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EXTENSION OF THE INTEGRAL EQUATION FORMULATION FOR LINEARIZED TIME DEPENDENT SUBSONIC FLOW

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This report describes the results of a study of the feasibility of extending the time domain formulation of the linearized aerodynamics of flexible bodies that had been developed earlier for planar wings to the general case of arbitrary, complete aircraft, configurations. The end products of that study are a formulation of the problem, an exact solution applicable to any geometry, a sample calculation for a simple nonplanar configuration, and some suggestions on how to proceed. The formulation is based on distributions of elementary sources and doublets over the mean solid surface. The problem is first formulated for arbitrary motion, and then specialized to the case of uniform translation.

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FOREWORD

Contract F33615-90-C-3206, entitled "Extension of the Integral Equation for Linearized, Time Dependent, Subsonic Flows," was intiated by the Aeroelasticity Group (WL/FIBRC), Flight Dynamics Directorate of the Wright Laboratory at Wright-Patterson Air Force Base. The objective of the contract was to incorporate the recently developed aerodynamic lifting surface formulae of Guderley into a numerical method for efficiently modelling the time dependent loads on a complete aircraft configuration.

Professor Marc H. Williams of Purdue University was the Principal Investigator. Dr. Max Blair was the Air Force Project Manager. The work reported here was performed from 3 May 1990 through 3 September 1991.

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Section (I)
Introduction

When a solid body moves nonuniformly through a compressible gas, compression and expansion waves are emitted from its surface, producing a reaction on the body. It is of interest to be able to predict the pressure forces exerted on the body in response to its motion. In general this problem is highly nonlinear, because the governing equations of a compressible gas are nonlinear. If the interest is in small amplitude perturbations of a rigid body in uniform motion, then the governing equations may rigorously be linearized about the steady flow produced by the rigid body. The deformation induced perturbations are then governed by a linear system with time invariant coefficients (in the mean body frame,) even though the mean flow field itself is still nonlinear. Most problems of aeroelastic interest fall in this category. The principal significance of this is that concepts of generalized force derivatives and aerodynamic transfer functions relating generalized deformation coordinates to their generalized forces are valid.

The problem is this: there are no good methods for solving the linearized equations of motion of the gas significantly more efficiently than the nonlinear equations. Therefore, there is little advantage (in most problems) to be gained by linearizing.

The reason for this is that the linearized equations have spatially variable coefficients. The principle of superposition applies, but there are no known solutions to superimpose. If, however, the mean flow itself is uniform, the governing equations have constant coefficients and elementary solutions are known. This is the realm of classical linearized aerodynamics, or equivalently classical acoustics, for which quite complicated solutions can be constructed simply by adding together many simple solutions.

The problem of constructing time dependent flow fields governed by classical acoustics will be reexamined here. It should be kept in mind, though, that the underlying assumption... namely that the mean flow is uniform... is fundamentally flawed unless the mean body is a (generalized) cylinder moving parallel to its generators, or a helical sheet moving tangent to itself, these being the only rigid bodies that can slice through a gas without disturbing it. Real flight vehicles do not look like this, and do produce significant distortions of the surrounding gas. For slender shapes, typical wings, tails and fuselages, these mean flow disturbances ought to be reasonably small except near stagnation points, and the assumption that the mean flow is uniform ought to be reasonably good away from stagnation points, at least in terms of predicting response to unsteady deformations.

Even if the results of a calculation based on linear theory are acceptably accurate, though, the approximation is justifiable only if those results can be obtained considerably more quickly than they could be from a full nonlinear analysis. A fast approximate solution can be useful for design or extended parametric studies which would be otherwise prohibitive. An approximate solution that is not faster than a more accurate method is useless. It is not clear at the moment whether the time dependent approach to these problems is in the "fast useful" or "slow useless" category, but that question must eventually be answered. For the case of rectilinear motion, the equations admit simple harmonic solutions, so that the problem can be solved for a single frequency of oscillation. Methods based on this have been around for many years and are a standard ("fast useful") tool in aeroelasticity[1]. A time domain method, in principle, can provide a complete spectrum of generalized forces from one transient calculation, or be coupled directly to the structural equations of motion (including nonlinearities) to look at aeroelastic response. There are, then, resons to believe that a time domain, rather than a frequency domain, solution method may be useful.

The approach that will be adopted in this report is to start from basics, the inviscid equations of motion of a gas and relevant boundary conditions. Issues of linearization will first be discussed. Then the elementary solutions for uniform mean flow will be described, followed by a presentation of the general method of superposition of those elementary solutions to form the desired solution for the flow about a given deforming body.

This work is intended as an extension of the planar wing time domain analysis performed earlier by M. Blair and the present author[2,5], to general body geometries. The approach taken here is quite similar to that developed by Morino[6,7], but with some differences in the integral formulation.

Section (II)
Linearized Inviscid Equations

The governing equations of fluid mechanics can, for our purposes, be written in the inviscid form (Euler equations):

$$D\rho/Dt = -\rho\nabla\cdot\vec{u} \tag{II.1}$$

$$D\vec{u}/Dt = -\frac{1}{\rho}\nabla p \tag{II.2}$$

$$Ds/Dt = 0 \tag{II.3}$$

where D/Dt is the material derivative, ρ the density, p the pressure, $s(p,\rho)$ the entropy, and $\vec{u}$ the flow velocity.

The relevant boundary condition on a solid surface is simply that the fluid shall not penetrate the surface. If the body surface velocity at some point is $\vec{u}_s$, the unit normal to the surface at that point is $\hat{n}$, then the gas velocity at the surface must obey:

$$\vec{u}\cdot\hat{n} = \vec{u}_s\cdot\hat{n} \tag{II.4}$$

These equations define the nonlinear problem.

Now suppose there is a steady flow, denoted by subscript 0, which obeys the Euler equations, and which is perturbed by a small disturbance flow, which we denote by subscript 1. The perturbation flow is governed by the linearized Euler equations:

$$D_0[\rho_1/\rho_0] = -\frac{1}{\rho_0}\nabla\cdot[\rho_0\vec{u}_1] \tag{II.5}$$

$$D_0\vec{u}_1 + \vec{u}_1\cdot\nabla\vec{u}_0 = -\frac{1}{\rho_0}\nabla p_1 + \frac{\rho_1}{\rho_0^2}\nabla p_0 \tag{II.6}$$

$$D_0 s_1 + \vec{u}_1\cdot\nabla s_0 = 0 \tag{II.7}$$

where D_0 denotes the linearized material derivative, based on the unperturbed flow velocity $\vec{u}_0$.

These equations are, of course, linear. However they generally have variable coefficients, which depend on the steady flow, and are not significantly easier to solve than the original equations.

If the mean flow is uniform, then we can drop all terms involving gradients of the mean flow. Suppose, in addition that we use a coordinate system which is fixed relative to the undisturbed gas, so that the mean flow velocity is zero. Then we recover the classical linear acoustic equations:

$$\rho_{1,t} = -\rho_0 \nabla \cdot [\vec{u}_1] \qquad \text{II.8}$$

$$\vec{u}_{1,t} = -\frac{1}{\rho_0} \nabla p_1 \qquad \text{II.9}$$

$$s_{1,t} = 0 \qquad \text{II.10}$$

Note that the difference between classical acoustics and linear aerodynamics lies not in the governing equations, but in the fact that in typical acoustic problems the sources are stationary with respect to the gas, while in aerodynamics the sources move with respect to the gas.

The surface boundary condition can be expressed simply as:

$$\hat{n}_0 \cdot \vec{u}_1 = \qquad \text{II.11}$$

where $\hat{n}_0$ is the unit normal top the undeformed body, since now $\vec{u}_1$ is a pure perturbation. The normal velocity of the body must, of course, be small compared to the speed of sound for the linear approximation to be meaningful. How small it has to be depends on the flight Mach number; the closer the Mach number is to 1, the smaller the normal velocity must be to avoid nonlinear steepening of waves. Note also that there will generally be some points on the body where the local body velocity is along the normal. These will usually be local points of failure of the linear theory.

Now it is apparent that in this approximation the entropy perturbation s_1 is fixed by the initial conditions and never changes. Hence an entropy disturbance, if one exists, has no effect on the velocity or pressure fields. The continuity and momentum equations can be written purely in terms of pressure and velocity:

$$(p_1/\rho_0)_{,t} = -a_0{}^2 \nabla \cdot \vec{u}_1 \qquad \text{II.12}$$

$$\vec{u}_{1,t} = -\nabla[p_1/\rho_0] \qquad \text{II.13}$$

where a_0 is the undisturbed speed of sound.

We can further split the problem into a potential wave field and a stationary rotational flow (gust). To do this, define a disturbance velocity potential ϕ, such that:

$$\phi_{,t} + p_1/\rho_0 = 0 \qquad \text{II.14}$$

which is a linear Bernoulli equation, and define a "rotational velocity" $\vec{u}_r$ by,

$$\vec{u}_r = \vec{u}_1 - \nabla\phi \qquad \text{II.15}$$

We are free to select this splitting so that the rotational velocity is initially solenoidal:

$$\nabla \cdot \vec{u}_r = 0 \text{ at } t=0 \tag{II.16}$$

It follows immediately that the rotational part of the velocity is independent of time:

$$\vec{u}_{r,t} = 0 \tag{II.17}$$

(and is, therefore, always solenoidal), and that the potential obeys the classical linear acoustic wave equation:

$$\Box^2\phi = \nabla^2\phi - \phi_{,t,t}/a^2{}_0 = 0 \tag{II.18}$$

Thus, the rotational flow is determined strictly by the initial conditions, like entropy. Unlike entropy, the rotational velocity is important, though, because it interacts with the wave field through the surface boundary condition:

$$\hat{n}_0 \cdot \nabla\phi = \hat{n} \cdot \vec{u}_s - \hat{n}_0 \cdot \vec{u}_r \tag{II.19}$$

The problem that must be solved, then, is the wave equation for the velocity potential ϕ, subject to the Neumann boundary condition on the undeformed body. Once this is done, the pressure can be determined from the linearized Bernoulli equation. Note though, that since the body moves with velocity $\vec{u}_s$ the time derivative in Bernoulli's equation should be expressed at a fixed point on the body:

$$p_1/\rho_0 = -(\dot{\phi} + \vec{u}_s \cdot \nabla\phi) \tag{II.20}$$

where $\dot{\phi}$ is the time derivative of surface potential at the point on the body which moves with velocity $\vec{u}_s$. The convective derivative can be decomposed into normal and tangential components, and the normal component is known from the surface boundary condition. Hence, it is sufficient to know the potential on the surface of the body to determine the pressure on the surface.

Section (III)
Elementary Solutions of the Wave Equation

The wave equation has a simple spherically symmetric solution:

$$\phi(\vec{x},t) = -Q(\tau)/(\,4\pi R\,) \qquad \text{III.1}$$

where $\vec{R} = \vec{x} - \vec{x}_s$ is the radius vector pointing from some fixed point $\vec{x}_s$ to the field point, R is its length and:

$$\tau = t - R/a_0 \qquad \text{III.2}$$

is the time at which a sound wave must leave $\vec{x}_s$ to arrive at $\vec{x}$ at time t, the so called "retarded time." This solution represents the potential due to a point source, emitting fluid from the source point $\vec{x}_s$ at the rate Q(t) in volume per unit time.

This solution is valid only if the source point is stationary, that is if $\vec{x}_s$ is independent of time. If the source moves with respect to the gas far away from it, the solution is modified by what are known as Doppler effects:

$$\phi(\vec{x},t) = -Q(\tau)/(\,4\pi R_c\,) \qquad \text{III.3}$$

where R_c is stretched by the relative motion:

$$R_c = R\,|\,1 - M_r\,| \qquad \text{III.4}$$

and M_r is the component of source Mach number directed at the receiving point:

$$M_r = \vec{R}/R \cdot \vec{M}_s \qquad \text{III.5}$$

$$\vec{M}_s = \dot{\vec{x}}_s\,/a_0 \qquad \text{III.6}$$

Note that if the source Mach number component is 1, then R_c is zero and the potential blows up. This corresponds to the field point being on the Mach cone emanating from the source when the source moves supersonically through the gas.

It is important to recognize that the retarded time is still determined by the same condition, but now the source position is that at the emission time τ:

$$t = \tau + |\vec{x} - \vec{x}_s(\tau)| \qquad \text{III.7}$$

If the retarded time is given, the reception time is explicit. If the reception time t is given, this relation is a nonlinear algebraic equation for τ, which could have any number of roots. If the source Mach number is always less than 1, then you can show that there is only one τ for every t.

The source solution can easily be used to generate other solutions by addition. In particular, since any derivative of a solution must be a solution, then we have that:

$$\phi = \nabla \cdot [-\vec{K}(\tau)/(4\pi R_c)] \qquad \text{III.8}$$

which is the potential of a dipole with strength and orientation fixed by $\vec{K}$.

Section (IV)
Source and Doublet Distributions

We first consider the representation of an arbitrary wave field by means of a distribution of sources and dipoles over a given, but general surface. The only real difficulty in doing this is that we must adopt some method of parameterizing the surface over which the singularities are to be distributed. Since the retarded time depends on time and position, using the spatial coordinates of the surface is not recommended. Instead let P_0 denote the pair (ξ,η), which shall uniquely span the surface. We need not further specify how these coordinates are to be defined, except that they shall not depend on time. Also, let $dA_0(P_0)$ denote a generalized area element attached at the point P_0. This need not represent a physical area. In fact, we may simply take it as $dA_0 = d\xi d\eta$. The presumption is, of course, that the coordinates of the surface are specified in the form:

$$\vec{x}_0 = \vec{x}_0\,(P_0,t) \qquad \text{IV.1}$$

Having adopted such a parameterization, it is then apparent that the potential field:

$$\phi(\vec{x},t) = -\frac{1}{4\pi}\iint \frac{q(P_0,\tau)}{R_c}dA_0(P_0) \qquad \text{IV.2}$$

$$+\frac{1}{4\pi}\nabla\cdot\iint \frac{\vec{\mu}(P_0,\tau)}{R_c}dA_0(P_0) \qquad \text{IV.3}$$

is a solution of the wave equation, since it is simply the sum of sources $q\ dA_0$, and dipoles $\vec{\mu}dA_0$. The reason that the "area element" dA_0 is arbitrary is that any scale change in it could be absorbed into a rescaling of the source and dipole densities q and $\vec{\mu}$. Note that at each integration point P_0 the retarded time must be found from the delay equation:

$$\tau = t - |\vec{x} - \vec{x}_0(P_0,\tau)|/a_0 \qquad \text{IV.4}$$

The orientation of the dipole density $\vec{\mu}$ is arbitrary. However, if it were tangential to the surface, then the dipole integral could, through an integration by parts, be converted into a source distribution and absorbed into q. It is sufficient, then, to take $\vec{\mu}$ as being normal to the surface. Let $\hat{n}$ be the local unit normal, so that:

$$\vec{\mu}(P_0,\tau) = \hat{n}(P_0,\tau)\mu(P_0,\tau) \qquad \text{IV.5}$$

where μ is the magnitude of the dipole strength.

Now the above ϕ is a solution. What are its properties? Consider first the case of a pure source distribution, μ=0. It can be shown that the potential field is then continuous but that the normal derivative is discontinuous across the surface. The jump in normal derivative determines the source density:

$$q(P_0,t) = \Delta(\hat{n}\cdot\nabla\phi)dA/dA_0 \qquad \text{IV.6}$$

where dA is the local physical area elements, and Δ is the jump across the surface, with positive side defined by the direction towards which $\hat{n}$ points. Note that since the jump is determined by local singularities, the retarded time does not come into play. The problem is locally incompressible.

Now consider the case of a pure dipole distribution, q=0. It can be shown that the normal derivatives are continuous, but that (since the dipole term is the derivative of a source distribution) the value of potential jumps across the surface. The size of the jump determines the dipole strength:

$$\mu = \Delta\phi dA/dA_0 \qquad \text{IV.7}$$

It is evident, then, that the general form, containing both sources and dipoles, will have jumps in both the value of ϕ and in its normal derivative across the surface, and that these jumps determine the source and dipole densities q and μ. If the surface is closed (and its exterior is of physical interest) then the densities q and μ have no direct physical significance, since they are then differences between exterior and interior values. If, however, the surface is open, and exposed to fluid on both sides, then the densities are meaningful.

Any solution of the wave equation for the wave field produced by a body can be represented by some distribution of sources and dipoles over its surface, as described above. But while arbitrary choices of q and μ yield a solution, that solution will not generally satisfy the flow impermeability condition on the surface. Our objective, then, is to determine suitable distributions of q and μ for which the field ϕ determined by them, satisfies the Neumann boundary condition at every point on the body. There are generally many ways to do this.

Note that an open surface has no interior, and the impermeability condition would usually imply equal normal velocities on either side. In such a case, the source density q would be identically zero, and μ would measure the jump in potential across the surface. This is the model of a zero thickness wing.

For a closed surface, which has an interior, one must generally make a choice for either q or μ, or set some constraints on them, simply because there is one boundary condition at each point on the surface, but two degrees of freedom (q and μ).

Finally, it may be noted that the foregoing discussion has presumed that the sources and dipoles are to be placed on the (undeformed) surface of the body. This is not necessary. In fact, singularities can clearly be placed inside a closed body if desired, or the entire physical surface could be replaced by some neighboring simplified simulacrum of it (in fact this is what would always be done, the only question is to what degree the model is faithful to the reality. Since linearization already has cost dearly in fidelity, there is little reason to be too religious about geometric accuracy.)

Section (V)
Generalized Green's Identity

In the previous section we looked at representations in terms of general source and doublet distributions. Usually the source and doublet densities involved have no direct physical significance. In this section, a formal derivation of Green's identity for an arbitrarily moving body will be presented. The result is a source/doublet representation containing only the known normal velocities and the unknown potentials on the exterior surface of the body.

We begin with an impulsive source:

$$G(\vec{x}\,, t\,; \vec{x}_0\,, t_0\,) = -\frac{\delta\,(\,t - t_0 + R\,/\,a_0\,)}{4\pi R} \qquad \text{V.1}$$

where δ is the Dirac delta function. This quantity G is the velocity potential of a "bang" source emitted from $\vec{x}_0 = \vec{x}$ at time $t = t_0$. It is a Green's function for the acoustic wave equation, in that:

$$\Box^2 G \;=\; \nabla^2 G - \frac{1}{a^2_0}\, G_{,t,t} \;=\; \delta(\vec{x} - \vec{x}_0)\delta(t - t_0) \qquad \text{V.2}$$

Note that since the source is impulsive, the field is zero everywhere except on the surface of the sound sphere radiating out from the point of origin. We may freely think of the source point as moving around in space along any path. There are no Doppler effects because the sound sphere is emitted at only one point on this trajectory. Any motion of the point when it's not emitting can have no influence on the disturbance generated.

For any ϕ which satisfies the wave equation, $\Box^2\phi = 0$, and any G that satisfies the inhomogeneous wave equation $\Box^2 G = \delta$, the following is a simple identity:

$$\nabla\cdot(\phi\nabla G - G\nabla\phi) = \phi\delta + \frac{1}{a_0{}^2}[\phi G_{,t} - G\phi_{,t}]_{,t} \qquad \text{V.3}$$

Let V be the volume exterior to some closed surface S(t) on which there is an outward normal $\hat{n}$, and a surface velocity $\vec{u}_s(t)$. Define the normal component of the surface velocity as $u_{sn} = \hat{n}\cdot\vec{u}_s$, and the Mach number of this normal velocity as $M_{sn} = u_{sn}/a_0$. Obviously we must have $M_{sn} \ll 1$ or the linear approximation would be ridiculous. We will keep terms proportional to M_{sn}, though, for completeness.

Integrate the above identity over the volume V, and apply Gauss's theorem to the divergence and Leibnitz' rule to the time derivative term. We get the following identity:

$$\phi(\vec{x}_0,t)\delta(t-t_0) = \iint_S dA[Gu_n-\phi G_n-M_{sn}/a_0(\phi G_{,t}-G\phi_{,t})]-\frac{d\Lambda}{dt} \qquad V.4$$

$$\Lambda = (\phi G_{,t}-G\phi_{,t})/a_0{}^2 \qquad V.5$$

$$u_n = \hat{n}\cdot\nabla\phi \quad ; \quad G_n = \hat{n}\cdot\nabla G \quad ; \qquad V.6$$

Here u_n is the normal derivative of ϕ, which is, of course, known from the boundary condition. u_{sn} is the normal component of surface velocity, which is also known, but not the same as u_n because the later includes the effects of deformation and gust, while the former includes only the undeformed body motion.

We now substitute in the particular choice of G, and group terms proportional to $\delta(t-t_0+R/a_0)$ and its derivative $\dot{\delta}$. The result can be expressed this way:

$$-4\pi\phi(\vec{x}_0,t)\delta(t-t_0) = \iint_S [\delta f_0+\dot{\delta}f_1] + 4\pi\frac{d\Lambda}{dt} \qquad V.7$$

where:

$$f_0=\frac{u_n}{R}-\phi\hat{n}\cdot\frac{\vec{R}}{R^3}+M_{sn}\frac{\phi_{,t}}{a_0R} \qquad V.8$$

$$f_1 = \frac{\phi}{a_0R}[\hat{n}\cdot\frac{\vec{R}}{R}-M_{sn}] \qquad V.9$$

We are now in a position to integrate with respect to time t. The term involving Λ will generally produce initial condition terms in the solution. These are of no interest, and we shall take this contribution to vanish, so that we get a representation of ϕ at the arbitrary point $\vec{x}_0$, t_0 as:

$$-4\pi\phi(\vec{x}_0,t_0) = \int dt\iint_S dA[\delta f_0+\dot{\delta}f_1] \qquad V.10$$

The problem is that S depends on t, so we cannot simply interchange the order of integration. Instead, use a marker parameterization of the surface $P_0 = (\xi,\eta)$, as in the last section, so that the instantaneous surface S(t) is defined by the collection of points $\vec{x}_s(P_0,t)$. We can define $\kappa(P_0,t)$ as the ratio of the area element dA to the generalized area $d\xi d\eta$, so that:

$$dA = \kappa d\xi d\eta \qquad V.11$$

Doing this allows us to interchange order of integration. Now we must perform the time integrals. To do this we require the following properties of the Dirac delta function:

$$\delta(g(t)) = \frac{\delta(t-\tau)}{|\dot{g}(\tau)|} \quad \text{V.12}$$

$$\dot{\delta}(g(t)) = \frac{\dot{\delta}(t-\tau)}{\dot{g}(t)\,|\dot{g}(\tau)|} \quad \text{V.13}$$

where $g(t)$ is any regular function, and τ is a root of g, so $g(\tau) = 0$. (If there are multiple roots, they must be summed.) In this application, we have $g(t) = t - t_0 + R/a_0$, where R depends on t through the motion of the surface coordinates.

Now perform the time integration to get:

$$-4\pi\phi(\vec{x}_0, t_0) = \iint d\xi d\eta [\kappa f_0 - (\phi\dot{\lambda} + \lambda\dot{\phi})]/|\dot{g}| \quad \text{V.14}$$

$$\lambda = \frac{\kappa f_1}{\phi \dot{g}} \quad \text{V.15}$$

The integrand is evaluated at the retarded time τ, the root of $g(t)$. This formula is still incomplete because it contains (in f_0) the partial time derivative $\phi_{,t}$. This can be replaced with the total time derivative of ϕ at the body fixed point P_0 by means of the chain rule identity:

$$\phi_{,t} = \dot{\phi} - u_n u_{sn} - \vec{u}_{st} \cdot \nabla\phi \quad \text{V.16}$$

where we have decomposed the surface velocity $\vec{u}_s$ into its normal and tangential components:

$$\vec{u}_s = \vec{u}_{st} + \hat{n} u_{sn} \quad \text{V.17}$$

The end result for the potential outside the body can now be written in the following somewhat more compact form:

$$-4\pi\phi(\vec{x}_{0,} t_0) = \iint [C_0 u_n + C_1 \phi + C_2 \dot{\phi} \quad \text{V.18}$$

Where the four coefficients C_j depend only on the surface geometry and state of motion, not on the properties of the wave field ϕ. These coefficients are given by:

$$C_0 = \frac{\kappa(1-M_{sn}^2)}{R_c} \qquad \text{V.19}$$

$$C_1 = -(\kappa\hat{n}\cdot\vec{N}/R^2+\dot{\lambda})/|\dot{g}| \qquad \text{V.20}$$

$$C_3 = -\frac{\kappa M_{sn}}{a_0 R_c} \qquad \text{V.21}$$

$$C_2 = -C_3-\lambda/|\dot{g}| \qquad \text{V.22}$$

$$\lambda = \frac{\kappa}{a_0 R\dot{g}}(\hat{n}\cdot\vec{N}-M_{sn}) \qquad \text{V.23}$$

$$\dot{g} = 1-\vec{u}_s\cdot\vec{N}/a_0 = 1-M_r \qquad \text{V.24}$$

$$\vec{R} = \vec{x}_0-\vec{x}(P_0,\tau) \qquad \text{V.25}$$

$$R_c = R\ |\dot{g}| \qquad \text{V.26}$$

$$\vec{N} = \vec{R}/R \qquad \text{V.27}$$

The above formula is perfectly general (aside from having ignored initial disturbances within the flow,) and makes no assumptions about the shape or state of motion of the surface S(t) over which the integration is performed. It is one possible choice of definition of the general source/doublet distribution formulas, which has the advantage that it contains only physically important quantities, the normal velocity at the surface u_n and the surface potential itself, ϕ. It appears more complicated than the earlier representation only because the derivatives have been carried through. Some small simplifications are possible in the special case of rectilinear unaccelerated motion of the surface S, but they are insignificant. For rectilinear motion, the retarded time calculation can be performed analytically. For other types of motion, it would have to be done numerically, though the additional cost would likely be small compared to the total cost of solving the problem.

Note that this identity applies everywhere in the exterior space surrounding the body, and gives us the potential there directly if the normal velocity and surface potential were both known. However, only the normal velocity is known a priori. To find the surface potential, we need only bring the field point $\vec{x}_0$ down onto the surface $S(t_0)$, at some point ξ_0, η_0, say. We then have an equation mapping the surface onto itself:

$$L\cdot\phi = b \qquad \text{V.28}$$

where b represents that part of the integral which is known at time t_0, and L is a linear Fredholm operator of the second kind. In fact, because of the finite time delay between points, L is very nearly a simple scalar multiplier.

Sec(V.1) Retarded Time

The retarded time τ is the solution of the delay equation $t=\tau+R/a_0$. This is equivalent to finding the positive roots of the equation:

$$F(R) = R^2 - |\vec{x}_0 - \vec{x}(t - R/a_0)|^2 = 0 \qquad \text{V.29}$$

Now for rectilinear motion this is just a quadratic. Other motions would be at least locally rectilinear. A general quadratic iteration is, thus, suggested, based on the Taylor series expansion about some point R:

$$F = F(R) + \delta R\, F_{,R} + .5[\delta R]^2\, F_{,R,R} \qquad \text{V.30}$$

$$F_{,R} = 2[R - \vec{R}\cdot\vec{M}_s] \qquad \text{V.31}$$

$$F_{,R,R} = 2[1 - M_s{}^2 + \vec{R}\cdot\dot{\vec{M}}_s/a_0] \qquad \text{V.32}$$

Solving the quadratic for the change δR gives the following solution for R :

$$R = [\,-B \pm \sqrt{D}\,]\,/\,A \qquad \text{V.33}$$

$$A = 1 - M_s{}^2 + \vec{R}\cdot\dot{\vec{M}}_s\,/\,a_0 \qquad \text{V.34}$$

$$B = R\,(1 - A) - \vec{R}\cdot\vec{M}_s \qquad \text{V.35}$$

$$D = (\,B + AR\,)^2 + A(|\vec{R}|^2 - R^2) \qquad \text{V.36}$$

The two roots for R are there simply because the equation is quadratic. If the flow is subsonic there can be but one positive root, and the other is simply discarded. At supersonic speeds, there may be two or no (real positive) roots, depending on whether the field point is inside the domain of influence of the source point or not. If there are multiple roots, and therefore multiple retarded times, the contribution of each to the integral is added in.

In rectilinear motion this gives the exact solution regardless of the initial guess for R. In nonrectilinear motion it would give the correct result if iterated starting from a nearly correct result. Since any real calculation would involve a continuous distribution of retarded time over the surface, one would always have a good initial guess in hand from the last point processed.

Section (VI)
Uniform Translation

In the important special case where the body translates with constant velocity, certain simplifications are possible. This special case will be examined herein, first for steady flow, then for unsteady flow.

Sec (VI.1) Steady Flow

Although this is a standard problem it will be described to establish the method and notation. If the Mach number is M, directed along the x axis, then the disturbance potential obeys the classical formula:

$$(1-M^2)\phi_{,x,x}+\phi_{,y,y}+\phi_{,z,z}=0 \qquad \text{VI.1}$$

If we define $\beta=\sqrt{(1-M^2)}$, and do a transformation of variables:

$$\bar{x}=x, \bar{y}=\beta y, \bar{z}=\beta z \qquad \text{VI.2}$$

then, as is well known, the equation reduces to Laplaces equation:

$$\bar{\nabla}^2\phi=0 \qquad \text{VI.3}$$

The Green's function is, of course, $-1/4\pi R_c$, where R_c is the radius in barred coordinates. This familiar devise is the steady state Lorentz transformation, which we will make use of again in the next section.

A direct application of Green's theorem to Laplace's equation in barred coordinates, followed by a transformation back to physical coordinates in the integrals, yields the following identity:

$$-4\pi\phi(\vec{x}_0)=\iint[\frac{q}{R_c}+\beta^2\frac{\vec{R}\cdot\hat{n}}{R_c{}^3}\phi]dA \qquad \text{VI.4}$$

which will be recognizable as a distribution of sources and dipoles over the surface. However the source density is:

$$q=\hat{n}\cdot\nabla\phi-M^2\phi_{,x}n_x \qquad \text{VI.5}$$

which is not known from the flow tangency condition on the solid surface, because it contains the streamwise velocity perturbation $\phi_{,x}$, which is part of the unknown. A similar term involving tangential velocity appeared in the general unsteady Green's theorem formulation.

There are a variety of ways to fix this. Here we use integration by parts to eliminate the tangential velocity. First we can decompose the velocity $\phi_{,x}$ into normal and tangential parts, with the result:

$$q=\beta_n{}^2 u_n - M^2\, n_x\, \gamma\, \hat{\tau}\cdot\nabla\phi \qquad \text{VI.6}$$

where $\hat{\tau}$ is the unit tangent vector in the flow plane, and:

$$\beta_n = \sqrt{1-M_n{}^2} \qquad \text{VI.7}$$

$$M_n = Mn_x \qquad \text{VI.8}$$

$$u_n = \hat{n}\cdot\nabla\phi \qquad \text{VI.9}$$

$$\gamma = \hat{i}\cdot\hat{\tau} \qquad \text{VI.10}$$

The part of q which contains tangential derivatives can be integrated by parts over the surface. We suppose that the surface is closed, so that:

$$\iint \nabla\cdot(M^2\, n_x\, \gamma\, \hat{\tau}\, \phi \,/\, R_c)\, dA = 0 \qquad \text{VI.11}$$

If this is true, then we obtain the representation:

$$-4\pi\phi(\vec{x}_0) = \iint [\beta_n{}^2 \frac{u_n}{R_c} + \phi K\,] dA \qquad \text{VI.12}$$

where K is a somewhat messy function of position and Mach number given by:

$$K = K_0/R_c{}^3 + K_1/R_c \qquad \text{VI.13}$$

$$K_1 = (M^2 - 3M_n^2)/R_s \qquad \text{VI.14}$$

$$K_0 = \beta^2(1+M_n{}^2)\vec{R}\cdot\hat{n} \qquad \text{VI.15}$$

The variable R_s is the radius of curvature of the surface in the flow plane. It appears in the result because of the differential geometry identity:

$$\nabla\cdot(n_x \gamma\hat{\tau}) = (1-3n_x{}^2)/R_s \qquad \text{VI.16}$$

This formula contains no tangential derivatives of the unknown, only a distribution of known sources, and a weighted integral of the unknown ϕ. It naturally contains many terms which, if they were not negligible, would invalidate the assumptions of the linear theory on which the formula is based. (For example, you must have $M_n \ll 1$.) This is because the result was derived for an arbitrary body shape, while not every body shape is really suitable for a small disturbance approximation.

Sec (VI.2) Unsteady Flow

Much of the complexity of the general unsteady formulation arises from the fact that the body is moving relative to the gas. For a stationary radiating body (acoustics), the formulation is quite simple, and gives the classical result:

$$-4\pi\phi(\vec{x}_0,t) = \iint[\frac{u_n(\vec{x},\tau)}{R}]dA+\nabla_0\cdot\iint[\frac{\hat{n}}{R}\phi(\vec{x},\tau)]dA \qquad \text{VI.17}$$

where u_n is the normal velocity at the surface (known), and $\tau = t-R/a_0$ is the retarded time.

In the special case of rectilinear motion of the surface, we can transform to Lorentz coordinates, which make the wave equation identical in form to the stationary case:

$$[\overline{\nabla}^2-\frac{1}{a_0^2}\frac{\partial^2}{\partial \overline{t}^2}]\phi = 0 \qquad \text{VI.19}$$

while leaving the body surface stationary (the over bars denote Lorentz variables). Now, since the bounding surface is independent of time, the formal solution is easily obtained just by replacing all variables in the stationary body integral formula with Lorentz coordinates.

The result is, however, awkward to deal with because the surface coordinates are deformed. Therefore, the integrals are transformed back to physical space. After some tedious manipulation, we get an integral identity with striking resemblance to the steady state formula:

$$-4\pi\phi(\vec{x}_0) = \iint[\frac{\beta_n{}^2}{R_c}\, u_n + \phi\, K\, + \dot{\phi}\, K_2]dA \qquad \text{VI.20}$$

which is just as in steady flow (K is the same), except that the integrand is evaluated at the retarded time, and there is a new term directly proportional to the local time derivative of ϕ, with a coefficient:

$$K_2 = M^2\frac{n_x\gamma\hat{\tau}\cdot\nabla\tau}{R_c}\vec{R}\cdot\hat{n}/R_c] \qquad \text{VI.21}$$

Note that if the Mach number is set to zero, the formula reduces to the stationary radiating body case.

This formula for ϕ may be taken as the basic result of this investigation. It give the integral representation of ϕ directly in terms of surface values of ϕ itself (with no tangential derivatives) and of a known source distribution. It is little harder to implement than a steady flow solver based on the same kind of approach. In fact the resemblance is close enough that the best way to implement it might be to take an existing steady panel code and modify it to account for the retarded time and the extra time derivative term in the equation.

Sec (VI.3) Discretization

The integral representation for ϕ can obviously be discretized and put into the canonical form, (on the surface itself):

$$[\phi] = C_0[u_n]_r + C_1[\phi]_r \qquad \text{VI.22}$$

where the brackets [ ] denote vectors spanning the surface and the coefficients C_k are square matrices with dimension equal to the number of elements of the surface representation. The values of these coefficients depends on the particular quadrature rules adopted. The important thing, though, is that for the case of rectilinear motion, these coefficient matrices are independent of time. In particular, C_0 and C_1 are the same matrices that would appear in a steady flow solver. (The coefficient matrix C_2 would, of course, not appear in a steady solver because it multiplies a term which vanishes in that case.) They, in principle, can be computed once and stored. The only drawback to doing this is that for a large problem, these storage requirements might be prohibitive. If there are 5000 elements on the surface, storage of all three coefficients would amount to $7.5\ 10^7$ matrix elements. For reasons which will be discussed below, only a small part of these matrices would have to be stored to do a direct solution for $[\phi]$ by time marching.

In steady flow, the C_2 term and the subscripts "r" can be dropped, leading to a linear algebra problem with the form:

$$AIC[\phi] = C_0[u_n] \qquad \text{VI.23}$$

$$AIC = I - C_1 \qquad \text{VI.24}$$

If there are N surface elements, then AIC is a full N by N matrix. This is the standard steady flow problem.

In an unsteady flow, the vector $[\phi]$ indicates the current time vector of ϕ's on the surface. The vector $[\phi]_r$ (and similarly for the other two vectors on the right hand side of the equation) indicates the retarded time realization of ϕ at each surface element. The farther away the sending element is from the receiving element, the farther back in time the signal will come from. Only the nearest elements would depend on the current time (unknown) solution, so that the size of the linear algebra problem to be solved is in fact far smaller than the number of elements on the surface, unless the time step is so large that a sound wave will span a large fraction of the surface in one step.

To make this concrete, suppose that the solution for ϕ is stored in an extended vector $[\phi]_e$ which spans the surface and as much of the time history as necessary to include all possible point to point interactions. (So this extended vector contains copies of $[\phi]$ at equal time intervals.) Then by the retarded vectors we must mean:

$$[u_n]_r = T[u_n]_e \tag{VI.25}$$

$$[\phi]_r = T[\phi]_e \tag{VI.26}$$

$$[\dot{\phi}]_r = T_1[\dot{\phi}]_e \tag{VI.27}$$

where T is an interpolation operator, and T_1 is a difference operator (i.e., matrix representations of such operators.) For example we could use $T_1 = T(1-E)/\Delta t$, where E is a unit back shift and Δt is the time step.

If there are N total surface elements, and a fixed N_t elements within the discrete domain of dependence of any given element, then the problem can be reorganized into a simple linear algebra problem:

$$\mathrm{AIC}\,[\phi] = [b] \tag{VI.28}$$

where AIC is an N by N matrix, with at most N_t nonzero entries on each row, and [b] is an N vector containing all of the known data. The AIC matrix is sparse, but its sparsity pattern is unknown and would vary depending on how the surface elements were numbered and on how big the time step is (and so on how big the domain of dependence of any element is). The system could be solved by a one-time LU decomposition of AIC, followed by a back solve at each time step.

Sec (VI.4) An Exact Solution

One difficulty with any numerical procedure is to decide whether the calculated results are correct or not. For the disturbance produced by a general three-dimensional time-dependent deforming body, this is a particular problem, since exact solutions for typical prescribed body motions are not available. This difficulty, however, can easily be circumvented.

In the Green's function based method, the surface source density, which is simply the normal velocity $\partial\phi/\partial n$, is set, and the resulting surface potential is to be determined through a numerical solution of the integral equation. However, consider a surface with finite volume but otherwise arbitrary shape. Somewhere in the volume enclosed by this surface place a moving point source of strength $\sigma(t)$. The velocity potential induced by this source is:

$$\phi(\vec{x},t) = -\sigma(\tau) / 4\pi R_c \qquad \text{VI.29}$$

From this elementary potential we can compute the velocity induced normal to the enclosing surface:

$$u_n = \frac{1}{4\pi R_c{}^2}[\, \hat{n}_0 \cdot \vec{R}\, (\, \dot{\sigma} / a_0 + \sigma\, (\, 1 - M_0^2\,) / R_c\,) - \sigma \hat{n}_0 \cdot \vec{M}_0\,] \qquad \text{VI.30}$$

where $\hat{n}_0$ is the unit outward normal to the bounding surface, $\vec{M}_0$ is the source Mach number, and all quantities are evaluated at the retarded time.

If we set the normal velocity on the surface according to this formula, then the exact solution for the surface potential is known: it is just the simple source potential. The advantage of this ruse is that it is applicable to bodies with completely arbitrary shape, moving along an arbitrary path. Obviously other test cases can be built similarly by placing more sources or other prescribed singularities within the body, and computing the resulting surface normal velocity. The simple source, however, would seem to be entirely sufficient for checking any numerical method.

We note, for future reference that for the special case of a spherical surface, with the source placed at the center of the sphere, the surface normal velocity simplifies slightly to:

$$u_n = [\, \hat{n}_0 \cdot \vec{R}\, \dot{\sigma} / a_0 \qquad \text{VI.31}$$

Sec (VI.5) An Example Problem

To test out the formulation on a simple example, we considered the following special case. Take a sphere. Inside the sphere, at the center, place a simple point source. The source, and the sphere surrounding it, translate along the x axis at some fixed subsonic Mach number. The normal velocity on the surface of the sphere is set, as indicated in the previous section, to the value:

$$u_n = [\ \hat{n}_0 \cdot \vec{R}\ \dot{\sigma} / a_0 \qquad \text{VI.32}$$

so that the exact solution for the potential should be:

$$\phi(\vec{x},t) = -\sigma(\tau) / 4\pi R_c \qquad \text{VI.33}$$

for any arbitrary choice of source strength $\sigma(t)$.

The problem is organized in the following way. The sphere is broken into N disjoint panels. The distance between the centers of panels i and j, $R_{i,j}$, defines a signal time delay:

$$R_{i,j} = a_0\ \Delta t\ (n_{i,j} + \varepsilon_{i,j}\ ) \qquad \text{VI.34}$$

where Δt is the time step, $n_{i,j}$ is the whole number of time steps, and $\varepsilon_{i,j}$ is the fractional part.

The surface potential is discretized as:

$$-4\pi\phi_i = \Sigma_j\ \ [C_{0,i,j} q_j{}^r] \qquad \text{VI.35}$$

where the superscript "r" quantities are defined by simple linear interpolation;

$$q_j{}^r = q_j^{n_{ij}} + \varepsilon_{ij}(q_j^{n_{ij}} - q_j^{n_{ij}+1}) \qquad \text{VI.36}$$

$$\phi_j{}^r = \phi_j^{n_{ij}} + \varepsilon_{ij}(\phi_j^{n_{ij}} - \phi_j^{n_{ij}+1}) \qquad \text{VI.37}$$

$$\dot{\phi}_j{}^r = (\phi_j^{n_{ij}} - \phi_j^{n_{ij}+1})/\Delta t \qquad \text{VI.38}$$

The superscripts n_{ij} are time counters, indicating where in the global history array, the data are to be found for panels i and j.

At the start of each time step, the arrays ϕ^n are advanced, and the current time values ϕ^1 are set to zero. The potential is then summed and the result of the summation is stored in a vector b_i.

The influence coefficient matrix is loaded based on the following:

$$AIC_{ii} = 4\pi \qquad \text{VI.39}$$

$$\text{if} \quad n_{ij} = 1, \text{then}$$

$$AIC_{ij} = AIC_{ij} + C_{1,ij}\varepsilon_{ij} + C_{2,ij}/\Delta t \qquad \text{VI.40}$$

No attempt was made to use the sparseness of AIC because the problems were all of small size.

The influence coefficients were evaluated in the crudest possible way by a simple midpoint rule, except on the diagonals, where a three-point average was taken. This method is unsatisfactory and was used only to test out the algorithm.

In the first example, the Mach number is 0.5 and the source strength is $\sigma(t)=1-e^{-t}$, in units where the speed of sound and radius of the sphere are both 1. The sphere was divided into 400 panels by 20 equal longitudinal and 20 latitudinal cuts. The time step was chosen as 0.2. The result for the time history of the potential at a point 45° over the leeward side of the sphere is shown in Figure 1, along with the exact solution. The qualitative features of the solution are reproduced, but the numerical results are rather jagged. This is not a special property of this point, as shown in Figure 2, which shows the longitudinal surface distribution at a sequence of times. It is believed that the noise in this calculation is caused primarily by the poor choice of near field influence coefficients, not by the low order temporal interpolations used. The fact that the general timing of the disturbances is correct indicates that the implementation was made correctly. All that remains to be done is to replace the steady-state influence coefficients by more accurate values.

This is shown somewhat more clearly in the second example, which is identical to the first except for a smoother source distribution, $\sigma(t) = (1-e^{-t})^2$. The distribution of surface normal velocity for this example is shown in Figure 3. It is quite smooth, and approaches a steady state at long times. The corresponding exact solution for surface potential is shown in Figure 4, and the numerically computed solution in Figure 5. Notice that the numerical solution starts out correctly, but eventually reaches an incorrect steady state. This is a clear indication of the inadequacy of the steady influence coefficients (which are what determine the steady solution.)

The numerical part of this work was not carried further for nontechnical reasons. Much remains to be done, principally with regard to the implementation of accurate steady-state influence coefficients.

Section (VII)
A Timing Study

An effort was made to examine the cost of this type of calculation. Admittedly, such costs are very machine dependent, but the results are suggestive. The major part of the calculation is simply the accumulation of the sums representing the surface integrals. Without time delays this amounts to simple vector multiplies and code vectorization gives a factor of 10 speed up. When the time delays are included, the basic operation is a summation of the type A(i,j)X(k(i,j)), which does not vectorize unless the vector X is gathered and scattered before and after the operations are performed. This was done, with an interesting result. The calculation became a trivial part of the calculation. The gathering and scattering operations themselves consumed the greatest part of the cpu time (by about a factor of 8). This means that only a factor of 2 savings could be obtained by vectorization, overall, since the gather/scatter time could not be reduced.

We can give a breakdown of the costs of a typical time step for each of six distinct phases of the vectorized code. The units are percent of the total cpu time for the step:

1) evaluation of source strengths 1%

2) source terms .... gather 42%
.... compute 7%

3) ϕ terms .... gather 38%
.... compute 10%

4) inversion .... <1%

5) shift stacks 1%

It is apparent that 80% of the overall cost involves no calculations.

It was estimated that in vector mode, the test code that was run took about 1.5 $10^{-5}N^2$ seconds of CPU time per time step (on a Gould NP1 machine.) Note that the cost is inherently quadratic in the number of panels and, therefore, would become prohibitive rather quickly. With this run time estimate, a 5000-panel model, run through 1000 time steps would take 100 hours. This is not reasonable given the physical limitations of the underlying theory. The Gould is about 1% Cray2 speed, depending on the problem, so this might translate into 1 hour on a Cray. It is quite likely, though not certain, that a direct finite difference solution to the same problem, with the same resolution, might be faster on such a machine. If this is true, then methods such as that discussed here ought to be useful only when N is small, that is for low resolution simulations. Since the resolution of the underlying theory is limited anyway, this may be appropriate.

Section (VII)
Conclusions

The present study provides a general formulation of the time dependent linearized aerodynamic problem for bodies moving along arbitrary paths, and in particular, for bodies moving rectilinearly at constant velocity (apart from small amplitude elastic deformations of the surface.) For the case of rectilinear motion, the formulation differs somewhat from previous work in that a "direct" Green's identity is given which contains only the surface potential and surface normal velocity. All tangential dipole terms were removed by an integration by parts.

A significant result of the study is a simple exact solution that can be used to check any general purpose time dependent code, at least for bodies with finite volume. This simple idea could be useful in the testing of other codes, though it does not, of course, provide results for problems of physical interest.

Implementation of the proposed formulation in a working code, for the most part, remains to be done. One example calculation was performed, using rather poor near field influence coefficients, to test out the basic ideas. The results of that calculation indicated the clear need for a better near field representation, if accurate solutions are to be obtained, but gave no suggestion that there would be any difficulty doing so. The example did allow a realistic assessment of the probable computational requirements of a full simulation. The costs would be quite high on a large problem (several thousand panels), though of course no method is likely to be able to handle a complete aircraft easily and cheaply.

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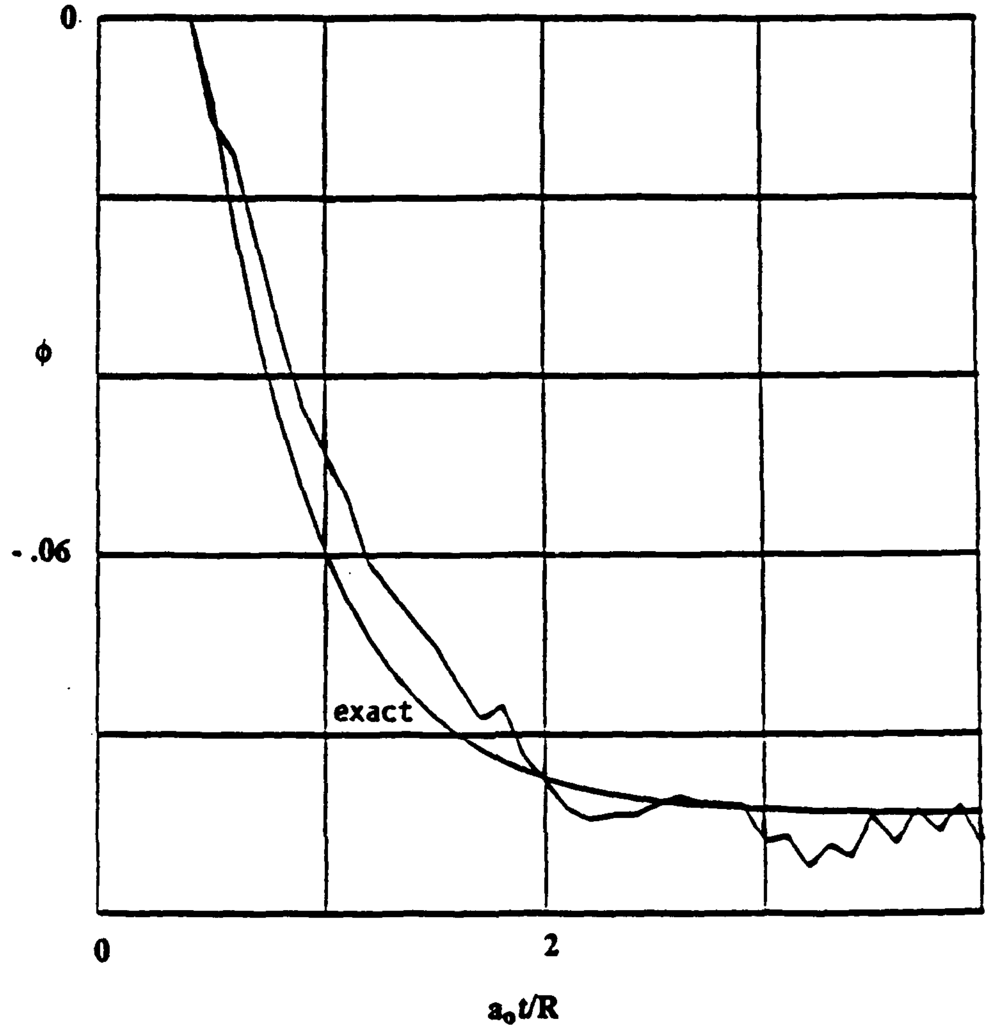

Figure 1

Computed Potential at 45 deg leeward, M = .5, $\sigma = 1 - e^{-t}$

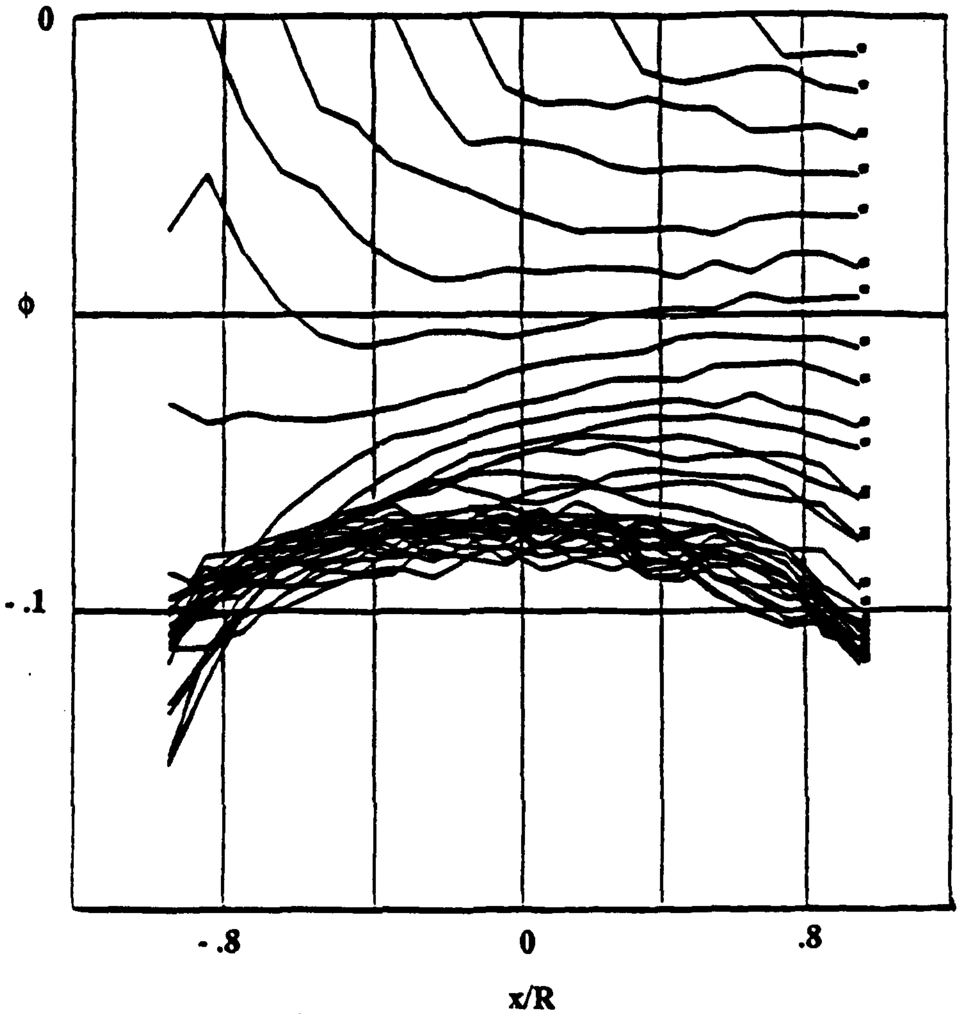

Figure 2

Distribution of potential with longitude, case of Figure 1

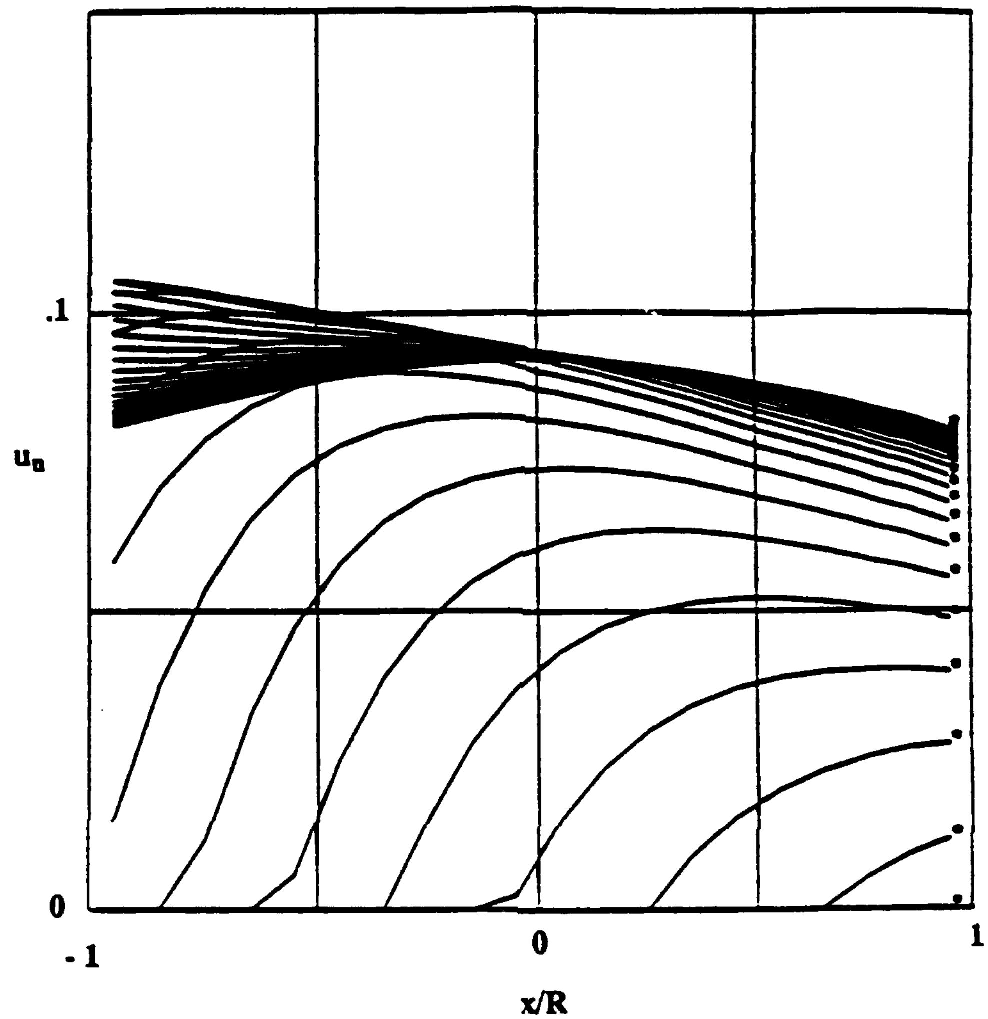

Figure 3

Distribution of Normal velocity, $M = .5$, $\sigma = (1 - e^{-t})^2$

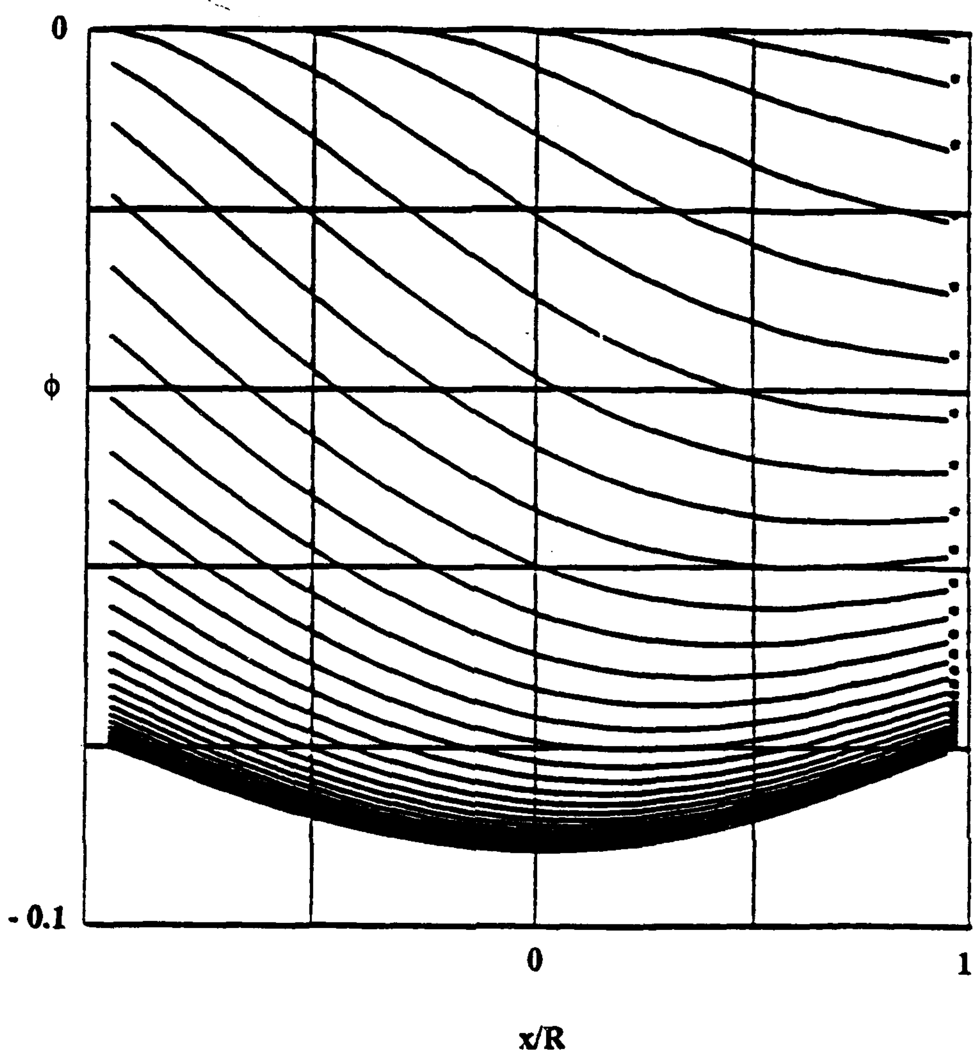

Figure 4

Exact distribution of surface potential, case of Figure 3

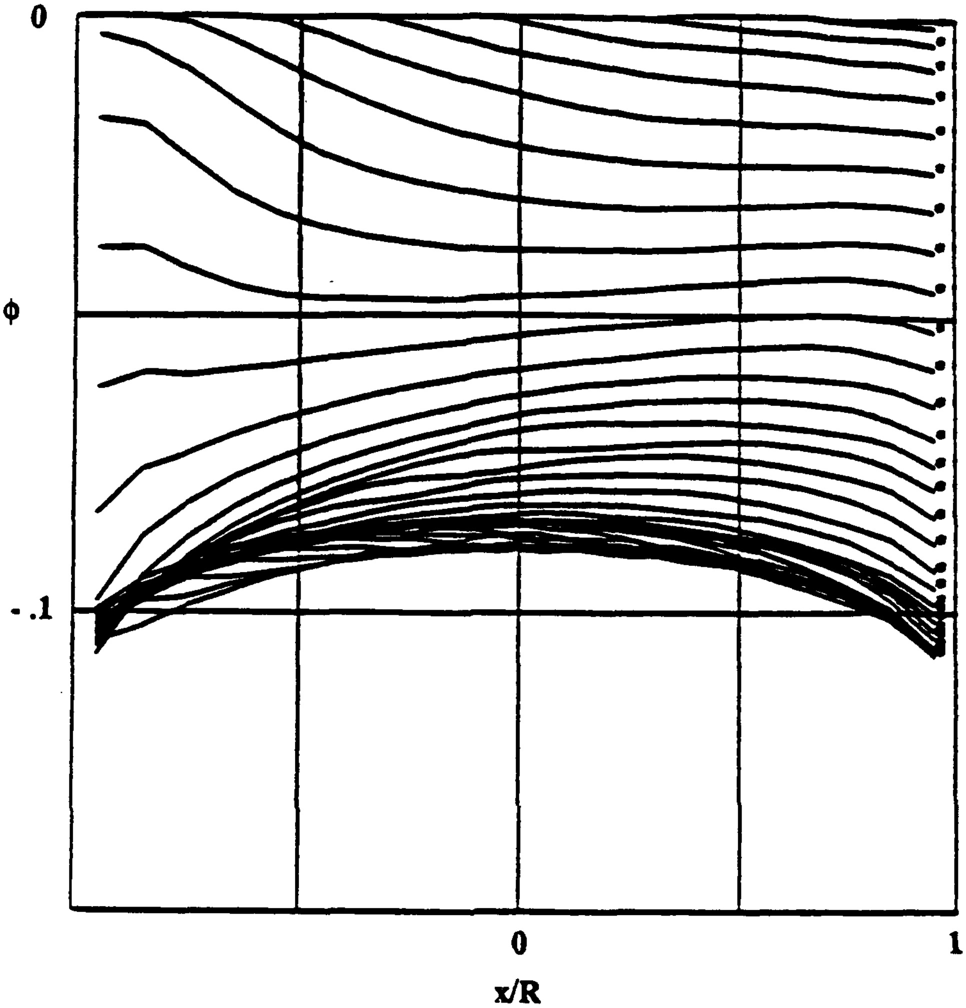

Figure 5

Numerical distribution of surface potential, case of Figures 3, 4