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13. ABSTRACT (Maximum 200 words)

The theme of the research has been "making modern control theory work." The product of the research has been theory, algorithms and software applicable to multivariable feedback control problems in which there are design constraints requiring robust attainment of stability and control performance objectives in the face of both structured and unstructured uncertainty. Advances in the past two years have included "relative-error" methods for system identification, model reduction and control, better algorithms for H_∞ and H_2 control computations and new results on the analysis of stability robustness in the presence of several uncertain real parameters. Although the research has been aimed primarily at developing basic concepts, theory and methodology for robust control design, the theory that is emerging from the research is already beginning to play a significant role in facilitating the control design process in a variety of aerospace engineering applications where robust performance is prerequisite, including aircraft stability augmentation systems, highly maneuverable aircraft design, missile guidance systems, and precision pointing and tracking systems.

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Practical Methods for Robust Multivariable Control

AFOSR Grant 89-0398

Final Technical Report
August 1, 1989 through October 31, 1991

I. SUMMARY

In terms of research activity and related publication activity, the twenty-seven months funded by this grant have been record-setting for us, with fifty-five AFOSR-supported publications appearing in print or submitted [1-55]. Enclosed are copies of those publications not previously forwarded. For your convenience, the reference number of each is pencilled in the upper right corner of each paper. Additionally, four Ph.D. theses supported in part by AFOSR Grant 89-0398 were completed recently [52,53,54,55] and a fifth is expected to be completed in February 1992.

Areas of significant progress made possible by AFOSR support include H^∞ robust control theory [1,3,5,7,11,16,21,27,28,29,43,49], structured stability margin analysis [8,9,10,17,25,26,33], relative-error model order reduction [4,12,15,25,32], and linear system theory [7,19,35]. Most of the theoretical developments have also been implemented in software and/or tested in several highly successful flexible space-structure control and supermaneuverable aircraft control design studies carried out with supplemental support from TRW [2,6] and Northrop Aircraft [16,25,26,32,37]. The critical question of how to go beyond the singular value in developing more precise, less conservative quantitative measures of stability and performance was examined in [47]; and some interesting negative results relating to the gap metric were described in [18,44]. Particularly beneficial in ensuring the effective and rapid transition from theory to practice has been the PRO-MATLAB *Robust Control Toolbox*, a robust control design software product developed without AFOSR support by Dr. R. Y. Chiang and Dr. M. G. Safonov and published by The MathWorks. The papers [24] and [42] describe some of the capabilities and uses of such software.

II. RESEARCH HIGHLIGHTS

Our four main areas of progress have been in H^∞ control theory and algorithms, relative error model reduction theory, linear system theory, and multivariable stability margin analysis. We summarize some of the highlights in the following, focusing principally on H^∞ control and model reduction where we feel the progress has been especially significant.

We have made significant progress in advancing H^∞ control theory. Our descriptor reformulation of the two-Riccati state-space H^∞ theory [1,3] made it practical to run the H^∞ γ -iteration to the optimal γ without singularity

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or numerical ill-posedness; a proof that the descriptor formula converge at the optimal γ was obtained in [5]. Also, H^∞ controller existence conditions $\lambda_i(A - B_2F) < 0$ and $\lambda_i(A - GC_2) < 0$ developed by us in [1] replaced numerically ill-posed positive-semidefiniteness conditions $P, Q \geq 0$ thereby making it practical to reliably perform the optimal H^∞ γ -iteration. The first published derivation of the 2-Riccati H^∞ control formula for the usual case in which the plant has a non-zero "D₁₁ matrix" was developed in [1] via a simple loop-shifting technique. We consider the foregoing developments to be fairly major breakthroughs, making it possible to do H^∞ optimal control computations routinely and reliably without the need for extensive human-supervised verification of stability and singular-values during the γ -iteration.

Our recent generalized eigenspace solution of H^∞ (and H^2) problems is another significant advance in the computation of H^∞ and H^2 (linear quadratic gaussian) controllers [49,53]. These results make it possible to directly compute limiting solutions corresponding to so-called "singular" H^∞ and H^2 control problems in which the plant is strictly proper, i.e., has more zeros than poles. These so-called singular H^∞ and H^2 problems are actually extremely common. It was proved in [49,53] that the optimal controllers for singular H^∞ and H^2 problems are usually of reduced order (i.e., have fewer states) than those for the nearby nonsingular problems that are solved by the conventional theories.

Other advances in the H^∞ control theory include a simpler derivation from first principles of the 2-Riccati H^∞ formula [11] for non-singular cases and a treatment of the singular "fat plant" case in which there is an overabundance of actuators and/or sensors [7,34]. Directions in which the H^∞ control theory can be generalized were explored in [21,28,34, and 48]. In [27], preliminary results on decentralized H^∞ control are developed and in [28,48] the optimal robust-performance problem for SISO plants with multiplicative plant uncertainty is treated exactly via function-space duality concepts and the Hahn-Banach theorem. The possibilities for robust adaptive H^∞ control theory and for iterative use of H^∞ control theory in meeting various non-standard frequency-domain inequality specifications are explored in the speculative survey paper [34].

Another area of significant progress has been **H^∞ relative-error model reduction**. The H^∞ relative error of a model is a particularly important measure of model accuracy: A sufficient condition for a plant model to be valid for control system design is that its H^∞ relative-error be smaller than one throughout the bandwidth over which disturbance attenuation is required. The simplest and most practical method for computing low-order approximants of high-order state-space models is presently the Balanced-Stochastic Truncation (BST) method. The immense practical impact of BST order reduction methods in control design was underscored by our flexible space structure design study [6] where a four-state approximate model generated via BST was proved to be as good for control design purposes as a NASTRAN finite-element model having more than 100 states inside the specified 2000 rad/sec disturbance attenuation bandwidth. We recently did a detailed theoretical study of BST,

developing a new, simpler, and much tighter H^∞ relative error bound for continuous-time BST [4,31] than had been previously available. An equivalent relative-error bound for discrete time BST was developed in [12]; improving on a weak preliminary result which we reported in [29], the paper [30] summarizes the theoretical underpinnings of the relative-error bound derivations.

Quantitative evaluation of **stability robustness** in the presence of **structured uncertainty** continues to be a difficult problem, but we have made some progress here. Robustness for "one-sided" complex parameter-variations has been shown to reduce to a tractable convex, but non-smooth, nonlinear programming problem [8,33]; nonlinear programs using a combination of generalized gradient techniques and Davidon-Fletcher-Powell scaling techniques were coded and demonstrated to converge reliably, even for difficult non-smooth examples [25]. In [26] we obtained an important "negative result" for real structured uncertainty, proving via an example that no analogue of the Kharitonov or edge theorems will be possible in general situations involving multiple real parameter variations. New multivariable generalizations of the classical stability robustness concepts of phase margin and gain margin are proposed in [9,10,22,38,45,54]. Yet another unified vision on structured stability margin problems using concepts of algebraic topology and simplicial geometry has been developed in [22,39,46,51].

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- [1] M. G. Safonov, D. J. N. Limebeer, and R. Y. Chiang, "Simplifying the H^∞ Theory via Loop Shifting, Matrix Pencil and Descriptor Concepts," Int. J. Control, Vol. 50, No. 6, pp. 2467-2488, 1989.
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- [4] W. Wang and M. G. Safonov, "A Tighter Relative-Error Bound for Balanced Stochastic Truncation," Systems and Control Letters, Vol. 14, pp. 307-317, 1990.
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- [18]* G. Hsieh and M. G. Safonov, "Conservatism of the Gap Metric," to appear, IEEE Trans. on Automatic Control, 1992.
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- [51] E. A. Jonckheere and J. Bar-on, Algebraic Topology in Robust Control, book project submitted to Oxford University Press, 1992.

Ph.D. Theses

- [52] W. Wang, "Relative-Error Model Reduction, Identification and Control," Ph.D. Thesis, University of Southern California, September 1990. Dr. Wang was appointed a Post-Doctoral Research Associate at Caltech, Pasadena, CA. He is presently a Post-Doctoral Research Associate at the University of Southern California, Los Angeles, CA.
- [53] B. R. Copeland, "Cheap and Singular H^2 and H^∞ Control Problems: A Generalized Eigenproblem Approach," Ph.D. Thesis, University of Southern California, December 1990. Dr. Copeland is presently a Professor at the University of the West Indies, Trinidad.
- [54] J. R. Bar-on, "Phase and Gain Margins for Multivariable Control Systems," Ph.D. Thesis, University of Southern California, May 1990. Dr. Bar-on left the Aerospace Corporation to join the School of Electrical Engineering and Computer Sciences at the University of Oklahoma, Norman, OK.
- [55] R. Li, "Model Reduction and H^∞ Control Over a Planar Domain," Ph.D. Thesis, University of Southern California, December 1989. After his graduation, Dr. Li spent one year as a Postdoctoral Fellow at the California Institute of Technology, and then moved to Lear Astronics Corp., Santa Monica.