On the Radial Component of the Electric Field for a Monopole Phased Array Antenna Focused in the Near Zone

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ON THE RADIAL COMPONENT OF THE ELECTRIC FIELD
FOR A MONOPOLE PHASED ARRAY ANTENNA
FOCUSED IN THE NEAR ZONE

A.J. FENN
Group 61

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LEXINGTON MASSACHUSETTS
ABSTRACT

The near-zone radial and principal spherical components of the electric field for a linear phased array antenna that is focused at one to two aperture diameters in the near zone are investigated. The phased array antenna, consisting of thin monopole elements, is analyzed by using the method of moments. A theoretical formulation is described and computer simulation results are presented. The results show that the radial component is negligible in the focused near-field region, which allows the principal component to be accurately computed from knowledge of only the tangential electric field on the focal plane.
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1. INTRODUCTION

The theoretical focused near-field adaptive nulling performance of phased array antennas has been studied recently, taking into account array element polarization and mutual coupling effects in Fenn, April 1989 and October 1990 [1,2] and for ideal isotropic point sources in Fenn, February 1990 [3]. In Fenn [1-3] it is shown that the adaptive array characteristics in the focused near-field region are equivalent to conventional far-field characteristics. Experimental verification of focused near-field adaptive nulling is presented in Fenn et al., May 1990 [4]. In Fenn, April 1989, October 1990, and February 1990 [1,2,4] the phased array antenna is focused at one to two aperture diameters and radiating test sources are positioned on the focal plane. Near-field main beam focusing is achieved with calibration and phase control at each active element of the array, producing an antenna near-field radiation pattern equal to a conventional far-field pattern [5]. As part of the near-field radiation pattern formulation in Fenn et al., April 1989, October 1990, and February 1990 [1,2,4], it is assumed that the radially-polarized component of the electric field is negligible compared to the principal component. The principal component was computed from the tangential component while ignoring the radial component. The radially-polarized electric field has been considered by several investigators for determining the cross-polarization level in reflector antenna systems [6,7]. No open-literature references are found that give quantitative data for the radially-polarized amplitude level of a phased array focused in the near field [8-11]. The purpose of the present report is to show by computer simulation that, for a near-field focused phased array antenna made up of perfect monopole elements, the assumption of a negligible radial component is valid.

The next section gives the design for a monopole phased array antenna used in the computer simulations. Section 3 describes the near-field formulation used to calculate the electric field components of the monopole phased array antenna. Section 4 presents the results, and Section 5 contains the conclusions.
2. MONOPOLE PHASED ARRAY ANTENNA DESIGN

Figure 1 shows the phased array antenna under evaluation. The array consists of thin monopole elements having a length of 0.264λ with wire radius 0.007λ and having element spacing 0.473λ at center frequency 1.3 GHz. This type of array has recently been used in evaluating planar near-field measurement techniques for low-sidelobe antenna applications [12]. A monopole phased array antenna is useful in achieving wide-angle scanning as described in Herper, 1975 and Fenn, October 1985 [13,14]. The array is assumed to have 180 elements arranged in 5 rows and 36 columns on a square lattice. For impedance matching purposes, two guard bands of passively terminated (50-ohm loaded) elements (unshaded elements in Figure 1) are assumed to surround the active portion (shaded elements) of the array that forms a linear array of 32 active elements in the center row. With 4.3-in interelement spacing, the length of the active array is 133.3 in. Near field distances \( z_0 = 133.3 \text{ in} \) (one aperture diameter) to \( z_0 = 266.6 \text{ in} \) (two aperture diameters) are considered in this report.
Figure 1. Monopole array geometry. The shaded elements are active and the unshaded elements are passively terminated.
3. THEORETICAL FORMULATION

In standard spherical coordinates, the far-field principal polarization of a monopole array, with \( \hat{z} \)-directed elements, is the \( E_\theta \) or vertically-polarized component. At a range distance of infinity, the radial component of the electric field, \( E_r \), is zero; however, in the near field \( E_r \) is nonzero. For a thin monopole, the \( E_\phi \) component of the electric field is theoretically zero both in the near field and in the far field and is ignored in this analysis.

The computation of the near-zone radial and vertical electric field components is accomplished by using expressions given in Schelkunoff, 1943 [15]. The method of moments [16] is used to include the effects of mutual coupling between the monopole array elements. The monopoles and infinite ground plane are replaced by an equivalent array of dipoles in free space as depicted in Figure 2. A piecewise-sinusoidal current distribution

\[
i_n(z) = i_{n,\text{term}} \frac{\sin[k(l - |z|)]}{\sin kl}
\]  

is assumed on the \( n \)th array element. In Equation (1), \( i_{n,\text{term}} \) is the terminal current for the \( n \)th element, \( k = 2\pi/\lambda \) is the propagation constant, with \( \lambda \) the wavelength, \( l \) is the monopole length (dipole half length), and \( |z| \leq l \).

The near-zone electric field components for a center-fed linear dipole antenna are given in cylindrical coordinates as [15]

\[
E_{nz} = \frac{j30i_{n,\text{term}}}{\sin kl} \left( \frac{e^{-jk\rho}}{\rho_0} \cos kl - \frac{e^{-jkr_1}}{r_1} - \frac{e^{-jkr_2}}{r_2} \right) 
\]  

\[
E_{n\rho'} = \frac{j30i_{n,\text{term}}}{\rho' \sin kl} \left( e^{-jk\rho} \cos \theta_1 + e^{-jk\rho} \cos \theta_2 - 2 \cos k \rho e^{-jk\rho} \cos \theta_0 \right) 
\]

where \( j = \sqrt{-1} \) and the parameters \( \rho_0, r_1, r_2, \rho' \) and \( \theta_0, \theta_1, \theta_2 \) are defined in Figure 2. The \( E_{nz} \) and \( E_{n\rho} \) rectangular coordinate system components are readily computed as

\[
E_{nz} = E_{n\rho'} \cos \phi' 
\]  

\[
E_{n\rho} = E_{n\rho'} \sin \phi' 
\]

where \( \phi' = \tan^{-1}((y - y_n)/(x - x_n)) \) is the angle between \( \hat{z} \) and \( \hat{\rho'} \) with the coordinates \((x_n, y_n)\) denoting the feed point of array element \( n \). To compute the antenna near field, including array mutual coupling effects, the array terminal currents are computed in the following manner:
Figure 2. Near-field geometry for equivalent dipole array antenna.
Let $Z$ represent the mutual impedance matrix for the equivalent dipole array, such that

$$Z = Z^{oc} + Z_L I$$

(6)

where $Z^{oc}$ is the open-circuit mutual impedance matrix for the array, $I$ is the identity matrix, and $Z_L$ is the load impedance at each element. The moment method mutual impedance matrix is evaluated by using computer subroutines developed in Richmond, 1974 [17].

Define $\mathbf{v}$ as the voltage excitation matrix of the array. Then the array element terminal currents, $i$, are related to the voltage excitation matrix and impedance matrix by

$$\mathbf{v} = Z \cdot \mathbf{i}.$$  \hspace{1cm} (7)

The $n$th element of the voltage excitation matrix for a phased array antenna is given by

$$v_n = A_n e^{j\psi_n}$$

(8)

where $A_n$ is the amplitude illumination and $\psi_n$ is the phase delay that scans the main beam to the near-field position $(z_0, \theta, \phi)$.

Using Equation (6) in Equation (7), the array terminal currents are found by solving the system of equations written in matrix form as

$$\mathbf{v} = [Z^{oc} + Z_L I] \cdot \mathbf{i}.$$  \hspace{1cm} (9)

The impedance matrix in Equation (9) is of block-Toeplitz form for which special-purpose computer subroutines are used in solving for the unknown currents [16,18]. Having computed the array terminal currents from Equation (9) and the $n$th-element near field by using Equations (2) to (5), the array near-zone field including mutual coupling effects is expressed by summing the contributions from all $N$ array elements as

$$E_x(x, y, z_0) = \sum_{n=1}^{N} E_{nx}$$

(10)

$$E_y(x, y, z_0) = \sum_{n=1}^{N} E_{ny}$$

(11)
\[ E_z(x, y, z_0) = \sum_{n=1}^{N} E_{nz}. \]  
(12)

After computing the rectangular components of the electric field, the equations for computing the exact near-field spherical components in the \( \phi = 0^\circ \) cut are given by a standard rectangular to spherical coordinates conversion,

\[ E_r = E_x \sin \theta + E_z \cos \theta \]  
(13)

\[ E_\theta = E_x \cos \theta - E_z \sin \theta. \]  
(14)

Note that in the \( \phi = 0^\circ \) cut, for a linear array of \( z \) monopoles the \( E_y \) component is theoretically zero and is ignored. If it is assumed that the radial component is zero (far-field (FF) assumption), then substituting \( E_r^{\text{FF}} = 0 \) in Equation (13) yields

\[ E_r^{\text{FF}} = -E_z \tan \theta. \]  
(15)

Therefore, the \( E_z \) component is dependent on the \( E_x \) component when the radial component is zero. From Equation (15) it is observed that \( E_r^{\text{FF}} \) is in phase with \( E_x \) for \( \theta < 0^\circ \) and is \( 180^\circ \) out of phase for \( \theta > 0^\circ \). Substituting Equation (15) in Equation (14) and simplifying yields

\[ E_\theta^{\text{FF}} = \frac{E_x}{\cos \theta} \]  
(16)

for the case of a zero radial component in the far field (or a negligible radial component in the near field). Thus, Equation (16) is an approximate expression for the vertical near-field component whereas Equation (14) is exact. Both Equations (14) and (16) are applied in the next section to the case of a near-field focused phased array antenna with variable focal range and variable scan angle.
4. RESULTS

As an example of the principal-plane ($y = 0$) radial and vertical electric field components radiated by the 32-active element monopole array shown in Figure 1, consider first a $-30^\circ$ scan angle from broadside and a Taylor, $\bar{\pi} = 10$ dB illumination function. The computed $E_r$ and $E_z$ components [from Equations (10) and (12), respectively] on the observation line at one aperture diameter distance ($z_o=133.3$ in) are shown in Figure 3(a). The peak $E_z$ (normal) component is down by $-4.5$ dB compared to the peak $E_x$ (tangential) component. Using the conversion from rectangular to spherical coordinates [Equations (13) and (14)], the radial and vertical field components are computed and are shown in Figure 3(b). Here, it is seen that the maximum radial component is down by $-19.1$ dB. Note that the near field data in Figure 3(b) have been presented as a function of angle with respect to the phase center $(x = 0, y = 0, z = 0)$ of the linear array. The corresponding data for near-field focusing and near-field observation at 1.5 aperture diameters are shown in Figure 4. In Figure 4(a), the peak $E_z$ component is down by $-4.6$ dB compared to the peak $E_x$ component. In Figure 4(b), the maximum radial component is $-22.2$ dB relative to the peak $E_\theta$. When the array is focused at two array diameters, the rectangular and spherical components are shown in Figure 5. In Figure 5(a) the peak $E_z$ component is down by $-4.7$ dB compared to $E_x$, and in Figure 5(b), the radial component has dropped to $-24.5$ dB below the maximum vertical component. These ratios are conveniently summerized in Table 1. Thus, as

\[
\begin{array}{|c|c|c|}
\hline
z_o (\text{inches}) & E^{\text{max}}_r/E^{\text{max}}_\theta & E^{\text{max}}_z/E^{\text{max}}_\theta \\
\hline
133.3 & -19.1 \text{ dB} & -4.5 \text{ dB} \\
199.95 & -22.2 \text{ dB} & -4.6 \text{ dB} \\
266.6 & -24.5 \text{ dB} & -4.7 \text{ dB} \\
\hline
\end{array}
\]

would be expected for a given scan angle, the relative radial component decreases as the focusing range increases. In Figure 6(a) (one diameter distance) and Figure 6(b) (two diameters distance) a comparison is made between the exact near-field calculation of $E_\theta$ by Equation (14) and the approximate near-zone $E_\theta$ obtained by Equation (16). Here it is demonstrated that there is little difference by the two methods of computation and so the radial component is demonstrated to be negligible. Next, in Figure 7 at one aperture diameter, scan angles of $-40$, $-45$, and $-50^\circ$
Figure 3. Simulated near-field amplitude for monopole array focused at one aperture diameter. The scan angle is $\theta_s = -30^\circ$. (a) Tangential $E_x$ and normal $E_z$ electric field components and (b) Radial $E_r$ and vertical $E_\theta$ electric field components.
Figure 4. Simulated near-field amplitude for monopole array focused at 1.5 aperture diameters. The scan angle is $\theta = -30^\circ$. (a) Tangential $E_x$ and normal $E_z$ electric field components and (b) Radial $E_r$ and vertical $E_\theta$ electric field components.
Figure 5. Simulated near-field amplitude for monopole array focused at two aperture diameters. The scan angle is $\theta_s = -30^\circ$. (a) Tangential $E_x$ and normal $E_z$ electric field components and (b) Radial $E_r$ and vertical $E_\theta$ electric field components.
Figure 6. Comparison of exact and approximate near field $E_\theta$ component at (a) one aperture diameter distance and (b) two aperture diameters distance.
are considered, for which the radial component is down by $-20.7\,\text{dB}$, $-21.8\,\text{dB}$, and $-22.9\,\text{dB}$, respectively. Thus, in Figure 7, as the scan angle increases from broadside, the near-field focal range increases and the computed maximum radial component decreases as expected. It is observed that the relative peak $E_z$ component at the scan angles $-40$, $-45$, and $-50^\circ$ has values $-1.8\,\text{dB}$, $-0.4\,\text{dB}$, and $1.0\,\text{dB}$ as shown in Figure 8. These ratios are summarized in Table 2. Expanded

### TABLE 2

Relative Radial and Normal Components for 32-Element Monopole Array
with Scan Angle $\theta_s$. Near Field Focusing Distance and Observation Distance
Is $z_o=133.3$ in (One Aperture Diameter).

<table>
<thead>
<tr>
<th>$\theta_s$</th>
<th>$E_z^{\text{max}} / E_\phi^{\text{max}}$</th>
<th>$E_z^{\text{max}} / E_{\theta}^{\text{max}}$</th>
</tr>
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<tr>
<td>$-30^\circ$</td>
<td>$-19.1,\text{dB}$</td>
<td>$-4.5,\text{dB}$</td>
</tr>
<tr>
<td>$-40^\circ$</td>
<td>$-20.7,\text{dB}$</td>
<td>$-1.8,\text{dB}$</td>
</tr>
<tr>
<td>$-45^\circ$</td>
<td>$-21.8,\text{dB}$</td>
<td>$-0.4,\text{dB}$</td>
</tr>
<tr>
<td>$-50^\circ$</td>
<td>$-22.9,\text{dB}$</td>
<td>$1.0,\text{dB}$</td>
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scale plots of the rectangular components in amplitude and phase across the main beam region for the $-50^\circ$ scan angle at one diameter distance are shown in Figure 9. As predicted earlier, the $E_x$ and $E_z$ components are nearly in-phase [Figure 9(b)] for this negative scan angle. For positive scan angles $E_x$ and $E_z$ components would be nearly $180^\circ$ out of phase in the main beam region.
Figure 7. Simulated near-zone radial and vertical electric field components. The monopole array is focused at one aperture diameter and the scan angle is (a) $\theta_s = -40^\circ$, (b) $\theta_s = -45^\circ$, and (c) $\theta_s = -50^\circ$. 
Figure 8. Simulated near-field amplitude for tangential and normal electric field components $E_x$ and $E_z$. The monopole array is focused at one aperture diameter and the scan angle is (a) $\theta_s = -40^\circ$, (b) $\theta_s = -45^\circ$, and (c) $\theta_s = -50^\circ$. 
Figure 9. Expanded scale display of the rectangular components $E_x$ and $E_z$ in amplitude and phase across the main beam region for $-50^\circ$ scan angle at one aperture diameter distance. (a) Amplitude and (b) Phase.
5. CONCLUSION

This report has shown moment method simulations of the principal and radial spherical components of the electric field for a monopole phased array antenna focused at a near-field range of one to two aperture diameters. For this low-sidelobe linear array, a negligible radial component exists in the focused near-field region. This fact means that only the computed or measured tangential electric field component is necessary in computing the focused near-field principal spherical component. A similar conclusion needs to be validated for other types of phased array radiating elements.
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On the Radial Component of the Electric Field for a Monopole Phased Array Antenna Focused in the Near Zone

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None

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