MEMORANDUM REPORT BRL-MR-3998

BRL

TARGET PRIORITIZATION TO OPTIMIZE EXPECTED UTILITY FOR A RANDOM BATTLE SCENARIO

DOUGLAS H. FRANK
INDIANA UNIVERSITY OF PENNSYLVANIA

ANN E. M. BRODEEN
U.S. ARMY BALLISTIC RESEARCH LABORATORY

SEPTEMBER 1992

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

NOTICES

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
September 1992

3. REPORT TYPE AND DATES COVERED
Final, July 1990 - September 1991

4. TITLE AND SUBTITLE
Target Prioritization to Optimize Expected Utility for a Random Battle Scenario

5. FUNDING NUMBERS
PR: 1L162618AH80
DA31 6059

6. AUTHOR(S)
Douglas H. Frank and Ann E. M. Brodeen

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
BRL-MR-3998

11. SUPPLEMENTARY NOTES
Dr. Frank is an associate professor in the Department of Mathematics at Indiana University of Pennsylvania.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The U.S. Army Ballistic Research Laboratory has been investigating the application of stochastic techniques to the problem of target value analysis (TVA) (i.e., assigning values to targets as an aid in the target selection process). Stochastic techniques were successfully applied when a friendly fire unit and two or more enemy targets were engaged in a simple battle. Here the simple battle has been extended to a random battle scenario. The problems associated with TVA are again examined from the standpoint of optimizing some utility function. Expected values of the utility functions are derived in terms of the parameters of interest, rate of fire and probability of kill, associated with a random battle. In the case of two enemy targets, an optimal target engagement ordering for all utilities can be obtained. Some special cases of utilities are considered, and the results generalized for an arbitrary target array.

14. SUBJECT TERMS
stochastic processes; utility function; optimization; threat; vulnerability; random battle scenario; algorithms; prioritization

15. NUMBER OF PAGES
22

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

INTENTIONALLY LEFT BLANK.

TABLE OF CONTENTS

INTENTIONALLY LEFT BLANK.

1. INTRODUCTION

The targeting process includes the development of a prioritized list specifying what targets are to be acquired and attacked, when they are to be acquired and attacked, and what resources are required to defeat the targets (U.S. Army Field Artillery School 1988).

In earlier research, target engagement orderings were designed to maximize a tactical outcome of a simple battle in which the friendly fire unit and each enemy target fired simultaneously (Frank 1988; Brodeen and Frank 1991). Here, battle outcome probabilities and optimal engagement orderings for a random battle are considered in a similar manner.

2. THE BATTLE

A single friendly fire unit and a group of enemy targets are engaged in a random battle. The random battle assumes each enemy target, as well as the friendly fire unit, fires independently and at a rate of fire that follows a single parameter negative exponential distribution but with possibly different mean rates and kill probabilities. Further, the single shot kill probabilities for the friendly fire unit and each enemy target, while constant from round to round, may also differ. Results given two enemy targets are derived first; these results are then extended to $T > 2$ enemy targets.

2.1 Two-Target Battle. Consider the following parameters for targets $i = 1,2$:

P_{B_i} = probability of friendly fire unit removing target i

P_{R_i} = probability of friendly fire unit being removed by target i

$B_i = \beta_i P_{B_i}$ = friendly fire unit firepower against target i (vulnerability of target i)

where β_i = mean rate of fire of friendly fire unit against target i

$R_i = \rho_i P_{R_i}$ = target i firepower against friendly fire unit (threat of target i)

where ρ_i = mean rate of fire of target i against friendly fire unit

It will be shown that the result of the classic stochastic duel can be extended to a battle between a single friendly fire unit and two enemy targets (Williams and Ancker 1963; U.S. Army Materiel and Development Readiness Command 1977). Assume target 1 is engaged until it is removed before target 2 is engaged. The battle concludes when either the friendly fire unit has been removed or it has removed both enemy targets. Removal is considered to be either the complete destruction or the infliction of a level of damage severe enough to abate the target's or the unit's contribution to its respective force. Victory is defined as the removal of both enemy targets regardless of whether the friendly fire unit survives.

If a weapon (which may be either an enemy target or the friendly fire unit), w, has a mean rate of fire, r, and a kill probability, p, then the probability of not killing the weapon in some time, h, is

$$Q_w(h) = e^{-rhp} . \tag{1}$$

Equation 1 will hereafter be referred to as **Lemma 1**. The following proof is offered.

Let X be the number of rounds fired during some time interval h, where X = 0, 1, 2, ..., ∞ (i.e., unlimited ammunition is assumed). Then, X has a Poisson distribution with mean rh. The single round probability of not being killed is (1 – p).*

$$\begin{aligned} Q_w(h) &= \sum_{x=0}^{\infty} \frac{e^{-rh}(rh)^x}{x!}(1-p)^x \\ &= e^{-rh} e^{rh(1-p)} \\ &= e^{-rhp} . \end{aligned} \tag{2}$$

In a battle with two enemy targets, the probability of the friendly fire unit removing target 1 before being removed is

---

* **Lemma 1** was derived by applying the series expansion $1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots = \sum_{x=0}^{\infty} \frac{m^x}{x!}$, which converges to e^m, for all values of m. Consider the function f(x) defined by $f(x) = \frac{m^x e^{-rh}}{x!}$, where $m^x = [(rh)(1-p)]^x$ for x = 0, 1, 2, ..., and f(x) = 0, elsewhere. For further discussion of the topic, the reader is referred to Hogg and Craig (1978).

$$P[1 \mid NOT] = \frac{B_1}{B_1 + R_1 + R_2.} \tag{3}$$

Equation 3 will be referred to as **Lemma 2** and is proved as follows. Divide the time of battle into units of length h and consider the Markov process formed (Bhattacharya and Waymire 1990; Kemeny and Snell 1960). From this process, the probability of the event occurring, where s is the number of rounds fired until a hit (i.e., kill) occurs, is

$$\begin{aligned} P_h[1 \mid NOT] &= \left([1 - Q_{B_1}(h)]Q_{R_1}(h)Q_{R_2}(h)\right) \sum_{s=0}^{\infty} \left(Q_{B_1}(h)Q_{R_1}(h)Q_{R_2}(h)\right)^s \\ &= \frac{[1 - Q_{B_1}(h)]Q_{R_1}(h)Q_{R_2}(h)}{1 - (Q_{B_1}(h)Q_{R_1}(h)Q_{R_2}(h))}. \end{aligned} \tag{4}$$

Applying **Lemma 1**,

$$P_h[1 \mid NOT] = \frac{e^{-h(\rho_1 P_{R_1} + \rho_2 P_{R_2})} - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})}}{1 - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})}}. \tag{5}$$

$$P[1 \mid NOT] = \lim_{h \to 0} P_h[1 \mid NOT] = \frac{\beta_1 P_{B_1}}{\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}} = \frac{B_1}{B_1 + R_1 + R_2}. \tag{6}$$

A detailed derivation of **Lemma 2** is presented in the appendix.

In a battle between the friendly fire unit and target 2, the probability of destroying target 2 is

$$P[2] = \frac{B_2}{B_2 + R_2}. \tag{7}$$

Equation 7 will be referred to as **Lemma 3** and is a well-known result (U.S. Army Materiel Development and Readiness Command 1977).

Theorem 1 defines f(w), where w is the number of targets destroyed. The density, f(w), is

$$f(0) = \frac{R_1 + R_2}{B_1 + R_1 + R_2}, \qquad f(1) = \frac{B_1 R_2}{(B_1 + R_1 + R_2)(B_2 + R_2)},$$
$$f(2) = \frac{B_1 B_2}{(B_1 + R_1 + R_2)(B_2 + R_2)}. \tag{8}$$

The event w = 0 is simply the complement of the event stated as **Lemma 2**. Similarly,

$$f(1) = P[1 \mid NOT] \cdot (1 - P[2]) \quad \text{and} \quad f(2) = P[1 \mid NOT] \cdot P[2]. \tag{9}$$

The results follow from **Lemmas 2** and **3**.

2.2 Multitarget Battle. Suppose there are T targets that are engaged in numerical order. Extend the definitions of B_i and R_i for i = 1, 2, ..., T. Define

$$S_i = \sum_{j=i}^{T} R_j, \quad S_0 = 0, \quad \text{and} \quad B_0 = 1. \tag{10}$$

The proof of **Theorem 1** is easily generalized. If w is the number of targets removed in a battle with T targets, **Theorem 2** states that the density, f(w), is

$$f(w) = \begin{cases} \left(\prod_{i=0}^{w} \frac{B_i}{B_i + S_i} \right) \cdot \frac{S_{w+1}}{B_{w+1} + S_{w+1}}, & \text{if } w < T \\ \prod_{i=1}^{T} \frac{B_i}{B_i + S_i}, & \text{if } w = T. \end{cases} \tag{11}$$

3. EVALUATION CRITERIA

Utility theory provides an apparatus for dealing with one-time decision making as well as a logical method for repetitive decision making. The term "utility," as conceived by von Neumann and Morgenstern (1947), is a measure of value used in the assessment of situations involving risk, which provides a basis for decision making. Different sets of axioms that imply the existence of utilities with the property that expected utility is an appropriate guide for consistent decision making are presented in von Neumann and Morgenstern (1947); Savage (1954); Luce and Raiffa (1957); Pratt, Raiffa, and Schlaifer (1965); and Fishburn (1970).

The objective of this study is to determine a value for each enemy target in terms of B_i and R_i such that if the targets are prioritized by their values, some optimal result is obtained. A target ordering is optimal when the expected value E[U] is maximized over all possible target orderings. Consider a utility function, U, defined on the variable, W, the number of enemy targets removed during the battle. In general, U(W) is nondecreasing, U(0) = 0, and U(T) = 1.

3.1 Utility Based on Total Victory. If the goal is to remove all enemy targets, where w = T, then maximize $f(T)$.

$$U1(W) = \begin{cases} 0, & \text{if } W < T \\ 1, & \text{if } W = T. \end{cases} \tag{12}$$

Recalling **Theorems 1** and **2**, $f(T) = \dfrac{B_1 \ldots B_T}{(B_1 + S_1)\ldots(B_T + S_T)}$. Since the numerator is the same for all permutations, the quantity, $\prod_{i=1}^{T} (B_i + S_i)$ must be minimized in order to maximize $f(T)$.

3.2 Utility Based on the Number of Hits. If hitting all the targets is not essential, and all targets seem equally important, then to maximize the number of targets removed

$$U2(W) = \frac{W}{T}. \tag{13}$$

For utility two, maximize $\sum_{w=0}^{T} \frac{W}{T} \cdot f(w)$, the expected value of U2(W), where f (w) was developed in Section 2.1.

3.3 Utility Based on a Reduction in Threat. **Lemma 2** shows that the combined threat of an array of targets acting as a single target is the sum of their individual threats. Consequently, in removing w targets, the overall threat is reduced from S_1 to S_{w+1}

$$U3(W) = 1 - \frac{S_{W+1}}{S_1}, \tag{14}$$

where $S_{T+1} = 0$ by definition. Note that U3(W) depends on the target engagement ordering chosen, whereas utilities 1, 2, and 4 do not.

3.4 Utility Based on a Reduction in Force. In many battles, the enemy can be halted when it loses only a small proportion of its forces. In these cases, an inflective utility seems appropriate. For convenience, consider an extreme example of an inflective function

$$U4(W) = \begin{cases} 0, & \text{if } W \le .3T \\ 1, & \text{if } W > .3T\ . \end{cases} \tag{15}$$

A commander may specify the type of effects he desires against specific target categories. The authors' current approach to TVA accounts for the complete removal of an enemy target; the selection of .3 was based accordingly on the definition of destruction, one of three target effects categories. Destruction is designed to put a target out of action permanently, and 30% casualties or materiel damage will usually render a target permanently ineffective (Headquarters Department of the Army FM 6-40 1984).

4. VALUES BASED ON TWO TARGETS

Given two enemy targets, there are only two possible orderings: 1,2 or 2,1. The utility, U, will be determined by U(1), the utility associated with removing one target. A generic target value algorithm for all utility functions will be derived, and the four special cases from Section 3 will be examined.

4.1 General Optimal Results. Suppose U(1) = c, is the same for either target ordering. Let $E_1[U]$ and $E_2[U]$ be the expected utilities for the orderings 1,2 and 2,1, respectively. We might ask, "Under what conditions is the initial ordering 1,2 better than the ordering 2,1?" From **Theorem 1**

$$E_1[U] = \frac{B_1(cR_2 + B_2)}{(B_1 + R_1 + R_2)(B_2 + R_2)}$$

and

$$E_2[U] = \frac{B_2(cR_1 + B_1)}{(B_2 + R_1 + R_2)(B_1 + R_1)}. \tag{16}$$

Setting $E_1[U] > E_2[U]$ and simplifying, an inequality concerning the threat and vulnerability of each target is obtained.

We now state **Theorem 3**. $E_1[U] > E_2[U]$ if and only if

$$\frac{B_1(B_1 + R_1)}{(B_1 + S_1)(cR_1 + B_1)} > \frac{B_2(B_2 + R_2)}{(B_2 + S_1)(cR_2 + B_2)}, \tag{17}$$

where $S_1 = R_1 + R_2$ represents total enemy firepower in the case of two enemy targets. Should the inequality not hold, that is, $E_2[U] > E_1[U]$, the ordering 2,1 becomes the new 1,2 target engagement ordering.

Theorem 3 provides a generic target value algorithm (i.e., all parameter subscripts removed) for some utility c. The value of a target with threat R and vulnerability B relative to a utility c is

$$\frac{B(B+R)}{(B+S)(cR+B)} . \tag{18}$$

The shortcomings of this definition are the presence of S, which depends on the entire enemy target array, and the limitation to utilities that are independent of target orderings. In the case of two enemy targets, one approach is to replace S by 2R, where S could be based upon knowledge about target 1 only, thereby degenerating to some average target threat. This is not recommended, in general, since optimality is not guaranteed. It will be shown that in some of the special cases that an equivalent value without S can be obtained.

To overcome the second objection, if the value of U(1) for an ordering could be expressed in terms of the second target to be removed, the target value algorithm could be redefined by interpreting the utility, c, as the utility when the target is the second one to be removed.

4.2 Value for Utility One. Recall the assumption U(1) = c. For utility 1, U1(W) = 0 if W < T. For example, when W = 1 and T = 2, U1(1) = 0. Thus, for utility 1, c = 0 when only one target is removed, and the generic target value algorithm reduces to $\frac{B+R}{B+S}$. It can be shown that $\frac{B_1+R_1}{B_1+S} > \frac{B_2+R_2}{B_2+S}$, if and only if $R_1(B_1 + R_1) > R_2(B_2 + R_2)$. This lends itself to the following definition of a value for utility 1:

$$VAL1 = R(B + R). \tag{19}$$

In the two target case, a target engagement ordering based on VAL1 will maximize E[U1].

Notice that threat, R, impacts VAL1 more than does the target's vulnerability, B. This is evident by the inclusion of a quadratic R term in the algorithm in addition to the linear R and B terms. From this, one might infer that if the objective were to remove both enemy targets, it is in one's interest to remove the more threatening target first.

4.3 Values for Utilities Two and Three. It is easily seen that for utility 2, $c = 1/2$, when $W = 1$ and $T = 2$. No simplification can be made of the generic target value algorithm for this case, therefore

$$VAL2 = \frac{B(B+R)}{(B+S)(1/2R+B)}. \tag{20}$$

Target orderings based on VAL2 will maximize E[U2]. VAL2 can also be written as

$$\frac{B}{B+S}\left(1+\frac{R}{R+2B}\right)$$

or

$$\frac{2B}{R+2B}\left(\frac{1}{1+\frac{S-R}{B+R}}\right),$$

but neither of these forms leads toward the elimination of S.

In the case of utility 3, the value of $c = 1 - (R/S)$; therefore, the generic target value can be written in the following manner:

$$VAL3 = \frac{BS(B+R)}{(B+S)(RS+BS-R^2)}. \tag{21}$$

No equivalent form without S is apparent.

4.4 Value for Utility Four. Given two targets, $c = 0$ for utility 4. The interest is in minimizing the probability of no hits. Intuitively, this occurs when $B_1 > B_2$, that is, when the most vulnerable, or easiest, target is removed first

$$VAL4 = B. \tag{22}$$

The case of two (or three) enemy targets should be regarded as a limiting case. If the functional form for utility four is applied directly, when T = 2 and W = 1, then 1 > .3(2) and U4 (W) should be equal to 1. However, since only the complete removal of an enemy target is accounted for, no fractional damage, the analyses of the utility functions are discrete. Therefore, it can be assumed that .3T has a lower limit of 1. Given this assumption, W ≤ .3T; that is, 1 ≤ 1 and U4 (W) is equal to 0. The same rationalization holds for T = 3.

5. CONCLUDING REMARKS

It was shown that general optimal results for a two target random battle could be derived from the expected values of the utilities associated with the two possible target engagement orderings. Theorem 3 outlined a generic target value algorithm relative to some utility c. Four target value algorithms based on a priori defined utility functions were reported.

The successful application of stochastic processes to the random battle TVA problem is an indication that this avenue of approach to TVA should be continued. Several areas warranting further investigation have been identified. First, what is the impact on the algorithms developed thus far if fractional damage to an enemy target is considered? The current research has concentrated solely on the complete removal, or kill, of a target. This also suggests future analysis of utility functions would no longer be discrete, but continuous. Second, what is the impact if the friendly fire unit's damage is also assessed in a fractional form, not simply as whether or not it survived? Third, and possibly the most important area of investigation, what is the impact of enemy target identification/ recognition on the target values algorithms?

Commander's guidance is to be respected; however, it should still be categorized as an arbitrary process. The primary objective of any sound TVA methodology should be to provide a means by which to minimize the judgmental aspect of this guidance. The selling point of the authors' research is its direct military wide application. For example, within the Army, the algorithms are as applicable to air defense as to the field artillery, and within the field artillery equally applicable whether the targets belong to the maneuver commander or to intelligence.

6. REFERENCES

Bhattacharya, R. N., and E. C. Waymire. Stochastic Processes With Applications. New York: John Wiley & Sons, Inc., 1990.

Brodeen, A. E. M., and D. H. Frank. "Target Prioritization to Optimize Expected Utility for a Simple Battle Scenario." BRL-MR-3945, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, October 1991.

Fishburn, P. C. Utility Theory for Decision Making. New York: Wiley, 1970.

Frank, D. H. "Target Value and Payoff." DAAL03-86-D-001, Battelle Memorial Institute, Columbus, OH, August 1988.

Headquarters Department of the Army. "Field Artillery Cannon Gunnery." FM 6-40, Washington, DC, December 1984.

Hogg, R. V., and A. T. Craig. Introduction to Mathematical Statistics. New York: MacMillan Publishing Co., Inc., 1978.

Kemeny, J. G., and J. L. Snell. Finite Markov Chains. Princeton: D. Van Nostrand Company, Inc., 1960.

Luce, R. D., and H. Raiffa. Games and Decisions. New York: Wiley, 1957.

Pratt, J. W., H. Raiffa, and R. O. Schlaifer. Introduction to Statistical Decision Theory. Preliminary ed., New York: McGraw-Hill, 1965.

Savage, L. J. The Foundations of Statistics. New York: Wiley, 1954.

von Neumann, J., and O. Morgenstern. Theory of Games and Economic Behavior. 2d ed., Princeton, NJ: Princeton University Press, 1947.

U.S. Army Field Artillery School. "Targeting Process." Coordinating Draft, FM 6-20-10 (and FM 34-118), Fort Sill, OK, February 1988.

U.S. Army Materiel Development and Readiness Command. Engineering Design Handbook: Army Weapon Systems Analysis, Part One. DARCOM-P 706-101, Alexandria, VA, November 1977.

Williams, T., and C. J. Anker, Jr. "Stochastic Duels." Operations Research, vol. 2, pp. 803–817, September–October 1963.

INTENTIONALLY LEFT BLANK.

APPENDIX:

DERIVATION OF LEMMA 2

INTENTIONALLY LEFT BLANK.

Lemma 2 states: given the friendly fire unit is engaging two enemy targets, the probability of the friendly fire unit removing the first target before being removed is

$$P[1 \mid NOT] = \frac{B_1}{B_1 + R_1 + R_2}.$$

The proof of Lemma 2 goes as follows. The time of battle is divided into units of length h and a Markov process is formed (Bhattacharya and Waymire 1990; Kemeny and Snell 1960). The geometric distribution is utilized to describe the probability of the event occurring, where s is the number of rounds fired before the first hit occurs and s = 0, 1, 2, ..., ∞.*

$$P_h[1 \mid NOT] = \Big([1 - Q_{B_1}(h)]Q_{R_1}(h)Q_{R_2}(h)\Big)\sum_{s=0}^{\infty}\Big(Q_{B_1}(h)Q_{R_1}(h)Q_{R_2}(h)\Big)^s.$$

To evaluate $\sum_{s=0}^{\infty}\Big(Q_{B_1}(h)Q_{R_1}(h)Q_{R_2}(h)\Big)^s$, consider the infinite series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$ which converges to $\frac{1}{1-x}$. *Therefore*, the summation converges to

$$\frac{1}{1-\Big(Q_{B_1}(h)Q_{R_1}(h)Q_{R_2}(h)\Big)},$$

and

$$P_h[1 \mid NOT] = \frac{[1 - Q_{B_1}(h)]Q_{R_1}(h)Q_{R_2}(h)}{1-\Big(Q_{B_1}(h)Q_{R_1}(h)Q_{R_2}(h)\Big)}.$$

Recalling Lemma 1, where $Q_w(h) = e^{-rhp}$ is the probability of killing a weapon (friendly or enemy) in some time frame h, then

* For this application, the geometric distribution refers to the number of failures observed before the first success (i.e., hit) is obtained.

$$Q_{B_1}(h) = e^{-h\beta_1 P_{B_1}}, \quad Q_{R_1}(h) = e^{-h\rho_1 P_{R_1}}, \quad \text{and} \quad Q_{R_2}(h) = e^{-h\rho_2 P_{R_2}}.$$

By substitution,

$$P_h[1 \mid NOT] = \frac{[1 - e^{-h\beta_1 P_{B_1}}]\, e^{-h\rho_1 P_{R_1}} \cdot e^{-h\rho_2 P_{R_2}}}{1 - \left(e^{-h\beta_1 P_{B_1}} \cdot e^{-h\rho_1 P_{R_1}} \cdot e^{-h\rho_2 P_{R_2}}\right)}$$

$$= \frac{e^{-h(\rho_1 P_{R_1} + \rho_2 P_{R_2})} - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})}}{1 - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{B_1} + \rho_2 P_{R_2})}}$$

$$= \frac{e^{-h(\rho_1 P_{R_1} + \rho_2 P_{R_2})}\left[1 - e^{-h(\beta_1 P_{B_1})}\right]}{1 - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{B_2})}} .$$

Then

$$\lim_{h \to 0} P_h[1 \mid NOT] = \frac{\left[\lim_{h \to 0} e^{-h(\rho_1 P_{R_1} + \rho_2 P_{R_2})}\right]\left[\lim_{h \to 0}(1 - e^{-h(\beta_1 P_{B_1})})\right]}{\lim_{h \to 0}\left(1 - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})}\right)} .$$

Since $\lim_{h \to 0} e^{-h(\rho_1 P_{R_1} + \rho_2 P_{R_2})} = 1$, the first term can be dropped leaving

$$\lim_{h \to 0} P_h[1 \mid NOT] = \frac{\lim_{h \to 0}\left(1 - e^{-h(\beta_1 P_{B_1})}\right)}{\lim_{h \to 0}\left(1 - e^{-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})}\right)} .$$

By the series expansion, $e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, then

$$\lim_{h\to 0} P_h[1 \mid NOT] = \frac{\lim_{h\to 0} -\left[(-h\beta_1 P_{B_1}) + \frac{(-h\beta_1 P_{B_1})^2}{2!} + \frac{(-h\beta_1 P_{B_1})^3}{3!} + \dots\right]}{\lim_{h\to 0} -\left[(-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})) + \frac{(-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}))^2}{2!} + \frac{(-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}))^3}{3!} + \dots\right]}$$

$$= \frac{\lim_{h\to 0}\left(-h\beta_1 P_{B_1} + \frac{h^2\beta_1^2 P_{B_1}^2}{2!} - \frac{h^3\beta_1^3 P_{B_1}^3}{3!} + \dots\right)}{\lim_{h\to 0}\left[(-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})) + \frac{(-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}))^2}{2!} + \frac{(-h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}))^3}{3!} + \dots\right]}.$$

Dividing through by $-h$ and taking the limits,

$$\lim_{h\to 0} P_h[1 \mid NOT] = \frac{\lim_{h\to 0}\left(\beta_1 P_{B_1} - \frac{h\beta_1^2 P_{B_1}^2}{2!} + \frac{h^2\beta_1^3 P_{B_1}^3}{3!} + \dots\right)}{\lim_{h\to 0}\left[(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}) - \frac{h(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})^2}{2!} + \frac{h^2(\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2})^3}{3!} + \dots\right]}$$

$$= \frac{\beta_1 P_{B_1}}{\beta_1 P_{B_1} + \rho_1 P_{R_1} + \rho_2 P_{R_2}}$$

$$= \frac{B_1}{B_1 + R_1 + R_2}.$$

INTENTIONALLY LEFT BLANK.

BIBLIOGRAPHY

Bharucha-Reid, A. T. Elements of the Theory of Markov Processes and Their Applications. New York: McGraw-Hill, 1960.

Breiman, L., J. Friedman, R. Olshen, and C. Stone. Classification and Regression Trees. Belmont, CA: Wadsworth Inc., 1984.

Dougherty, W., and R. Kaste. "Knowledge Acquisition Survey and Analysis: Firepower Control Experiment Part 11 of 12." BRL-MR-3731, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, December 1988.

Harvey, C. M. Operations Research: An Introduction to Linear Optimization and Decision Analysis. New York: North Holland, 1979.

Kaste, R., W. Dougherty, W. Yeakel, F. Lannon, and B. Reichard. "Fire Advisor Concept Paper." U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, October 1986.

U.S. Army Field Artillery School. "Targeting and Target Value Analysis." Coordinating Draft, FC 6-20-2, Fort Sill, OK, October 1984.

U.S. Army Materiel Development and Readiness Command. Engineering Design Handbook: Army Weapon Systems Analysis, Part Two. DARCOM-P 706-102, Alexandria, VA, October 1979.

Winner, W. A., A. E. M. Brodeen, and J. H. Smith. "Test Design and Analysis: Firepower Control Experiment Part 12 of 12." BRL-MR-3612, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, June 1987.

Winner, W. A., and W. Dougherty. "Scenario Development and Tactical Input Data: Firepower Control Experiment Part 10 of 12." BRL-MR-3578, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, April 1987.

INTENTIONALLY LEFT BLANK.

No. of Copies	Organization
2	Administrator Defense Technical Info Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22304-6145
1	Commander U.S. Army Materiel Command ATTN: AMCAM 5001 Eisenhower Ave. Alexandria, VA 22333-0001
1	Commander U.S. Army Laboratory Command ATTN: AMSLC-DL 2800 Powder Mill Rd. Adelphi, MD 20783-1145
2	Commander U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-IMI-I Picatinny Arsenal, NJ 07806-5000
2	Commander U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-TDC Picatinny Arsenal, NJ 07806-5000
1	Director Benet Weapons Laboratory U.S. Army Armament Research, Development, and Engineering Center ATTN: SMCAR-CCB-TL Watervliet, NY 12189-4050
(Unclass. only)1	Commander U.S. Army Rock Island Arsenal ATTN: SMCRI-TL/Technical Library Rock Island, IL 61299-5000
1	Director U.S. Army Aviation Research and Technology Activity ATTN: SAVRT-R (Library) M/S 219-3 Ames Research Center Moffett Field, CA 94035-1000
1	Commander U.S. Army Missile Command ATTN: AMSMI-RD-CS-R (DOC) Redstone Arsenal, AL 35898-5010

No. of Copies	Organization
1	Commander U.S. Army Tank-Automotive Command ATTN: ASQNC-TAC-DIT (Technical Information Center) Warren, MI 48397-5000
1	Director U.S. Army TRADOC Analysis Command ATTN: ATRC-WSR White Sands Missile Range, NM 88002-5502
1	Commandant U.S. Army Field Artillery School ATTN: ATSF-CSI Ft. Sill, OK 73503-5000
(Class. only)1	Commandant U.S. Army Infantry School ATTN: ATSH-CD (Security Mgr.) Fort Benning, GA 31905-5660
(Unclass. only)1	Commandant U.S. Army Infantry School ATTN: ATSH-CD-CSO-OR Fort Benning, GA 31905-5660
1	WL/MNOI Eglin AFB, FL 32542-5000
	Aberdeen Proving Ground
2	Dir, USAMSAA ATTN: AMXSY-D AMXSY-MP, H. Cohen
1	Cdr, USATECOM ATTN: AMSTE-TC
3	Cdr, CRDEC, AMCCOM ATTN: SMCCR-RSP-A SMCCR-MU SMCCR-MSI
1	Dir, VLAMO ATTN: AMSLC-VL-D
10	Dir, USABRL ATTN: SLCBR-DD-T

No. of Copies	Organization
2	Director U.S. Army Harry Diamond Laboratories ATTN: SLCHD-TA-AS, P. Emmerman L. Tokarcik 2800 Powder Mill Road Adelphi, MD 20783-1197
1	Director U.S. Army Harry Diamond Laboratories ATTN: SLCHD-ST-RC, D. Slife 2800 Powder Mill Road Adelphi, MD 20783-1197
2	Commander U.S. Army Electronics Technology and Devices Laboratory ATTN: SLCET-SD-A, Elizabeth Tuttle Fort Monmouth, NJ 07703-5000
4	Commander U.S. Army Communications Electronics Command ATTN: AMSEL-RD-C3-IR, Dr. Klose (3 cys) Mr. John Strozyk Fort Monmouth, NJ 07703-5000
1	Project Manager PM-ADCCS ATTN: SPIC-CC-ADCCS Redstone Arsenal, AL 35098-5600
1	Project Manager PM-FAAD C2 ATTN: SPIC-CC-ADCCS-FA Redstone Arsenal, AL 35898-5600
2	Project Manager PM-ASAS ATTN: JTFDO-ASAS JTFDO-AE-C 1500 Planning Research Drive McLean, VA 22102

No. of Copies	Organization
2	Project Manager PM-CHS ATTN: SSAE-CC-CHS Fort Monmouth, NJ 07703-5000
2	Project Manager PM-OPTADS ATTN: SPIC-CC-OTDS Fort Monmouth, NJ 07703-5000
4	PEO-Command & Control Systems ATTN: SPIS-CC, Mr. Giordano Mr. Albarelli Fort Monmouth, NJ 07703-5000
4	PM-FATDS ATTN: AMCPM-TF Bldg 457 Fort Monmouth, NJ 07703-5027
2	Commander U.S. Army Communication Electronics Command ATTN: AMSEL-RD-SI, Robert Ruth Fort Monmouth, NJ 07703-5001
1	Commander U.S. Army Armament Research, Development, and Engineering Command ATTN: Technical Director, Dr. Thomas E. Davidson Picatinny Arsenal, NJ 07806-5000
2	Commander U.S. Army Armament Research, Development, and Engineering Command ATTN: SMCAR-AR Picatinny Arsenal, NJ 07806-5000
1	Commander U.S. Army Armament Research, Development, and Engineering Command ATTN: SMCAR-FSS, J. Brooks Picatinny Arsenal, NJ 07806-5000

No. of Copies	Organization
6	Commander U.S. Army Armament Research, Development, and Engineering Command ATTN: SMCAR-FSP, Bernard Mack Al Graf Mike Bortak Fred Scerbo Floyd Ribe Mr. Reisman Picatinny Arsenal, NJ 07806-5000
2	Commander U.S. Army Armament Research, Development, and Engineering Command ATTN: SMCAR-ASH, Richard Rhinesmith Larry Ostuni Picatinny Arsenal, NJ 07806-5000
1	Commander U.S. Army Materiel Command ATTN: AMCDRA, Dr. Lucy Hagan 5001 Eisenhower Avenue Alexandria, VA 22333-0001
2	Commander U.S. Marine Corps Research, Development, and Acquisition Command ATTN: PM, Ground C2 Quantico, VA 22134-5080
1	Sandia National Laboratories ATTN: Dr. Larry Choate P.O. Box 5800 Albuquerque, NM 87185
2	Commander USACADA ATTN: TPIO-HFM, Duane Skelton Fort Leavenworth, KS 66027-5000
1	HQDA (SARD-T-N, Mr. Hunter Woodall) The Pentagon, Room 3E360 WASH DC 20310-0103
1	HQDA (SAUS-OR, Mr. W. Hollis) WASH DC 20310-0001

No. of Copies	Organization
2	Director TRAC-WSMR ATTN: ATRC-WFB, Dr. Deason White Sands Missile Range, NM 88002-5502
2	Director HQ, TRAC RPD ATTN: Dir Firepower, COL John Rogers Ch, FA Div Fort Monroe, VA 23651-5000
1	Commandant U.S. Army Field Artillery School ATTN: ATSF-TSM-FS, Mr. R. Willis Fort Sill, OK 73503-5000
1	Commandant U.S. Army Field Artillery School ATTN: ATSF-TSM-C3, COL W. Sanchez Fort Sill, OK 73503-5000
2	Commandant U.S. Army Field Artillery School Director of Combat Development ATTN: ATSF-CD Fort Sill, OK 73503-5000
1	Commandant U.S. Army Field Artillery School ATTN: ATSF-CD, Mr. Dublisky Fort Sill, OK 73503-5000
1	Commandant U.S. Army Field Artillery School ATTN: President FA BD Fort Sill, OK 73503-5000
2	Commander U.S. Army Missile Command ATTN: SFAE-FS Redstone Arsenal, AL 35898-5000
2	Commander U.S. Army Intelligence Center and School Fort Huachuca, AZ 85613-7000

No. of Copies	Organization
1	Commander U.S. Army Missile Command ATTN: AMSMI-RD-SD-UV Redstone Arsenal, AL 35898-2456
1	Commander U.S. Army Missile Command PM-Non-Line-of-Sight (NLOS) Redstone Arsenal, AL 35898-5000
2	Institute for Defense Analysis 1801 N. Beauregard St. Alexandria, VA 22311-1772
1	Association of the U.S. Army 2425 Wilson Blvd Arlington, VA 22210-0860
2	Director Defense Advance Research Projects Agency ATTN: Asst. Dir. TTO 1400 Wilson Blvd Arlington, VA 22209-2308
1	BTI Program Office ATTN: Dr. John Transue 1901 North Beauregard St. Suite 380 Alexandria, VA 22311
1	ODDRE/R&AT/ET ATTN: Mr. Fredrick L. Menz The Pentagon, Room 3D1089 Washington, DC 20301-3080
1	Director, Unmanned Aerial Vehicles Joint Project Office Naval Air Systems Command Washington, DC 20361-1014
1	Naval Underwater Systems Center ATTN: Jeffrey A. Manickas Operations Research Analyst Newport, RI 02841

No. of Copies	Organization
1	U.S. Coast Guard Department of Transportation ATTN: J. Smith Avery Pt. Groton, CT 06340
5	Indiana University of Pennsylvania ATTN: Dr. Douglas H. Frank 208 Stright Hall Department of Mathematics Indiana, PA 15705-1072
1	GE ATCCS SE&I ATTN: Neil Vestermark Building 97 P.O. Box 8048 Philadelphia, PA 19101
1	General Dynamics Land Sys Div 38500 Mound Road MZ 436-10-61 Sterling Heights, MI 48310-3268
1	Magnavox ATTN: Mike Meier (10-06) 1313 Production Road Fort Wayne, IN 46808
1	ATCCS SE&I ATTN: Frank Pumillo 1301 Virginia Drive 4th Floor, Maplewood Plaza Fort Washington, PA 19034
1	M.I.T. ATTN: Allen G. Dors Aerospace Division Lincoln Laboratory 244 Wood Street Lexington, MA 02173-0073
1	FMC Corporation Naval Systems Division 4800 E. River Rd. Minneapolis, MN 55421-1498

No. of Copies	Organization
1	Defense Analysis Corporation ATTN: Fritz Laux 1213 Jefferson Davis Highway Suite 805 Arlington, VA 22202
1	Alliant Techsystems, Inc. ATTN: Ralph Badger 10400 Yellow Circle Drive Minnetonka, MN 55343
1	Alliant Techsystems, Inc. ATTN: P.M. Narendra 5901 Lincoln Drive Edina, MN 55436
2	Command Systems Inc. ATTN: Devin R. Willis Mark McCleary 1025 Goshen Road Fort Wayne, IN 46808
1	United Technologies - Sikorsky ATTN: Maynard Marquis Chief, Requirements Analysis Advanced Design and Business Development North Main Street Stratford, CT 06601
1	Dr. Ralph E. Beatty, Jr. Consultant (TASC) 922 Timberbranch Pkwy. Alexandria, VA 22302

No. of Copies	Organization
	Aberdeen Proving Ground
14	Dir, USAMSAA ATTN: AMXSY-C, Mr. Phil Beavers Mr. Hal Burke Mr. Pete Reid AMXSY-G, Mr. Wilbert Brooks Mr. Julian Chernick Mr. H. Cheever Mr. Ben King Mr. John Kramer Mr. Tom Ruth Mr. Doug Smith Mr. Floyd Wofford Mr. Alex Wong Mr. William Yeakel AMXSY-A, Mr. Walter Clifford JTCG/ME, Mr. Art Lagrange
5	Dir, HEL ATTN: SLCHE-D SLCHE-AD, Mr. Clarence Fry SLCHE-FT, Mr. Gary Horley Mr. Bill Dousa Mr. Doug Tyrol

INTENTIONALLY LEFT BLANK.

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. BRL Report Number BRL-MR-3998 Date of Report September 1992

2. Date Report Received ____________________

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) ____________________

4. Specifically, how is the report being used? (Information source, design data, procedure, source of ideas, etc.) ____________________

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided, or efficiencies achieved, etc? If so, please elaborate. ____________________

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.) ____________________

CURRENT ADDRESS

Name

Organization

Address

City, State, Zip Code

7. If indicating a Change of Address or Address Correction, please provide the New or Correct Address in Block 6 above and the Old or Incorrect address below.

OLD ADDRESS

Name

Organization

Address

City, State, Zip Code

(Remove this sheet, fold as indicated, staple or tape closed, and mail.)

DEPARTMENT OF THE ARMY
Director
U.S. Army Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

OFFICIAL BUSINESS

BUSINESS REPLY MAIL
FIRST CLASS PERMIT No 0001, APG, MD

Postage will be paid by addressee.

Director
U.S. Army Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES