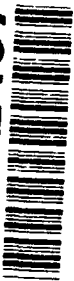AD-A254 478

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ANGLE OF ARRIVAL FLUCTUATIONS FOR
MILLIMETER WAVES PROPAGATING IN A TURBULENT
ATMOSPHERE NEAR THE EARTH

Irvin W. Kay

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I. INTRODUCTION

In recent years the possible use of a millimeter wave radar for guidance of a direct-fire, surface-to-surface, anti-tank missile has created an interest in the effect of atmospheric turbulence on millimeter wave propagation near the ground. Despite well planned measurement programs (Refs. 1-2) followed by analysis of the resulting data (Refs. 3-4), based on much experience with other wavelengths and propagation at higher altitudes (Refs. 5-7), the likely contribution of turbulence to the radar tracking error remains uncertain.

This uncertainty is the result of several factors associated with the particular nature of the tactical requirements imposed on the system. It is assumed here that the radar frequency of interest is 94 GHz and that the propagation path is parallel to, and 1 to 2 meters above, the ground. It is also assumed that target ranges of interest are from 1 to 5 km. For this combination of parameters, questions arise concerning the validity of certain approximations used (cf. Ref. 5) in the theory of propagation in turbulence. In addition, some errors and misinterpretations of the theory in prior analyses (Refs. 1-4) have contributed to the problem.

The objective of the present document is to estimate the likely angle-of-arrival error based on data reported in References 2 and 3 and to assess the reliability of the estimate. The conclusion based on that assessment leads to a recommendation that another experimental program be conducted under a greater variety of temperature and humidity conditions than those encountered in the Flatville, Illinois, experiment (Ref. 2), and which would include direct angle-of-arrival measurements.

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II. SOME DEFINITIONS AND BASIC THEORY

A. LOCAL HOMOGENEITY AND THE STRUCTURE FUNCTION

Reference 5 and its updated version (Ref. 8) by the same author, V.I. Tatarski, provide the fundamental theoretical results used in the analyses of the millimeter wave turbulence propagation experiments mentioned in the introduction. The theory is based on characterizing the turbulent atmosphere as one with an index of refraction that is a spatial, i.e., three dimensional, random process.[1]

In some applications such a process may be statistically homogeneous,[2] which, if it were true in the present case, would imply that the auto-correlation of the index of refraction is a function satisfying

$$\overline{\left[n\left(\vec{r}_1\right) - \overline{n\left(\vec{r}_1\right)}\right]\left[n\left(\vec{r}_2\right) - \overline{n\left(\vec{r}_2\right)}\right]} = K\left(\vec{r}_1 - \vec{r}_2\right) \quad ,$$

$$\overline{n\left(\vec{r}_1\right)} = \overline{n\left(\vec{r}_2\right)} = \text{const.} \quad , \tag{1}$$

where the overline indicates an ensemble average.

If the process is also isotropic then

$$K\left(\vec{r}_1 - \vec{r}_2\right) = K(\rho) \quad ,$$

where

$$\rho = \left|\vec{r}_1 - \vec{r}_2\right| \quad .$$

However, for the turbulent atmosphere the index of refraction is *not* homogeneous in general. It is more often the case that the index is *locally* homogeneous, which is defined by a weaker condition, valid over some restricted region,

---

[1] Reference 5 refers to the spatial random process as a random field.

[2] The term "homogeneous" is a generalization to three dimensions of the term "stationary," which refers to a one-dimensional process.

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$$\overline{n(\vec{r}_1) - n(\vec{r}_2)} = M(\vec{r}_1 - \vec{r}_2) \quad ,$$

$$\overline{\left[n(\vec{r}_1) - n(\vec{r}_2) \right]^2} = D_n(\vec{r}_1 - \vec{r}_2) \quad .$$

The first equation states that the mean of the difference between the values of the index of refraction at two points is a function of the displacement vector from one point to the other. The quantity defined on the left-hand side of the second equation is called a structure function.

In particular,

$$\overline{\left[n(L, y_1, z_1) - n(L, y_2, z_2) \right]^2} = D_n(\rho) \quad , \tag{2}$$

where

$$\rho = \sqrt{\left(y_2 - y_1 \right)^2 + \left(z_2 - z_1 \right)^2} \quad .$$

That is, ρ is the distance between two points lying in the same y, z plane, defined by x = L. In (2) it is assumed that propagation is in the direction of increasing x and that the structure function is defined by (2) over the fixed orthogonal plane at x = L. Reference 5 derives similar structure functions for the phase and for the log amplitude fluctuations of a plane wave and of a spherical wave propagating through a turbulent medium characterized by an index of refraction satisfying a relation of the form (2).

The turbulence process has two fundamental parameters: l_0, which is the inner scale size and lower bound of the so-called inertial subrange where the Kolmogoroff turbulence model predicts a $\kappa^{-11/3}$ dependence (Ref. 6), and L_0, which is the outer scale size and upper bound of the inertial subrange. Temperature fluctuation observations near the ground suggest that the quantity L_0 is roughly one third the height of the observation point above ground (Ref. 6). Other measurements (Ref. 6) indicate that the quantity l_0 is of the order of millimeters.

B. THE WAVE EQUATION AND THE RYTOV APPROXIMATION

It is assumed that the scalar wave equation

$$\nabla^2 u + k^2 n^2 (\vec{r}) u = 0 \tag{3}$$

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governs the electromagnetic propagation, where n is the index of refraction for the assumed inhomogeneous atmosphere and, because the inhomogeneities are small, has the form

$$n(\vec{r}) = 1 + n_1(\vec{r}) \; , \quad |n_1(\vec{r})| \ll 1 \; .$$

Tatarski (Ref. 5, Ch. 7) uses what he calls "the method of smooth perturbations," but which is more commonly known as the Rytov method (cf. Ref. 7), to find approximate solutions of a pair of differential equations that are equivalent to (3),

$$\Delta\psi_0 + \left(\nabla\psi_0\right)^2 + k^2 = 0 \; , \tag{4a}$$

$$\Delta\psi_1 + \nabla\psi_1 \cdot \left(2\nabla\psi_0 + \nabla\psi_1\right) + 2k^2 n_1(\vec{r}_1) = 0 \; , \tag{4b}$$

where

$$\psi_0 + \psi_1 = \psi \; ,$$

$$\psi = \log u \; ,$$

and a term $k^2 n_1^2$ has been dropped because it is small compared to the term $2k^2 n_1$. With the assumption that ψ_1 does not change much over a distance of the order of a wavelength, the quadratic term in the gradient of ψ_1 can also be dropped, leaving

$$\Delta\psi_1 + 2\nabla\psi_0 \cdot \nabla\psi_1 + 2k^2 n_1(\vec{r}) = 0 \tag{5}$$

in place of (4b).

Choosing

$$u_0 = A_0 e^{ikx} \Rightarrow \psi_0 = ikx \; ,$$

which provides a solution to equation (4a) to use in (5), and neglecting the second derivative of ψ_1 with respect to x in (5), leads to the equation

$$\frac{\partial^2 \psi_1}{\partial y^2} + \frac{\partial^2 \psi_1}{\partial z^2} + 2ik \frac{\partial \psi_1}{\partial x} + 2k^2 n_1(\vec{r}) = 0 \; . \tag{6}$$

The differential equation (6) embodies the Rytov approximation, sometimes called the parabolic approximation because of its form.

Reference 5 (p. 128) shows that the Rytov approximation is valid, given the condition

$$L \ll l_0^4 / \lambda^3 \ , \tag{7}$$

where λ is the wavelength. Obviously, the parameters associated with the systems of interest in this paper will not satisfy (7). However, it is clear from its derivation in Reference 5 that the condition is overly restrictive. The derivation of the Rytov approximation starts with the integral representation

$$\psi_1(\vec{r}) = \frac{k^2}{2\pi} \int_V n_1(\vec{r}') e^{-ik(x-x')} \frac{e^{ik|r-r'|}}{|\vec{r}-\vec{r}'|} \ dV' \ ,$$

which leads to the approximate relation for ψ_1 that satisfies (6):

$$\psi_1(\vec{r}) \sim \frac{k^2}{2\pi} \int_V n_1(\vec{r}') \frac{\exp\left(ik\frac{\rho^2}{2(x-x')}\right)}{x-x'} \ dV' \ ,$$

where

$$\rho = \sqrt{(y-y')^2 + (z-z')^2} \ .$$

This result is obtained by introducing the series expansion

$$k|\vec{r}-\vec{r}'| = k(x-x') + \frac{k\rho^2}{2(x-x')} + \frac{k\rho^4}{(x-x')^3} + \dots$$

and dropping all terms of order higher than quadratic in ρ. Reference 5 obtains the condition (7) by treating the neglected term of 4th order as an estimate of the error and using the argument that

$$\rho \sim \theta L \sim \frac{\lambda L}{l_0} \ ,^3$$

and

$$x - x' \sim L \ ,$$

in the important region of integration. In deriving (7) Reference 5 does not take into account the directionality of the antenna beam, however, a narrow beamwidth would justify dropping terms of higher order in powers of θL without the necessity of imposing the

---

3 The lateral distance ρ is the chord of a circular arc with radius L and subtending an angle θ.

condition (7). An argument analogous to that of Reference 5 would lead to the condition with the antenna aperture diameter D replacing l_0 in (7):

$$L \ll \frac{D^4}{\lambda^3} ,$$

which is much weaker than (7) for the systems of interest here.

Another justification for asserting that the atmospheric inhomogeneities contributing to the forward propagating field will be primarily those lying in a narrow cone about the forward direction is the general principle that multiple scattering is attenuated in a random medium. The translator's note $\underline{d}$ on p. 265 of Reference 5 gives an argument, as well as a reference [I. Kay and R.A. Silverman, "Multiple Scattering by a Stack of Dielectric Slabs," *Nuovo Cimento*, V. 9, Series X, Supplemento No. 2,626 (1958)], lending support to this principle.

In terms of a turbulent atmospheric index of refraction, assumed to be a locally homogeneous and isotropic random variable, Reference 5 uses the Rytov approximation, i.e., a solution of (6), to derive the log amplitude and phase structure functions for a wave propagating through the medium. From these results, based on the Kolmogorov-Obukhov model of turbulence for the index of refraction structure function (cf. Ref. 5, p. 31), Reference 5 derives an expression for the variance of the log amplitude χ of the propagating wave (Ref. 5, p. 153)

$$\overline{\chi^2} = 0.31 \, C_n^2 k^{7/6} L^{11/6} , \tag{8}$$

where C_n^2 is the structure parameter characterizing the index of refraction structure function in the Kolmogoroff turbulence model. The corresponding log intensity variance can be obtained from (8) by multiplying the right hand side by 4.

Experimental evidence, based on a measurement of intensity, indicates (cf. Ref. 9) that the Reference 5 prediction of the log amplitude fluctuation variance, calculated by means of the Rytov approximation, saturates (i.e., levels off and ceases to grow) with increasing distance, breaking down when, according to Ref. 7, the Rytov value for the log intensity variance corresponding to (8),

$$\sigma_I^2 = 1.23 k^{7/6} C_n^2 L^{11/16} ,$$

exceeds 0.3.[4] On the other hand, Reference 7 shows that even when

$$\sigma_I^2 \gg 1$$

the structure function for phase fluctuations changes only slightly.

C. THE KOLMOGOROV-OBUKHOV TURBULENCE MODEL AND ANGLE-OF-ARRIVAL FLUCTUATIONS

Reference 5 uses the Kolmogorov-Obukhov model of turbulence (cf. Ref. 5, p. 21) for the structure function of the index of refraction (Ref. 5, p. 150):

$$D_n(r) = \begin{cases} C_n^2 r^{2/3} & \text{for } l_0 \ll r \ll L_0 \ , \\ C_n^2 l_0^{2/3} \left(\dfrac{r}{l_0}\right)^2 & \text{for } r \ll l_0 \ , \end{cases} \qquad (9)$$

which implies a locally isotropic medium. Associated with the structure function D_n is the spectral density Φ_n; they are related by (cf. Ref. 5, p. 47)

$$D_n(\vec{r}) = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(1 - \cos\vec{\kappa}\cdot\vec{r}\right) \Phi_n(\vec{\kappa}) \, d\vec{\kappa} \ .$$

For a locally isotropic medium this becomes

$$D_n(r) = 8\pi \int_0^{\infty} \left(1 - \frac{\sin\kappa r}{\kappa r}\right) \Phi_n(\kappa) \, \kappa^2 \, d\kappa \ . \qquad (10)$$

For the structure function given by (9) Reference 5 suggests[5] a spectral density given by

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \ , \qquad (11)$$

where it is assumed that $\Phi_n(\kappa)$ vanishes for values of κ larger than some finite value. Reference 8 (p. 76) uses a more precisely defined variation of (11):

---

4   According to Reference 9, the threshold for saturation is 1.5 for σ_I rather than 0.3 for σ_I^2. However, Reference 9 states that saturation occurs for propagation distances "as little as 250 m," while Reference 7 refers to saturation for propagation distances over 1.0 km.

5   N.b., the structure function given by (9) is defined only for values of r in a region that, as is evident from the restrictions placed on r in (9), is not, itself, precisely defined. Therefore, some ambiguity exists in the corresponding spectral density.

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$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp\left(-\kappa^2/\kappa_m^2\right) \quad , \tag{12}$$

where

$$\kappa_m = \frac{5.92}{l_0} \quad .$$

Reference 5 (pp. 136-137) gives the equation

$$D_s(\rho) = 4\pi^2 k^2 L \int_0^\infty \left[1 - J_0(\kappa\rho)\right] \left(1 + \frac{k}{\kappa^2 L} \sin \frac{\kappa^2 L}{k}\right) \Phi_n(\kappa)\kappa d\kappa \tag{13}$$

for the phase structure function of an electromagnetic wave propagating in a locally homogeneous, isotropic turbulent medium. In addition, Reference 5 (p. 226-227) (cf. also Refs. 6-8) gives the equation

$$\overline{(\Delta\alpha)^2} = \frac{D_s(b)}{k^2 b^2} \tag{14}$$

for the variance of the corresponding angle-of-arrival fluctuations, where b is an interferometer separation or a telescope aperture diameter.

Assuming the Obukhov-Kolmogorov turbulence law, for the case of a plane wave, Reference 5 (p. 155) gives

$$D_s(\rho) = 2.91 \, k^2 \, L C_n^2 \, \rho^{5/3} \quad , \tag{15}$$

which is valid when

$$\rho \gg \sqrt{\lambda L} \quad ,$$

and (p. 156)

$$D_s(\rho) = 1.46 \, k^2 \, L C_n^2 \, \rho^{5/3} \quad , \tag{16}$$

which is valid when

$$l_0 \ll \rho \ll \sqrt{\lambda L} \quad .$$

For the circumstance that neither of these conditions is satisfied, Reference 5 (p. 155) gives the more general equation (which reduces to (15) or (16) when the appropriate condition is satisfied)

$$D_s(\rho) = 2.91 \, k^2 \, LC_n^2 \, \rho^{5/3} - 0.62C_n^2 \, L^{11/6} \, k^{7/6} \left[1 - b_A(\rho) \right] \quad , \tag{17}$$

where

$$b_A(\rho) = B_A(\rho)/B_A(0)$$

and

$$B_A(\rho) = 2\pi^2(0.033)C_n^2 \, k^2 \, L \int_0^\infty J_0 \, (\kappa\rho) \left(1 - \frac{k}{\kappa^2 L} \sin \frac{\kappa^2 L}{k} \right) \kappa^{-8/3} d\kappa \quad .$$

D. PHASE AND ANGLE-OF-ARRIVAL TEMPORAL POWER SPECTRUM

According to the so-called frozen turbulence hypothesis (attributed by Refs. 6 and 8 to G. Taylor) the temporal variations of a quantity observed at a point in the atmosphere can be attributed to the uniform motion of the atmosphere past the point. Assuming the frozen turbulence hypothesis, the phase structure function defined by (2) becomes

$$D_s(\rho) = D_s(v_\perp \tau) \quad , \tag{18}$$

where $v_\perp$ is the wind velocity component transverse to the electromagnetic propagation direction and τ is time. According to Reference 6, with the frozen turbulence assumption when $v_\perp \approx 0$, it is justifiable to neglect the influence of random motions of the medium occurring during the evolution of the turbulence only if the condition

$$L_0 \gg \sqrt{\lambda L}$$

is satisfied.[6]

Corresponding to the temporal function defined by (18) is the spectral function $w_S(f)$ defined by

$$D_s(v_\perp \tau) = 2 \int_0^\infty \left[1 - \cos(2\pi f\tau) \right] w_s(f) \, df \quad . \tag{19}$$

In terms of the quantity defined by

$$\Omega = f/f_0 \quad ,$$

---

[6] However, this condition will not usually be satisfied in the cases of interest here. On the other hand, according to Reference 10, the frozen turbulence model may still be valid for describing the high frequency part of the intensity fluctuation spectrum.

where

$$f_0 = \frac{v_\perp}{\sqrt{2\pi\lambda L}} \quad ,$$

Reference 8 (p. 268), derives[7]

$$w_s(f) = 8.2 \cdot 10^{-3} C_n^2 k^2 L v_\perp^{5/3} f^{-8/3} \quad , \quad \Omega \ll 1 \quad ,$$

$$w_s(f) = 4.1 \cdot 10^{-3} C_n^2 k^2 L v_\perp^{5/3} f^{-8/3} \quad , \quad \Omega \gg 1 \quad . \tag{20}$$

Using (20), Reference 8 also derives the spectral function $w_{\delta s}(f)$ of the spatial phase difference between two points in the plane $x = L$, separated by the distance ρ. The basic relation for this purpose is (p. 269)

$$w_{\delta s}(f) = 4 \sin^2\!\left(\frac{\pi\rho f}{v_\perp}\right) w_s(f) \quad , \tag{21}$$

from which Reference 8 obtains (p. 271)

$$w_{\delta s}(f) = 0.033\, C_n^2 k^2 L v_\perp^{5/3} \sin^2\!\left(\frac{\pi\rho f}{v_\perp}\right) f^{-8/3} \quad , \rho \gg \sqrt{\lambda L} \quad ,$$

$$w_{\delta s}(f) = 0.016\, C_n^2 k^2 L v_\perp^{5/3} \sin^2\!\left(\frac{\pi\rho f}{v_\perp}\right) f^{-8/3} \quad , \rho \ll \sqrt{\lambda L} \quad , \tag{22}$$

The spectral function corresponding to the angle-of-arrival fluctuations can be obtained from $w_{\delta s}(f)$ by dividing by $k^2\rho^2$. Figure 1 illustrates the curve corresponding to a normalized version $W(f)$ of the spectral function defined by (22).

---

[7] The numerical coefficients on the right-hand sides of the equations in (20) differ from those in Reference 6 by a factor of 0.25. The difference appears to be due to errors introduced into the Reference 6 equations (29) and (44), in which a factor of $\pi/2$ in the corresponding equations (15) and (34) of Reference 8 is replaced by the factor 2π.
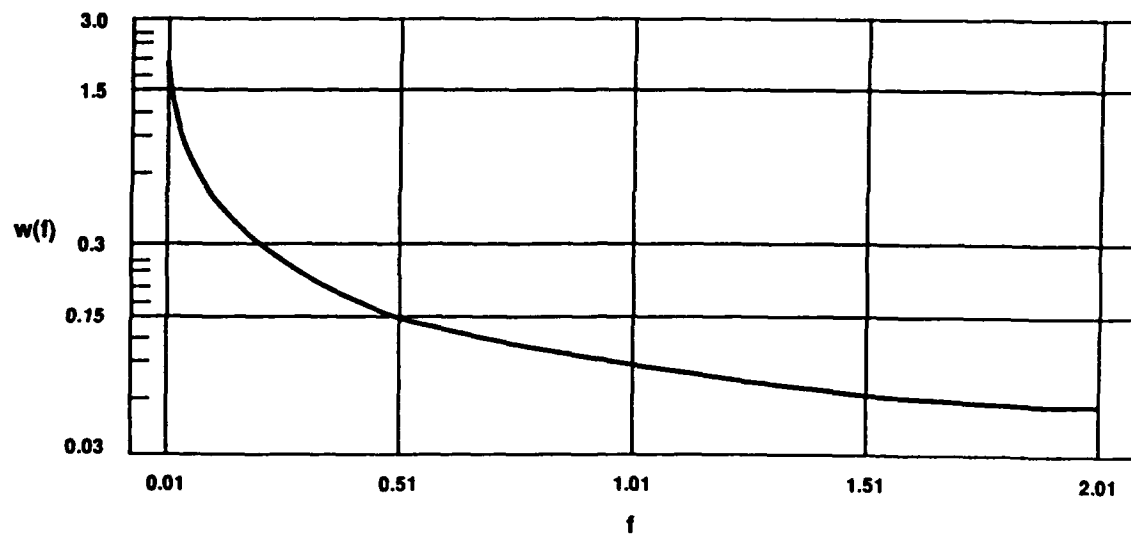
Figure 1.   Normalized Phase Difference Spectrum--Propagation In Turbulence

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III. MEASUREMENTS AND DATA ANALYSIS

A. THE REDSTONE ARSENAL MEASUREMENTS

Reference 1 describes an experiment at the Redstone Arsenal airport to measure atmospheric scintillation effects on electromagnetic wave propagation at 94 GHz 1.1 m above the ground over a range of 1232 m. To measure the scintillation at optical wavelengths the authors used a He-Neon laser over a range of 300 m.

The authors, who apparently assumed that the structure parameter is the same at optical as at millimeter wave frequencies, observe that values of the structure parameter C_n^2 reported in the literature at the time of their experiment differed widely, ranging from $4 \times 10^{-14} \, m^{-2/3}$ to $7 \times 10^{-12} \, m^{-2/3}$. They regard the higher value as suspect because it is so much higher than the other values. However, the measurements that led to the smallest value were made 50 m above the ground, whereas those leading to the largest were made only 2 m above the ground, where the turbulence is normally much larger.[8]

As for their own estimate of C_n^2: the optical value may be too small because it was based on measurements of log amplitude fluctuations over a distance of 300 m in the optical wavelength region, for which Reference 9 reports that saturation can be expected at a distance as short as 250 m. The effect of saturation is to reduce the measured intensity fluctuations, leading to an underestimate of C_n^2. In addition, according to Reference 3, in the Flatville experiment the structure parameters observed at the optical frequency were two orders of magnitude smaller than those observed at the millimeter wave frequency.

B. THE FLATVILLE, ILLINOIS, MEASUREMENTS

Reference 2 describes an experiment conducted by the National Oceanic and Atmospheric Administration (NOAA) and the Georgia Institute of Technology (GIT) near Flatville, Illinois, in five separate data-taking sessions: (1) June 3-July 15, 1983; (2) October 30-December 11, 1983; (3) February 5-March 9, 1984; (4) May 29-July 5,

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[8] Cf. Reference 6.

1984; (5) January 29-March 26, 1985. NOAA collected micrometeorological data and GIT millimeter wave propagation data.

Propagation observations included some with a CO_2 laser beam for the purpose of measuring the index of refraction structure parameter C_n^2 as well as the wind component orthogonal to the propagation path. Both the optical and millimeter wave paths were at levels about 3.7 meters above the ground.

The NOAA temperature and humidity data were used to calculate temperature and humidity structure and cross-structure parameters, which, in turn, were used to estimate C_n^2 for both the optical and millimeter wave frequencies. Reference 2 mentions several problems with the micrometeorological measurements during the first session as well as attempts to cure the problems by means of software.

Reference 2 gives an example of log intensity fluctuation variance determined during the first session, in July, 1983, for millimeter wave (173 GHz) propagation over a range of 1.374 km as a function of C_n^2. The values of C_n^2 were estimated by means of the micrometeorological and optical data at six different times of the day, with the results, as reported in Reference 2, shown in Table 1.

Table 1. Values of C_n^2 at Various Times During a Day in July 1983 at Flatville, Illinois (Ref. 2)

Time	8:40 a.m.	10:16 a.m.	11:30 a.m.	1:40 p.m.	3:50 p.m.	6:50 p.m.
C_n^2 (10^{-12} m$^{-2/3}$)	4.3	5.8	5.9	5.4	4.7	1.9

Reference 2 presents a plot of the log intensity fluctuation variance[9] obtained from millimeter wave observations at the same times versus the theoretical values of the variance as a function of C_n^2. The log intensity fluctuation variance calculations are based on the equation

$$\overline{l^2} = .5 C_n^2 k^{7/6} L^{11/6} \ ,$$

the right hand side of which is, as it should be, four times that given in Reference 5 (p. 188) for the log amplitude fluctuation variance of a spherical wave.

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[9] References 2 and 3 refer incorrectly to the quantity plotted as intensity instead of log intensity.

However, Reference 3, discussing the same experiment, reports somewhat different values for C_n^2, as shown in Table 2. Some values of the measured intensity fluctuation variance also appear to have been changed in the Reference 3 plot of the measured intensity fluctuation variance versus the theoretical based on C_n^2.

Table 2. Values of C_n^2 at Various Times During a Day In July 1983 at Flatville, Illinois (Ref. 3)

Time	8:40 a.m.	10:16 a.m.	11:30 a.m.	1:40 p.m.	3:50 p.m.	6:50 p.m.
$C_n^2 (10^{-12} m^{-2/3})$	4.3	5.3	6.2	5.2	4.2	1.9

Since the authors of Reference 3 include most of those who contributed to Reference 2 and the publication date of Reference 3 is three years later than that of Reference 2, it seems reasonable to suppose that the revisions are corrections of presumptive errors in the original work. Perhaps they were based on changes in the software that Reference 2 mentions was used for correcting measurement errors.

The abscissas for the points in the two plots, shown in Figs. 2 and 3, are the theoretical values of the intensity fluctuation variance based on the values of C_n^2 shown in Tables 1 and 2. The corresponding Reference 2 ordinates are: 0.006, 0.013, 0.016, 0.017, 0.018, 0.020. For Reference 3 they are the same except for the first and last, which change to 0.007 and 0.0225.

A linear regression (least square error) line fit to the points in the Reference 2 plot has a slope of 0.759 and an intercept of -3.83×10^{-4}, so that the line, in effect, passes through the origin of the coordinate system. For the revised Reference 3 points, the regression line has a slope of 0.796 and an intercept of -2.315×10^{-5}, so that the Reference 3 regression line passes even more nearly through the origin than the Reference 2 line. Evidently, in either case the measured data agree quite well with the Reference 5 theory, except that the values of C_n^2 appear to have been underestimated consistently.

The explanation for the low (off by 20 percent) C_n^2 estimates may be connected with the saturation phenomenon discussed in Reference 9: experimental data indicate that for propagation through turbulence at some optical wavelengths the growth of the log amplitude fluctuation standard deviation σ_χ as a function of distance levels off when $\sigma_\chi \approx 1.5$, which occurs at distances as small as 250 m. Reference 2 gives 1 km as the propagation path length of the laser beam used to estimate C_n^2, while Reference 3 gives the

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revised value of 670 m for the same path length. In either case, it may well be that the propagation distance is above what is needed for saturation to occur, even allowing for some difference in the CO_2 laser frequency and that of the optical beam in the experiment discussed in Reference 9.
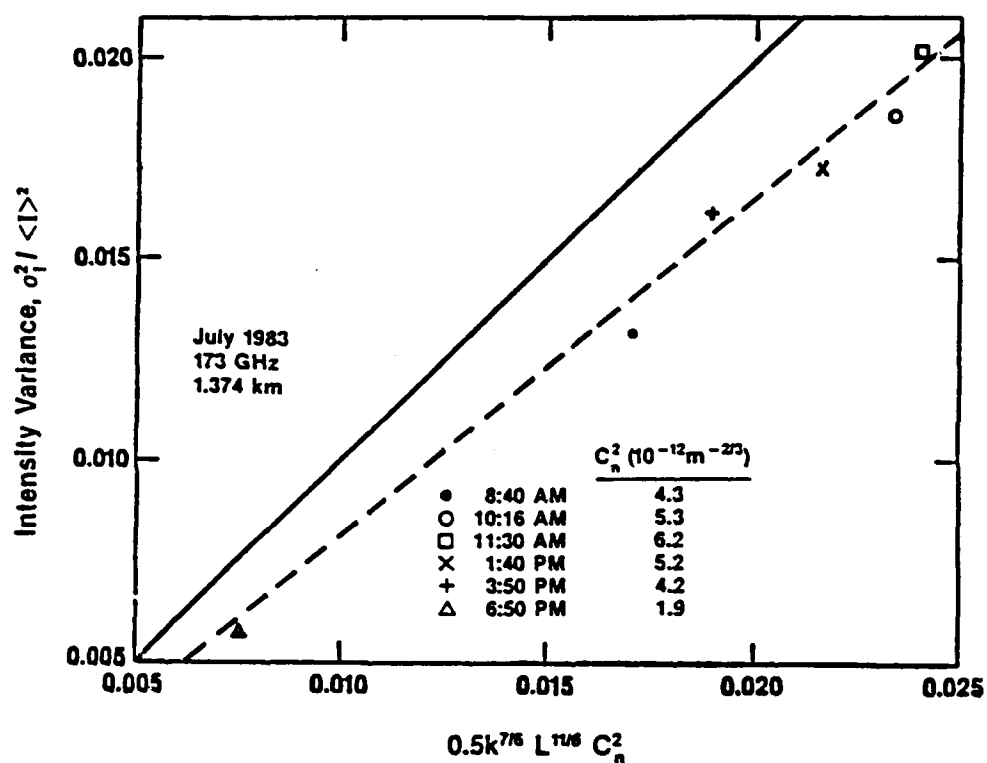

Figure 2. The Normalized Variance of Intensity, Measured at Flatville, Plotted as a Function of Its Theoretical Value. The solid line shows calculated values. The dashed line is a least square fit to the data.
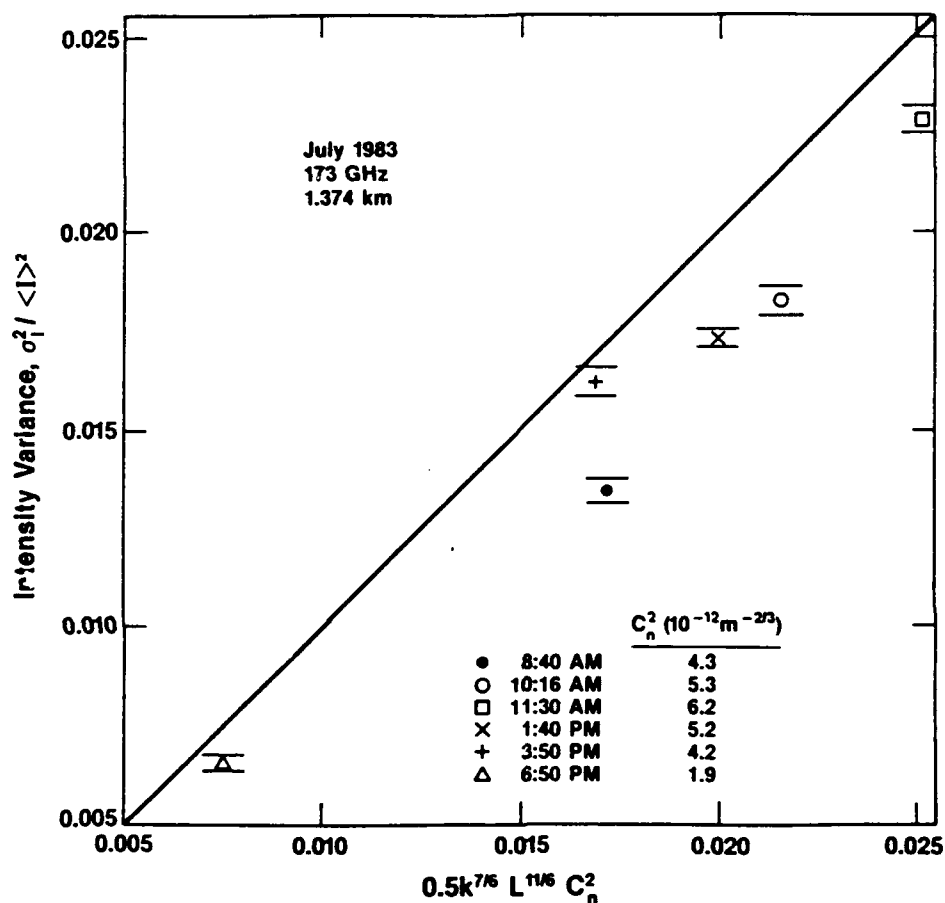
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Figure 3. The Normalized Variance of Intensity Versus Its Inertial-Range Formula. The straight line shows agreement with the inertial range formula. The symbol gives the average of the variances from antennas 1, 2, and 3. The vertical separation of the horizontal bars lying above and below each symbol is the average of the absolute value of the difference between the average variance and the variance from each individual antenna.
(Source *IEEE Transactions on Geoscience and Remote Sensing*,Vol. 26, No. 3, May 1988)

from one fifth to one third the height of the propagation path above the ground, while, on the other hand, according to Reference 2, the Flatville millimeter wave path was only about 3.7 m above the ground, which would make L_0 about 1.0 m.

Reference 4, using the values $C_n^2 = 5.9 \times 10^{-12}$ m$^{-2/3}$, $r = 1$ m, and a hypothetical range $L = 5000$ m in (23), finds that the associated standard deviation of the angle-of-arrival fluctuations is 126 μrad, and that extrapolating the Flatville data to the same conditions gives standard deviations ranging from 85 to 200 μrad. However, assuming the Flatville frequency of 173 GHz, which is equivalent to a wavelength of 1.73 mm, using the same parameter values in (17), and calculating the angle-of-arrival fluctuation variance by means of (14) gives a different range of angle-of-arrival fluctuation standard deviations corresponding to the Flatville measurements. This is evident from an examination of Table 3, which displays, for both the Flatville range of 1374 m and the 5000 m range used in the Reference 4 example, the standard deviations calculated this way using the C_n^2 values given in Table 2.

Table 3. Angle-of-Arrival Fluctuation Standard Deviations Based on Table 2 C_n^2 Values for $\rho = 1$ m, Wavelength = 3 mm

Time		8:40 a.m.	10:16 a.m.	11:30 a.m.	1:40 p.m.	3:50 p.m.	6:50 p.m.
Angle of Arrival	L=1374 m	125	139	150	137	123	82
σ μrad	L=5000 m	229	254	275	252	226	152

Reference 4, having remarked that the fluctuations observed at night were at least an order of magnitude smaller than those observed during the day, points out that for millimeter waves the worst case conditions for large angle-of-arrival fluctuations should occur on a hot, humid day. An example cited is measurements made at the White Sands Missile Range in the fall of 1987, a few days after the occurrence of unseasonable heavy rain. Apparently, larger than expected fluctuations of 94 and 140 GHz waves occurred. This was attributed to additional contribution to the turbulence due to an increased humidity structure constant resulting from strong sunlight evaporating the water in the wet ground.

Reference 4 concludes that amplitude fluctuations due to turbulence should not be a serious problem for the kind of millimeter wave radar application of interest but that, under worse case conditions, angle-of-arrival fluctuations may be. Otherwise, the conclusion is

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that in most circumstances even angle-of-arrival fluctuations should be less than a serious problem.

Reference 4 discusses the possibility of reducing the angle-of-arrival fluctuations by time averaging. However, the discussion is based on the assumption that the statistical behavior of the angle-of-arrival depends on phase fluctuation statistics, which, as remarked earlier, is erroneous.

IV. CONCLUSIONS

This document argues that Reference 2 underestimates certain index of refraction structure parameters derived from the Flatville measurements and that the Reference 4 method of calculating angle-of-arrival fluctuation statistics is incorrect theoretically. Nevertheless, the conclusion of Reference 4 that millimeter wave amplitude fluctuations should have a negligible effect on the performance of a direct fire, anti-tank missile system appears to be correct. On the other hand, the conclusion of this document concerning angle-of-arrival fluctuations is somewhat more pessimistic than that of Reference 4, which predicts that they will have an important effect on tracking accuracy only in hot, humid weather. This conclusion is based on its estimate of angle-of-arrival values corresponding to the Flatville structure parameter measurements and the observation that those values are large only near midday when both the temperature and humidity were high. However, the angle-of-arrival standard deviation values given in Table 3 of the present document, at least for the 5000 m range, are unacceptably large throughout the entire day.

Reference 4 recommends a measurement program to get values of the index of refraction structure parameter C_n^2 in other geographical locations, under different environmental conditions. This could settle the question of how much atmospheric turbulence will affect millimeter wave angle-of-arrival fluctuations in other than hot, humid weather, providing very useful data pertinent to the design and application of millimeter wave radar systems having propagation paths near the ground.

But, because the nature of the index of refraction spectrum is uncertain at the low frequency end when (cf. Ref. 5, p. 150)

$$\rho \approx \sqrt{\lambda L} \ ,$$

and because of the possibility of saturation at some ranges of interest, perhaps an even more useful addition to the program would be to accompany the measurements of intensity fluctuations, like those done at Flatville for the purpose of estimating angle-of-arrival statistics indirectly, with direct angle-of-arrival measurements. Indeed, Reference 10 reports on just such an experiment involving millimeter waves and outlines the method used for measuring angle-of-arrival fluctuations.

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