(2)

Image Acquisition & Processing Equipment for Machine Vision

Final Technical Report

Donald E. McClure, Principal Investigator

Grant Period: 1 December 1988 - 30 September 1990

Contracting Unit:

Office of Naval Research

for the

Department of Defense:

University Research Instrumentation Program

Grant No. N00014-89-J-1285

Contractor:

Brown University

Providence, Rhode Island 02912

Abstract

This report describes the equipment acquired with the support of Grant No. N00014–89–J–1285 to Brown University and some of the research projects carried out during the grant period, projects which have benefited from the availability of the scientific equipment.

Contents

1 Introduction **1**

2 Laboratory Configuration **3**

3 Research Projects **5**

3.1 3D Object Recognition 5

3.2 Deformable Templates 6

3.3 Computed Tomography and Related Inverse Problems 11

3.4 Radiation Therapy Treatment Planning 13

3.5 Film Restoration 17

3.6 Neural Networks 18

3.7 Speech Processing and Hidden Markov Models 19

1 Introduction

The Grant titled *Image Acquisition and Processing Equipment for Machine Vision* (N00014–89–J–1285) funded under the Defense University Research Instrumentation Program was used to obtain a sensor and state-of-the-art special-purpose computer equipment for research on problems of image analysis and computer vision. The basic research has been supported by contracts with the Mathematics Divisions of the U.S. Army Research Office and the Office of Naval Research. Specific projects include ONR contracts *A Mathematical Framework for Image Analysis* and *Mathematical Methods for Object Recognition* (just beginning) and ARO-supported *A Renormalization Group Approach to Image Processing, A New Computational Method for 3-dimensional Robot Vision, and The Complexity of the Cooling Algorithms* and

Center for Intelligent Control Systems. Principal Investigators of these projects include S. Geman, B. Gidas, U. Grenander and D.E. McClure at Brown University and D. Geman at the University of Massachusetts (ONR subcontract).

Our approach to problems in image analysis is strongly rooted in a mathematical statistical foundation. The foundation builds from Pattern Theory and adopts a Bayesian formulation of image inference problems.

While a primary objective of the research is to develop a unified *theoretical* framework to encompass low-level image processing and high-level computer vision, the work has continually been driven by *applications* and real data. Examples of motivating applications have included enhancement and segmentation of FLIR imagery, texture segmentations for SAR imagery, image reconstruction methods for single photon emission tomography, visual microdefect inspection of semiconductor wafers, and identification of biological shapes. We always bring the emerging theory back to the motivating application and attempt to implement algorithms for the driving problem.

A current thrust of our research is on vision problems, broadly speaking, the understanding of scenes in a three-dimensional world based on two-dimensional digital imagery. The items funded by the instrumentation grant extended our ability to study the three-dimensional problems. The hardware has been installed in an experimental laboratory for the acquisition of images, the computational analysis of image data and of possible approaches to vision problems, and the design and implementation of parallel algorithms for computationally intensive inference procedures.

In the next section, we shall give a brief overview of the equipment acquired and how the laboratory has been configured.

The third section includes capsule summaries of completed and ongoing research.

2 Laboratory Configuration

The main equipment items proposed and acquired include (i) an imaging laser radar for research experiments in 3D object recognition from range data, (ii) a SUN host system for user control of the laser radar and for research in computer vision, (iii) a parallel image and array processing board to support research in computer vision, including design of coarse-grained parallel algorithms, (iv) assorted networking hardware and supplies for connecting the laser radar system to other research computing systems, and (v) a medium duty-cycle PostScript printer for hard copy output of image analysis experiments. Each of these items is described in detail below.

Laser Radar. The laser radar is designed by the Advanced Intelligent Machines Division of Odetics. It is an imaging device that captures range and amplitude images of a field of view of moderate scale. The amplitude-modulated imaging laser radar includes a scanning head transmitter/receiver, radar output and digital storage device, and power supply. It produces Digital Equipment Corporation standard DRV11–W output for 128×128 reflectance (amplitude) and range (phase) images. Each image subtends a $60° \times 60°$ field of view and has a 30.7-foot ambiguity interval, with either 1.44-inch or 0.72-inch range resolution. A frame is acquired in about 0.8sec.

SUN Host Computer. The laser radar (ladar) delivers output image data through a DRV11 interface. In order to control the functions of the laser radar and to receive and store the acquired images, the ladar is connected via a local area network to a SUN host system. The primary host is a SUN 4/360 server, with VME bus, 32MB main memory, 688MB disk, 60MB cartridge tape drive, 19-inch color monitor and two client SPARCstation I desktop workstations. The SUN network provides immediate access for the graduate students, post-doctoral fellows and faculty affiliated with our research team. Since its initial acquisition, the SUN system has been upgraded, using funds from other sources, with additional hard disks, a 150MB tape drive, CD reader, and important software (Matlab, APL II, compilers, ...). The network is interconnected with other research computer networks giving ready access to additional shared software and hardware resources (SGI workstations, Abekas Digital Disk Recorder, VTR equipment, Mathematica, TeX, ...).

Parallel Image and Array Processing Board. The SUN VME-bus server also hosts a vision performance-enhancement board. The Androx ICS–400 image and array multi-processor board contains four Analog Devices ADSP2100 8MHz digital-signal-processing (DSP) nodes, each with 16K $\times$ 24 bits code RAM, 16K $\times$ 24 bits coefficient RAM, and two 16K-words data caches. The board also has a Texas Instruments TMS34010 graphics system processor, which serves as a data-bus master and gives the system enhanced graphics capabilities in addition to an ideal architecture for image analysis. There is 8MB of high-speed video RAM (an increase from the 2MB available at the time the proposal was prepared), which is reconfigurable for a variety of image resolutions and for color images. Other features include: 1MB scratch memory; 4 input channels (RS170, RS330, PAL) with 2K $\times$ 8 bit LUTs; 3 output channels with 16K $\times$ 8 bit LUTs. Software support includes a C-callable image processing library and DSP assembler programming tools.

Networking. Essential for the installation of the ladar and SUN host is networking hardware permitting its integration with other research computing resources. Network hardware and supplies includes ethernet tranceivers, one multiport tranceiver, thinnet cable, PC-ethernet cards, and SUN PC-NFS software.

PostScript Printer. One QMS PS820 TURBO PostScript laser printer is available on the SUN system. It's main purpose is to permit good quality half-tone images for work with the ladar and other image analysis work. Examples of the image copy produced by the printer are included in §3.1.

Brown University renovated and made available two large rooms at the main building of the Division of Applied Mathematics for housing the ladar and SUN system. The facility is conveniently located for immediate access by students and faculty.

3 Research Projects

In this section, we describe some of the completed and ongoing research projects which have used the equipment provided by this grant. The SUN host and Androx board were installed early in the grant period (Fall 1989) and have had a greater impact on research up to this time than has the laser radar. The laser radar was received in late spring 1990 and is currently being configured for easy experimental use through the implementation of a convenient, menu-driven user interface.

3.1 3D Object Recognition

3D object recognition is a major focus of a research project commencing in October 1990 and funded by the Office of Naval Research [2]. An abstract of the project follows:

Abstract. *Mathematical Methods for Object Recognition*, Principal Investigators: S. Geman, B. Gidas, U. Grenander, D.E. McClure, and D. Geman (subcontract), ONR Contract N00014-91-J-1021.

Probabilistic models are now widely used as a basis for developing algorithms for low-level image analysis problems. This research group has played a leading role in laying the mathematical foundations for the use of Markov random field models and in demonstrating the effectiveness of model-based approaches. The proposed research focuses on high-level vision problems and the development of appropriate mathematical frameworks for designing effective algorithms. Four areas to be explored are: rigid object recognition, nonrigid object recognition, two and three dimensional global shape models, and recovery of surface and shape information from optical and remotely sensed images. The mathematical research will be done in combination with controlled experiments with real data, including optical, laser radar, SAR, and X-ray images.

The laser radar will be used as one component of the rigid body recognition work. We shall study effective algorithms for the use of range data for object recognition and for the use of multi-sensor data. Figures 1 and 2 depict registered range and amplitude images acquired using the ladar. In the range image, displayed intensity is proportional to distance of a point from the ladar. The amplitude image resembles normal low-light video.

3.2 Deformable Templates

Deformable templates are being developed by several members of our group as a tool for global analysis of images in which objects with identifiable structure, but highly variable forms from one instance to another, occur. Common examples are biological shapes, such as a human hand; the structure with a thumb and four fingers is immediately recognizable, at least to a human observer, but one individual's right hand can vary greatly from another's.

A recent monograph [8] lays the groundwork for probabilistically deformable templates.

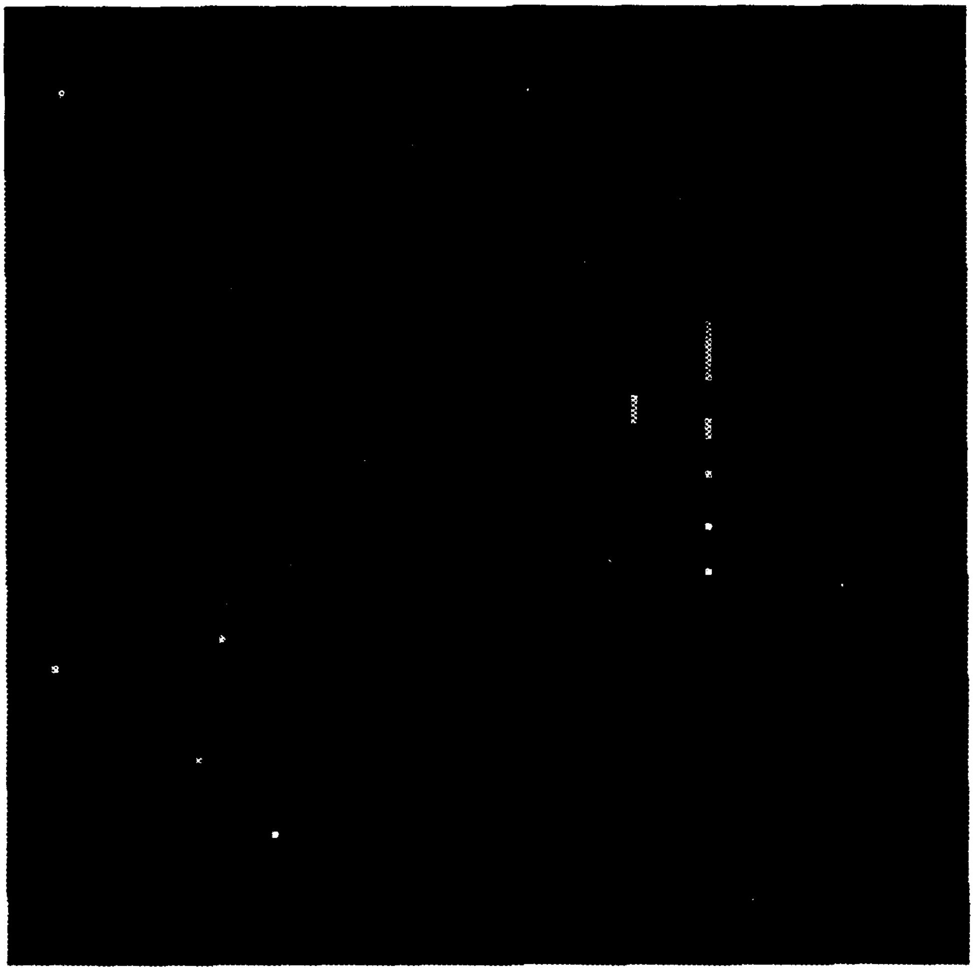

Figure 1: Range Image from Laser Radar

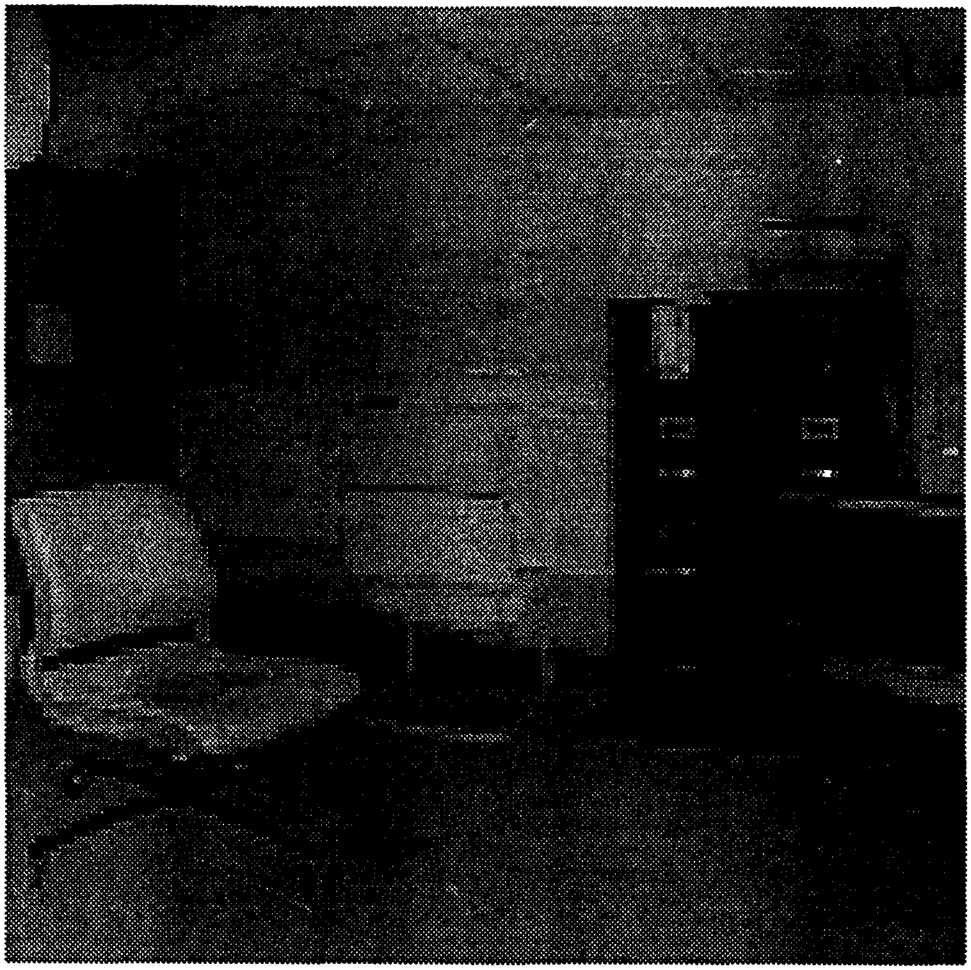

Figure 2: Amplitude Image from Laser Radar

One recently completed project [1] and one ongoing project have made extensive use of the SUN/ANDROX system for research on deformable templates.

Abstract. *Structural Image Restoration through Deformable Templates*, Y. Amit, U, Grenander and M. Piccioni, *JASA*, submitted for publication.

Prior knowledge on the space of possible images is given in the form of a function or template in some domain. The set of all possible true images is assumed to be formed by a composition of that function with continuous mappings of the domain into itself. A prior Gaussian distribution is given on the set of continuous mappings. The observed image is assumed to be a degradation of the true image with additive noise. Given the observed image, a posterior distribution is then obtained and has the form of a nonlinear perturbation of the Gaussian measure on the space of mappings. We present simulations of the posterior distribution that lead to structural reconstructions of the true image in the sense that it enables us to determine landmarks and other characteristic features of the image, as well as to spot pathologies in it. Moreover, we show that the reconstruction algorithm is relatively robust when the images are degraded by noise that is not necessarily additive.

Abstract. *The Machine Recognition of Human Coronary Arteries*, (Invited Lectures) by S. Geman, J.Elion and K.M. Manbeck, June 1990.

Arteriography is a medical technique where an X-ray absorbing dye is injected into a patient's arteries. When X-rays are passed through the patient and collected, a medical specialist is able to visually examine the data and diagnose abnormalities such as blocked arteries. Arteriography is often used by cardiologists in order to examine a patient's coronary arteries. Arteriography has proven to be a valuable tool, however, one problem is particularly troublesome. There is a high variability among cardiologists in the interpretation of arteriograms. For this reason, it is desirable to have a consistent, reliable, machine interpretation of the coronary arteriogram.

There are two sources of variability present in human coronary arteries. The first is the natural variability one would expect between different people, the second comes from the stretching and twisting that occurs as the heart contracts. Even though there is variability, there is also a certain degree of *invariance*. This idea of variability superimposed on an invariant structure brings to mind Grenander's notion of the *deformable template*.

The notion of a deformable template is exploited to design an arteriogram recognition package based on a powerful mathematical model. The modeling process begins by specifying several "prototypical" coronary artery structures. The prototype arterial structures, or templates, are studied in a effort to quantify the variability between patients. The prototype images are also used to model the effects of the ribs and lungs which tend to degrade arteriograms.

Once a workable mathematical model of an arteriogram has been developed, a recognition algorithm is applied to real patient data. Prototypical templates are deformed into the observed data, and the arteriogram is recognized as that template which most closely matches the data. The process of determining the optimal match is accomplished by a dynamic programming algorithm similar to the ones that have been successfully implemented on the hidden Markov model approach to speech recognition. Based on the optimal template, the arteriogram recognition package segments and labels the observed arteriogram into its component parts.

3.3 Computed Tomography and Related Inverse Problems

Previously, members of our research group have pioneered the use of Bayesian statistical methods for emission computed tomography [5], [6]. In the earlier work, however, only limited use was made of data from real clinical emission-tomography systems and no experiments were carried out with practical methods for accommodating the effects of photon scattering.

Kevin Manbeck's dissertation [10] emphasizes the use of data from real single-photon-emission-tomography systems and, in his extensions of the Bayesian approach, he develops a semi-empirical approach based on well-understood models of the underlying physics for compensating for effects of Compton scattering. Computational experiments, which were intensive, made substantial use of the SUN/ANDROX system.

Abstract. *Bayesian Statistical Methods Applied to Emission Tomography with Physical Phantom and Patient Data*, by Kevin Monroe Manbeck, Ph.D., Brown University, May 1990.

In single photon emission computed tomography (SPECT), projection data Y are used to reconstruct an unknown isotope concentration map X. In this thesis, the physics of SPECT—including photon attenuation, photon scatter, and the Poisson nature of radioactive decay—are modelled in a probabilistic framework, and a physical phantom is used in experiments designed to measure the magnitude of these effects. The experimental results are incorporated into the entries of the discrete attenuated Radon transform $\mathcal{A}$. The matrix $\mathcal{A}$ specifies the process by which the original image X is transformed into the data Y. The model, studied from a Bayesian perspective, gives rise to a posterior probability distribution $P(X|Y)$. A convenient algorithmic

method of using the data Y to estimate X is presented. The algorithm, which iterates conditional expectations (ICE) of the posterior distribution, is implemented in order to reconstruct isotope concentrations. Isotope concentrations are estimated for both patient data and for physical phantom data.

Building on the work started in this thesis, S. Geman, K. Manbeck and D.E. McClure are writing a Chapter for the forthcoming volume on theory and applications of Markov random fields in image analysis [4]. Computational experiments were done on the SUN/ANDROX system.

Abstract. *A Comprehensive Statistical Model for Single Photon Emission Tomography*, by S. Geman, K.M. Manbeck and D.E. McClure, in *Markov Random Fields: Theory and Applications*, R. Chellappa and A. Jain, editors, Academic Press, 1991(?), in preparation.

A Bayesian approach to the reconstruction of SPECT images is presented. The approach requires two separate mathematical models: one model (called the likelihood or data model) describes the process which generates the data; the second model (called the prior model) captures certain prior knowledge about the image under study, via a probability distribution on the ensemble of possible reconstructions. This paper presents a general degradation model for the SPECT problem. In addition, experimental methods for measuring critical parameters in this model are presented. Concerning the prior model, a probability distribution is constructed that enforces a degree of smoothness while accommodating the discontinuities associated with boundaries. Experiments with physical phantoms and patient data demonstrate the effectiveness of the approach. Finally, the isotropic properties of the prior model are examined analytically in a suitable continuum limit.

Dr. Manbeck also participated in research projects concerned with deblurring of Hubble telescope images [11]. The work on recovering quantitative information about faint point sources in images affected by aberrations of the Hubble telescope was done in cooperation with scientists at the University of Massachusetts. The inverse problems encountered in deblurring the images are formally similar to the inverse problems encountered in dealing with Compton scattering in the context of computed tomography. Computational experiments were done on the SUN/ANDROX system.

Abstract. *Hubble Telescope Image Restoration by Statistical Methods*, by K.M. Manbeck, Technical report, Division of Applied Mathematics, Brown University, October 1990.

Two statistical methods are presented for removing blur from Hubble Space Telescope images. Images reconstructed by maximizing likelihood have some desirable characteristics, but suffer from a lack of regularization, as is expected. Images reconstructed in a Bayesian framework capture the necessary regularization with a prior distribution. The statistical methods are compared to conventional Fourier inversion techniques for deconvolution.

3.4 Radiation Therapy Treatment Planning

Only recently has there been any attempt to carefully model the physical processes in radiation therapy for the purpose of using the models for treatment planning. The type of mathematics that arises can be interpreted as a *dual* version of the tomography reconstruction problem. In treatment planning, however, the system of equations that describes a treatment plan may not have a physically realizable solution. Thus one is led to the goal of trying to find a "best" approximate solution among those that are physically realizable.

Christopher Raphael's thesis [12] carefully formulates the treatment planning problem as an optimization problem. The criteria of optimality are considered in terms of their clinical interpretations, as opposed to the mere mathematical convenience. The thesis research *per se* is now completed. Extensive computational experiments were done on the SUN/ANDROX system. Currently, Raphael is writing and revising drafts of the thesis.

Abstract (Thesis Plan). *Mathematics in Radiation Therapy Treatment Planning*, by Christopher Raphael, PhD Dissertation, Brown University, 1991 (expected).

> Ever since Marie Curie's discovery of the value of radium in treating cancer, radiation therapy has been a field of great importance in medicine. In its earlier days, application techniques were crude and the goals of treatment vague. Since then, CAT scanning procedures have developed which allow the precise differentiation of healthy tissue and tumor. In addition the modern teletherapy machines enable the clinician to deliver clearly delineated and carefully contoured fields of radiation. Now that so much more information and control are available, we feel acutely the need for a mathematical context which supports some notion of optimality in treatment. The main goal of this dissertation is to present this context and examine a wide variety of optimization problems ranging from the mathematically expedient to the most current and biologically motivated.
>
> The basic operator of this thesis, the *dose operator*, describes the relationship between a *treatment plan* and the *dose distribution* it produces. The treatment plan consists of a collection of sources of radiation and an associated collection of intensity profiles—one for each source. A source can be thought of as a point lying somewhere

outside the patient while an intensity profile describes the intensities of the various rays which emanate from the source. In practice this profile is controlled by constructing a lead shield of variable thickness. Once the treatment plan has been fixed, various physical phenomena such as attenuation, divergence, and scatter of radiation combine to produce a distribution of dose inside the patient. The dose operator which governs this relationship is linear.

Our first chapter depicts the dose operator in the context of $\mathcal{L}^2$. We develop the adjoint operator which is quite similar to the fundamental operator in tomographic medical imaging. The interesting relationship between these two operators leads to some simple yet extremely useful identities which are essential to the later chapters. Also in this chapter we construct our discrete model which is the framework in which we present our computer experiments. Due to the generality of this chapter, the discrete model is just a special case of the $\mathcal{L}^2$ theory already treated.

Next we treat the problem of optimization using the ambient $\mathcal{L}^2$ distance as a measure of optimality. That is, we formulate our notion of an "ideal" dose distribution and try to minimize the $\mathcal{L}^2$ distance between our potential treatment and this target. The optimization is complicated by the natural positivity constraint on our treatments since it is not possible to deliver a negative quantity of radiation. We look at this optimization first in the case where the sources are *known*. In this case the dose operator is linear and we develop several interesting algorithms which solve the minimization and prove convergence. When the sources are unknown, the dose operator has linear dependence on the intensity profiles but nonlinear dependence on the sources. In this situation we have developed a projection-pursuit-like algorithm for simultaneously choosing the sources and profiles. This algorithm guarantees convergence of the associated dose distributions to the target when we drop the positivity constraint. In the more realistic situation of constrained optimization with *unknown* sources we present a useful and reasonable algorithm but provide no asymptotic results.

Our next effort is to consider optimization of more realistic measures of error than $\mathcal{L}^2$ distance. We examine two of these. The first formulates the problem as a collection of dose constraints which are not mutually satisfiable. In this setup the object is to satisfy as many constraints as possible (or more accurately a weighted sum over the satisfied constraints). The second formulation attempts to quantify the *probability* of success of a given treatment. This is computed in terms of the probability of tumor control and the probability of serious complication, both of which we describe in terms of simple and interesting biological models. For both of these measures we combine relaxation techniques with gradient-descent-like algorithms to achieve our optimization. These formulations provide encouraging practical results on the computer by successfully choosing the "correct" treatment philosophy under various assumptions.

This thesis also examines some more theoretical issues pertaining to radiation therapy. In one chapter we consider the consequences of using the familiar Backprojection Operator from medical tomography as a model for the dose operator. This assumption corresponds to disregarding any degradation of the beam as it moves away from its source and assuming that the collection of sources consists of the entire continuum of possibilities. We discuss the inversion theorem of the Radon Transform which gives our ideal dose distribution as a backprojection of a certain treatment. Unfortunately this treatment contains negative values in general. We have examined some possible compromises although the chapter offers more *insight* into the fundamental mathematics of radiation therapy than it offers practical solutions.

A final chapter develops the null spaces of several simple models of the dose operator. In particular we develop the null spaces of the parallel beam model (no divergence), with no attenuation and then with constant attenuation. These null spaces are useful because they allow us to modify a treatment without changing the distribution of dose it produces. This can be used, for instance, to convert a treatment involving negative values into a more positive one, or to distribute dose more evenly over the different sources in a treatment.

3.5 Film Restoration

Recently, work has been done on extending the Bayesian paradigm for image restoration to frame sequences, such as occur in film, standard video, fluoroscopy, etc. A report is currently being prepared [7] that shows what can be achieved in using temporal as well as spatial information for removing various forms of degradation from a frame sequence. Computational experiments were done on the SUN/ANDROX system.

Abstract. *A Nonlinear Filter for Film Restoration and Other Problems in Image Processing*, by D. Geman, S. Geman and D.E. McClure, Technical Report, Division of Applied Mathematics, Brown University, 1990, in preparation.

A filter is proposed for removing noise and other types of degradation. An application is explored to enhancement of frame *sequences*, motivated by the problem of film restoration for the movie industry. Experiments are performed on sequences from a degraded black-and-white copy of a recently released movie. Both *temporal* and *spatial* information are used in the restoration. Temporal information is obtained from the preceding unprocessed frame. The smoothing must be *nonlinear* in order to preserve boundaries.

3.6 Neural Networks

At the Snowbird, Utah conference on neural networks in April, S. Geman and E. Bienenstock reported on experiments with various learning algorithms and highlighted the theoretical foundation that is already in place in the area of nonparametric statistics. A paper is in preparation [3], for publication in *Neural Computation*, reporting on further experiments and on the tradeoffs between the number of parameters to be learned and the size of a training set, viz. the "bias/variance dilemma" familiar in nonparametric statistics. Extensive computational experiments were done on the SUN/ANDROX system.

Abstract. *Neural Networks and the Bias/Variance Dilemma*, by S. Geman, E. Bienenstock and R. Doursat, Technical Report, Division of Applied Mathematics, Brown University (to be submitted to *Neural Computation*), 1990 (in preparation).

Feedforward neural networks trained by error backpropagation are examples of nonparametric regression estimators. We present a tutorial on nonparametric inference and its relation to neural networks, and we use the statistical viewpoint to highlight strengths and weaknesses of neural models. We illustrate the main points with some recognition experiments involving artificial data as well as handwritten numerals. In way of conclusion, we suggest that current-generation feedforward neural networks are largely inadequate for difficult problems in machine perception and machine learning, regardless of parallel-versus-serial hardware or other implementation issues. Furthermore, we suggest that the fundamental challenges in neural modeling are about representation rather than learning per se. The last point is supported by additional experiments with handwritten numerals.

3.7 Speech Processing and Hidden Markov Models

A question of great interest and of fundamental importance for understanding how generic is the practice of using hidden Markov models (HMMs) has recently been addressed by S. Geman, H. Künsch, and A. Kehagias. They have developed theoretical results on weak approximation of stationary, ergodic stochastic processes by HMMs. The results will be reported in the PhD dissertation of A. Kehagias [9] and a subsequent article. Kehagias's thesis plan describes the nature of these results. Ongoing computational experiments are making extensive use of the SUN/ANDROX system.

Abstract (Thesis Plan). *Approximation of Stochastic Processes by Hidden Markov Models*, by Athanasios Kehagias, PhD Dissertation, Brown University, 1991 (expected).

> In this thesis we restrict ourselves to stationary and discrete valued stochastic processes.
>
> A pair of stochastic processes (X, Y) is a *Hidden Markov Model* (HMM) if X (the state process) is a Markov process and Y (the observable process) is an incomplete observation of X. The observation can be deterministic or noisy and the observable can be a state or a state transition. Hence we have four possible types of HMM's.
>
> First we establish that *all types of HMM's are equivalent*, in the sense that, given any HMM of arbitrary type we can construct a HMM of any other arbitrary type, such that the two models have identical observable processes. Therefore all types of HMM's have the same modeling power.
>
> Second, we consider the problem of *Representation*: what kind of stochastic processes can we approximate with Hidden Markov Models? To make the question meaningful we define two types of stochastic process approximation: (a) *weak approximation*, based on the weak convergence of probability measures and (b) *cross entropy approxi-*

mation, based on the Kullback-Leibler informational divergence. Then we prove that *there is a sequence of HMM's (of increasing size) that approximate any ergodic stochastic process in the weak and cross entropy sense.*

Third, we consider the problem of *Consistent Estimation.* To approximate an ergodic process we need a sequence of HMM's of increasing size. For a fixed size Hidden Markov Model we can use the very efficient Baum algorithm to find the Maximum Likelihood parameters estimate. But will the sequence of estimates be consistent (i.e. will it converge to the true process)? The answer is: *the sequence of Maximum Likelihood Estimates will be consistent if the original process is ergodic, has strictly positive probability and conditional probability bounded away from zero.*

Fourth, we develop HMM models of the *raw speech signal* and demonstrate numerically consistency of Maximum Likelihood estimation.

Finally, we develop *Hidden Gibbs Models*, an analogue of HMM, and use these to model one dimensional speech signals and two dimensional images.

References

[1] Y. Amit, U. Grenander, and M. Piccioni. Structural image restoration through deformable templates. *J. Amer. Statis. Assoc.*, submitted for publication.

[2] D. Geman, S. Geman, B. Gidas, U. Grenander, and D.E. McClure. *Mathematical Methods for Object Recognition.* Technical Report, Division of Applied Mathematics, Brown University (Proposal to the Office of Naval Research, Contract No. N00014–91–J–1021), 1990.

[3] S. Geman, E. Bienenstock, and R. Doursat. *Neural Networks and the Bias/Variance Dilemma.* Technical Report, Division of Applied Mathematics, Brown University, to be submitted to *Neural Computation*, 1990 (in preparation).

[4] S. Geman, K.M. Manbeck, and D.E. McClure. A comprehensive statistical model for single photon emission tomography. In R. Chellappa and A. Jain, editors, *Markov Random Fields: Theory and Applications*, Academic Press, 1991(?) in preparation.

[5] S. Geman and D.E. McClure. Bayesian image analysis: An application to single photon emission tomography. In *1985 Proceedings of the Statistical Computing Section, American Statistical Association*, pages 12–18, 1985.

[6] S. Geman and D.E. McClure. Statistical methods for tomographic image reconstruction. *Bulletin of the International Statistical Institute*, 52:5–21, 1987.

[7] S. Geman, D.E. McClure, and D. Geman. *A nonlinear filter for film restoration and other problems in image processing.* Technical Report, Division of Applied Mathematics, Brown University, 1990 (in preparation).

[8] U. Grenander, Y.S. Chow, and D. Keenan. *HANDS, A Pattern Theoretic Study of Biological Shapes.* Springer-Verlag, 1991 (to appear).

[9] A. Kehagias. *Approximation of Stochastic Processes by Hidden Markov Models.* PhD thesis, Division of Applied Mathematics, Brown University, 1991 (expected).

[10] K.M. Manbeck. *Bayesian Statistical Methods Applied to Physical Phantom and Patient Data.* PhD thesis, Division of Applied Mathematics, Brown University, 1990.

[11] K.M. Manbeck. *Hubble telescope image restoration by statistical methods.* Technical Report, Division of Applied Mathematics, Brown University, 1990.

[12] C.S. Raphael. *Mathematics in Radiation Therapy Treatment Planning.* PhD thesis, Division of Applied Mathematics, Brown University, 1991 (expected).