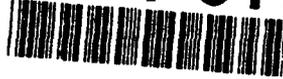


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TECHNICAL REPORT ARCCB-TR-92029

**NOTCH DIMENSIONS FOR THREE-POINT
BEND FRACTURE SPECIMENS BASED
ON COMPLIANCE ANALYSES**

FRANCIS L. BARATTA
JOHN H. UNDERWOOD

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**US ARMY ARMAMENT RESEARCH,
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NOMENCLATURE

a	total length of crack plus notch*
a_f	extension of notch by fatigue crack*
a_N	notch dimension*
B	specimen thickness*
b, b_c	specimen dimensions*
E, E'	elastic modulus; (E' for plane strain)
H	notch dimension*
L	notch length*
L_c , N_c	cutout dimensions*
N	notch width*
P	applied load*
U_a , U_b	elastic strain energy; case a and b, see Figure 3
S	specimen span*
W	specimen depth*
α	normalized total length of notch plus crack (a/W)
α_N	normalized notch dimension (a_N/W)
β	crack envelope angle*
τ	included notch-tip angle*
δ	load-line displacement of beam*
Ω	normalized notch dimension (H/W)
Ω_c	normalized notch dimension (L_c/W)
Δ	normalized load-line compliance ($\delta EB/P$)
Δ_c	normalized load-line compliance due to cutout ($\delta_c EB/P$)
μ	Poisson's ratio

*see Figs 1-3

INTRODUCTION

Technical committees within ASTM Committee E24 on Fracture currently are developing a comprehensive fracture toughness test method which will include many of the existing fracture testing procedures. The intent is to provide a "common" fracture toughness test method for any of four basic types of fracture behavior which are currently investigated using the following methods: E399 for Plain-Strain Fracture Toughness of Metallic Materials; E813 for J_{Ic} , A Measure of Fracture Toughness; E1152 for Determining J-R Curves; and E1290 for Crack-Tip Opening Displacement (CTOD) Fracture Toughness Measurement.

The specimen configuration which is included in each of the four test methods is the three-point bend specimen. Two of the methods, E813 and E1152, require load-line compliance determination by both experimental means and theoretical analysis. These test methods refer to a formulation of load-line compliance based on results generated in the referenced authors' work [Ref.1]. This method represents any given combination of machined notch plus crack extension as an ideal zero-width crack, and ignores the effect of the finite width notch on the beam compliance. The compliance contributed by the finite width of the notch, although not the major contributor to compliance, can be substantial, depending on the notch and crack dimensions. In many cases this additional compliance attributable to the notch width can be ignored; but if accurate load-line compliance results are desired, the notch configuration should be considered in the analysis. Baratta [Ref.2] recently provided relevant results, wherein he determined that in some instances errors resulting from the use of reference 1 as applied to fracture testing were considerable.

In response to the above determination, the objective of this text is to use Baratta's method and results [i] to calculate load-line compliance for various notch and crack configurations, and [ii] to provide guidance to ASTM technical committees in defining appropriate and practical geometry limits to minimize load-line compliance errors in fracture testing with three-point bend specimens.

ANALYSIS

Because the method for obtaining load-line compliance of a three-point bend beam is well-documented in reference 2, little detail is necessary in this document. Some general comments about the method appear in the following paragraphs.

A simple yet accurate way of calculating compliance for stepped structural elements has been provided by Bluhm [Ref. 3]. Engineering strength of materials analysis was combined with elastic fracture mechanics to obtain deflections of geometrically discontinuous structures. The method was based on the work of Paris [Ref. 4] and subsequently Tada et al. [Ref. 5], which suggested techniques for computing certain displacements in crack-related problems. The approach used in reference 3 as applied to stepped structures was adapted in reference 2 to various V-notched configurations using superposition. Specifically, the configuration examined in reference 2, which is also appropriate to the topic here, appears in Figure 1. Using the methods from reference 2 outlined above for a three-point bend specimen with $S/W = 4$, the normalized load-line compliance including the effects of notch configuration is¹:

$$\begin{aligned} \Delta = & \left[(b/W)^3/4 \right] + \left[\{3(1+\mu)/5\} [b/W - 2 \tan(\tau/2) \ln(1-\alpha_w/\{1-\Omega\})] \right] \\ & + \left[3 \tan(\tau/2) \right] \left[\{S/(2W\{1-\Omega-\alpha_N\})\}^2 - \{b/(2W\{1-\Omega\})\}^2 \right] \\ & - \left[2 \tan(\tau/2) \right] \left[\{S/(2W\{1-\Omega-\alpha_N\})\} - \{b/(2W\{1-\Omega\})\} \right] \\ & + \tan(\tau/2) \ln(1-\alpha_w/\{1-\Omega\}) \left] + \left[f(\alpha) \right] \end{aligned} \tag{Eq. 1}$$

¹Note that this equation in reference 2 has a typographical error; Eq. 1 above is correct.

According to the work of Wu [Ref. 6]:

$$f(\alpha) = 18 (S/2W)^2 [-0.365 \alpha^5 + 1.326 \alpha^4 - 2.71 \alpha^3 + 3.87 \alpha^2 - 8.614 \alpha - 2.268 + 6.018 \ln(1+2\alpha) - 1.015 \ln(1-\alpha) + (2.829 \alpha^2 - 4.437 \alpha + 2.268) / ((1+2\alpha)(1-\alpha)^2)]$$

(Eq. 2)

Equation 1 applies to a plane-stress condition; simply multiply $f(\alpha)$ by the quantity $(1-\mu/2)$ to realize the plain-strain condition. Equation 1 does not account for the compliance due to the radius at the apex of the V-notch nor for the local discontinuities at the junction of the V-notch and the straight sides of the notch. However, it is expected that the effect of these subtle geometric details of the notch on the load-line compliance will be negligible.

Equation 1 also does not account for a displacement gage cutout, such as that shown in Figure 2. The displacement and associated compliance due to the cutout can be readily accounted for by superposition of the two cases shown in Figure 3. This superposition (see the Appendix) results in an additional normalized compliance, Δ_c , based on the following:

$$\Delta_c = 3 [(b/2W)^2 - (b_f/2W)^2] / [(1-\Omega_c)^2 - 2(b/2W - b_f/2W) ((1+\mu)/5 + 1/(1-\Omega_c))]$$

(Eq. 3)

The total load-line compliance, Δ_T , is simply the sum of Eqs. 1 and 3:

$$\Delta_T = \Delta + \Delta_c$$

(Eq. 4)

In the results to follow, load-line compliance for various notched configurations are presented and compared with the ideal crack results [Ref. 1]. The expression for load-line compliance for the case of an ideal crack, Δ_i , from reference 1, is the following:

$$\Delta_i = [(S/W)/(1-\alpha)]^2 [1.193 - 1.980 \alpha + 4.478 \alpha^2 - 4.443 \alpha^3 + 1.739 \alpha^4]$$

(Eq. 5)

RESULTS

Notch and cutout configurations were selected to show what were believed to be the key factors which affect load-line compliance and also to show the more significant differences in compliance for the finite-width notch and the ideal crack, Eqs. 4 and 5, respectively. The range of configurations selected covered those of interest in the fracture test methods mentioned earlier. Figures 4-7 and Tables 1 and 2 present the results.

Figure 4 compares the normalized load-line compliance, $\Delta_i = \delta EB/P$, for the ideal zero-width crack (Eq. 5) with results for notches of various widths (Eq. 4), over a range of a/W . Note that the increase in compliance due to the finite width of the notch becomes more significant for deeper notches. The relative change in compliance, compared to that for an ideal crack, can be more directly considered when plotted as a ratio, Δ/Δ_{ideal} (see Figure 5). These results can be used to show the upper bound differences in compliance between three-point bend specimens with a finite thickness notch and specimens with an idealized crack, for a variety of notch and crack lengths. For example, consider a beam having a notch length, $L = 0.425 W$ (the lower curve) and a fatigue crack length of $a_f = 0.025 W$ (the smallest fatigue crack considered here) giving a total notch-plus-crack length of $a = 0.45 W$. For this configuration the difference in compliance from that of the idealized crack is 7.3 %. However, as a_f is allowed to increase this difference diminishes to a value of 2.0 % when $a_f = 0.325 W$ and $a = 0.75 W$. Thus, a large notch depth with a small fatigue crack produces a significant increase in compliance over that of the ideal crack. The end points of the family of curves for a range of L/W values produce an upper bound description of the increase in compliance (see the dashed line in Figure 5). For most real testing

situations the difference in compliance will be less than these maximum values, because $a_p > 0.025W$.

Figures 6 and 7 show the effect on compliance of important configurational variables. Figure 6 shows the Δ/Δ_{ideal} values for three finite notch widths (solid lines with symbols) compared with the ideal crack, with $\Delta/\Delta_{ideal} = 1$ (the solid line). The notches each have $a_p/W = 0.025$ and $\tau = 90^\circ$. Note the significant effect on compliance due to notch width, with a value of $N/W = 0.10$ resulting in a 10-19 % increase over that of the ideal crack, for the range $0.45 \leq a/W \leq 0.75$. The lesser effect of two other configurational features on compliance can also be seen. The amount of crack extension from the notch tip, aF/W , has less affect on compliance than notch width; note that the dashed curve for $a_p/W = 0.050$ is reduced as would be expected (the additional crack extension makes the notch behave a bit more like an ideal crack), but the reduction is only about 2 %. The effect of the cutout on compliance can be judged from the dotted curve. The cutout with $L_c/W = 0.1$ and $N_c/W = 0.2$ adds only about 1 % to compliance for $a/W = 0.45$, and its addition diminishes as a/W increases.

The effect of notch-tip included angle, τ , on compliance is considered in Figure 7. If τ were 30° rather than 90° , about one third of the compliance increase due to the notch would be eliminated (for this notch $N/W = 0.10$; $a_p/W = 0.025$). However, the fabrication difficulties associated with a 30° notch-tip angle would be significant for many users. In addition, the effect of a small τ in eliminating some of the compliance increase due to the notch will be greatly diminished for notches with smaller N/W and larger a_p/W .

Values of normalized compliance for various notch and crack configurations, including many of those of Figs. 4-7, are listed in Tables 1 and 2. Also shown are the values of the crack and notch envelope angle, β , for each configuration, see Figure 2. Historically, a limitation on this envelope angle has been used to insure that a given notch configuration is a reasonable simulation of an ideal crack (see Appendix). Although a general trend can be noted in Table 1, in which a small envelope angle is associated with a small difference between notch and ideal crack compliance, the trends already described between notch dimensions and compliance are better defined.

DISCUSSION

The results of solid mechanics analyses described here were used to suggest two sets of notch and cutout dimensions for use in fracture tests with the three-point bend specimen. We believe that these same dimensions are also applicable to other configurations which are subjected to predominantly bending stresses, including the compact specimen used in many fracture tests and the arc and disk-shaped specimens used in ASTM Method E399. In addition to analytical results, some engineering experience and judgement were used in arriving at the suggested specimen dimensions, particularly as related to specimen fabrication and test procedures in common use today.

Table 3 gives the two suggested notch and cutout configurations, as a list of five required dimensions: the maximum allowed notch width, N/W ; the maximum allowed notch-tip included angle, τ ; the minimum required crack extension, a_p/W ; the maximum allowed cutout length and width, L_c and N_c . The current requirements in ASTM Methods E399 and E813 are listed in this table for reference.

The most significant change in specimen configuration involves notch width, where a wide notch with $N/W = 0.063$ is suggested for tests in which specimen fabrication requirements are controlling, and a narrow notch with $N/W = 0.01$ is suggested for tests in which a close modeling of the ideal crack compliance is important. The wide notch can be easily cut in relatively large specimens using conventional machining, whereas the narrow notch requires a quite narrow slitting process, such as electric-discharge machining. The other notch and crack dimensions are unchanged from existing methods. Although some narrowing of the difference in compliance of the real notch and cutout compared to the ideal crack could have been accomplished with tighter dimensions, the user would have paid dearly in fabrication and testing difficulties.

The final result of the suggested notch, crack and cutout dimensions is: [i] for the wide notch the compliance can be 7-12 % above that of the ideal crack, for $0.45 \leq a/W \leq 0.75$, respectively; [ii] for the narrow notch the compliance is 3 % above that of the ideal crack, for the range $0.45 \leq a/W \leq 0.75$. It should be noted that generally only the lower end of the possible 7-12 % increase in compliance mentioned above would be experienced in fracture testing because, although R-curve type tests are often performed for $a/W = 0.7$, the notch length is generally at $a/W = 0.6$ or less.

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Table 1 Load-Line Compliance for Three-Point Bend Specimens with Various Notch Widths and Crack Configurations

Notch Width N/W	Cutout Dimensions L _c /W N _c /W	Tip Angle τ deg	Crack Extension a _p /W	Envelope Angle β deg	Notch+Crack Length a/W	Normalized Compliance δEB/P
0.100	0.10 0.20	90	0.050	53.1	0.475	55.49
					0.525	68.51
					0.575	86.75
					0.625	113.20
					0.675	153.34
					0.725	218.03
0.063	0.10 0.20	90	0.025	58.1	0.45	49.67
					0.50	60.51
					0.55	75.49
					0.60	96.88
					0.65	128.65
					0.70	178.44
0.010	0.10 0.20	90	0.025	18.9	0.45	47.46
					0.50	57.39
					0.55	71.07
					0.60	90.49
					0.65	119.16
					0.70	163.77
0.010	0.0 0.0 (No Cutout)	90	0.025	18.9	0.45	46.87
					0.50	56.80
					0.55	70.48
					0.60	89.90
					0.65	118.57
					0.70	163.19
0.0	(Ideal Crack; Eq.5)				0.45	46.29
					0.50	56.05
					0.55	69.41
					0.60	88.28
					0.65	116.01
					0.70	159.08
					0.75	231.10

Table 2 Load-Line Compliance for Three-Point Bend Specimens with Various Notch-Tip Included Angles

Notch Width N/W —	Cutout Dimensions L _c /W N _c /W		Tip Angle τ deg	Crack Extension a _p /W —	Envelope Angle β deg	Notch+Crack Length a/W —	Normalized Compliance δEB/P —
0.100	0.10	0.20	30	0.025	26.6	0.45	49.63
						0.50	60.49
						0.55	75.45
						0.60	96.70
						0.65	128.09
						0.70	176.93
						0.75	258.52
0.100	0.10	0.20	45	0.025	37.9	0.45	50.20
						0.50	61.26
						0.55	76.54
						0.60	128.09
						0.65	130.48
						0.70	180.72
						0.75	264.92
0.100	0.10	0.20	60	0.025	48.3	0.45	50.55
						0.50	61.75
						0.55	77.24
						0.60	99.31
						0.65	132.07
						0.70	183.30
						0.75	269.43
0.100	0.10	0.20	90	0.025	67.4	0.45	50.99
						0.50	62.37
						0.55	78.12
						0.60	100.64
						0.65	134.16
						0.70	186.78
						0.75	275.69

Table 3 Suggested Notch Dimensions for Three-Point Bend Fracture Specimens; see Figs. 1 and 2

		Existing Standards		Suggestions from Compliance	
		E399	E813	Wide Notch	Narrow Notch
<u>max width;</u>	N	0.100W	0.063W	0.063W	0.010W
<u>max tip angle;</u>	τ	90°	-	90°	90°
<u>min extension;</u>	a_p	0.025W	0.050a	0.025W	0.025W
<u>max cutout;</u>	L_c	-	0.10W	0.10W	0.10W
<u>max cutout;</u>	N_c	-	0.20W	0.20W	0.20W

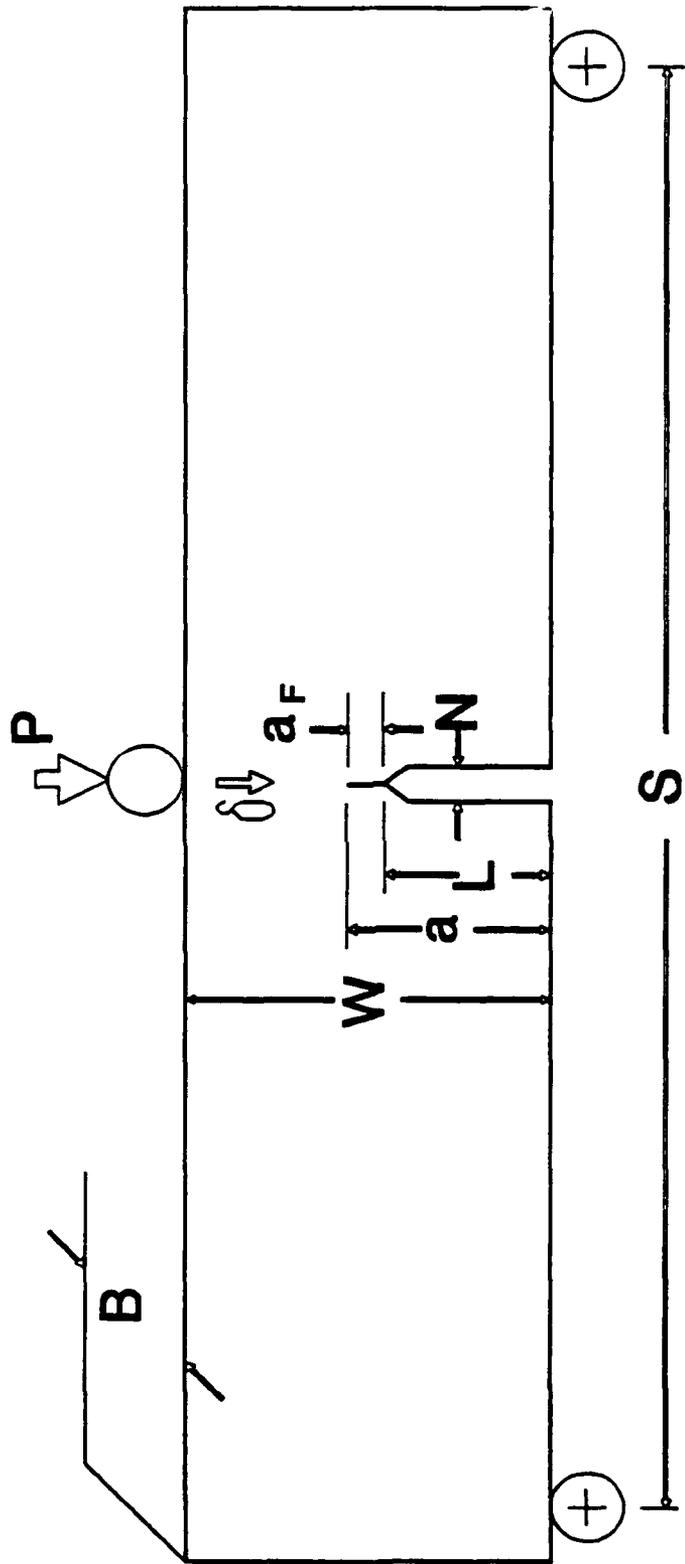


Figure 1 Three-Point Bend Specimen Configuration

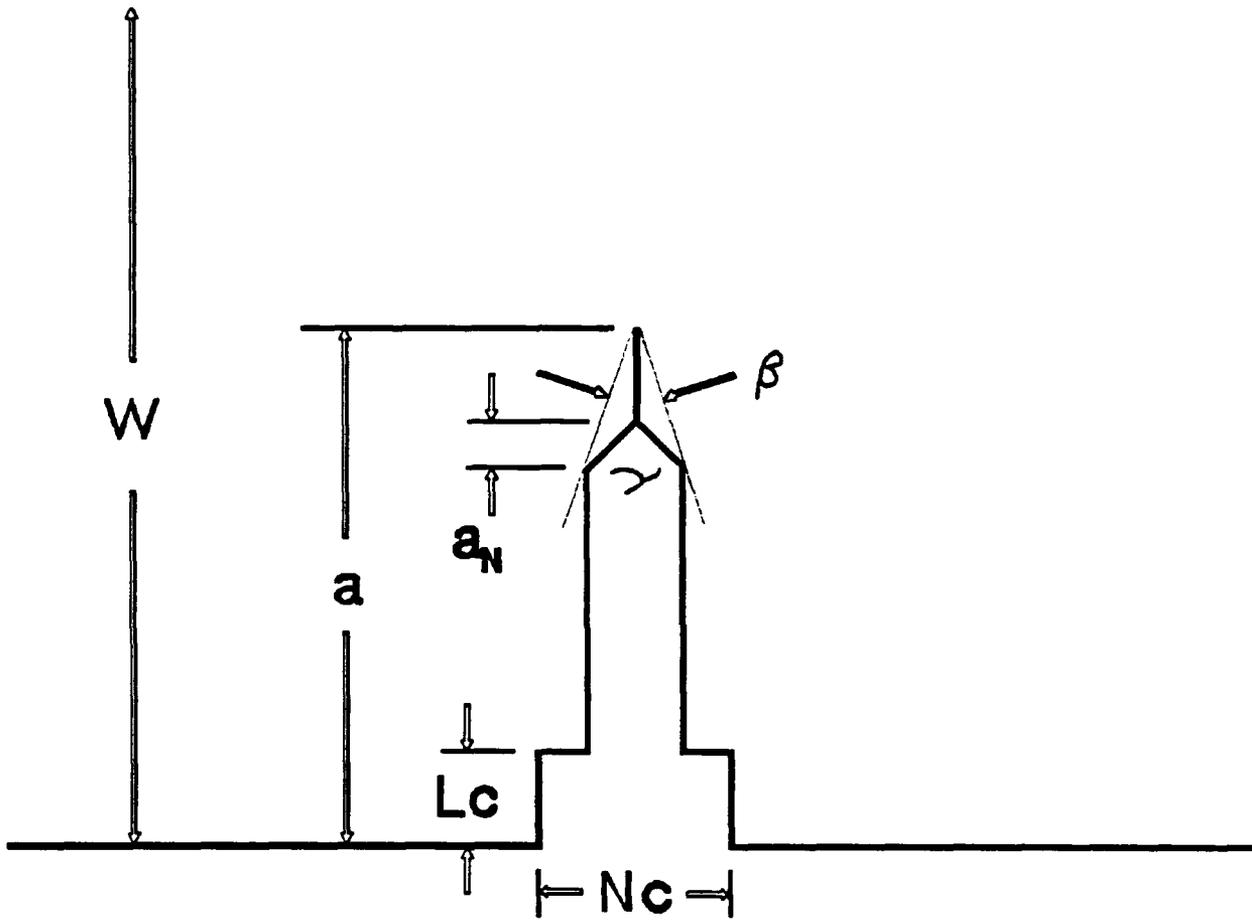


Figure 2 Notch and Cutout Configuration

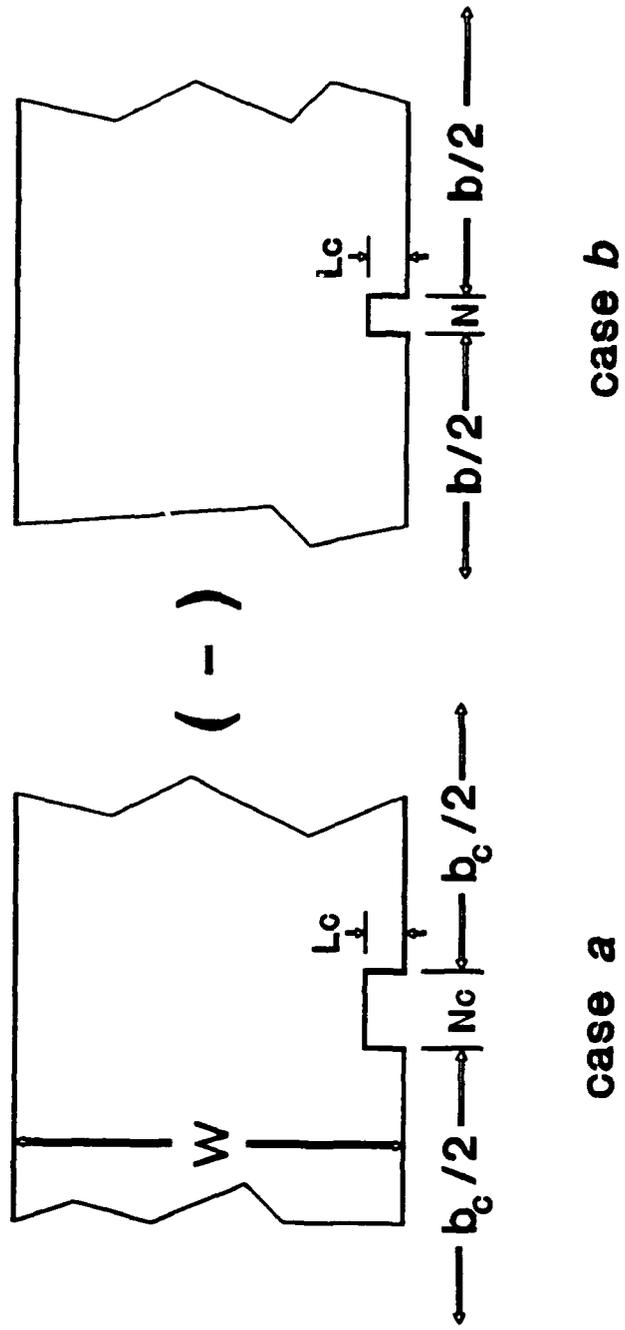


Figure 3 Cutout Configurations for Superposition

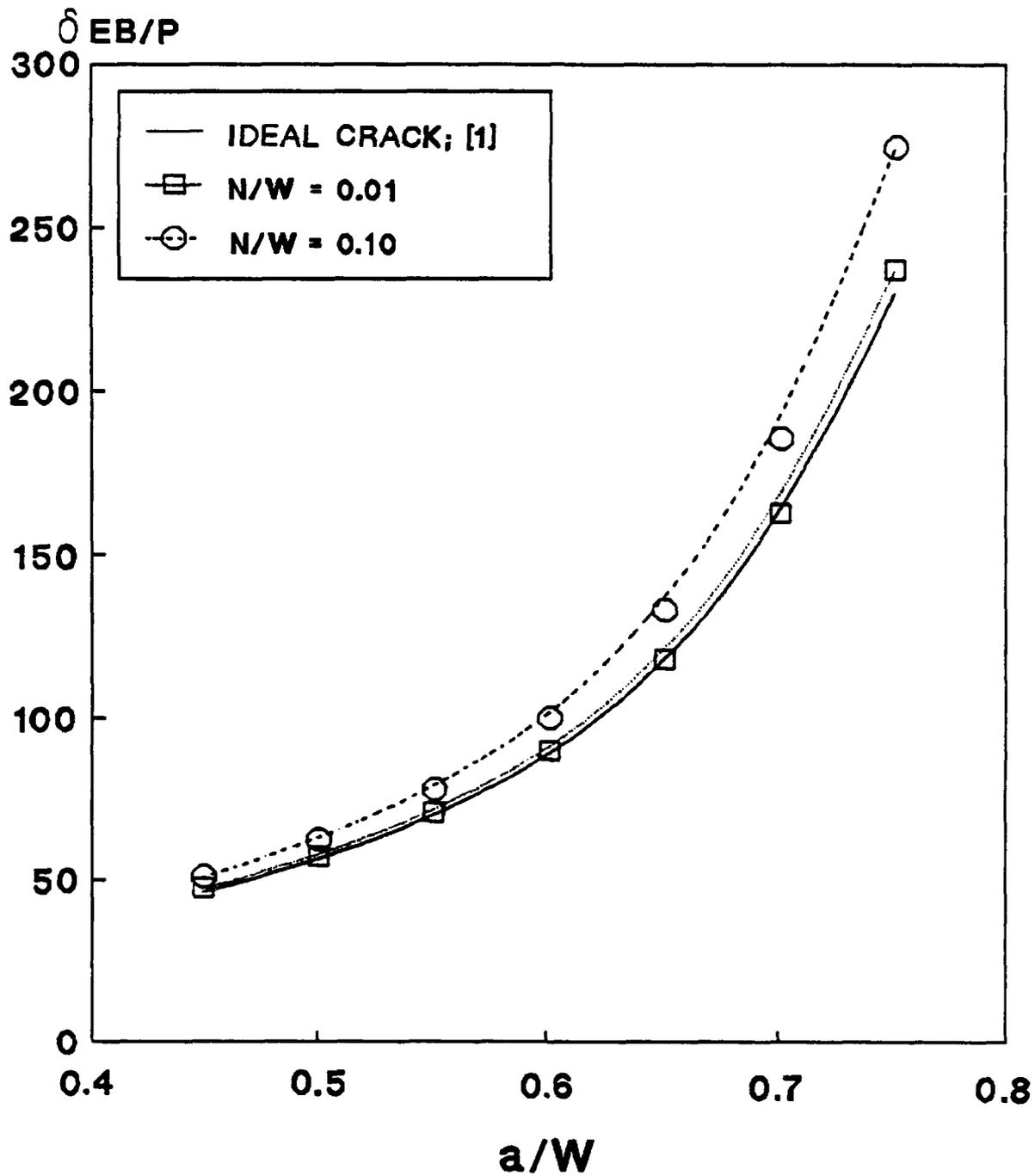


Figure 4 Comparison of Normalized Load-Line Compliance for Three-Point Bend Specimens with Notches and Ideal Crack; notched configurations are: $\tau=90^\circ$, $a_p/W=0.025$, $N_p/W=0.2$, $L_p/W=0.1$

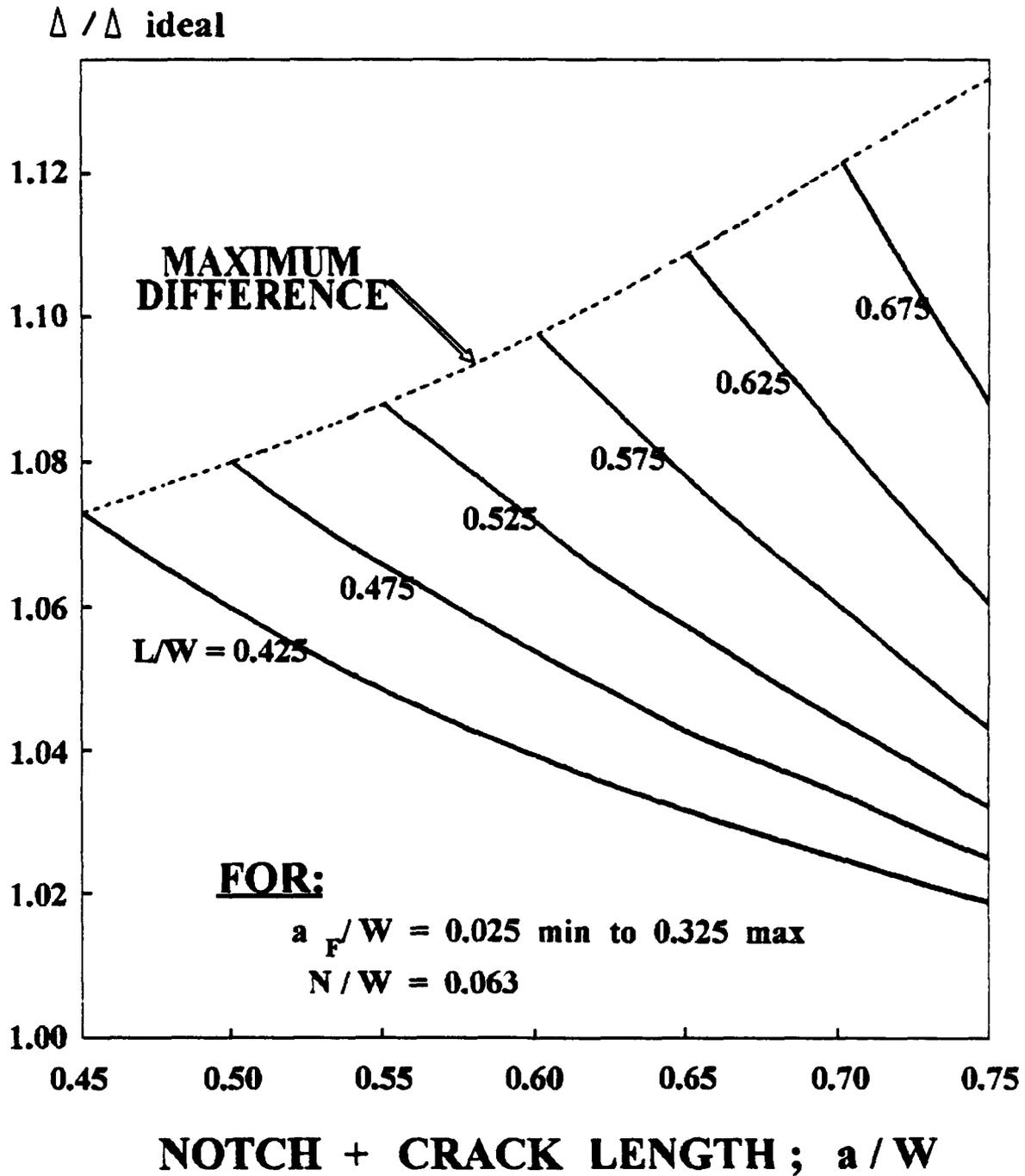


Figure 5 Compliance Differences for Three-Point Bend Specimens with Various Lengths of Idealized Crack and Notch Plus Crack; notched configurations are: $\tau=90^\circ$, $N_c/W=0.2$, $L_c/W=0.1$

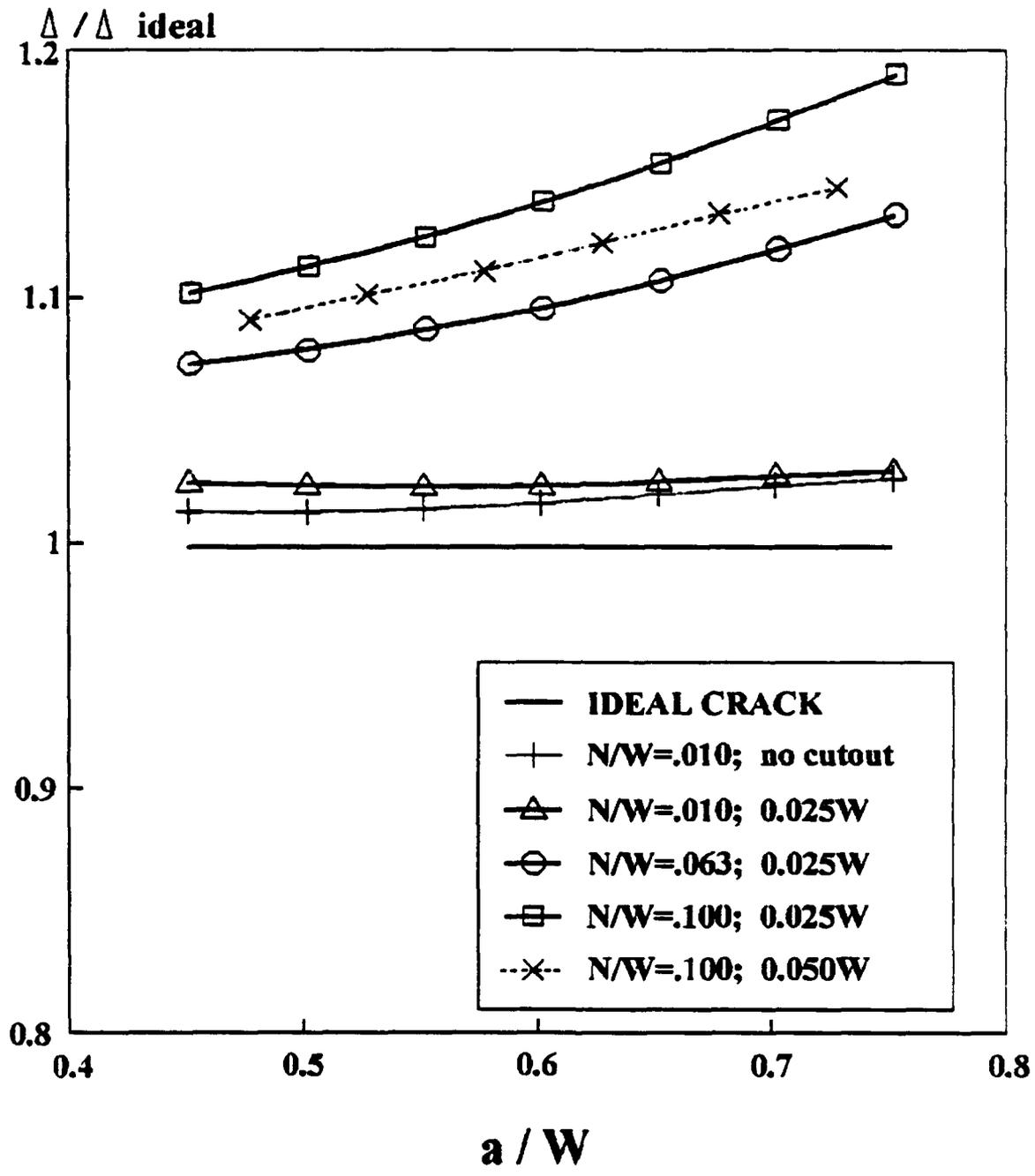


Figure 6 Effects of Notch Width (N), Cutout Dimensions (N_c , L_c), and Crack Extension (a) on Load-Line Compliance for Three-Point Bend Specimens; notched configurations are: $\tau=90^\circ$, $N_c/W=0.2$, $L_c/W=0.1$

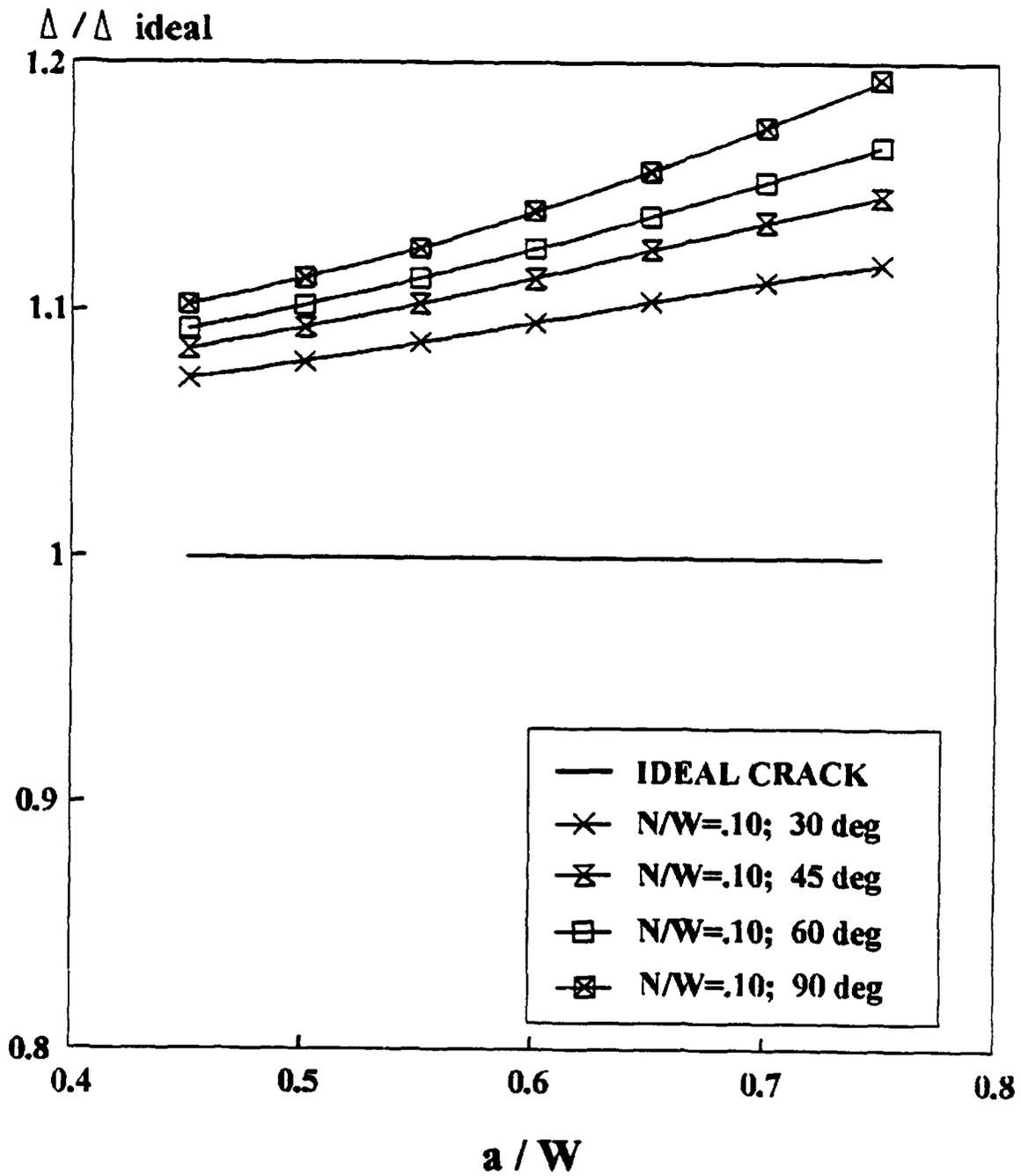


Figure 7 Effects of Notch-Tip Angle (α) on Load-Line Compliance for Three-Point Bend Specimens; notched configurations are: $a_p/W=0.025$, $N_p/W=0.2$, $L_p/W=0.1$

APPENDIX

Compliance Due to Cutout

The displacement due to an additional cutout, such as that shown in Figure 2 to accommodate a displacement gage, is required to obtain the total displacement of the beam configuration. All that is needed is the superposition of the two cases shown in Figure 3. This contribution is then added to the compliance given by Eq. 1.

The increase in strain energy of a notched beam due to shear loading and beam bending can be obtained from reference 2, Eqs. 40 and 42, respectively. With $\tau = \pi$, $\alpha_n = 0$, and redefining $\Omega = \Omega_c = L_c/W$, then for case *a*, $b = b_c$ and for case *b*, $b = b$. Substitution of these values into the appropriate equations cited above and subtracting the strain energy due to case *b* from that due to case *a* results in the following:

$$U_a - U_b = \frac{[3P^2/2BE] [(b/(2W(1-\Omega_c)))^2 - (b_j/(2W(1-\Omega_c)))^2 - 2(b/(2W(1-\Omega_c)) - b_j/(2W(1-\Omega_c)))] - [3(1+\mu)P^2 (b/W - b_j/W) / (10BE)]}{(Eq. 6)}$$

Since the displacement of the beam due to the cutout, δ_c , is

$$\delta_c = d/dP (U_a + U_b) \quad (Eq. 7)$$

Then

$$\Delta_c = \frac{3[(b/2W)^2 - (b_j/2W)^2]/(1-\Omega_c)^2 - 2(b/2W - b_j/2W)((1+\mu)/5 + 1/(1-\Omega_c))}{(Eq. 8)}$$

which is added to Eq. 1 to obtain the total compliance Δ_T .

Equation 8 accounts for the additional compliance due to a cutout. Although this equation ignores the compliance due to discontinuities at the corners of the cutout, this variance should be relatively small.

Crack Envelope Angle

Historically, the crack envelope angle, β , shown in Figure 2, has been used (see ASTM Methods E561 and E647) to insure that the fatigue crack extension to the notch is sufficiently large enough that the stress intensity factor (or compliance) is not overly influenced by the notch configuration. With the aid of Figure 2 the envelope angle is readily defined in terms of the notch configuration as follows:

$$\beta = 2 \tan^{-1}[N/2(a_n + a_p)]$$

and since $a_n = N/2(\tan(\tau/2))$, then

$$\beta = 2 \tan^{-1}[1/(1/\tan(\tau/2) + 2a/N)] \quad (Eq. 9)$$

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