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# Minimization of the Vibration Energy of Thin-Plate Structure

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# MINIMIZATION OF THE VIBRATION ENERGY OF THIN-PLATE STRUCTURES

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## ABSTRACT

An optimization method is proposed to reduce the vibration of thin-plate structures. The method is based on a finite-element shell analysis, a modal analysis, and a structural optimization method. In the finite-element analysis, a triangular shell element with 18 degrees of freedom is used. In the optimization, the overall vibration energy of the structure is adopted as the objective function, and it is minimized at the given exciting frequency by varying the thickness of the elements. The technique of modal analysis is used to derive the sensitivity of the vibration energy with respect to the design variables. The sensitivity is represented by the sensitivities of both eigenvalues and eigenvectors. The optimum value is computed by the gradient projection method and a unidimensional search procedure under the constraint condition of constant weight.

A computer code, based on the proposed method, is developed and is applied to design problems using a beam and a plate as test cases. It is confirmed that the vibration energy is reduced at the given exciting frequency. For the beam excited by a frequency slightly less than the fundamental natural frequency, the optimized shape is close to the beam of uniform strength. For the plate, the optimum shape is obtained such that the changes in thickness have the effect of adding a stiffener or a mass.

## INTRODUCTION

The machine design process generally has several steps. A key step is the evaluation of a preliminary design to determine whether or not the planned performance features are achieved. A decision to change or approve the design is then made. Although these steps depend on the designer, an optimization technique can aid in obtaining a satisfactory solution. The process of design optimization consists of two stages: the first is the formulation of the design mathematically, for example, the minimization of weight. The second stage is the process of solving the problem. Considerable research has been done in these fields (Atrek, 1989), and the significance of optimum design has increased. In connection with analysis, the first author has reported the optimum statical design of a plate based on maximization of its bending rigidity (Inoue, 1988).

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Takatsu (1991) and Lim et al. (1989) conducted research on the vibration of gear systems. The gearboxes used in these studies were made of steel plates. One technique that is often used to avoid resonance shifts the natural frequency for the purpose of controlling and reducing the vibration of structures like gearboxes. A mass or stiffener is added at some position on the structure to change the resonance. Although effective, it is difficult to select the position and to estimate a suitable shift of natural frequency. Another technique to avoid resonance reduces the dynamic response, or transfer function. The dynamic response is used as the index of vibration in this technique. However, it is not always the best index for evaluating the overall vibration of the structure because it is usually evaluated at a specific point. The first author adopted the vibration energy as the index and proposed a method to design a plate (Inoue et al., 1990). In this research, the method was refined and a computer program was developed for the optimum design of thin-plate structures. The program primarily uses finite-element and modal analyses. The latter is used successfully to derive the sensitivity of the vibration energy with respect to the design variable. The proposed method is applied herein to some beam and plate design problems in order to demonstrate the optimization process. The application to the design problem of shell structure is discussed in another paper (Inoue et al., 1992).

## OPTIMIZATION BASED ON THE MINIMIZATION OF VIBRATION ENERGY

### Modal Analysis of Forced Vibration

An elastic structure is divided into finite elements and discretized to  $N$  degrees of freedom. The equation of motion of the structure is given in matrix form as

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{f(t)\} \quad (1)$$

where  $[M]$  is the mass matrix,  $[C]$  is the damping matrix,  $[K]$  is the stiffness matrix,  $\{u(t)\}$  is the displacement vector, and  $\{f(t)\}$  is the exciting force vector. When proportional damping is assumed, the damping matrix is represented by the linear combination of the mass matrix and the stiffness matrix

$$[C] = \alpha[M] + \beta[K] \quad (2)$$

where  $\alpha$  and  $\beta$  are the coefficients that define the proportional damping matrix. The solutions of the eigenvalue problem are obtained easily from the eigenvalues and eigenvectors of the undamped vibration problem with the same matrices  $[M]$  and  $[K]$ .

The eigenvector has orthogonality in a broad sense. Let  $\{\phi\}$  be the eigenvector normalized with respect to mass matrix

$$\{\phi_r\}^T [M] \{\phi_s\} = \delta_{rs} \quad (3)$$

where  $\delta_{rs}$  is Kronecker's delta, and  $r$  and  $s$  define the  $r^{\text{th}}$  and  $s^{\text{th}}$  mode. The modal matrix  $[\Phi]$  is defined by the assembly of these eigenvectors

$$[\Phi] = [\{\phi_1\}, \{\phi_2\}, \dots, \{\phi_n\}] \quad (4)$$

where  $n$  indicates the number of eigenvectors considered. From the normalized orthogonality of eigenvectors, any displacements  $\{u(t)\}$  can be represented approximately by their linear combination

$$\{u(t)\} = \sum_{r=1}^n \xi_r(t) \{\phi_r\} = [\Phi] \{\xi(t)\} \quad (5)$$

When the structure concerned is excited by a harmonic force of angular frequency  $\omega$ ,

$$\{f(t)\} = \{F\} e^{j\omega t} \quad (6)$$

where  $\{F\}$  is the exciting force amplitude. The vibration of the structure is written by the same form

$$\{u(t)\} = \{U\} e^{j\omega t} = [\Phi] \{\xi\} e^{j\omega t} \quad (7)$$

where  $\{U\}$  indicates the amplitude and is represented by the modal matrix. Substituting Eqs. (6) and (7) into Eq. (1) and premultiplying it by  $[\Phi]^T$ ,  $n$  independent algebraic equations are obtained to evaluate the vector  $\{\xi\}$ . The  $r^{\text{th}}$  component is given in the following form by solving the equation:

$$\xi_r = \frac{\{\phi_r\}^T \{F\}}{(\lambda_r - \omega^2) + j\omega(\alpha + \beta\lambda_r)} \quad (r = 1, 2, \dots, n) \quad (8)$$

In the expression,  $\lambda_r$  denotes the  $r^{\text{th}}$  eigenvalue. Consequently, the amplitude  $\{U\}$  for the vibration displacement is obtained by substituting  $\{\xi\}$  into Eq. (7).

#### Sensitivity of Vibration Energy

The velocity  $\{\dot{u}(t)\}$  can be expressed in a form similar to Eq. (5). The vibration velocity is given by

$$\{\dot{u}(t)\} = \{\dot{U}\} e^{j\omega t} = [\Phi] \{\dot{\xi}\} e^{j\omega t} \quad (9)$$

However, the velocity can be derived directly by differentiating the displacement in Eq. (7):

$$\{\dot{u}(t)\} = j\omega [\Phi] \{\xi\} e^{j\omega t} \quad (10)$$

From Eqs. (9) and (10), the vector  $\{\dot{\xi}\}$  is given by

$$\{\dot{\xi}\} = j\omega \{\xi\} \quad (11)$$

Therefore, the real and imaginary parts of  $r^{\text{th}}$  component are obtained as follows:

$$\begin{aligned} \xi_{rR} &= \frac{(\alpha + \beta\lambda_r) \{\phi_r\}^T \{F\}}{\frac{1}{\omega^2} (\lambda_r - \omega^2)^2 + (\alpha + \beta\lambda_r)^2} \\ \xi_{rI} &= \frac{\frac{1}{\omega} (\lambda_r - \omega^2) \{\phi_r\}^T \{F\}}{\frac{1}{\omega^2} (\lambda_r - \omega^2)^2 + (\alpha + \beta\lambda_r)^2} \end{aligned} \quad (12)$$

where  $R$  and  $I$  are the real and imaginary parts and  $\{\dots\}^T$  denotes the transpose matrix.

The vibration energy  $T$  of the structure is given by

$$\begin{aligned} T &= \frac{1}{2} \{\dot{U}^*\}^T [M] \{\dot{U}\} \\ &= \frac{1}{2} \{\dot{\xi}^*\}^T [\Phi]^T [M] [\Phi] \{\dot{\xi}\} \\ &= \frac{1}{2} \{\dot{\xi}^*\}^T \{\dot{\xi}\} \end{aligned} \quad (13)$$

where  $\{\dot{U}^*\}$  and  $\{\dot{\xi}^*\}$  show the conjugate complex of  $\{\dot{U}\}$  and  $\{\dot{\xi}\}$ . The energy is, therefore, represented by the following expression:

$$T = \frac{1}{2} \sum_{r=1}^n (\xi_{rR}^2 + \xi_{rI}^2) \quad (14)$$

The sensitivity of the vibration energy with respect to the design variable  $x_i$  ( $i = 1, \dots, m$ ) is given by

$$\frac{\partial T}{\partial x_i} = \sum_{r=1}^n \left( \xi_{rR} \frac{\partial \xi_{rR}}{\partial x_i} + \xi_{rI} \frac{\partial \xi_{rI}}{\partial x_i} \right) \quad (15)$$

Since  $\alpha$ ,  $\beta$ ,  $\omega$ , and  $\{F\}$  in Eq. (12) are independent of the design variable, the sensitivity of the vibration energy is represented by the function of the sensitivity of the eigenvalue and the sensitivity of the eigenvector. The evaluation of the sensitivities of both eigenvalue and eigenvector is based on the method proposed by Fox and Kapoor (1968).

### Search for the Optimum Value

The optimization design problem is expressed as follows:

$$\begin{aligned} \text{Minimize} & \quad T(\{x\}) \\ \text{Behavior constraint} & \quad g(\{x\}) = \bar{W} - W(\{x\}) = 0 \\ \text{Side constraint} & \quad h_1(\{x\}) = x_i - x_{\min} \geq 0 \\ & \quad h_2(\{x\}) = x_{\max} - x_i \geq 0 \\ \text{Design variables} & \quad \{x\} = (x_1, x_2, \dots, x_m)^T \end{aligned} \quad (16)$$

where  $\bar{W}$  and  $W(\{x\})$  are the prescribed weight and the function of the structure weight, respectively. The behavior constraint gives the design of constant weight. The element thickness is adopted as the design variable. The side constraints limit the thickness within the given minimum and maximum.

In the optimization process, the design variables are modified along the feasible direction vector  $\{d^{(k)}\}$  to reduce the objective function, and the new design variables are obtained in the following form:

$$\{x^{(k+1)}\} = \{x^{(k)}\} + s\{d^{(k)}\} \quad (17)$$

The superscript (k) means the  $k^{\text{th}}$  iteration in the optimization process, and  $s$  is the step. If the variables are in the feasible region, the feasible direction vector is given by the steepest descent vector and is composed of the sensitivity of vibration energy with a minus sign. After the design variables reach the intersection formed by the constraint surfaces, the vector is projected on the intersection. This procedure is the gradient projection method (Rosen, 1961). The method can be applied to problems with linear or nonlinear constraints. Since the constraints for the design problem discussed in this research are linear, the feasible direction is easily determined. In order to obtain the step  $s$  in Eq. (17), the region which includes the relative minimum of the energy is searched by an iterative interpolation process. Once the region is found, the energies are evaluated at the limits and the middle point of the region. Next a quadratic expression is obtained so that it goes through these points. The step size is determined to obtain the minimum of the expression. The region cannot be found every time because the objective function frequently decreases monotonously. In this case, the search procedure is stopped if the minimum is not obtained after 10 iterations. Then the step at the last iteration is used, the sensitivity is calculated again, and the optimum is searched. The value of the energy at the last iteration is stored and used to determine convergence. The convergence criterion is 1/1000 relative variation.

### OPTIMUM DESIGN PROGRAM

A computer program was developed based on the procedures described in the previous sections. The program consists of two main parts. One part is the analyzer, which is composed of the finite-element method, the eigenvalue analysis, and the evaluation of the vibration energy by means of modal analysis. The other part is the optimizer which includes the sensitivity analysis and search procedure for the optimum value.

Several finite-element programs are available; however, in this research, the program is written to make the program as compact and flexible as possible. A triangular shell element with 18 degrees of freedom is used. The element is formed by the combination of elements for plate bending and plane problem analyses, and both are given in Zienkiewicz and Cheung (1967). The consistent mass matrix is evaluated by using the method of numerical integration.

The program was implemented on Digital Equipment Corporation VAX computers at the NASA Lewis Research Center. It is executed interactively for small models. In the case of a larger model like a three-dimensional structure, the program is submitted to the Cray Research Corporation XMP 4/28 computer. The program is written in FORTRAN 77.

### APPLICATION TO THE OPTIMUM DESIGN OF BEAM AND PLATE

In order to demonstrate the optimization process, the proposed method is applied to the optimum design problem of beams and plates. These constants were used in this research: modulus of elasticity in tension = 206 GPa, Poisson's ratio = 0.3, mass density = 8000 kg/m<sup>3</sup>, and the coefficients for composing the proportional damping matrix are  $\alpha = 1.0 \text{ sec}^{-1}$  and  $\beta = 5.0 \times 10^{-7} \text{ sec}$ .

#### Optimum Shape of Simply Supported Beams

The element used in this research is not a beam element but a shell element, and it is not best suited for the analysis of beams. However, a sufficiently narrow-width plate may be approximated as a beam. The pattern of the mesh is shown in Fig. 1. The element numbers are from 1 to 40 as shown in the figure. The beam has dimensions of length  $L = 200 \text{ mm}$ , width  $B = 10 \text{ mm}$ , and the initial thickness  $t_0 = 5 \text{ mm}$ .

The calculated lower four natural frequencies and their errors (compared to the theoretical values for a simply supported beam) are 288.9 Hz (0.5 percent), 1172 Hz (1.9 percent), 2704 Hz (4.5 percent), and 4982 Hz (8.3 percent), respectively. When the beam center is excited by a unit harmonic force, the frequency response of the vibration energy is evaluated at every 10 Hz, based on these lower four modes, and is illustrated by the dashed curve in Fig. 2. The abscissa indicates the exciting frequency. The peak in the figure approximately represents the resonance at the fundamental natural frequency of 288.9 Hz. If this beam is excited by a unit harmonic force (1 N) at 270 Hz, the energy level of about  $0.207 \times 10^{-3} \text{ J}$  is considerably high because of the influence of the resonance. The optimum design program was executed to minimize the vibration energy at the resonant frequency. The upper and lower limits of the thickness are given as  $1 \leq t \leq 10$ . The process of energy convergence, which is represented by the ratio of the initial energy, is shown in Fig. 3. The energy decreases rapidly and converges to about one-tenth of the initial energy after four iterations. The frequency response of the optimally designed beam is indicated by the solid curve in Fig. 2. The fundamental natural frequency shifts to 319.7 Hz.

The optimum shape of the beam is illustrated in Fig. 4. The thickness is maximum at the center and decreases gradually toward the supported ends. The shape is compared with the shape of a beam of uniform strength in Fig. 5. The uniform-strength beam is well known as the optimum shape of the static maximum stiffness design (Huang, 1968). A similar design performed by using a fine mesh of 20 by 2 by 2 gave a shape approximately the same as the aforementioned optimum shape in Fig. 5.

In the optimization, the thicknesses of four elements in every section are forced to be equal. If the constraint is eliminated, the optimum shape illustrated in Fig. 6 is obtained. Most of the elements reach the higher and lower thickness limits. However, the tendency for the element thickness to decrease toward the supporting ends is the same as in the previous example. The element thickness variation is summarized in Table 1. The element thickness in each section is very close to each other in the first few steps of the process and then the difference increases gradually. The frequency response and the energy convergence are shown in Figs. 7 and 8, respectively. Compared with the former example, more iterations are needed until the energy converges, and the energy greatly reduces. The beneficial effect of this method on the reduction of vibration is confirmed by the vibration amplitude shown in Fig. 9.

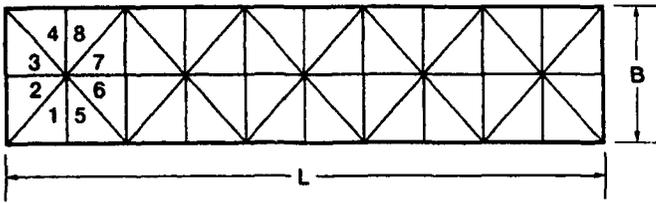


Figure 1.—Pattern of mesh used for the optimum design of beam. Length  $L = 200$  mm, width  $B = 10$  mm, initial thickness  $t_0 = 5$  mm.

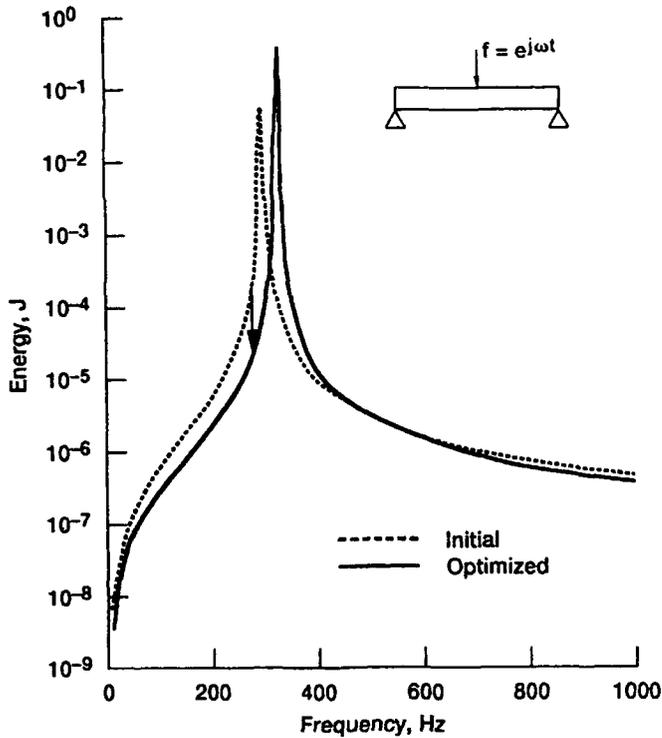


Figure 2.—Frequency response of simply supported beam due to unit exciting force. Excitation frequency  $f_{ex} = 270$  Hz.

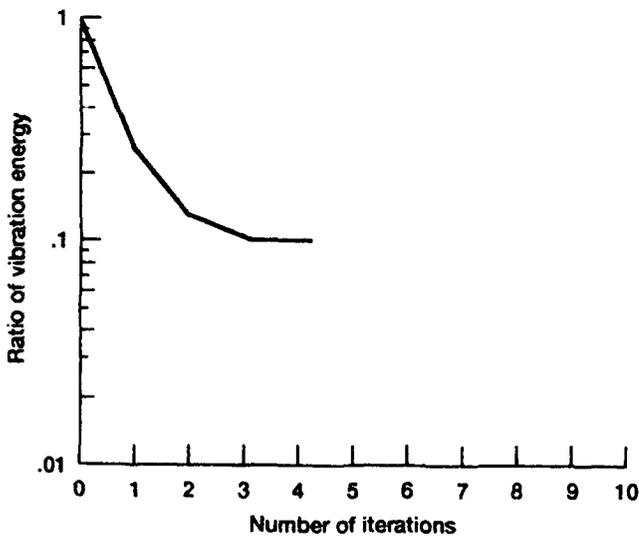


Figure 3.—Convergence of vibration energy.

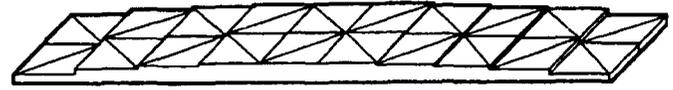


Figure 4.—Optimum shape of simply supported beam. Excitation frequency  $f_{ex} = 270$  Hz. (The thicknesses of four elements in every section are forced to be equal.)

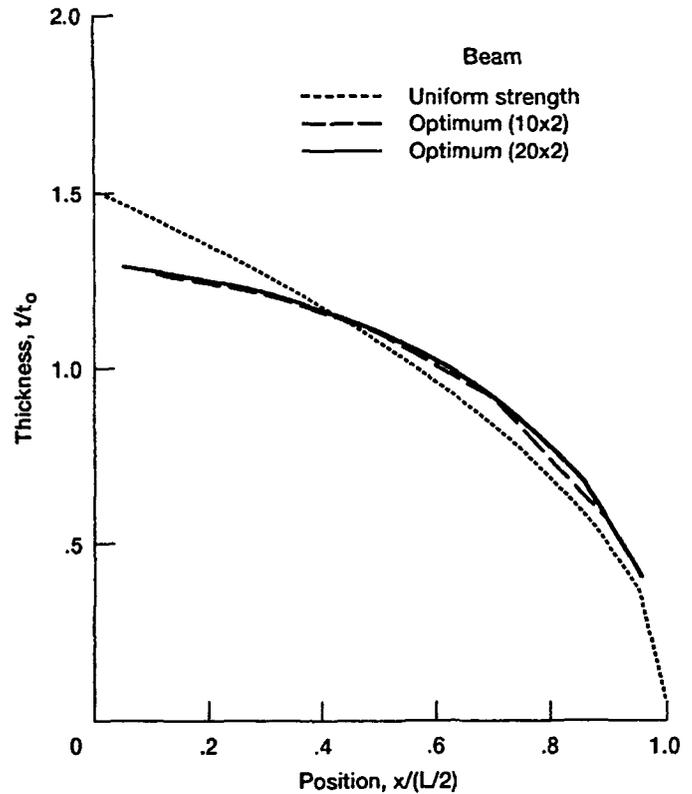


Figure 5.—Comparison of the optimum shape with the shape of the beam of uniform strength.

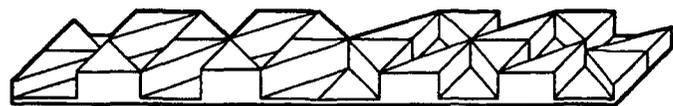


Figure 6.—Optimum shape of simply supported beam. Excitation frequency  $f_{ex} = 270$  Hz.

TABLE 1.—VARIATION OF ELEMENT THICKNESS OF SIMPLY SUPPORTED BEAM  
WITH VARIABLE THICKNESS AND WIDTH (cf. Fig. 6)

Element	Iteration										
	0	1	2	3	4	6	8	10	12	14	15
1	5.00	3.63	2.95	2.68	1.96	1.42	1.00	1.00	1.00	1.00	1.00
2	5.00	3.66	2.98	2.72	2.65	4.24	4.36	4.46	4.49	5.72	5.56
5	5.00	4.18	4.53	4.59	5.26	6.83	7.16	7.39	7.58	9.28	9.44
6	5.00	4.11	4.41	4.44	3.47	1.95	1.46	1.00	1.00	1.00	1.00
9	5.00	4.93	5.36	5.37	3.74	1.04	1.00	1.00	1.00	1.00	1.00
10	5.00	5.01	5.49	5.56	6.89	8.52	8.84	9.07	9.28	10.00	10.00
13	5.00	5.86	5.97	6.09	7.65	9.26	9.70	10.00	10.00	10.00	10.00
14	5.00	5.79	5.86	5.93	4.95	2.58	1.93	1.35	1.00	1.00	1.00
17	5.00	6.40	6.21	6.29	6.22	5.69	5.53	5.31	4.65	1.00	1.00
18	5.00	6.43	6.24	6.34	7.20	8.48	9.00	9.42	10.00	10.00	10.00

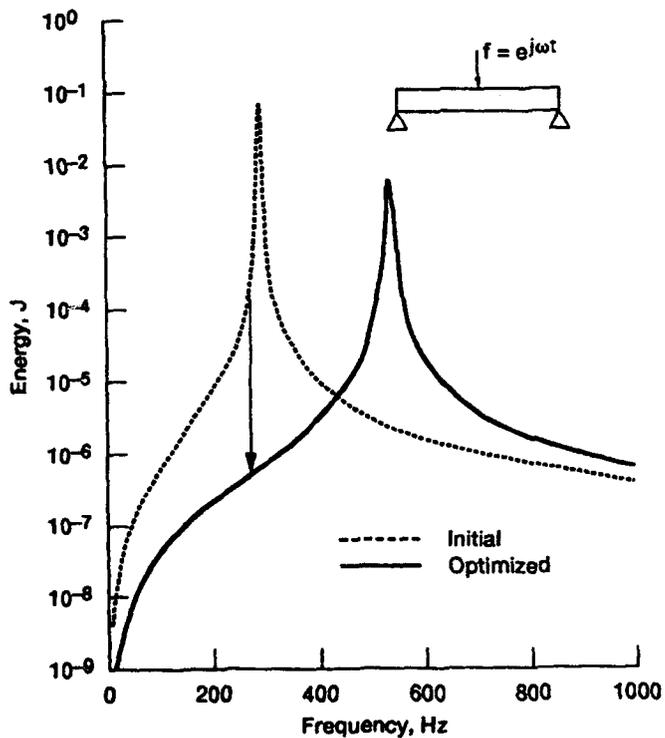


Figure 7.—Frequency response (due to unit exciting force) of simply supported beam with variable thickness and width. Excitation frequency,  $f_{ex} = 270$  Hz.

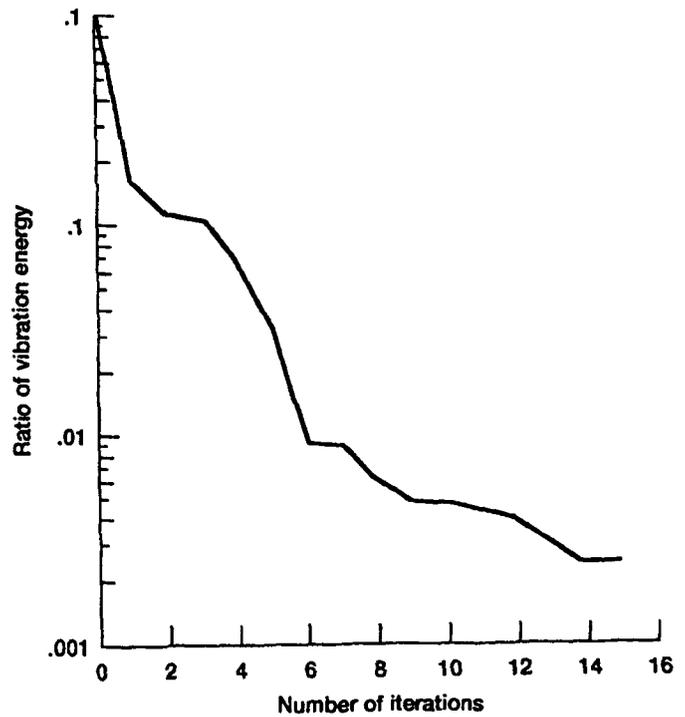


Figure 8.—Convergence of vibration energy with variable thickness and width.

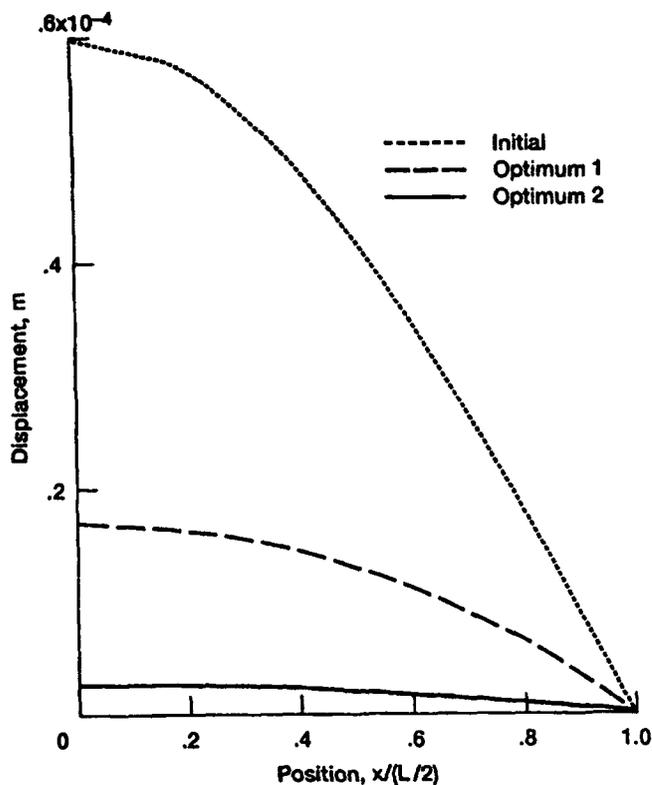


Figure 9.—Displacements due to continuative vibration (initials; uniform thickness, optimum 1, beam in fig. 4, optimum 2, beam in fig. 6).

#### Optimum Shape of Simply Supported Plate

The plate (whose dimensions are  $a = 600$  mm,  $b = 400$  mm, and initial thickness  $t_0 = 5$  mm) is divided into 6 by 2 by 2 elements as shown in Fig. 10. The thickness constraint is given as  $2 \leq t \leq 10$ . The eigenvalue problem is solved first, and the lower 12 modes are used for the optimization because they fully cover the frequency response range considered in the optimization.

In the first example, the plate is excited by a central unit harmonic force (1 N) with a frequency of 370 Hz, which is slightly lower than the natural frequency of mode (3,1), 376 Hz. Mode (3,1) means the mode shape has two nodal lines parallel to the  $y$ -axis. The frequency response of the plate with initial thickness is evaluated at every 10 Hz up to 1000 Hz and is shown by the dashed curve in Fig. 11. The energy at the exciting frequency is about  $0.338 \times 10^{-4}$  J. The energy is considerably high because of the resonance influence. The energy convergence process is shown in Fig. 12. The energy decreases sharply by the first optimization and converges to about one-thousandth of the initial energy after eight iterations. The final frequency response is also shown by the solid curve in Fig. 11. The resonance peak under consideration shifts to about 520 Hz, which causes a large vibration reduction at the excitation frequency. The optimum shape of the plate is illustrated in Fig. 13. A marked change of the shape is the increase in thickness of the elements along the longer centerline which may function as a stiffener and lead to the vibration reduction of mode (3,1).

In the second example, the same plate is excited by 390 Hz, which is a little higher than the natural frequency under consideration. The solution converges to the optimum shape as illustrated in Fig. 14. The frequency response and the energy convergence are shown in Figs. 15 and 16, respectively. In this example, the thickness increases at the central portion of the plate and appears to function as a mass. Indeed, the natural frequency under consideration is reduced by the optimization and shifts to about 314 Hz.

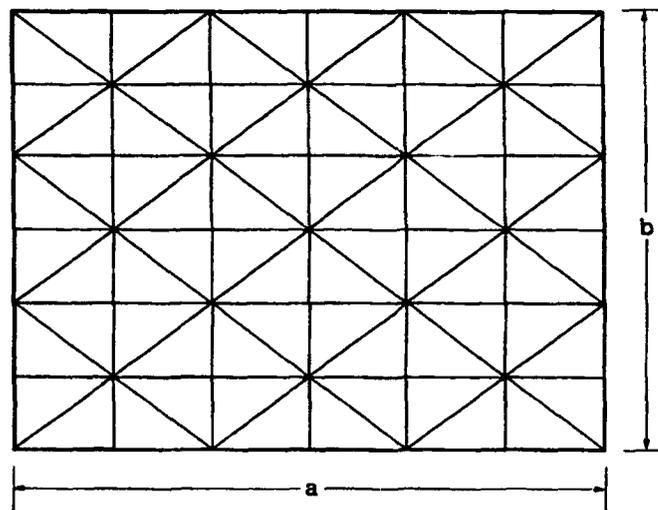


Figure 10.—Pattern of mesh used for the optimum design of plate ( $a = 600$  mm,  $b = 400$  mm, initial thickness  $t_0 = 5$  mm).

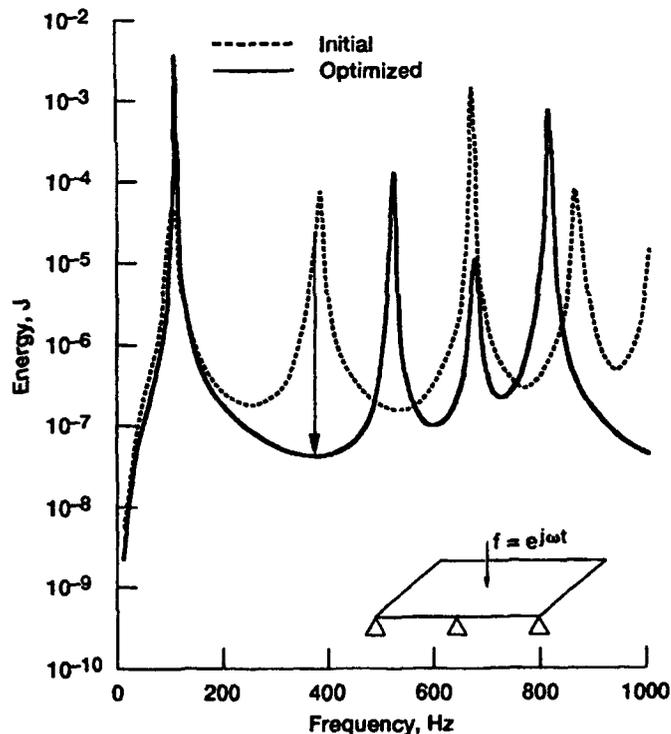


Figure 11.—Frequency response of simply supported plate due to unit exciting force. Excitation frequency  $f_{ex} = 370$  Hz.

In comparison with the first example, the energy at 390 Hz is higher than the former energy at the same frequency, thus demonstrating that the minimum energy obtained in these examples is a local minimum. The comparison also indicates that the natural frequency shift to a higher frequency region is better than the shift to a lower frequency region for the purpose of reducing the vibration.

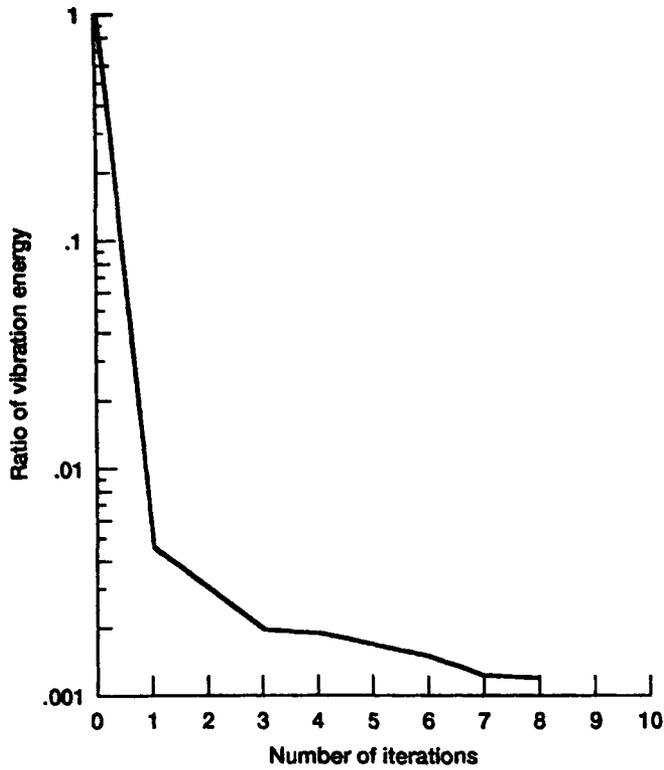


Figure 12.—Convergence of vibration energy.

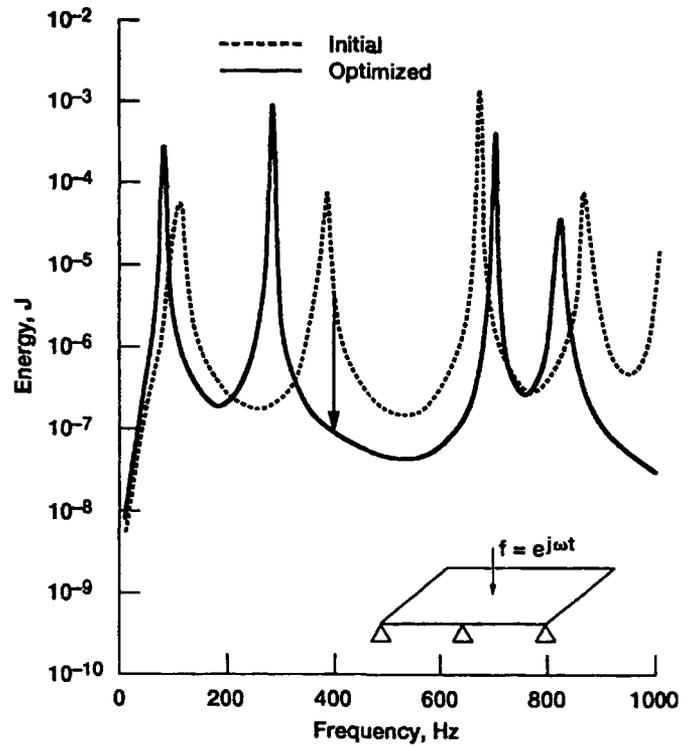


Figure 15.—Frequency response of simply supported plate due to unit exciting force. Excitation frequency  $f_{ex} = 390$  Hz.

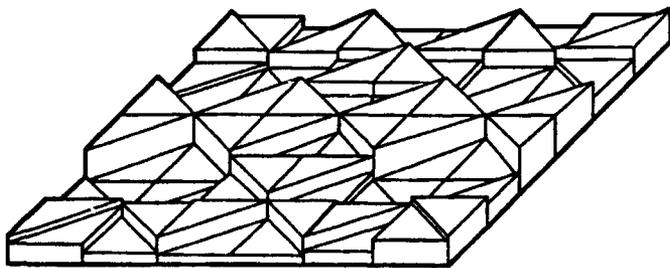


Figure 13.—Optimum shape of simply supported plate. Excitation frequency  $f_{ex} = 370$  Hz.

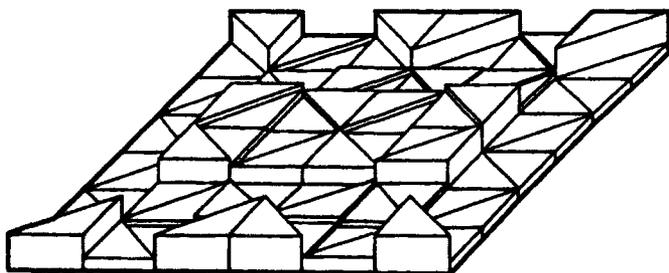


Figure 14.—Optimum shape of simply supported plate. Excitation frequency  $f_{ex} = 390$  Hz.

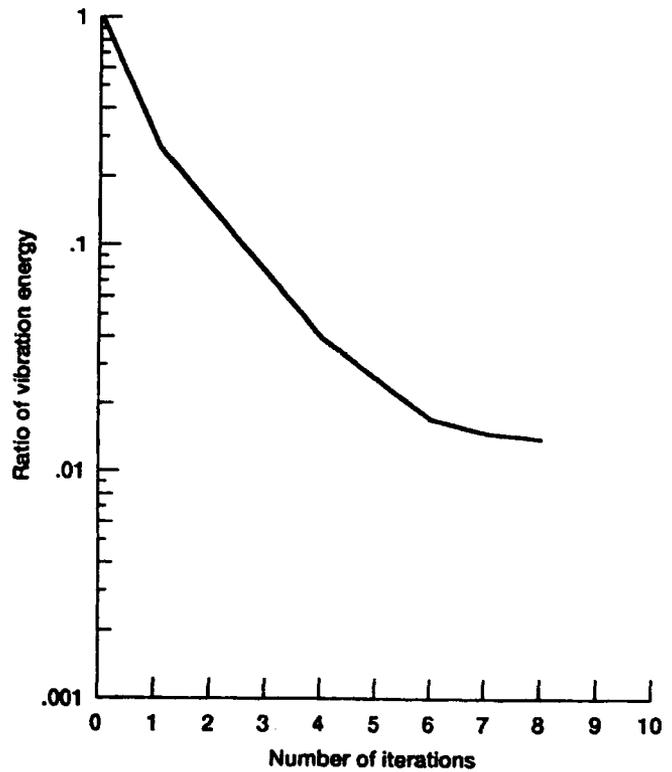


Figure 16.—Convergence of vibration energy.

## CONCLUSION

The vibration energy was selected as the index for vibration evaluation for the purpose of designing low-vibration thin-plate structures. A method was proposed to minimize the energy. The method is based on a finite-element shell analysis, a modal analysis, and a structural optimization technique. The vibration energy sensitivity with respect to the design variable was derived and shown to be expressed by the sensitivities of both eigenvalues and eigenvectors.

The computer program was developed and applied to the analysis of a beam and a plate to study the optimization process. This method was demonstrated to be effective in reducing the energy at the given frequency. In the design of a plate, the optimum shape was obtained such that the increase in thickness functioned as a stiffener or a mass. If the optimization frequency is slightly lower than the natural frequency, then this method shifts the natural frequency under consideration to a higher frequency region. The opposite is also true. Consequently, this method can control the direction of shift of the natural frequency.

## REFERENCES

- Atrek, E., ed., 1989, New Directions in Optimum Structural Design, John Wiley & Sons, New York.
- Fox, R.L., and Kapoor, M.P., 1968, "Rates of Change of Eigenvalues and Eigenvectors," AIAA Journal, Vol. 6, No. 12, pp. 2426-2429.
- Huang, N.C., 1968, "Optimal Design of Elastic Structures for Maximum Stiffness," International Journal of Solids and Structures, Vol. 4, pp. 689-700.
- Inoue, K., Kato, M., and Ohnuki, K., 1988, "Maximization of Bending Rigidity of Plates," Preprint of JSDE No. 4, pp. 17-24 (in Japanese).
- Inoue, K., Kato, M., and Ohnuki, K., 1990, "Optimum Design of a Plate on the Minimization of the Vibration Energy," Transactions, JSME, Vol. 56, No. 529, pp. 2361-2366 (in Japanese).
- Inoue, K., Townsend, D.P., and Coy, J.J., 1992, "Optimum Design of a Gearbox for Low Vibration," to be presented at the 6th International Power Transmission and Gearing Conference, Phoenix, AZ, Sept. 13-16, 1992.
- Lim, T.C., Singh, R., and Zakrajsek, J.J., 1989, "Modal Analysis of Gear Housings and Mounts," Proceedings of the 7th International Modal Analysis Conference, Society for Experimental Mechanics, Inc., Bethel, CT, Vol. II, pp. 1072-1078.
- Mota Soares, C.A., ed., 1987, Computer Aided Optimal Design: Structural and Mechanical Systems, Springer-Verlag, New York.
- Rosen, J.B., 1961, "The Gradient Projection Method for Non-linear Programming: Part II Nonlinear Constraints," SIAM Journal on Applied Mathematics, Vol. 9, No. 4, pp. 514-532.
- Takatsu, N., et al., 1991, "Building Block Approach of the Vibration Transmission in a Single Stage Gearbox," Transactions, JSME, Vol. 57, No. 538, pp. 2126-2131 (in Japanese).
- Zienkiewicz, O.C., and Cheung, Y.K., 1967, The Finite Element Method in Structural and Continuum Mechanics, McGraw-Hill, New York.

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