appendix III summarizes 33 states with respect to the patterns of items grouped by their underlying attribute patterns.

Insert Table 3 about here

Knowledge state No. 10 is the mastery state; that is, all answers are correct while No 21 represents the state in which all answers are wrong.

Remediation Instruction and a Routing Method based on Probabilities

Fourteen basic skills were defined and their instructional units were written on the same computerized instructional system as that on which tests were prepared. For example, if an examinee is classified into State No. 25, then an automated adaptive routing system sends the examinee to the units A_3,-- teaching what a common denominator is and how to get it- and A_1, -reminding the student that F+M type can be separated into 0 and the whole number part of the second number and 0+d=d because students often overlook this identity principle. The following figure shows an example of our remediation unit.

Insert Figure 1 about here

The first box shows a wrong answer by a student and starts teaching each step to reach the correct answer. If the student cannot get the least common multiple of 8 and 10 correctly, then a specific feedback based on the particular answer will appear on the screen. This example shows that the student selected a common denominator but did not choose the least common denominator. Therefore an instruction that teaches multiples of 8 and 10 appears on the screen for the student. The box below in Figure 1 indicates that the student selected the right answer, 40 and got the feedback of "correct!". Then the computation screen goes to the next step, making equivalent fractions of 1/8 and 7/10.

The top of the remediation instruction in Figure 1 shows the routing index of 14 basic units. Type 11 means item type 11, which is characterized by the attributes A_1, A_2, A_3 and mixed number addition problems with different denominators. Any examinee who is classified into one of the cognitive states which includes "cannot do A_1" as a subset would be routed to study a series of instructions indexed by this label.

However, there are several states to which the examinees diagnosed "cannot do A_1, A_2 and A_3" belong. Since Boolean algebra defines a partial order among states derived from a given incidence matrix, the relationships among the cognitive states can always be expressed by a tree such as the example given in Figure 2.

Insert Figure 2 about here

The mastery state means Group 10, all attributes are mastered. The states, "cannot do A_4" and "cannot do A_9" mean, respectively, that they cannot do A_4 but can do the remaining attributes A_1, A_2, A_3, A_5, A_6, A_7, A_8 and A_9 and that they can do A_1 through A_8 only.

Suppose an examinee was classified into the cognitive state "cannot do A_3, A_4, A_5 and A_9" which is shown at the bottom of the tree given in Figure 2. The issue that arises here is that whether A_3 should be remediated first and then A_5 , or A_5 first and then A_3.

Tatsuoka and Tatsuoka (1987) introduced what they called a bug distribution, which is a multivariate normal distribution with centroid (θ_R, ζ_R) and covariance matrix,

$$\begin{bmatrix} 1/I(\theta) & 0 \\ 0 & 1 \end{bmatrix}. \qquad (8)$$

Bayes' decision rule to determine whether an examinee's point **X** in the rule space belongs to State $\underline{A}$ or to $\underline{B}$ is equivalent to comparing the Mahalanobis distance between **X** and A versus that between **X** and B. Mahalanobis distance in the rule space context is the same as considering the negative loglikelihood ratio of the two posterior probabilities of $\underline{A}$ and $\underline{B}$ given $\underline{\mathbf{X}}$ (Lachenbruch 1975),

$$-\ln\left(\text{Prob}(A \mid \mathbf{X})/\text{Prob}(B \mid \mathbf{X})\right) \qquad (9)$$

By taking the position of State "cannot do A_3, A_4, A_5 and A_9" as **X** and computing the two Mahalanobis distances, that between **X** and the State "cannot do A_3, A_4 and A_9", would provide a plausible rule for a computerized routing

system. However, in this study, the order for selecting an attribute from two connected states comes from the flowchart that was constructed earlier for a task analysis (Klein et al., 1982; Birenbaum & Shaw, 1986). Attributes A_3 (getting the least common denominator) and A_5 (reducible before getting the common denominator) can be located in the flowchart in Figure 3.

Insert Figure 3 about here

It is now obvious that A_5 should be remediated before A_3.

As for the Mahalanobis distances, that between States "cannot do A_3, A_4, A_5 and A_9" and "cannot do A_4 A_5, A_9" is larger than that between States "cannot do A_3, A_4, A_5 and A_9" and "A_3, A_4, and A_9".

Computational comparison of these relationships between the order from the flowchart and the order based on Mahalanobis distances in rule space confirmed that pairs of attributes closer on the flowchart have smaller Mahalanobis distances than do pairs of attributes farther apart.

The result suggests that a potentially good routing criterion for remediation is to compare the negative loglikelihood ratio of the posterior probabilities of two targeted states versus that of the state into which an examinee is currently classified.

<u>Data collection: Pretest, Remediation and Posttest in 1988 and 1989</u>

Three fraction diagnostic tests, pre- and post-tests and a retention test were given in 1988 to students in the seventh and eighth grades of a junior high school in a small city in Illinois. The pretest classified each student into one of the 33 states. Since each state is expressed as the mastery and non-mastery of given cognitive attributes, the examinee was

assigned to the instructional units which teach the examinee his/her non-mastered skills. When examinees were classified into No. 10, the mastery state, they did not participate in the remediation and posttest parts but they took the retention test three months later.

The examinee was given a series of exercises at the end of each remediation unit. After correctly doing the exercises at all the remediation units which he/she had to complete, the examinee was given the posttest, which was also adaptive, and a cognitive diagnosis was carried out. The posttest also used the same "bug bank" consisting of 33 states.

Insert Figure 4 about here

Three months later, the retention test was administered to the examinees who took the pretest and posttest.

Figure 4 shows an example of an adaptive test that was administered to a real examinee, whose final classification was in State 6. The same test was given to the examinees three months later, and the retention of examinees' states was examined.

In the next year of 1989, the same study was replicated with 191 students.

Results

Table 4 summarizes the results of the six tests given in 1988 and 1989.

Insert Table 4 about here

The two independent studies in 1988 and 1989 show a considerable resemblance in the classification results. In 1988, 57 examinees achieved the mastery state, while 39 failed in all the cognitive attributes and ended up in the non-mastery state (No. 21). In 1989, 34 were classified in the mastery state and 13 were in the non-mastery state.

State No. 26 is the most popular knowledge state after No. 10 throughout the six tests. The examinees in No. 26 are also very high achievers and their errors are "cannot do F + M type but can do all other cognitive attributes required in the other types of problems". Their erroneous rules are often "append 1 to F type and proceed with all the computations correctly" (2/5 + 4 3/7 becomes 1 2/5 + 4 3/7) or " omitting the whole number part in the answer" ({0 + 4} + {2/5 + 3/7} = 29/35). The frequencies of such bugs were reported in Tatsuoka (1984) and Shaw and Tatsuoka (1982).

Insert Table 5, 6 and 7 about here

However, Table 4 does not show how the examinees improved after the remediation lessons were given. The transition of examinees' knowledge states before and after remediation are summarized in Table 5 for the transition between pre-and post-tests, in Table 6 for that between posttest and retention tests, and in Table 7 for that between pretest and retention test. The states are listed in descending order of the number of tasks mastered, starting from state No. 26 (13 skills mastered) down to state No. 21 (none of the tasks mastered).

Table 5 shows that twenty-five examinees moved from No. 26 to No. 10, (all tasks mastered). Eight from No. 25 and No. 16 moved to No. 10. Further,

11 from No. 6, 7 from No. 33 and No. 21 moved to No. 10. In the posttest, 89 examinees (39% of the 226 students who took the posttest) were classified into No. 10. Fifteen examinees stayed in No. 26 on the posttest. However, four examinees who were in No. 26 moved back to lower-level states. In all, 16 (7% of N = 226) moved to lower-level states, but a majority of the examinees (93%) moved to higher-level states. Similar trends were found in the replication study in 1989.

These changes are graphically expressed in Figure 5. The points, 21, 16, 25, 24, 33, 6, 9, 26 and 10 are the centroids of distributions associated with these states. The arrows indicate that examinees' changes in their states between the pre-and posttests are as indicated in Table 10 and 11. The locations of 33 states in the rule space are listed in Appendix IV.

Insert Figure 5 about here

Insert Tables 8 and 9 about here

As for changes from the posttest to the retention test, quite a few examinees moved back to their pretest states. Twenty-five examinees maintained their mastery states in the retention test, while 23 did not take the retention test. Forty-one examinees (48%) regressed toward lower-level states from their posttest state, No. 10. The examinees who were classified in No. 26, retained their skill level better than did those in No. 10; 43% stayed in either No. 10 or No. 26 while 34% moved back to lower-level states. Overall, 48% of the examinees (N = 185) regressed toward lower-level states

between the posttest and retention tests. Twenty-one percent of the examinees maintained their posttest states, and 31% moved up to higher-level states than the posttest states.

However, changes between the pretest and the retention test are encouraging. (see Table 7). Fifty-five examinees moved from various states to the mastery state, No. 10. Thirty moved to No. 26, and 25 moved to No. 6. If we add up these numbers, 110 out of 185 examinees (59%) were classified into either No. 10, 26 or 6 which are near-mastery states. However, about 6% of examinees regressed toward lower-level states from the pretest state to the retention state.

Since quite a few examinees dropped out of our experiment before taking the retention test, analysis of transition states lacks in statistical power. Therefore, the 33 states were grouped into two categories: those with serious vs. non-serious error types. Of the states that have mastery tasks, more than eight were categorized as non-serious error states, and the remaining states were categorized as having serious errors. Table 10 summarizes the categorization of the states into which at least one examinee was classified.

Insert Table 10 about here

Nine states were categorized in the non-serious-error group while 17 were classified into the serious-error group.

Insert Tables 11 and 12

Tables 11 and 12 show 2x2 contingency tables of serious vs. non-serious

error groups for the pre- and post-test in the 1988 and 1989 data. Table 13 is the corresponding table for pre-test and retention test in 1988. Tables 11 and 12 indicate that diagnoses using the rule-space model are very efficient. Indeed, we carefully designed our remediation instruction so that if cognitive diagnosis by the rule-space model is not correct, then remediation should not work well.

Therefore we consider the results shown in Tables 11 and 12 to be a strong indication of the reliability of rule-space diagnoses.

Insert Tables 13

However, Table 13 shows that 38 examinees moved from non-serious to serious while 4 moved from serious to non-serious error groups. The number of examinees who remained in non-serious error groups was 110, which is 68% of the examinees who participated in the retention test.

Proportion Correct for Cognitive Attributes

A unique feature of the rule-space model for cognitive diagnosis is that item-response patterns for examinees who are successfully classified can be converted into attribute-mastery patterns, which are usually unobservable. This means that one can obtain the p-values (proportions correct) for cognitive attributes as well as for items. If classification rates are high, say 90%, then the p-values can provide researchers with valuable information about the underlying cognitive models.

Figure 6 shows the proportion correct scores for eight

Insert Figure 6 about here

cognitive attributes, for the pretest (N - 287), for the posttest (N - 225) and for the retention test (N - 185) in 1988. The three line graphs indicate that the proportions correct over eight attributes for the retention test maintain values higher than the pretest values and lower than the posttest values. Since the examinees classified in the mastery state (No. 10) in the pretest (N - 58) were not routed to study the remediation instruction, they took neither the posttest nor the retention test. Hence, the given proportion correct scores for 8 attributes in Figure 5 do not reflect the same total sample as that of the pretest, and they could be much higher than the current values if these mastery examinees would have studied the remediation instruction and took both the posttest and retention test.

Time Required for Completing Remediation Instruction

The average times, in minutes, for students in various states to complete the remediation treatment are summarized in Table 14.

Insert Table 14 about here

The overall average time for completing required remediation units is 37.21 minutes across the knowledge states listed in Table 14. 49.8% of the examinees reached mastery or near-mastery while 5% of the examinees remained in the serious-error category.

Conclusion and Summary

Our motivation for this study was to validate the results of cognitive diagnoses by the rule-space model and to demonstrate the usefulness of cognitive diagnoses for instruction. This empirical validation study strongly indicates that the rule-space model can diagnose students' knowledge states effectively and remediate the students' errors quickly with minimum effort. The lack of knowledge in particular domains can also be diagnosed, and the knowledge can be supplied by instruction.

Designing instructional units for remediation can be effectively navigated by the rule-space model because the determination of all the possible ideal item-score patterns (universal set of knowledge states) given an incidence matrix is based on a tree structure of cognitive attributes, knowledge states and items (Tatsuoka, 1990). Remediation should start at the states whose probability of mastery for diagnosed deficiency of skills is as high as possible.

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Table 1

Fifteen Basic Cognitive Tasks required in the Fraction Addition Problems

Tasks	Description of Cognitive Tasks
1.	Separate the whole number*, a from the fraction part*, b/c.
2.	Separate the whole number part, d from the fraction part, e/f.
3.	Get the common denominator CD, CD = c x f.
4.	Get the common denominator CD when c is a multiply of f.
5.	Get the common denominator CD when f is a multiply of c.
7.	Get the common denominator CD when f and c are factors of CD.
8.	Convert b/c before getting CD, b>c.
9.	Convert e/f before getting CD, e>f.
10.	Reduce b/c before getting CD.
11.	Reduce e/f before getting CD.
12.	Answer to be simplified, reduce the fraction part.
13.	Answer to be simplified, convert the fraction part to a mixed number.
14.	Add a whole number to the whole number part of the answer after converting the original fraction part.
15.	Add two numerators.
*	Item a(b/c) + d(e/f) ; F = a simple fraction, M is a mixed number such as 5 3/5

Table 2

Description of Items by Various Combinations of Attributes

Attributes	items 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$a \neq 0$ or $d \neq 0$	1	0	0	1	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0
$a \neq 0$ and $d \neq 0$	1	0	0	1	1	1	0	0	1	0	0	1	1	0	0	1	0	0	0
$c \neq f$	0	1	0	1	1	0	0	1	0	0	1	0	1	0	1	0	0	1	0
convert before getting CD	1	1	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
reduce before getting CD	1	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
answer to be simplified	1	1	1	1	1	1	1	0	1	0	1	1	0	0	0	0	0	0	0
add two numeratos	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
adjust a whole number part	1	0	0	1	1	1	0	0	1	0	0	0	0	0	0	1	0	0	0
Mixed numbers with $c \neq f$	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0

*Items 20 through 38 are parallel to Items 1 through 19.

1. a is zero if the first term is a fraction.
2. d is zero if the second term is a fraction.
3. $c=f$ if two numbers have the same denominator.
4. If $e>f$ or/and $b>f$ then the number(s) can be converted before addition.
5. If $(e,f) \neq 1$ or/and $(b,c) \neq 1$, the numerators can be reduced before addition.
6. Answer can be simplified by converting or/and reducing.
7. Add two numeratos.
8. When a mixed number answer is simplified by converting the fraction part, the whole number part must be adjusted.

Table 3

Description of the top ten, the most popular Knowledge States among the thirty three states in the "bug list"

	Attribute Mastery pattern								
States	1	2	3	4	5	6	7	8	9
21	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
9	1	1	1	0	1	0	1	0	1
16	1	1	0	0	0	0	1	0	1
24	1	1	0	1	0	1	1	1	1
25	0	1	0	1	1	1	1	0	1
26	0	1	1	1	1	1	1	1	1
33	1	0	0	0	0	1	1	1	1
4	1	1	0	1	1	1	1	1	1
11	0	0	0	1	0	1	1	1	1

Table 4

Frequencies of ^Students Classified in Each Class

Groups	1988 Data			1989 Data		
	Pretest N=287	Posttest N=226	Retention N=185	Pretest N=191	Posttest N=175	Retention N=171
1	3	0	1	1	0	1
2	2	0	0	1	0	0
3	1	1	0	1	2	0
4	3	2	7	4	2	7
5	3	0	0	1	0	1
6	26	25	25	20	16	16
7	5	5	5	6	6	3
8	0	0	1	0	1	0
9	12	13	5	10	14	8
10	57	89	55	34	69	51
11	15	2	6	12	2	2
12	1	0	0	1	0	0
13	4	0	1	3	2	1
15	0	0	2	0	0	0
16	25	7	10	17	3	13
17	0	0	1	0	1	0
18	1	2	1	1	0	3
19	4	3	0	5	1	0
20	1	0	0	0	0	0
21	39	1	2	13	1	7
23	1	1	6	5	1	7
24	9	1	6	9	2	10
25	12	6	11	7	4	6
26	45	44	30	27	33	24

(*table continues*)

Groups	1988 Data Pretest N=287	1988 Data Posttest N=226	1988 Data Retention N=185	1989 Data Pretest N=191	1989 Data Posttest N=175	1989 Data Retention N=171
27	0	1	0	0	1	0
28	1	0	0	3	0	0
30	0	1	0	0	1	0
31	6	9	4	3	5	6
32	2	8	2	0	5	1
33	9	4	4	7	3	4

Table 5

Transition Frequencies of Students who were classified in the groups for Pre-and-Post Tests, 1988 Data

Test 2	# of Tasks	14	13	11	11	10	10	9	8	8	8	8	7	7	6	6	4	3	3	3	0	0	
Test 1	Post Group	10	26	6	7	9	19	33	4	18	32	31	25	3	16	24	30	23	11	27	21	0	#
# of Tasks	Pre Group																						
13	26	25	15	1		2								1								1	45
11	6	11	7	4		2		1				1											26
11	7		1	1							1	2											5
10	9	3	4	2	1	2																	12
10	19	3						1															4
9	33	7		2																			9
8	4	1											1							1			3
8	18	1																					1
8	32	1		1																			2
8	31	3		1		1						1											6
7	25	8	1	1								1	1										12
7	3	1																					1
7	5	1		1											1								3
6	16	8	2	4	1	1	1	1			2	1			1		1					2	25
6	24	3		2				1	1	1					1								9
5	2		1			1																	2
4	1	1		1							1												3
4	12					1																	1
3	11	3	3	1	2	2	1		1		1					1							15

(table continues)

Test 2	# of Tasks	14	13	11	11	10	10	9	8	8	8	8	7	7	6	6	4	3	3	3	0	0	
Test 1	Post Group	10	26	6	7	9	19	33	4	18	32	31	25	3	16	24	30	23	11	27	21	0	#
# of Tasks	Pre Group																						
3	23		1																				1
3	13	1		1							1								1				4
3	28		1																				1
1	20	1																					1
0	21	7	8	2	1	1	1		1	2	3	4			4			1	1		1	2	39
		89	44	25	5	13	3	4	2	2	8	9	6	1	7	1	1	1	2	1	1	5	230

Table 6

Transition Frequencies of Students who were classified in the groups for Post and Retention Tests, 1988 Data

Test 3	# of Tasks	14	13	11	11	10	10	9	8	8	8	8	7	6	6	5	5	4	3	3	3	0	?	
Test 2	Post Post	10	26	6	7	9	8	33	4	18	32	31	25	16	24	15	17	1	11	23	13	21	0	#
# of Tasks	Post Group																							
14	10	25	14	9	4	1			2			1	1	3	1		1		2	1		1	23	89
13	26	12	7	7		1					1		2	1						2	1		10	44
11	6	3	5	3				2				2	1	1					1				7	25
11	7	1												1						1			2	5
10	9	2	1	1			1		1						1	1				1			4	13
10	19												2										1	3
9	33		1	1															1				1	4
8	4	1																		1				2
8	18								1					1										2
8	32	1											1	2	2								2	8
8	31			2							1	1	2		1								2	9
7	25								2				1										3	6
7	3		1																					1
6	16				1					1				1				1	1				2	7
6	24														1									1
4	30																		1					1
3	23								1															1
3	11						1														1			2
3	27											1												1

(table continues)

Test 3	# of Tasks	14	13	11	11	10	10	9	8	8	8	8	7	6	6	5	5	4	3	3	3	0	?	
Test 2	Post Post	10	26	6	7	9	8	33	4	18	32	31	25	16	24	15	17	1	11	23	13	21	0	#
# of Tasks	Post Group																							
0	21																						1	1
?	0	10	1	2		3		1								1							44	62
		55	30	25	5	5	1	4	7	1	2	4	11	10	6	2	1	1	6	6	1	2	102	287

Table 7

Transition Frequencies of Students who were classified in the groups for Pre-and Retention Tests, 1988 Data

Test 3	# of Tasks	14	13	11	11	10	10	9	8	8	8	8	7	6	6	5	5	4	3	3	3	0	0	
Test 1	Post Post Group	10	26	6	7	9	8	33	4	18	32	31	25	16	24	15	17	1	11	23	13	21	0	
# of Tasks	Pre Group																							
14	10	10	1	2		3		1								1							39*	57
13	26	16	12	2		2								1									12	45
11	6	6	3	6			1	2			1	1											6	26
11	7		1	1								1			1								1	5
10	9	4	1	1																			6	12
10	19	1	1		1													1						4
9	33	5		2																			2	9
8	4							1				1											1	3
8	18																	1						1
8	32																						2	2
8	31	2	1												1								2	6
7	25	1	1	1	1					1									1				6	12
7	3	1																						1
7	5				1						1								1					3
6	16	3	2	1	1			1			1	1	2	1		1		2					9	25
6	24		1	1				1				1							1				4	9
5	2								1			1							1					2
4	1		1											1									1	3

(*table continues*)

Test 3	# of Tasks	14	13	11	11	10	10	9	8	8	8	8	7	6	6	5	5	4	3	3	3	0	0	
Test 1	Post Post Group	10	26	6	7	9	8	33	4	18	32	31	25	16	24	15	17	1	11	23	13	21	0	
# of Tasks	Pre Group																							
4	12																						1	1
3	11	2	2	2	1			1		1		1	2	1									2	15
3	23	1																						1
3	13			1			1						1	1										4
3	28																		1					1
1	20	1																						1
0	21	2	3	5				3	1			5	4	1			1	2	1	1	2		8	39
		55	30	25	5	5	1	4	7	1	2	4	11	10	6	2	1	1	6	6	1	2	102	287

*39 students who took Test 1 and wre classified as Group 10 did not take Test 3.

Table 8

Percentage of Transition Frequencies of Students Who are Classified in States 21, 16, 24, 25, 33, 6, 9, 26, & 10.

	Posttest states											
pretest states	10	26	6	9	33	25	16	24	11	21	other states	
10	–	–	–	–	–	–	–	–	–	–	–	
26	57	**33**	2	4	0	0	0	0	0	0	4	100
6	42	27	**15**	8	4	0	0	0	0	0	4	100
9	25	33	17	**17**	0	0	0	0	0	0	8	100
33	78	0	22	0	**0**	0	0	0	0	0	0	100
25	68	8	8	0	0	**8**	0	0	0	0	8	100
16	32	8	16	4	4	0	**4**	0	0	0	32	100
24	33	0	22	0	11	0	11	**0**	0	0	23	100
11	20	20	7	13	0	0	0	7	**0**	0	33	100
21	18	21	5	3	0	0	10	0	3	**3**	37	100

Table 9

Percentage of Transition Frequencies of Students Who are Classified in States 21, 16, 24, 25, 33, 6, 9, 26, & 10.

	Retention test states										
pretest states	10	26	6	9	33	25	16	24	11	21	did not take test
10	**18**	2	4	5	2	0	0	0	0	0	68*
26	36	**27**	4	4	0	0	2	0	0	0	27
6	23	12	**23**	0	8	0	0	0	0	0	23
9	33	8	8	**0**	0	0	0	0	0	0	50
33	56	0	22	0	**0**	0	0	0	0	0	22
25	8	8	8	0	0	**8**	0	0	0	0	50
16	12	8	4	0	0	4	**8**	4	8	0	36
24	0	1	1	0	0	1	0	**0**	0	0	44
11	13	13	13	0	0	7	13	7	**0**	0	13
21	5	8	13	0	0	13	10	3	5	**5**	21

Note. * 39 out of 57 examinees who took the pretest and were classified as No. 10 state did not take the retention test. (287 examinees at Pretest but 102 dropped out of this study without taking the retention test)

Table 10

Two Categories of Error Types: Serious vs. Nonserious

Serious Errors			Nonserious Errors		
Classes		# of mastered tasks	Classes		# of mastered tasks
1	4	8	1	26	13
2	18	8	2	6	11
3	3	7	3	7	11
4	5	7	4	9	10
5	25	7	5	8	10
6	24	6	6	19	10
7	16	6	7	33	9
8	2	5	8	31	8
9	30	4	9	32	8
10	17	4			
11	1	4			
12	27	3			
13	23	3			
14	28	3			
15	11	3			
16	13	3			
17	21	0			

Table 11

2 x 2 Contingency Table of Seriousness of Error Classes and Pre-Posttests, 1988 Data

	Posttest		
Pretest	Serious	Nonserious	Total
Serious	21	93	114 (51%)
Nonserious	1	107	108 (49%)
Total	22	200	222
	(10%)	(90%)	

Table 12

2 x 2 Contingency Table of Seriousness of Error Classes and Pre-Posttests, 1989 Data

	Posttest		
Pretest	Serious	Nonserious	Total
Serious	19	75	94 (55%)
Nonserious	1	77	78 (45%)
Total	20	152	172
	(12%)	(88%)	

Table 13

2 x 2 Contingency Table of Seriousness of Error Classes and Post-Retention Tests, 1988 Data

	Posttest		
Pretest	Serious	Nonserious	Total
Serious	15	4	19 (11%)
Nonserious	38	110	148 (89%)
Total	53	114	167
	(32%)	(68%)	

Table 14

Time required for completing the remediation instructional units.

States	Time in minutes
2	70.3
5	26.0
6	3.0
7	12.0
9	1.2
11	75.8
13	79.7
16	82.1
18	33.0
19	21.0
21	116.4
25	32.5
26	2.9
31	

Appendix I.
38 fraction addition problems

1. $2\frac{8}{6} + 3\frac{10}{6} =$

2. $\frac{2}{5} + \frac{12}{8} =$

3. $\frac{8}{5} + \frac{6}{5} =$

4. $2\frac{1}{2} + 4\frac{2}{4} =$

5. $\frac{1}{2} + 1\frac{10}{7} =$

6. $3\frac{5}{7} + 4\frac{6}{7} =$

7. $\frac{3}{5} + \frac{7}{5} =$

8. $\frac{1}{3} + \frac{1}{2} =$

9. $1\frac{4}{7} + 1\frac{12}{7} =$

10. $\frac{3}{5} + \frac{1}{5} =$

11. $\frac{3}{4} + \frac{1}{2} =$

12. $2\frac{5}{9} + 1\frac{1}{9} =$

13. $3\frac{1}{6} + 2\frac{3}{4} =$

14. $\frac{15}{35} + \frac{10}{35} =$

15. $\frac{1}{2} + \frac{3}{8} =$

16. $1\frac{2}{5} + \frac{3}{5} =$

17. $\frac{1}{4} + \frac{3}{4} =$

18. $\frac{4}{15} + \frac{1}{10} =$

19. $\frac{4}{5} + \frac{3}{5} =$

20. $3\frac{10}{4} + 4\frac{6}{4} =$

21. $\frac{2}{7} + \frac{18}{12} =$

22. $\frac{9}{7} + \frac{11}{7} =$

23. $1\frac{1}{3} + 2\frac{4}{6} =$

24. $\frac{1}{5} + 2\frac{5}{3} =$

25. $3\frac{4}{5} + 5\frac{3}{5} =$

26. $\frac{4}{7} + \frac{5}{4} =$

27. $\frac{1}{5} + \frac{1}{4} =$

28. $1\frac{3}{5} + 1\frac{8}{5} =$

29. $\frac{4}{7} + \frac{1}{7} =$

30. $\frac{5}{6} + \frac{1}{3} =$

31. $3\frac{5}{8} + 1\frac{1}{8} =$

32. $2\frac{1}{8} + 3\frac{5}{6} =$

33. $\frac{16}{36} + \frac{10}{36} =$

34. $\frac{1}{3} + \frac{4}{9} =$

35. $2\frac{5}{7} + \frac{2}{7} =$

36. $\frac{1}{5} + \frac{4}{5} =$

37. $\frac{5}{6} + \frac{1}{8} =$

38. $\frac{6}{7} + \frac{3}{7} =$

Appendix II

Values of Item Parameters (N=595)

item	a values	b values
1	.5961	-.6818
2	1.3826	.3310
3	.9763	-.6910
4	2.0791	.1002
5	1.8043	.4495
6	.7118	-.7733
7	1.2399	-.5675
8	2.5406	-.0420
9	.7030	-.4912
10	1.2994	-1.1418
11	2.4550	.0477
12	.8223	-.9559
13	2.7216	.2354
14	.8753	-.9561
15	3.4782	.0881
16	1.3121	-.8332
17	1.1248	-1.2200
18	2.2354	.2188
19	1.1463	-.9001
20	1.0034	-.5024
21	1.8327	.8211
22	1.1226	-.5687
23	2.6170	.1793
24	2.7318	.4305
25	1.1263	-.4051
26	1.2935	-.6279
27	5.5522	.1568
28	1.1126	-.4178

(appendix continues)

item	a values	b values
29	1.4587	-1.0376
30	3.2306	.1782
31	1.1067	-.6542
32	3.4223	.3703
33	.8436	-.5876
34	4.1896	.1788
35	1.1425	-.6935
36	1.1988	-.9417
37	4.1909	.2970
38	.8676	-.8572

Appendix III

Description of States by Basic Types given in Appendix IV

	States													
1	B_1	B_2	B_3	B_6										
2	B_4	B_5	B_7	B_8										
3	B_1	B_2	B_3	B_6	B_9	B_{10}	B_{13}							
4	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8						
5	B_1	B_2	B_3	B_6	B_{11}	B_{12}	B_{14}							
6	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}				
7	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_{11}	B_{14}				
8	B_4	B_5	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}				
9	B_1	B_2	B_3	B_6	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}				
10	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}
11	B_1	B_2	B_3											
12	B_1	B_2	B_3	B_6										
13	B_4	B_5	B_7											
14	B_{11}	B_{14}												
15	B_1	B_2	B_3	B_9	B_{10}									
16	B_1	B_2	B_3	B_4	B_5	B_7								
17	B_1	B_2	B_3	B_1	B_{14}									
18	B_1	B_2	B_3	B_4	B_5	B_7	B_9	B_{10}						
19	B_1	B_2	B_3	B_4	B_5	B_7	B_9	B_{10}	B_{11}	B_{14}				
20	B_9													
21														
22	B_1													
23	B_1	B_2	B_3											
24	B_1	B_2	B_3	B_4	B_5	B_7								
25	B_1	B_2	B_3	B_4	B_6	B_7	B_8							
26	B_1	B_2	B_3	B_4	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{13}	B_{14}		
27	B_1	B_9	B_{10}											

(appendix continues)

	States									
28	B_1	B_8	B_{10}							
29	B_1	B_2	B_9	B_{10}						
30	B_1	B_2	B_3	B_9						
31	B_1	B_2	B_3	B_4	B_5	B_7	B_9	B_{12}		
32	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	
33	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}

APPENDIX IV

Location of Groups in Rule Space, (θ, ζ)

Group	θ	ζ	Standard Error of Estimates
1	-.666	-.738	.210
2	-.984	.906	.233
3	.245	1.953	.097
4	-.168	-3.042	.137
5	-.020	2.876	.112
6	.421	-.296	.114
7	.150	-.028	.096
8	.467	5.221	.122
9	.580	2.147	.144
10	1.818	.507	.778
11	1.184	.165	.249
12	-.833	-.688	.223
13	-1.343	.949	.267
14	-.877	6.117	.226
15	-.042	4.574	.115
16	-.639	-.333	.208
17	-.353	4.430	.171
18	.078	1.739	.100
19	.250	1.806	.097
20	-1.316	3.670	.264
21	-2.979	.597	.863
22	-1.902	.709	.373
23	-.802	-.327	.221
24	-.292	-1.890	.160
25	-.227	-2.354	.148
26	.848	.378	.221
27	-.483	6.187	.191

(appendix continues)

Group	θ	ζ	Standard Error of Estimates
28	-.947	2.858	.231
29	.054	5.428	.102
30	.076	2.332	.100
31	.277	.088	.099
32	.213	-.669	.096
33	.305	-1.022	.101

Figure Captions

Figure 1. Examples of remediation instruction prepared on PLATO system.

Figure 2. A tree representation of nine cognitive states.

Figure 3. A flowchart for solving fraction addition problems.

Figure 4. An example of a student's performance (Student No. 48).

Figure 5. A map showing transition of states in 1988 data.

Figure 6. Percentage of correct scores for eight attributes described in Table 2.

Question type 11 of 14.

$3\frac{1}{8}+4\frac{7}{10}=$ 7 8/40 Your answer is not correct.

Let's do this problem step-by-step.

First, add the whole number parts: 3 + 4 = 7 ok

Are the two denominators the same? n

The denominators are unequal. They are 8 and 10.

Choose the least common denominator of 8 and 10: > 80

80 is a common denominator, but let's use 40

because it's the LEAST common denominator.

The multiples of 8 are: 8 16 24 32 40 48 56 64 72 80

The multiples of 10 are: 10 20 30 40 50 60 70 80

For the denominator, choose the SMALLEST number

that is a multiple of BOTH 8 and 10.

Question type 11 of 14.

$3\frac{1}{8}+4\frac{7}{10}=$ 7 8/40 Your answer is not correct.

Let's do this problem step-by-step.

First, add the whole number parts: 3 + 4 = 7 ok

Are the two denominators the same? n

The denominators are unequal. They are 8 and 10.

Choose the least common denominator of 8 and 10: 40

Correct!

Now let's make equivalent fractions:

$\frac{1}{8} = \frac{5}{40}$ $\frac{7}{10}$ => $\frac{28}{40}$

Figure 1 Examples of remediation instruction prepared on PLATO system

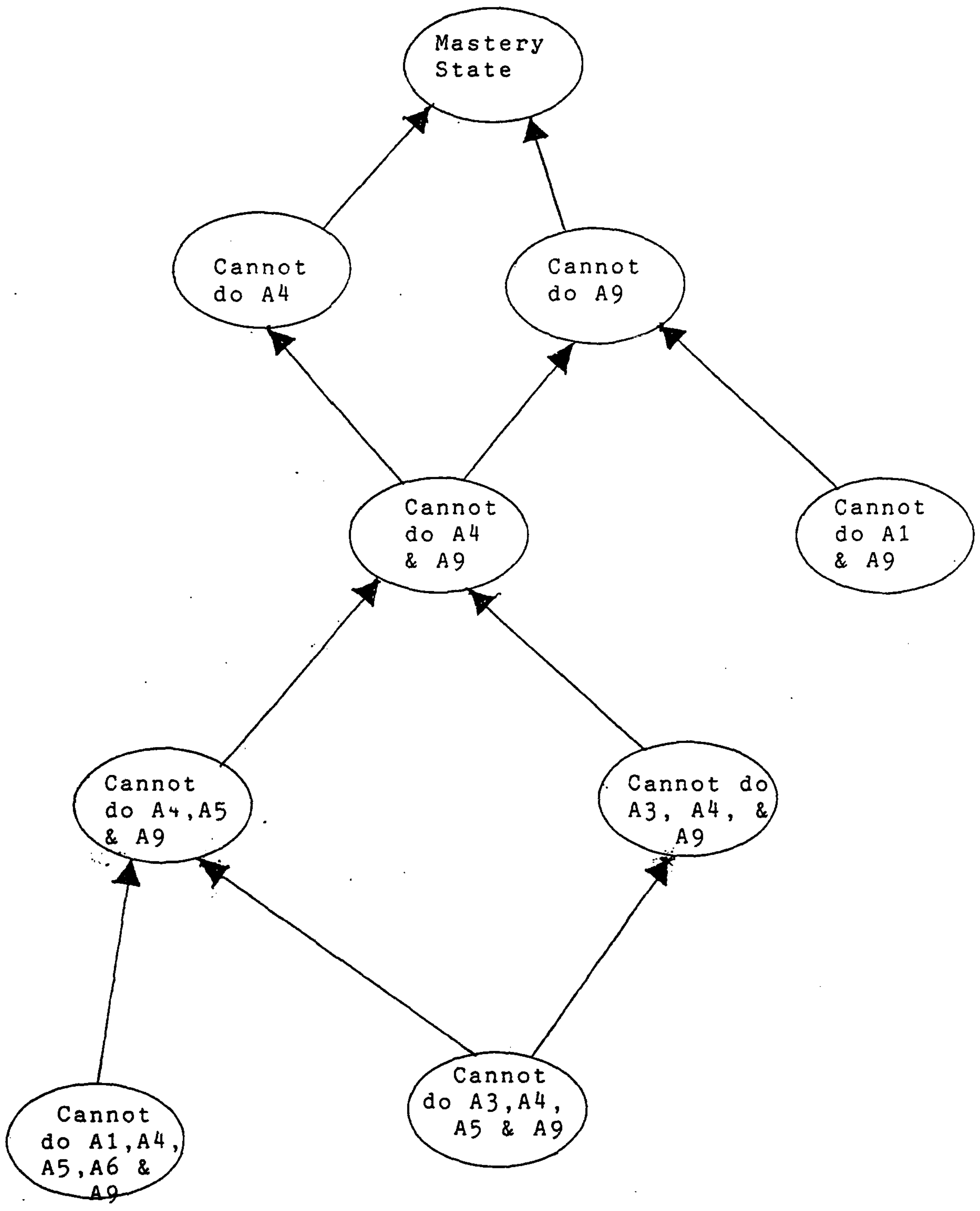
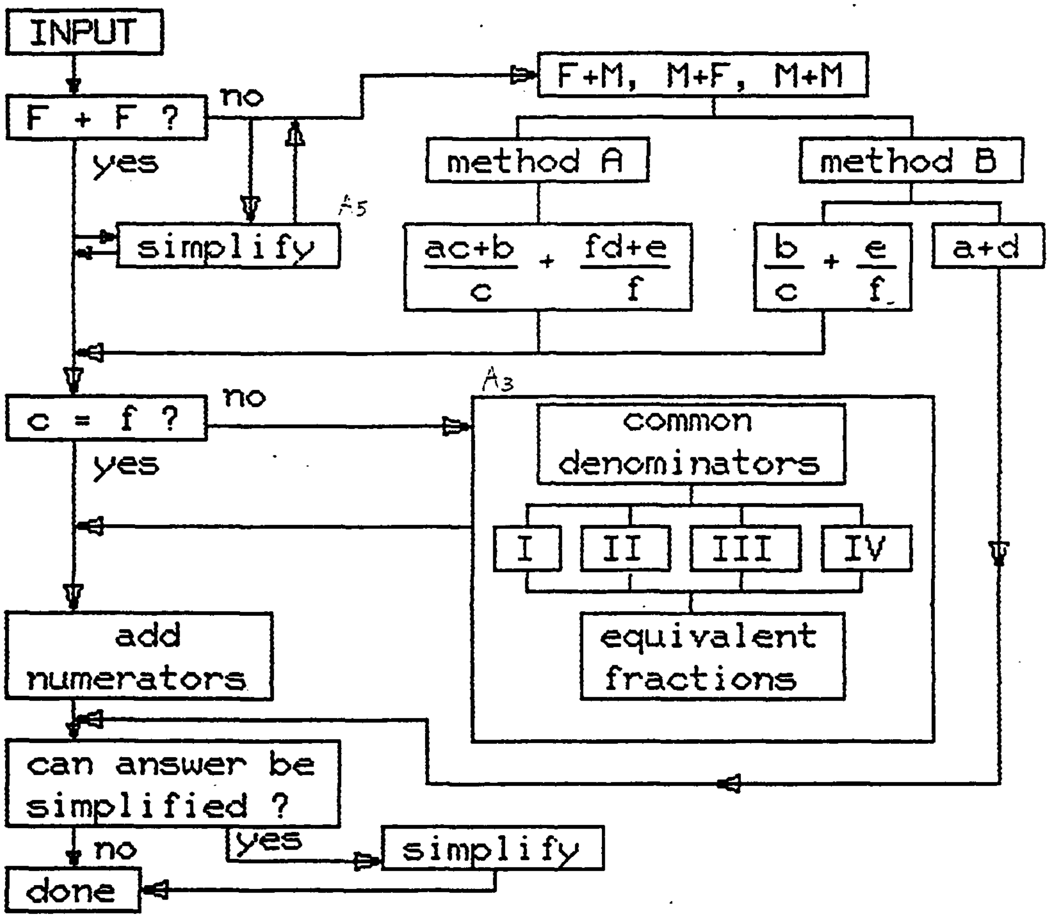


Figure 2 A tree representation of nine cognitive states



F = fraction (b/c)
M = mixed number (a b/c)
a,d = 1st, 2nd whole numbers
b,e = 1st, 2nd numerators
c,f = 1st, 2nd denominators
Method A: convert improper fraction
Method B: separate fraction and whole number part

I: LCD prime factoring
II: multiples mc = nf
III: one denom = multiple of another
IV: automatic cf

Figure 3 A flow chart for solving fraction addition problems

```
student  48:                    /group2"
 17 items done, total score  =  13
   *                       **
dj1=  Ø.1537, grp1= 6 ; dj2=  2.5Ø58, grp2=31
       *** +
  1)  1Ø 1      3/ 5 +      1/ 5
  2)   5 Ø      1/ 2 + 1  1Ø/ 7
  3)  25 1   3  4/ 5 + 5   3/ 5
  4)  27 1      1/ 5 +      1/ 4
  5)  29 1      4/ 7 +      1/ 7
  6)  32 Ø   2  1/ 8 + 3   5/ 6
  7)  21 1      2/ 7 +     18/12
  8)  24 Ø      1/ 5 + 2   5/ 3
  9)   2 1      2/ 5 +     12/ 8
 1Ø)  18 1      4/15 +      1/1Ø
 11)  34 1      1/ 3 +      4/ 9
 12)  13 Ø   3  1/ 6 + 2   3/ 4
 13)  37 1      5/ 6 +      1/ 8
 14)  15 1      1/ 2 +      3/ 8
 15)   8 1      1/ 3 +      1/ 2
 16)   7 1      3/ 5 +      7/ 5
 17)   9 1   1  4/ 7 + 1  12/ 7
```

* Mahalanobis distance between the student's position and the centroid of State 6 in Rule Space.

**Mahalanobis distance between the student's position and the centroid of State 31 which is the second closest.

***Items given .

+ Item scores

Figure 4 An example of a student's performance (Student No. 48)

Unusualness of Response Patterns

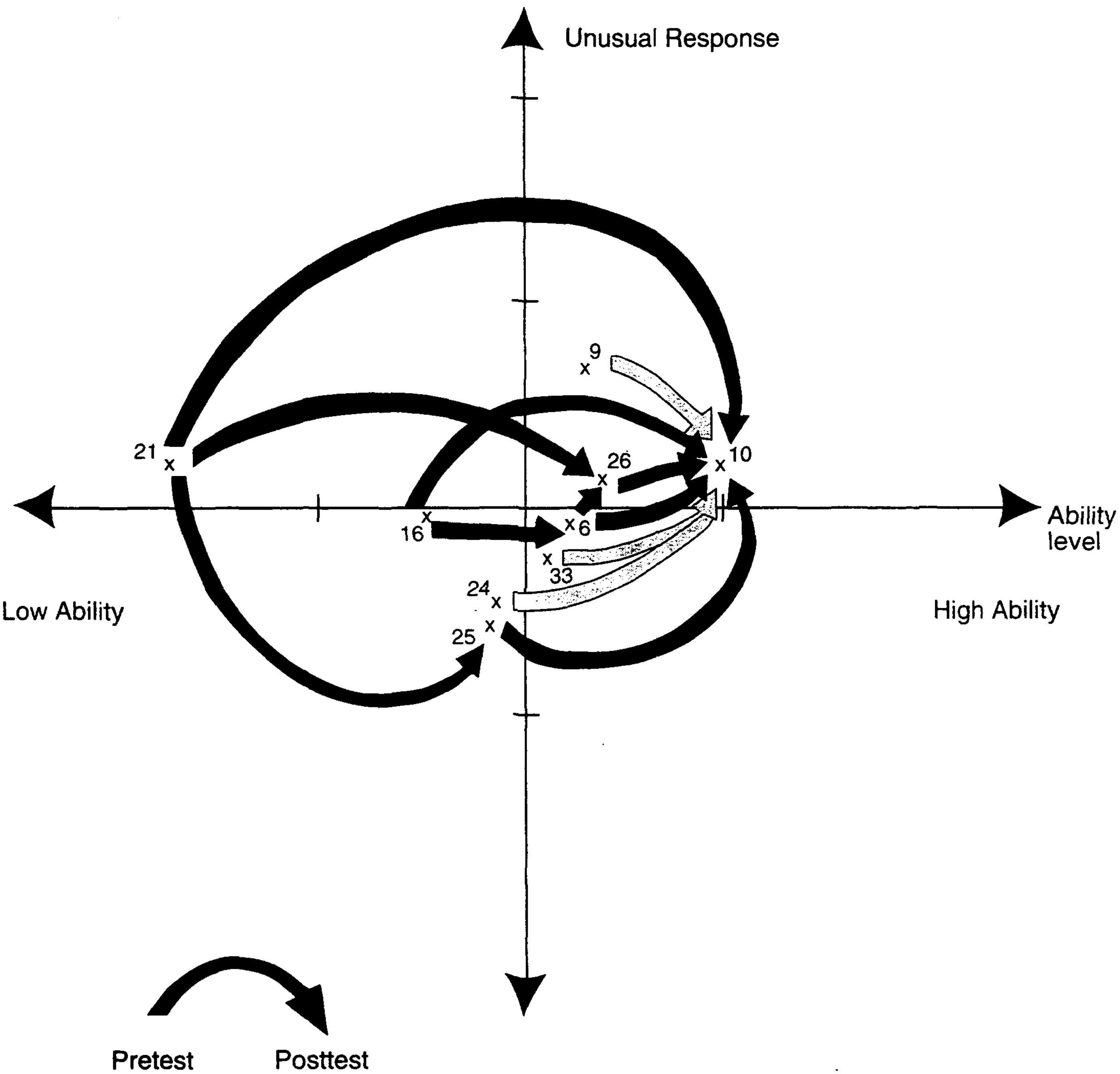


Figure 5. A map showing transition of states in 1988 data

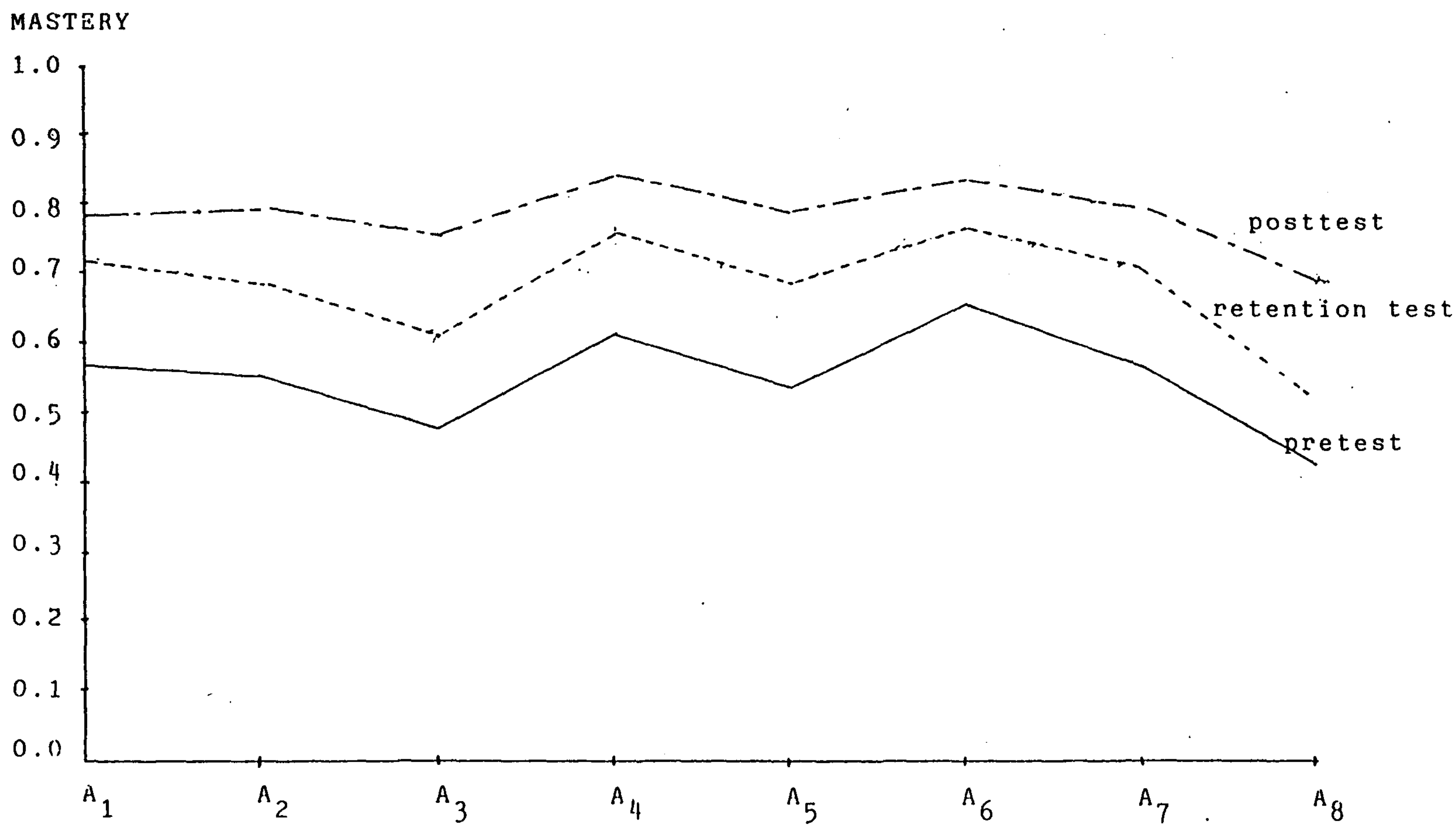


Figure 6 Percentage of correct scores for eight attributes described in Table 2

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