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## Velocity Analysis by Residual Moveout

*Zhenyue Liu and Norman Bleistein*

### ABSTRACT

Velocity analysis by normal moveout encounters problems in handling dipping reflectors or lateral variation of velocity. Prestack migration provides a powerful tool to do velocity analysis, which is based on the following principle: the imaged depths at a common location are independent of source-receiver offset when the correct velocity is used. Conventional approaches, such as depth focusing analysis, generally involve iteration, which requires repeated prestack migration. In this paper, a residual moveout method for velocity analysis on multi-offset data is presented that needs only a single prestack migration. A number of theoretical problems in this method are studied. When the velocity has a lateral anomaly, we derive a formula to calculate the interval velocity from the stacking velocity by perturbation theory. A suggested data processing technique based on our method is composed of prestack migration with a constant velocity, velocity analysis, residual moveout, stacking, velocity conversion, and poststack residual migration.

### INTRODUCTION

Normal moveout, typically used to do velocity analysis in seismic data processing, is robust when reflectors are flat and velocity is laterally invariant. However, this method encounters difficulty when reflectors are dipping or velocity varies laterally. Some geophysicists, therefore, have concentrated on doing velocity analysis by migration, such as the focusing analysis and common location imaging methods (Jeannot, 1986; Al-Yahya, 1989). When the background velocity is correct, the imaged depths at a common location are independent of source-receiver offset. Otherwise, if an incorrect background velocity is used, the imaged depths at a common location change with offset. In this situation, a residual moveout is observed in common-location images of migrated data. The principle of velocity analysis is to choose a background velocity so that the common-location images are close to a horizontal alignment.

Conventional methods use iteration to correct velocities. In each step of iteration, prestack migration is required. Consequently, the total computation is so huge that application of this method is limited.

It would be desirable to estimate velocity directly from the residual moveout. If this is successful, only one prestack migration is required. To make this estimate, we need a quantitative relationship between the residual moveout and the error in background velocity. Some geophysicists applied this idea to velocity analysis (Doherty, 1976; Deregowski, 1990). However, no general formula was derived.

In this paper, the residual moveout is used to do velocity analysis on multi-offset migrated data. By means of the reflector equation, we derived a general formula for the residual moveout. Theoretically, an arbitrary background velocity can be used to do velocity analysis. However, we prefer using a constant background velocity so that the residual moveout is an explicit function of the velocity error. Furthermore, we may use Stolt migration for constant velocity; this is much faster than other algorithms. Under the assumption of a small offset, we obtain an analytic expression for the residual moveout that is an explicit function of background velocity, curvature of unmigrated data, and slope of unmigrated data. Furthermore, this residual moveout is independent of the dip of the reflectors when the true velocity is a constant. Parallel to NMO velocity analysis, we can use a semblance and velocity scans to do RMO velocity analysis. Therefore, the program for RMO velocity analysis is similar to the existing program for NMO velocity analysis, except in the residual term.

After velocity analysis, we obtain the stacking velocity that is assumed to equal RMS velocity for a laterally invariant medium. However, when velocity has an anomaly, this assumption is invalid. Lynn and Claebout(1982) proposed a formula that gives a relationship between the anomaly of the stacking velocity and the anomaly of the interval velocity. But this formula requires a stratified medium. In this paper, we derive a formula that is valid for arbitrary velocity. This formula computes the anomaly of the interval velocity from the anomaly of the stacking velocity by an integral equation.

We have two ways to do migration: residual prestack migration or residual post stack migration. We prefer the latter because poststack migration runs much faster than prestack migration. The residual moveout, stacking, and residual poststack migration are recommended to obtain the final structural image.

## REFLECTION EQUATION

Seismic signals consist of amplitude information and phase information. Under the high frequency assumption, phase information is simplified to travelttime information. Therefore, seismic signals can be described approximately by travelttime-offset curves. A travelttime-offset curve in seismic record corresponds to a specific reflector in the earth. Given a reflector, we can compute the travelttime-offset curve by modeling. On the other hand, given a travelttime-offset curve, we also can compute the reflector

position (subsurface) by migration. Now, we will show how the geometry of reflector is determined by the geometry of the traveltine-offset curve.

We shall denote by  $X$  a 2-D vector,  $X = (x, z)$ . Let  $x_s$  be source position and  $x_r$  be receiver position located on the horizontal datum surface  $L$  with  $y$  the midpoint and  $h$  the half-offset:

$$x_s = y - h, \quad x_r = y + h.$$

For any point  $X$  below the surface,  $\tau_s(x_s, X)$  and  $\tau_r(X, x_r)$ , respectively, denote traveltimes from  $x_s$  to  $X$ , and from  $X$  to  $x_r$ .

Suppose we know the total reflection traveltine  $T(y, h)$  and a background velocity  $c(x, z)$ . Then, for each half-offset  $h$ , the reflector is determined such that

$$\tau_s(x_s, X) + \tau_r(X, x_r) = T(y, h), \quad (1)$$

$$\frac{\partial \tau_s}{\partial y} + \frac{\partial \tau_r}{\partial y} = \frac{\partial T}{\partial y}, \quad (2)$$

where  $X = (x, z)$  is the point on the reflector. Equations (1) and (2) show: if we know the traveltine-offset curve  $T(y, h)$  (therefore,  $\partial T/\partial y$ ) for some  $h$ , we can compute a imaged depth  $z$  for a fixed location  $x$ . If  $c(x, z)$  equals the true velocity, then the imaged depth  $z$  is independent of offset  $h$ ; otherwise, for wrong background velocity,  $z$  varies with offset  $h$ . Consequently, the imaged depth provides us information on velocity.

Equations (1) and (2) are for the common offset case. Similarly, we have the reflector equation

$$\tau_s(x_s, X) + \tau_r(X, x_r) = T(y, h), \quad (3)$$

$$\frac{\partial \tau_r}{\partial x_r} = \frac{\partial T}{\partial x_r}, \quad (4)$$

for the common shot case; the equation

$$\tau_s(x_s, X) + \tau_r(X, x_r) = T(y, h), \quad (5)$$

$$\frac{\partial \tau_s}{\partial x_s} = \frac{\partial T}{\partial x_s}, \quad (6)$$

for the common receiver case; the reflector equation

$$\tau_s(x_s, X) + \tau_r(X, x_r) = T(y, h), \quad (7)$$

$$\frac{\partial \tau_r}{\partial h} - \frac{\partial \tau_s}{\partial h} = \frac{\partial T}{\partial h}, \quad (8)$$

for the common midpoint case. Among these cases, only common offset has a symmetric imaged-depth function, which allows for the possibility of approximating the imaged-depth function by a hyperbola.

Specifically, when the background velocity is the constant,  $c$ , then

$$\tau_s(X, x_s) = \rho_s/c, \quad \tau_r(X, x_r) = \rho_r/c,$$

where

$$\rho_s = \sqrt{(x_s - x)^2 + z^2}, \quad \rho_r = \sqrt{(x_r - x)^2 + z^2}.$$

For this case, equations (1) and (2) are simplified to

$$\rho_s + \rho_r = cT(y, h), \quad (9)$$

$$\frac{\partial \rho_s}{\partial y} + \frac{\partial \rho_r}{\partial y} = c \frac{\partial T}{\partial y}. \quad (10)$$

Next, we will study the quantitative properties of  $z(h)$  when  $c(x, z)$  differs from the true velocity.

## RESIDUAL MOVEOUT

When the background velocity  $c(x, z)$  differs from the true velocity, there is a deviation between the imaged depths of different offsets; i.e., a residual moveout from a horizontal alignment is observed. We expect that this residual moveout can be used to measure the error of the velocity and be independent of the dip of a reflector.

### Constant Velocity and Horizontal Reflector

Suppose that the true velocity and background velocity are constants and the reflector is horizontal. In this special situation,

$$T^2(y, h) = T^2(y, 0) + 4h^2/v^2,$$

$$\rho_s = \rho_r = \sqrt{(z^2 + h^2)} = \frac{c}{2}T(y, h),$$

so that

$$z^2(h) = \frac{c^2}{4}T^2(y, h) - h^2 = \frac{c^2}{4}T^2(y, 0) + (c^2/v^2 - 1)h^2 = z^2(0) + (c^2/v^2 - 1)h^2.$$

That is, the moveout is the hyperbola

$$z^2(h) = z^2(0) + (c^2/v^2 - 1)h^2. \quad (11)$$

Equation (11) shows that the residual moveout is the exact hyperbola for constant velocity  $v$  and horizontal reflector. This result is parallel to that of moveout in unmigrated data.

### General Case

For general velocity or arbitrary reflector, one should not expect a simple expression such as (11). Instead of that, we consider the asymptotic expression under the assumption of small offset.

First, because  $z$  is a symmetric function of  $h$ , we get

$$\frac{dz}{dh} \Big|_{h=0} = 0, \quad \frac{d^3z}{dh^3} \Big|_{h=0} = 0, \text{ etc.}$$

This implies the Taylor series expansion

$$z^2(h) = z^2(0) + \frac{1}{2} \frac{d^2z^2}{dh^2} \Big|_{h=0} h^2 + O(h^4). \quad (12)$$

Now let us try to estimate the second derivative,

$$\frac{d^2z^2}{dh^2} \Big|_{h=0}.$$

For fixed  $x$ , the midpoint  $y$  and the imaged depth  $z$  are functions of offset  $h$ . Differentiating equation (1) with respect to  $h$ , we have

$$\left[ \frac{\partial \tau_s}{\partial y} + \frac{\partial \tau_r}{\partial y} \right] \frac{dy}{dh} + \left[ \frac{\partial \tau_s}{\partial h} + \frac{\partial \tau_r}{\partial h} \right] + \left[ \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right] \frac{dz}{dh} = \frac{\partial T}{\partial y} \frac{dy}{dh} + \frac{\partial T}{\partial h}. \quad (13)$$

Using (2), we get

$$\left[ \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right] \frac{dz}{dh} = \frac{\partial T}{\partial h} - \left[ \frac{\partial \tau_s}{\partial h} + \frac{\partial \tau_r}{\partial h} \right]. \quad (14)$$

Notice that  $y$  is symmetric in  $h$ , so

$$\frac{dy}{dh} \Big|_{h=0} = 0.$$

**Lemma.** Suppose that  $f(x, y, z)$  is a smooth enough function. For any function  $y = y(x)$  and  $z = z(x)$ , if

$$\frac{dy}{dx} \Big|_{x=0} = 0, \quad \frac{dz}{dx} \Big|_{x=0} = 0,$$

then

$$\frac{d}{dx} f(x, y(x), z(x)) \Big|_{x=0} = \frac{\partial f}{\partial x} \Big|_{x=0}.$$

**Proof.** By the definition of the total differential,

$$\frac{df}{dx} \Big|_{x=0} = \frac{\partial f}{\partial x} \Big|_{x=0} + \left[ \frac{\partial f}{\partial y} \frac{dy}{dx} \right] \Big|_{x=0} + \left[ \frac{\partial f}{\partial z} \frac{dz}{dx} \right] \Big|_{x=0} = \frac{\partial f}{\partial x} \Big|_{x=0}.$$

This completes the proof.

Using this lemma, and

$$\frac{dy}{dh} \Big|_{h=0} = 0, \quad \frac{dz}{dh} \Big|_{h=0} = 0,$$

we get

$$\frac{d}{dh} \left( \frac{\partial T}{\partial h} - \left[ \frac{\partial \tau_s}{\partial h} + \frac{\partial \tau_r}{\partial h} \right] \right) \Big|_{h=0} = \frac{\partial^2 T}{\partial h^2} \Big|_{h=0} - \left[ \frac{\partial^2 \tau_s}{\partial h^2} + \frac{\partial^2 \tau_r}{\partial h^2} \right] \Big|_{h=0}. \quad (15)$$

Also, from  $\frac{dz}{dh} \Big|_{h=0} = 0$ , we have

$$\frac{d}{dh} \left[ \left( \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right) \frac{dz}{dh} \right] \Big|_{h=0} = \left[ \left( \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right) \frac{d^2 z}{dh^2} \right] \Big|_{h=0}. \quad (16)$$

Differentiating equation (14) with respect to  $h$ , and using (15) and (16), we set up the following equation for  $d^2 z / dh^2 \Big|_{h=0}$

$$\left[ \left( \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right) \frac{d^2 z}{dh^2} \right] \Big|_{h=0} = \frac{\partial^2 T}{\partial h^2} \Big|_{h=0} - \left[ \frac{\partial^2 \tau_s}{\partial h^2} + \frac{\partial^2 \tau_r}{\partial h^2} \right] \Big|_{h=0}. \quad (17)$$

Equation (17) holds for any velocity function  $v(x, z)$ , any background velocity  $c(x, z)$ , and an arbitrary reflector. Now we will simplify equation (17) for the laterally invariant background velocity.

Suppose the background velocity is the constant,  $c$ . Then

$$\tau_s = \sqrt{z^2 + (x - y + h)^2} / c, \quad \tau_r = \sqrt{z^2 + (x - y - h)^2} / c.$$

After calculation,

$$\begin{aligned} \frac{\partial \tau_s}{\partial z} \Big|_{h=0} &= \frac{\partial \tau_r}{\partial z} \Big|_{h=0} = \frac{z}{c\rho}, \\ \frac{\partial^2 \tau_s}{\partial h^2} \Big|_{h=0} &= \frac{\partial^2 \tau_r}{\partial h^2} \Big|_{h=0} = \frac{z^2}{c\rho^3/2}, \end{aligned}$$

where

$$\rho = \sqrt{z^2 + (x - y)^2}.$$

We have

$$\begin{aligned} \left[ \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right] \Big|_{h=0} &= \frac{2z}{c\rho}, \\ \left[ \frac{\partial^2 \tau_s}{\partial h^2} + \frac{\partial^2 \tau_r}{\partial h^2} \right] \Big|_{h=0} &= \frac{2z^2}{c\rho^3} = \frac{2}{c\rho} - \frac{2(x - y)^2}{c\rho^3}. \end{aligned}$$

Using these results, equation (17) is simplified by

$$\left[ \frac{2z}{c\rho} \frac{d^2 z}{dh^2} \right] \Big|_{h=0} = \frac{\partial^2 T}{\partial h^2} \Big|_{h=0} + \frac{2(x - y)^2}{c\rho^3} - \frac{2}{c\rho}. \quad (18)$$

Furthermore,

$$\left[ 2z \frac{d^2 z}{dh^2} \right] \Big|_{h=0} = \frac{d^2 z^2}{dh^2} \Big|_{h=0},$$

$$\rho = \frac{c}{2} T \Big|_{h=0},$$

$$\frac{(y-x)}{\rho} = \frac{c}{2} \frac{\partial T}{\partial y} \Big|_{h=0}.$$

Thus, we obtain the result

$$\frac{d^2 z^2}{dh^2} \Big|_{h=0} = \frac{c^2}{2} \left[ T \frac{\partial^2 T}{\partial h^2} + \left( \frac{\partial T}{\partial y} \right)^2 \right] \Big|_{h=0} - 2, \quad (19)$$

or

$$\frac{d^2 \tau^2}{dh^2} \Big|_{h=0} = 2 \left[ T \frac{\partial^2 T}{\partial h^2} + \left( \frac{\partial T}{\partial y} \right)^2 \right] \Big|_{h=0} - \frac{8}{c^2}, \quad (20)$$

where  $\tau$  is the migration time.

From equation (19), we conclude that after migration, the the main part of the residual moveout is determined by the background velocity  $c$ , traveltime  $T$ , curvature of unmigrated data  $\partial^2 T / \partial h^2$ , and slope of unmigrated data  $\partial T / \partial y$ . This is true for a constant background velocity  $c$ , any velocity function  $v(x, z)$ , and arbitrary reflector. In addition, the stacking velocity is defined by

$$\frac{1}{[c_{stk}]^2} = \frac{1}{4} \left[ T \frac{\partial^2 T}{\partial h^2} + \left( \frac{\partial T}{\partial y} \right)^2 \right] \Big|_{h=0}, \quad (21)$$

which can be directly estimated from the residual moveout. Compared to NMO, we have the new term,  $\partial T / \partial y$ . In the follow examples, one will see this term removes the dip effect in RMO velocity analysis.

For a small gradient velocity  $c(z)$  or a small dip of the reflector, equation (20) can be modified to

$$\frac{d^2 \tau^2}{dh^2} \Big|_{h=0} = 2 \left[ T \frac{\partial^2 T}{\partial h^2} + \left( \frac{\partial T}{\partial y} \right)^2 \right] \Big|_{h=0} - \frac{8}{[c_{rms}]^2}, \quad (22)$$

where  $c_{rms}$  is the root-means-squared velocity of  $c(z)$ .

### Examples For Constant Background Velocity

We use constant background velocity for these examples. For arbitrary  $c(x, z)$ , the residual moveout formula will become too complex to do velocity analysis. Furthermore, prestack migration is fastest for constant velocity. In addition to the normal moveout formula (11), we now compute residual moveout by the formula (19) and (20) for several special cases. These results are similar to normal moveout, except that the former is insensitive to the dip of the reflector.

### 1. Constant velocity and dipping reflector

Suppose that the true velocity is a constant,  $v$ , and the reflector dip is  $\theta$ . In this case,

$$T^2(y, h) = 4y^2 \sin^2 \theta / v^2 + 4h^2 \cos^2 \theta / v^2.$$

We have,

$$\left[ T \frac{\partial^2 T}{\partial h^2} \right] |_{h=0} = 4 \cos^2 \theta / v^2,$$

$$\frac{\partial T}{\partial y} |_{h=0} = 2 \sin \theta / v.$$

Therefore,

$$\frac{d^2 z^2}{dh^2} |_{h=0} = 2(c^2/v^2 - 1). \quad (23)$$

$$z^2(h) = z^2(0) + (c^2/v^2 - 1)h^2 + O(h^4). \quad (24)$$

or

$$\tau^2(h) = \tau^2(0) + (1/v^2 - 1/c^2)4h^2 + O(h^4). \quad (25)$$

Equation (25) shows that the residual moveout is independent of the reflector dip and the stacking velocity is  $v$ , when the true velocity  $v$  is a constant.

### 2. Constant velocity and diffraction from a scattering point

Suppose that the true velocity is a constant,  $v$ , and a scatterer is located at the point  $(x^*, z^*)$ . In this case,

$$T(y, h) = \left[ \sqrt{(y - h - x^*)^2 + (z^*)^2} + \sqrt{(y + h - x^*)^2 + (z^*)^2} \right] / v$$

We have,

$$\left[ T \frac{\partial^2 T}{\partial h^2} \right] |_{h=0} = \frac{4}{v^2} \frac{(z^*)^2}{(y - x^*)^2 + (z^*)^2},$$

$$\left( \frac{\partial T}{\partial y} \right)^2 |_{h=0} = \frac{4}{v^2} \frac{(y - x^*)^2}{(y - x^*)^2 + (z^*)^2}.$$

Therefore,

$$\frac{d^2 z^2}{dh^2} |_{h=0} = 2(c^2/v^2 - 1). \quad (26)$$

$$z^2(h) = z^2(0) + (c^2/v^2 - 1)h^2 + O(h^4). \quad (27)$$

or

$$\tau^2(h) = \tau^2(0) + (1/v^2 - 1/c^2)4h^2 + O(h^4). \quad (28)$$

Again, equation (28) shows that the residual moveout is independent of the lateral offset from the point scatterer and the stacking velocity is  $v$ , when the true velocity  $v$  is a constant.

### 3. Laterally invariant velocity and horizontal reflector

Suppose that the true velocity is a laterally invariant function,  $v(z)$ , and the reflector is horizontal, with depth  $z^*$ . In this case,

$$T^2(y, h) = T^2(y, 0) + 4h^2/[v_{rms}(z^*)]^2 + O(h^4).$$

We have,

$$\left[ T \frac{\partial^2 T}{\partial h^2} \right]_{h=0} = 4/[v_{rms}(z^*)]^2,$$

$$\frac{\partial T}{\partial y} \Big|_{h=0} = 0.$$

Therefore,

$$\frac{d^2 z^2}{dh^2} \Big|_{h=0} = 2 \left( \frac{c^2}{[v_{rms}(z^*)]^2} - 1 \right). \quad (29)$$

$$z^2(h) = z^2(0) + \left( \frac{c^2}{[v_{rms}(z^*)]^2} - 1 \right) h^2 + O(h^4). \quad (30)$$

Notice that  $z(0) \neq z^*$ . That is, in depth migration, the imaged depth is inconsistent with the desired point at which the root-mean-square (RMS) velocity is determined from the residual moveout. However, if we let

$$\tau^* = 2 \int_0^{z^*} \frac{ds}{v(s)},$$

then

$$\tau^2(h) = \tau^2(0) + \left( \frac{1}{[v_{rms}(\tau^*)]^2} - \frac{1}{c^2} \right) 4h^2 + O(h^4), \quad (31)$$

and  $\tau(0) = \tau^*$ . Therefore, time migration can give us the correct location at which the RMS velocity is determined by the residual moveout.

Equation (31) shows that when the true velocity is laterally invariant, the stacking velocity determined from RMO is consistent with RMS velocity at the imaged time.

### 4. Laterally invariant velocity and dipping reflector

Suppose that the true velocity is a laterally invariant function,  $v(\tau)$ , and the reflector is a dip, with angle  $\theta$ . In this case,

$$T(y, 0) = \int_0^{\tau^*} (1 - p^2 v^2(\sigma))^{-1/2} d\sigma,$$

$$\frac{\partial^2 T}{\partial h^2} \Big|_{h=0} = \frac{4}{\int_0^{\tau^*} v^2 (1 - p^2 v^2)^{-3/2} d\sigma},$$

$$\frac{\partial T}{\partial y} \Big|_{h=0} = 2p,$$

where  $\tau^*$  is the vertical time at the reflection point and

$$p = \frac{\sin \theta}{v(\tau^*)}.$$

Therefore,

$$\left[ T \frac{\partial^2 T}{\partial h^2} + \left( \frac{\partial T}{\partial y} \right)^2 \right] \Big|_{h=0} = 4 \frac{\int_0^{\tau^*} (1 - p^2 v^2)^{-3/2} d\sigma}{\int_0^{\tau^*} v^2 (1 - p^2 v^2)^{-3/2} d\sigma}. \quad (32)$$

The stacking velocity is estimated by

$$[c_{stk}]^2 = \frac{\int_0^{\tau^*} v^2(\sigma) (1 - p^2 v^2(\sigma))^{-3/2} d\sigma}{\int_0^{\tau^*} (1 - p^2 v^2(\sigma))^{-3/2} d\sigma}. \quad (33)$$

Notice that  $c_{stk}$  does not equal the RMS velocity  $v_2$  that is defined by

$$v_2^2 = \frac{1}{\tau^*} \int_0^{\tau^*} v^2(\sigma) d\sigma. \quad (34)$$

In fact,

$$[c_{stk}]^2 = v_2^2 + \frac{3}{2}(v_4^4 - v_2^4)p^2 + O(p^4), \quad (35)$$

where

$$v_4^4 = \frac{1}{\tau^*} \int_0^{\tau^*} v^4(s) ds.$$

$v_4$  is always greater than  $v_2$  and they are equal only if  $v(z)$  is a constant. Therefore,  $c_{stk}$  is always greater than  $v_2$  and they are close for small  $p$  or a small gradient of  $v(z)$ . Furthermore, the imaged time,  $\tau(0)$ , is different from  $\tau^*$ , and

$$[v_2(\tau(0))]^2 = v_2^2(\tau^*) + \frac{1}{2}(v_2^2(\tau^*) - c^2)(v^2(\tau^*) - v_2^2(\tau^*))p^2 + O(p^4). \quad (36)$$

Usually,  $v$  is bigger than  $v_2$ , so  $v_2(\tau(0))$  is bigger than  $v_2(\tau^*)$  when  $c$  is smaller than  $v_2(\tau^*)$ . If  $c$  is chosen suitably small,  $c_{stk}$  may be a good approximation to  $v_2(\tau(0))$ .

we conclude that for a suitable background velocity, the stacking velocity from RMO velocity analysis may approximate the RMS velocity at the imaged time.

### Higher Residual Terms

The residual moveout is hyperbola-like only for small offset. In fact, offsets should not be too small so that we can have a high resolution in velocity analysis. (See Liu and Bleistein.) Therefore, we require an error estimate for formulas in nonzero offset. This work is partially implemented in Mathematica.

Using the rule of differential for a compound function in equation (14) and setting  $c$  constant, the fourth order derivative of  $z$  with respect to  $h$  satisfies

$$\begin{aligned} \frac{d^4 z}{dh^4} \Big|_{h=0} \left[ \frac{\partial(\rho_s + \rho_r)}{\partial z} \right] \Big|_{h=0} &= c \left[ \frac{\partial^4 T}{\partial h^4} + 3 \frac{d^2 y}{dh^2} \frac{\partial^3 T}{\partial h^2 \partial y} \right] \Big|_{h=0} - \frac{\partial^4(\rho_s + \rho_r)}{\partial h^4} \Big|_{h=0} \\ &- 6 \left[ \frac{d^2 z}{dh^2} \frac{\partial^3(\rho_s + \rho_r)}{\partial h^2 \partial z} \right] \Big|_{h=0} - 3 \left[ \frac{d^2 y}{dh^2} \frac{\partial^3(\rho_s + \rho_r)}{\partial h^2 \partial y} \right] \Big|_{h=0}. \quad (37) \end{aligned}$$

For a constant velocity  $v$  and a dipping reflector with angle  $\theta$ , we can obtain

$$\frac{d^4 \tau^2}{dh^4} \Big|_{h=0} = -\frac{24(1/v^2 - 1/c^2) \sin^2 2\theta}{(z^*)^2}, \quad (38)$$

where  $\tau$  is the migration time and  $z^*$  is the reflection depth. Therefore, from (38) and (23), a more accurate expression for the residual moveout is

$$\begin{aligned} \tau^2(h) &= \tau^2(0) + 4 \left( \frac{1}{v^2} - \frac{1}{c^2} \right) h^2 - \frac{(1/v^2 - 1/c^2) \sin^2 2\theta}{(z^*)^2} h^4 + O(h^6) \\ &= \tau^2(0) + 4 \left( \frac{1}{v^2} - \frac{1}{c^2} \right) h^2 \left( 1 - \frac{1}{4} \left( \frac{h}{z^*} \right)^2 \sin^2 2\theta \right) + O(h^6). \end{aligned} \quad (39)$$

For a constant velocity  $v$  and a scattering point at  $(x^*, z^*)$ ,

$$\frac{d^4 \tau^2}{dh^4} \Big|_{h=0} = -\frac{24(1/v^2 - 1/c^2) \sin^2 2\theta}{(z^*)^2}, \quad (40)$$

where

$$\theta = \arctan \frac{y - x^*}{z^*}.$$

By the way, for the unmigrated data, the higher residual term is

$$\frac{\partial^4 T^2}{\partial h^4} \Big|_{h=0} = \frac{24 \sin^2 2\theta \cos^2 \theta}{v^2 (z^*)^2}. \quad (41)$$

Equations (38) and (40) show that when the true velocity is a constant, small higher-residual terms of moveout are obtained for a closed background velocity, small ratio of offset to the imaged depth, and the dipping angle that is near 0 or 90 degree.

For laterally invariant velocity and a horizontal reflector with depth  $z^*$ ,

$$\frac{d^4 \tau^2}{dh^4} \Big|_{h=0} = \frac{24(v_2^4 - v_4^4)}{t_0^2 v_2^8}, \quad (42)$$

where  $t_0$  is the zero-offset time, and

$$v_2^2 = \frac{1}{t_0} \int_0^{t_0} v^2(s) ds,$$

$$v_4^4 = \frac{1}{t_0} \int_0^{t_0} v^4(s) ds.$$

The result in (42) is the same as in the unmigrated data.

Equation (42) shows that when the true velocity is laterally invariant, a small higher-residual term is obtained for small gradient of the velocity.

## VELOCITY CONVERSION

In the previous chapter, we show how stacking velocity is obtained by moveout methods. However, an interval velocity function is required for migration and inversion. Usually, we assume that the stacking velocity equals the RMS velocity. Then the interval velocity is computed by Dix equation or other algorithms. Unfortunately, this assumption fails when the velocity has a lateral anomaly. Now we will give an equation to solve for the laterally varying interval velocity. This equation requires the small lateral variation and horizontal reflector.

Suppose the true slowness  $w(x, z)$  can be written as

$$w(x, z) = \bar{w}(z)(1 + \alpha(x, z)), \quad (43)$$

where  $\bar{w}(z)$  is a reference slowness and  $\alpha(x, z)$  is a small perturbation. We obtain the equation

$$\delta w_s(y, z) = \frac{1}{\bar{T}_0 \bar{v}_s(z)} \int_0^z \left[ \frac{\partial^2 \alpha}{\partial x^2} \left( \frac{\int_\sigma^z \bar{v} ds}{\bar{v}_s(z)} \right)^2 + \alpha \left( 1 + \frac{\bar{v}^2(\sigma)}{\bar{v}_s^2(z)} \right) \right] \frac{d\sigma}{\bar{v}} \quad (44)$$

where  $\delta w_s$  is the anomaly of the stacking slowness,  $\bar{T}_0$  is the zero offset time, and  $\bar{v}_s$  is the RMS velocity from the referenced velocity. When  $\alpha$  and  $\bar{w}$  are depth independent, the result in equation (44) is the same as that of Lynn and Claerbout. Equation (44) shows that the second derivative of  $\alpha$  determines the anomaly of the stacking velocity; the anomaly of the stacking velocity at a depth results from the anomaly of the interval velocity above this depth. Furthermore,  $\int_\sigma^z \bar{v} ds$  increases as  $\sigma$  decreases, so the anomaly of the interval velocity near the surface has the largest effect.

Applying Fourier transform, with respect to  $y$ , to equation (44), we obtain

$$\delta w_s(k_x, z) = \frac{1}{\bar{T}_0 \bar{v}_s(z)} \int_0^z \alpha(k_x, \sigma) \left[ -k_x^2 \left( \frac{\int_\sigma^z \bar{v} ds}{\bar{v}_s(z)} \right)^2 + \left( 1 + \frac{\bar{v}^2(\sigma)}{\bar{v}_s^2(z)} \right) \right] \frac{d\sigma}{\bar{v}}. \quad (45)$$

Equation (45) is a First-kind Volterra integral equation that is ill-posed. Therefore, the recursive algorithm for equation (45) is unstable. To obtain a stable solution, we may apply the damping least-squared method to equation (45).

## SYNTHETIC DATA EXAMPLE

To test the residual moveout method, we applied this method to synthetic seismograms computed for a subsurface model in which velocity increases linearly with depth  $z$ , according to  $v(z) = 1.5 + 0.8z$  km/s. The model, shown in Figure 1, consists of five reflectors, each with a dipping and horizontal segment. Dips for the dipping segments range from 30 to 90 degrees in 15-degree increments. The seismograms contain 10 offsets, ranging from 100 m to 1900 m in 200 m increments. Because of dipping reflectors and depth dependent velocity, the stacking velocity in equation (33)

and the RMS velocity are not same but close each other. The error between the both, shown in Figure 2, increases with depth and dip.

After prestack migration with the constant velocity,  $c = 1.5$  km/s, one of the CDP gathers is plotted in Figure 3. Because the background velocity is lower than the true velocity, all event locations increase with offset. The velocity scan for this CDP is plotted in Figure 4. Unlike the velocity scans in NMO, velocity peaks here are single-valued. After residual moveout, all events are corrected to horizontal ones (in Figure 5). The stacking result is shown in Figure 6, which is equivalent to the poststack migration with the constant velocity,  $c$ . By using the interval velocity converted from the stacking velocity, poststack residual migration gives the correct reflector positions (in Figure 7).

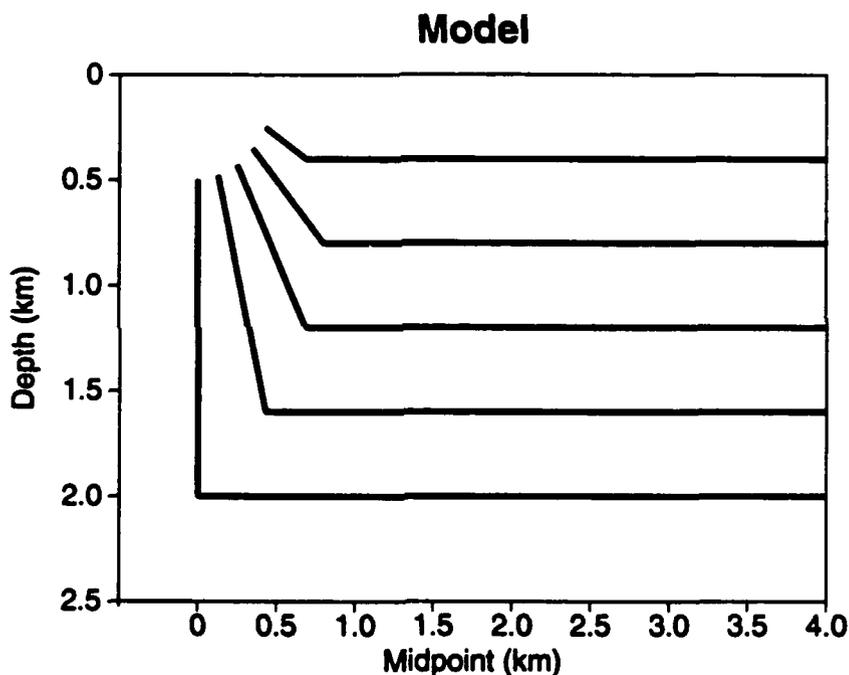


FIG. 1. Subsurface model used to generate synthetic seismic traces.

## CONCLUSION

Velocity analysis by prestack migration can handle dipping reflectors. Conventional approaches use iteration, which results in larger computation than the method proposed in this paper. Using the relationship between the residual moveout and the error in the background velocity, we can estimate directly the true velocity without iteration. Furthermore, stacking after the residual moveout provides a partial migration output. With this output, a residual post migration should yield a more accurate structural image. Using a more general background velocity, we may handle

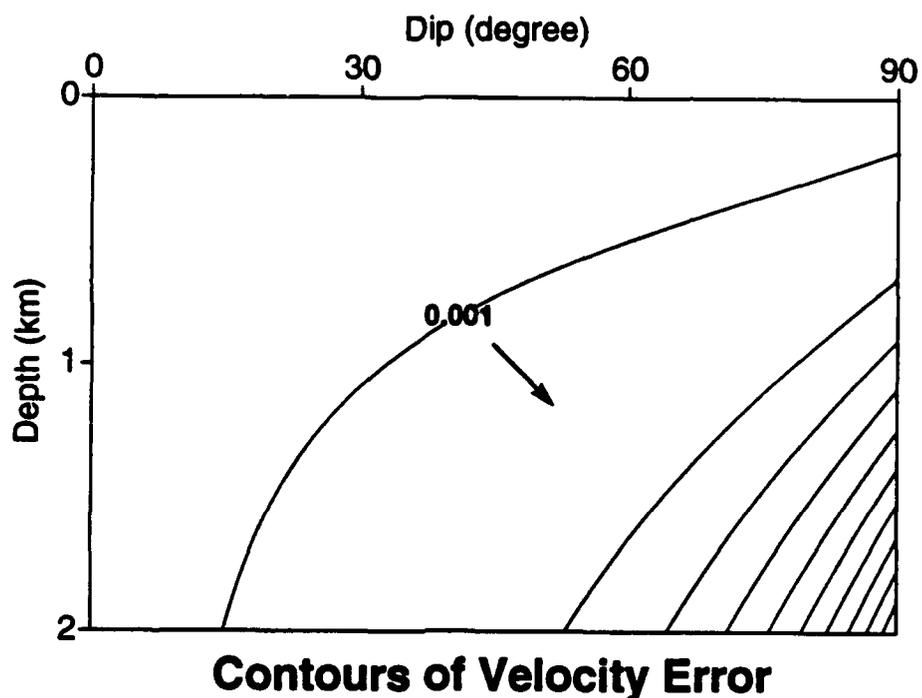


FIG. 2. The relative error between the stacking velocity and RMS velocity. The difference of contours is 0.002. The arrow direction indicates increase of the error.

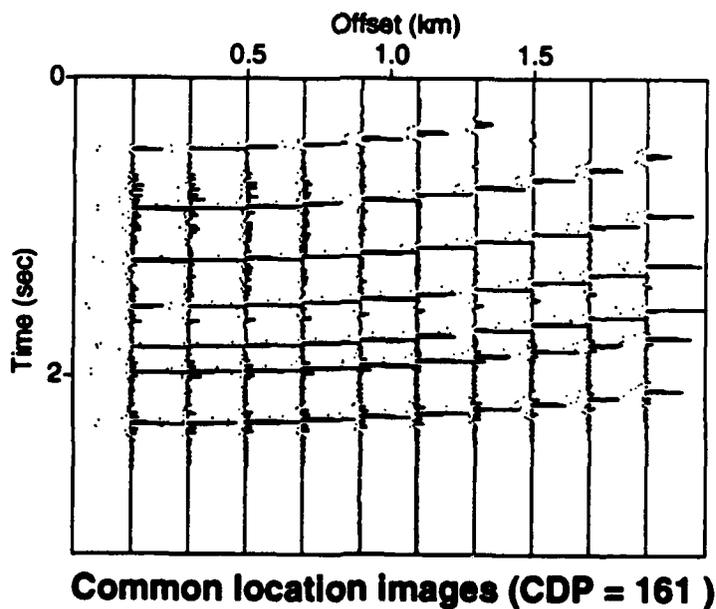


FIG. 3. One of the CDP gathers after the migration with the constant velocity.

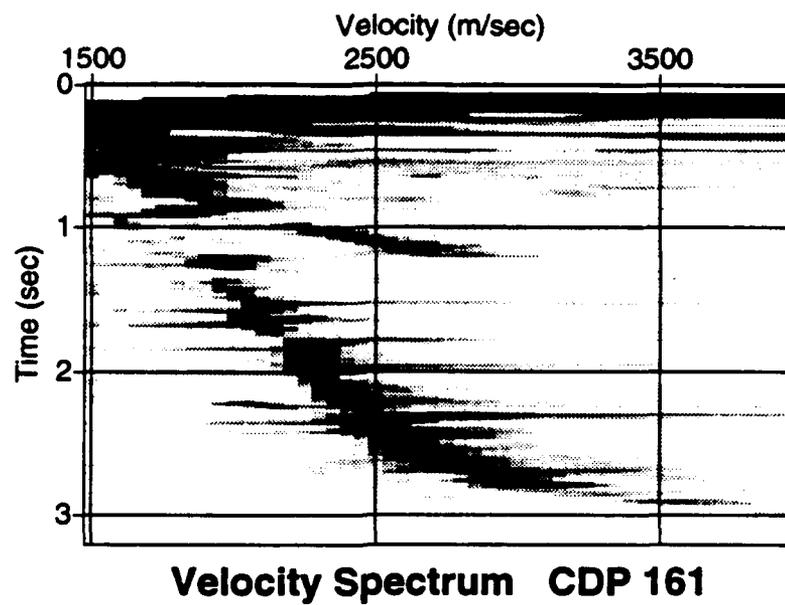


FIG. 4. Velocity analysis for the CDP gather in Fig. 2.

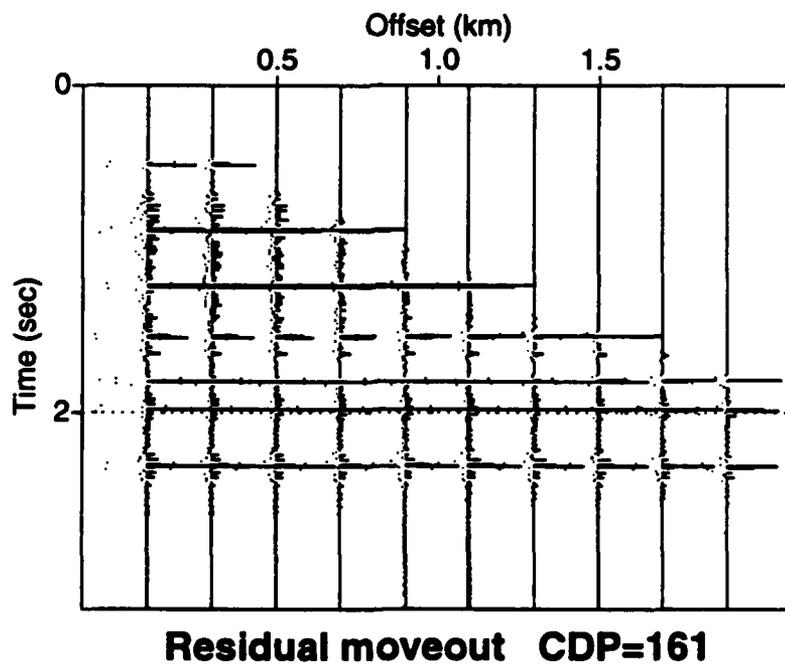


FIG. 5. Residual moveout for the CDP gather in Fig. 2.

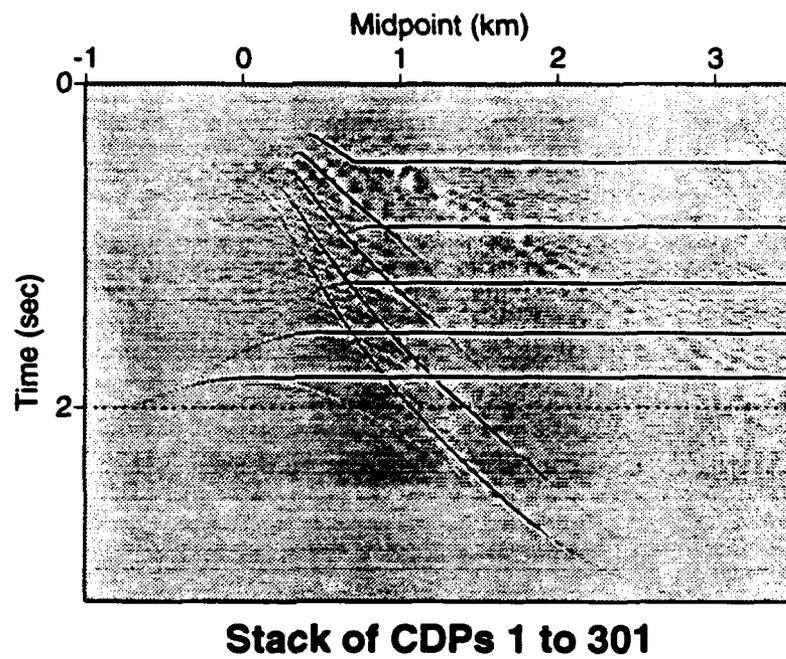


FIG. 6. Stacking for the ten offsets.

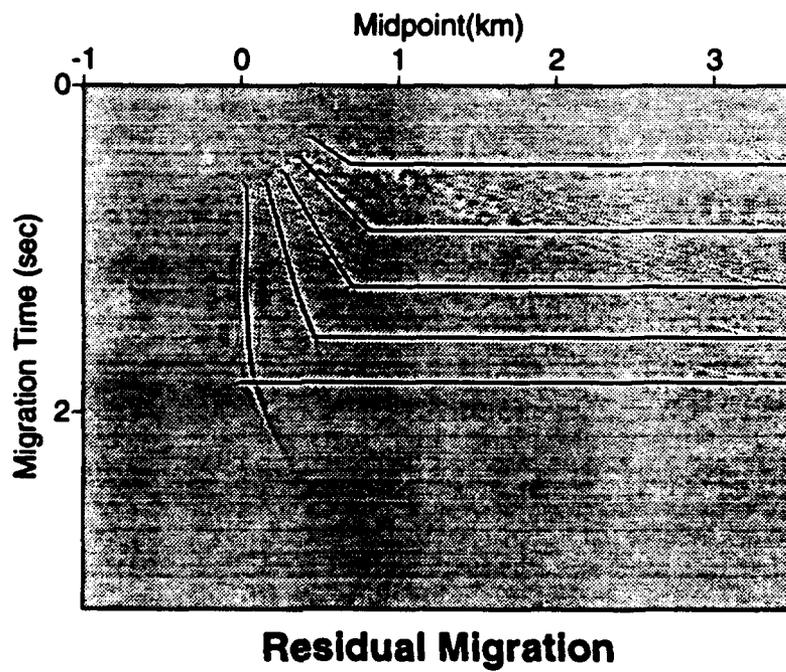


FIG. 7. Poststack residual time migration.

the lateral variation of velocity, but it will require a complicated algorithm. The perturbation method here is suggested to handle the lateral variation of velocity.

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### REFERENCES

- Al-Yahya, K., 1989, Velocity analysis by iterative profile migration: *Geophysics*, **54**, 718-729.
- Deregowski, S. M., 1990, Common-offset migrations and velocity analysis: *First break*, **8**, 6, 225-234.
- Dothry, S., and Claerbout, J., 1976, Structure independent velocity estimation: *Geophysics*, **41**, 850-881.
- Jeannot, J. P., Faye, J. P., and Dennelle, E., 1986, Prestack migration velocities from depth-focusing analysis: 56th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 438-440.
- Liu, Z. and Bleistein, N., 1991, Velocity analysis by inversion: Project Review In CWP Consortium Meeting.
- Lynn, W. S. and Claerbout, J. F., 1982, Velocity estimation in laterally varying media: *Geophysics*, **47**, 884-897.

## APPENDIX A: DERIVATION OF EQUATIONS (35) AND (36)

From Taylor's expansion,

$$(1 - p^2 v^2(\sigma))^{-3/2} = 1 + \frac{3}{2} v^2(\sigma) p^2 + O(p^4),$$

so that

$$\int_0^{\tau^*} v^2(\sigma) (1 - p^2 v^2(\sigma))^{-3/2} d\sigma = \int_0^{\tau^*} v^2(\sigma) d\sigma + \frac{3}{2} p^2 \int_0^{\tau^*} v^4(\sigma) d\sigma = \tau^* (v_2^2 + \frac{3}{2} v_4^2 p^2),$$

$$\int_0^{\tau^*} (1 - p^2 v^2(\sigma))^{-3/2} d\sigma = \tau^* (1 + \frac{3}{2} v_2^2 p^2).$$

Using these results and the definition in (33), we obtain

$$[c_{stk}]^2 = \frac{\tau^* (v_2^2 + 3/2 v_4^2 p^2)}{\tau^* (1 + 3/2 v_2^2 p^2)} = v_2^2 + \frac{3}{2} (v_4^2 - v_2^2) p^2 + O(p^4). \quad (\text{A-1})$$

This completes the proof of equation (35).

From the Taylor's expansion,

$$\begin{aligned} v_2^2(\tau) &= v_2^2(\tau^*) + (\tau - \tau^*) \frac{dv_2^2}{d\tau} \Big|_{\tau=\tau^*} + O((\tau - \tau^*)^2) \\ &= v_2^2(\tau^*) + (\tau/\tau^* - 1)(v^2(\tau^*) - v_2^2(\tau^*)) + O((\tau - \tau^*)^2), \end{aligned} \quad (\text{A-2})$$

where we use the fact that

$$\frac{dv_2^2}{d\tau} \Big|_{\tau=\tau^*} = (v^2(\tau^*) - v_2^2(\tau^*)) / \tau^*.$$

From

$$\tau(0) = T(y, 0) \sqrt{1 - c^2 p^2} = T(y, 0) (1 - \frac{1}{2} c^2 p^2) + O(p^4),$$

and

$$T(y, 0) = \int_0^{\tau^*} (1 - p^2 v^2(\sigma))^{-1/2} d\sigma = \tau^* (1 + \frac{1}{2} v_2^2 p^2) + O(p^4),$$

we have

$$\tau(0) = \tau^* (1 + \frac{1}{2} (v_2^2(\tau^*) - c^2) p^2) + O(p^4).$$

Substituting the above formula into equation (A-2) and setting  $\tau = \tau(0)$ , we obtain

$$v_2^2(\tau(0)) = v_2^2(\tau^*) + \frac{1}{2} (v_2^2(\tau^*) - c^2) (v^2(\tau^*) - v_2^2(\tau^*)) p^2 + O(p^4). \quad (\text{A-3})$$

This completes the proof of equation (36).

## APPENDIX B: DERIVATION OF EQUATION (44)

Suppose reflectors are horizontal and the true velocity  $v(x, z)$  can be written by

$$\frac{1}{v(x, z)} = \frac{1}{\bar{v}(z)}(1 + \alpha(x, z)), \quad (\text{A-4})$$

where  $\bar{v}(z)$  is a reference velocity and  $\alpha(x, z)$  is a small perturbation. Under the assumption of the small perturbation, we can calculate the two-way traveltime by

$$T(h, y) = \bar{T}(h, y) + 2 \int_0^z \frac{\alpha(\xi(\sigma), \sigma)}{\bar{v}(\sigma) \cos \theta} d\sigma \quad (\text{A-5})$$

where  $\theta$  is the angle of the ray path from the vertical,  $(\xi, \sigma)$  is a point on the ray path, and

$$\xi(\sigma) = y + \int_{\sigma}^z \tan \theta ds. \quad (\text{A-6})$$

Therefore,

$$\frac{\partial^2 T(h, y)}{\partial h^2} \Big|_{h=0} = \frac{\partial^2 \bar{T}(h, y)}{\partial h^2} \Big|_{h=0} + 2 \int_0^z \frac{\partial^2}{\partial h^2} \left( \frac{\alpha(\xi, \sigma)}{\cos \theta} \right) \Big|_{h=0} \frac{d\sigma}{\bar{v}}. \quad (\text{A-7})$$

From

$$\frac{\partial^2}{\partial h^2} \left( \frac{\alpha(\xi, \sigma)}{\cos \theta} \right) = \frac{\partial^2 \alpha}{\partial h^2} \frac{1}{\cos \theta} + 2 \frac{\partial \alpha}{\partial h} \frac{\partial}{\partial h} \left( \frac{1}{\cos \theta} \right) + \alpha(\xi, \sigma) \frac{\partial^2}{\partial h^2} \left( \frac{1}{\cos \theta} \right), \quad (\text{A-8})$$

and

$$\begin{aligned} \frac{\partial}{\partial h} \left( \frac{1}{\cos \theta} \right) \Big|_{h=0} &= 0, \\ \cos \theta \Big|_{h=0} &= 1, \end{aligned}$$

we have

$$\frac{\partial^2}{\partial h^2} \left( \frac{\alpha(\xi, \sigma)}{\cos \theta} \right) \Big|_{h=0} = \frac{\partial^2 \alpha}{\partial h^2} \Big|_{h=0} + \alpha(y, \sigma) \frac{\partial^2}{\partial h^2} \left( \frac{1}{\cos \theta} \right) \Big|_{h=0}. \quad (\text{A-9})$$

Notice that  $\xi$  is a function of  $h$ , so that

$$\frac{\partial^2 \alpha}{\partial h^2} = \frac{\partial^2 \alpha}{\partial \xi^2} \left( \frac{\partial \xi}{\partial h} \right)^2 + \frac{\partial \alpha}{\partial \xi} \frac{\partial^2 \xi}{\partial h^2}. \quad (\text{A-10})$$

From (A-6),  $\xi - y$  is an odd function of  $h$ ; hence

$$\frac{\partial^2 \xi}{\partial h^2} \Big|_{h=0} = \frac{\partial^2 (\xi - y)}{\partial h^2} \Big|_{h=0} = 0.$$

This result and formula (A-10) give

$$\frac{\partial^2 \alpha}{\partial h^2} \Big|_{h=0} = \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\partial \xi}{\partial h} \right)^2 \Big|_{h=0}. \quad (\text{A-11})$$

Here we use the fact that  $\xi = y$  at  $h = 0$ . To do the further calculation, we introduce the slope parameter

$$p = \frac{\sin \theta}{\bar{v}(\sigma)},$$

which is independent of  $\sigma$ . Then

$$\xi(\sigma) = y + \int_{\sigma}^z \frac{\bar{v}p}{\sqrt{1 - (\bar{v}p)^2}} ds,$$

$$h = \int_0^z \frac{\bar{v}p}{\sqrt{1 - (\bar{v}p)^2}} ds.$$

These formulas imply that

$$\frac{\partial \xi}{\partial p} = \int_{\sigma}^z \frac{\bar{v}}{(1 - (\bar{v}p)^2)^{3/2}} ds,$$

$$\frac{dh}{dp} = \int_0^z \frac{\bar{v}}{(1 - (\bar{v}p)^2)^{3/2}} ds. \quad (\text{A-12})$$

$$\frac{\partial \xi}{\partial h} \Big|_{h=0} = \frac{\partial \xi}{\partial p} \Big|_{h=0} / \frac{dh}{dp} \Big|_{h=0} = \frac{\int_{\sigma}^z \bar{v} ds}{\int_0^z \bar{v} ds}. \quad (\text{A-13})$$

From

$$\frac{1}{\cos \theta} = \frac{1}{\sqrt{1 - (\bar{v}p)^2}},$$

we have

$$\begin{aligned} \frac{\partial^2}{\partial h^2} \left( \frac{1}{\cos \theta} \right) &= \frac{\partial^2}{\partial h^2} \left( \frac{1}{\sqrt{1 - (\bar{v}p)^2}} \right) \\ &= \frac{\partial^2}{\partial p^2} \left( \frac{1}{\sqrt{1 - (\bar{v}p)^2}} \right) \left( \frac{\partial p}{\partial h} \right)^2 + \frac{\partial}{\partial p} \left( \frac{1}{\sqrt{1 - (\bar{v}p)^2}} \right) \frac{\partial^2 p}{\partial h^2}. \end{aligned} \quad (\text{A-14})$$

Again,  $p$  is an odd function of  $h$ , so that

$$\frac{\partial^2 p}{\partial h^2} \Big|_{h=0} = 0.$$

This result, formulas (A-12), and (A-14) give

$$\frac{\partial^2}{\partial h^2} \left( \frac{1}{\cos \theta} \right) \Big|_{h=0} = \frac{\partial^2}{\partial p^2} \left( \frac{1}{\sqrt{1 - (\bar{v}p)^2}} \right) \Big|_{h=0} \frac{1}{\left( \int_0^z \bar{v} ds \right)^2} = \frac{\bar{v}^2(\sigma)}{\left( \int_0^z \bar{v} ds \right)^2}. \quad (\text{A-15})$$

By formulas (A-9), (A-11), (A-13) and (A-15), we obtain

$$\frac{\partial^2}{\partial h^2} \left( \frac{\alpha(\xi, \sigma)}{\cos \theta} \right) \Big|_{h=0} = \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\int_{\sigma}^z \bar{v} ds}{\int_0^z \bar{v} ds} \right)^2 + \alpha(y, \sigma) \frac{\bar{v}^2(\sigma)}{\left( \int_0^z \bar{v} ds \right)^2}. \quad (\text{A-16})$$

Substituting this formula into equation (A-7), we have

$$\frac{\partial^2 T(h, y)}{\partial h^2} \Big|_{h=0} = \frac{\partial^2 \bar{T}(h, y)}{\partial h^2} \Big|_{h=0} + 2 \int_0^z \left[ \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\int_0^z \bar{v} ds}{\int_0^z \bar{v} ds} \right)^2 + \frac{\alpha \bar{v}^2(\sigma)}{(\int_0^z \bar{v} ds)^2} \right] \frac{d\sigma}{\bar{v}}. \quad (\text{A-17})$$

The stacking velocity  $v_s$  is defined by

$$\frac{1}{v_s^2(y, z)} = \frac{1}{4} \left[ T(y, h) \frac{\partial^2 T(h, y)}{\partial h^2} \right] \Big|_{h=0}. \quad (\text{A-18})$$

By using formula (A-5) and (A-17), we have

$$\begin{aligned} \left[ T \frac{\partial^2 T}{\partial h^2} \right] \Big|_{h=0} &= \left[ \bar{T} \frac{\partial^2 \bar{T}}{\partial h^2} \right] \Big|_{h=0} + 2 \frac{\partial^2 \bar{T}}{\partial h^2} \Big|_{h=0} \int_0^z \alpha(y, \sigma) \frac{d\sigma}{\bar{v}} + \\ &2 \bar{T}(0, y) \int_0^z \left[ \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\int_0^z \bar{v} ds}{\int_0^z \bar{v} ds} \right)^2 + \frac{\alpha \bar{v}^2(\sigma)}{(\int_0^z \bar{v} ds)^2} \right] \frac{d\sigma}{\bar{v}}. \end{aligned} \quad (\text{A-19})$$

From

$$\left[ \bar{T} \frac{\partial^2 \bar{T}}{\partial h^2} \right] \Big|_{h=0} = \frac{4}{\bar{v}_s^2(z)},$$

(A-19) becomes

$$\frac{4}{v_s^2(y, z)} - \frac{4}{\bar{v}_s^2(z)} = \frac{8}{\bar{T} \bar{v}_s^2} \int_0^z \frac{\alpha d\sigma}{\bar{v}} + 2 \bar{T} \int_0^z \left[ \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\int_0^z \bar{v} ds}{\int_0^z \bar{v} ds} \right)^2 + \frac{\alpha \bar{v}^2(\sigma)}{(\int_0^z \bar{v} ds)^2} \right] \frac{d\sigma}{\bar{v}},$$

or

$$\delta w_s(y, z) = \frac{1}{\bar{T} \bar{v}_s} \int_0^z \frac{\alpha d\sigma}{\bar{v}} + \frac{\bar{T} \bar{v}_s}{4} \int_0^z \left[ \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\int_0^z \bar{v} ds}{\int_0^z \bar{v} ds} \right)^2 + \frac{\alpha \bar{v}^2(\sigma)}{(\int_0^z \bar{v} ds)^2} \right] \frac{d\sigma}{\bar{v}} \quad (\text{A-20})$$

where we use the relation

$$\delta w_s(y, z) = \frac{1}{v_s(y, z)} - \frac{1}{\bar{v}_s(z)} = \frac{\bar{v}_s(z)}{2} \left( \frac{1}{v_s^2(y, z)} - \frac{1}{\bar{v}_s^2(z)} \right).$$

From

$$\int_0^z \bar{v} ds = \frac{\bar{T}}{2} \bar{v}_s^2(\tau),$$

equation (A-20) becomes

$$\delta w_s(y, z) = \frac{1}{\bar{T} \bar{v}_s(z)} \int_0^z \left[ \frac{\partial^2 \alpha}{\partial y^2} \left( \frac{\int_0^z \bar{v} ds}{\bar{v}_s(z)} \right)^2 + \alpha \left( 1 + \frac{\bar{v}^2(\sigma)}{\bar{v}_s^2(z)} \right) \right] \frac{d\sigma}{\bar{v}}. \quad (\text{A-21})$$

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