(2)

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

MODELLING STRATEGIC STRIKES
AGAINST
TRANSPORTATION NETWORKS

by

Kok-Hua Loh

September, 1991

Thesis Advisor: R. Kevin Wood

Approved for public release; distribution is unlimited.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b OFFICE SYMBOL (if applicable) OR	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000	7b ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS Program Element No. / Project No. / Task No. / Work Unit Accession Number

11 TITLE (Include Security Classification)
MODELLING STRATEGIC STRIKES AGAINST TRANSPORTATION NETWORKS

12 PERSONAL AUTHOR(S) Loh, Kok-Hua

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (year, month, day)	15 PAGE COUNT
Master's Thesis	From To	1991, September	100

16 SUPPLEMENTARY NOTATION
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 COSATI CODES			18 SUBJECT TERMS (continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUBGROUP	

19 ABSTRACT (continue on reverse if necessary and identify by block number)

This thesis presents deterministic and probabilities models for the analysis of strategic strikes against transportation networks. The deterministic models use integer programming to solve problems on single and multicommodity networks. The aims of a network interdictor are (a) to minimize the maximum network flow with a fixed amount of interdiction resources, or (b) to minimize the total effort and mission turnaround time if given sufficient resources to stop the flow completely. In the case of a multicommodity network, the interdictor also aims to utilize minimum resources to achieve a disconnecting set which severs the paths connecting all sources to their respective sinks. In the probability model, arc capacity is not a factor and the objective of a single interdictor is to minimize the probability of infiltration by a single evader through a network while the objective of the evader is just the opposite.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS REPORT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL R. Kevin Wood	22b TELEPHONE (Include Area code) (408)646-2523 22c OFFICE SYMBOL OR/Wd

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

Approved for public release; distribution is unlimited.

Modelling Strategic Strikes
Against Transportation Networks

by

Kok-Hua Loh
Lieutenant Colonel, Republic of Singapore Air Force
B.S., University of Manchester, United Kingdom

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN (Operations Research)

from the

NAVAL POSTGRADUATE SCHOOL
September 1991

Author: Loh Kok Hua
Kok-Hua Loh

Approved by:
R. Kevin Wood, Thesis Advisor

Gerald Brown, Second Reader

Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

We present deterministic and probabilistic models for the analysis of strategic strikes against transportation networks. The deterministic models use integer programming to solve problems on single and multicommodity networks. The aims of a network interdictor are (a) to minimize the maximum network flow with a fixed amount of interdiction resources, or (b) to minimize the total effort and mission turnaround time if given sufficient resources to stop the flow completely. In the case of a multicommodity network, the interdictor also aims to utilize minimum resources to achieve a disconnecting set which severs the paths connecting all sources to their respective sinks. In the probabilistic model, arc capacity is not a factor and the objective of a single interdictor is to minimize the probability of infiltration by a single evader through a network while the objective of the evader is just the opposite.

TABLE OF CONTENTS

I. INTRODUCTION

A strategic interdiction campaign against the logistics supply system of an enemy may take many forms: it may consist of attacks on the key installations such as military industrial bases, supply depots, or attacks on the vital links between these installations and the front-line troops in the battlefield. The key installations are normally well protected through hardening and the deployment of air defenses and troops. Compared to the installations, the land transportation network, as it traverses through a large expense of territories, is harder to protect against concerted and determined efforts by the air, naval and land forces.

A. OBJECTIVE

The objective of this thesis is to develop models for the optimum allocation of multiple types of strike resources to interdict a land transportation network in order to degrade the performance of an enemy's logistics supply system. In particular, we will be formulating mathematical programs for the strikes against single commodity and multicommodity networks.

B. OVERVIEW

Strategic strike is a complex task for the Armed Services to undertake. It involves the setting of campaign goals, development of attack plans and coordinated execution by the participating strike forces.

1. Types of Strike assets

The resources available to the operational planners at the command headquarters for the interdiction campaign consist of:

a. Air power such as a fleet of fighter aircraft and attack helicopters,

b. Naval gunfire support,

c. Special forces trained for deep strike, sabotage and reconnaissance missions, and,

d. Long range missiles launched from land, air and sea platforms.

In developing a campaign plan, the strike planner would estimate the weapons efforts required to interdict the targets based on a desired confidence level of mission success. The available strike assets should then matched in an optimum manner to the targets according to the weapon suitability and economy of efforts, which in turn depend on a host of factors such as the ease of target detection and identification, enemy defenses, target vulnerability, the potential collateral damages.

2. Campaign Objectives

The central task of the strike planner is the optimum allocation of the multi-dimensional forces to interdict the following types of networks:

a. Single Commodity Network. To degrade the performance of a road network, a viable strategy is one of counter-capacity. The aim is to minimize the throughput of network by allocating a fixed amount of resources to interdict the most "profitable" arcs. Another approach is to minimize the demands of the front-line units, or the demands prioritized according to geographical locations. Yet another type of single commodity network is one faced in an unconventional warfare setting. Here, the enemy attempts to infiltrate through the network by avoiding detection; arc capacity is not a factor. The aim of the strike planner is to minimize the probability of enemy infiltration.

b. Multicommodity Network. The attacker would want to reduce as much as possible the weighted flow of multiple commodities from certain nodes in the supply network to other nodes.

C. LITERATURE REVIEW

The network interdiction problem has been studied for quite some time now. There are two common approaches to modelling the problem; either a deterministic model or stochastic / game theoretic model.

Mustin and McMasters 1967 develop an algorithm for optimally interdicting a transportation network using limited assets. They consider a model in which the capacity of an arc (i,j) can be reduced by one unit by the expenditure of e_{ij} units of resources. The objective is to minimize the maximum flow obtainable between two nodes s and t subject to the consumption of no more than E units of resources. Their methodology requires that the network be planar so that the topological dual can be taken for the enumeration of many cut-sets. This is an undesirable requirement. The source-sink planar graph requires not only planarity but also requires that the source and the sink lie on the outer face of the graph, which is quite a strong assumption. Furthermore, the algorithm is not easy to generalize to handle multiple resources with which to interdict arcs, or multiple resources which must be applied together to interdict an arc.

Nugent and McMasters 1969 develop an embellished version of the above model in that the damage function is now assumed to be exponential. Associated with each arc (i,j), there is a measure of vulnerability b_{ij}. The proportional reduction in the arc capacity with e_{ij} units of resources allocated is $\{1 - \exp(-b_{ij}e_{ij})\}$. The object is to maximize the reduction in the maximum flow. The algorithm also requires the topological dual of the network to be taken, so that the "shortest" routes, which represent minimum capacity cut-sets in the primal network, can be determined. Since the damage function is now exponential, the allocation of weapons in one cut-set is determined using a Lagrangian method. The algorithm always produces non-integer solutions. This model suffers the same weaknesses as the previous model. The method of taking a topological dual and enumerating cut-sets is inefficient if the network becomes too large.

Wollmer 1970 presents two algorithms for targeting strikes against a network. The enemy is assumed to have the policy of either maximizing flow between a source node s and a sink node t or meeting a given flow at minimum cost. For his first algorithm, the cost of traversing arc (i,j) is assumed to be a linear function of flow. After the arc has been struck, there is a resulting cost increase per unit time Δc of a minimum cost circulation flow and repair time t_{ij}. Associated with each arc, there is a strike value v_{ij} which is defined as the repair cost k_{ij} plus $t_{ij}\Delta c$. The model is a one-strike algorithm in that, at any one time, only one arc which is that arc with the maximum strike value will be interdicted. Multiple strikes are approximated by repeated applications of the same algorithm. The second algorithm treats the arc costs as piece-wise linear functions with one break point. Wollmer's algorithms do not produce optimal results in a multiple strike scenario.

Capps and Taylor 1970 develop a model similar to the above model. The damage function is now assumed to be exponential. Against an arc (i,j) with a vulnerability measure b_{ij}, the proportional reduction in capacity d_{ij} due to e_{ij} units of effort is (1-exp$\{b_{ij}e_{ij}\}$). The arc repair time t_{ij} and repair cost k_{ij} are assumed in turn to be linear functions of d_{ij}. As in Wollmer's model, this model selects and interdicts the arc with the maximum strike value which is the sum of k_{ij} and $t_{ij}\Delta c$. The model allows for arc repairs and allocates the resources on a daily basis. This model suffers the same weaknesses as the previous model.

Preston and Howard 1970 develop a procedure for determining the optimum allocation of aircraft against a transportation network system using the technique of dynamic programming. An exponential relationship between arc capacity and interdiction effort is assumed. The model requires a topological dual of the network to be constructed and a range of cut-sets enumerated. The minimum capacity for each level of available resources $E = 1,2,3,...$ is determined by dynamic programming. The algorithm will increase the value of E until either the maximum value of E is reached or when the benefit to be gained from assigning the last aircraft is less than the cost of assigning the aircraft. The model entails the enumeration of many cut-sets and does so for a large number of E. It is also rather difficult to produce for a particular war-time scenario a reasonable estimate on the cost of assigning one aircraft and the benefit of each mission.

Helmbold 1971 develops a model for the allocation of limited strike assets among the arcs of a transportation network by analyzing its topological dual. The damage function is assumed to be a step function. A particular arc (i,j) would have its capacity c_{1ij} reduced to a lower level of capacity c_{2ij} by an interdiction effort e_{ij}. The optimum allocation of strike assets to the arcs in a cut-set is solved by dynamic-programming. He claims that the algorithm can be extended to handle the problem of multiple resources, but this would be possible only in a limited way. Furthermore, the model is again restricted to planar networks.

Ghare, Montgomery and Turner 1971 develop a branch and bound algorithm for the interdiction of a set of arcs in order to minimize the maximum flow of a network subject to fixed amount of resources. For each arc (i,j) with capacity c_{ij}, its destruction requires e_{ij} units of resource; no partial destruction of an arc is allowed. The model computes the lower bounds on the maximum flow once an arc is destroyed, which affects the decision whether to include or exclude a particular arc from the branching procedure. This choice of lower bound for the branch and bound procedure is not ideal. As a whole, this model is based on a rather weak characterization of an interdicted network, and would be difficult to implement, for instance, with multiple strike resources.

Wollmer 1968 develops a non-deterministic model for determining where to place interdicting forces in order to maximize the probability of preventing an opposing force (the evader) from proceeding from one node in a network to another. The interdictor determines a probability π_{ij} of placing a single interceptor on arc (i,j) given a known probability of p_{ij} of stopping a single evader traversing the arc given that the evader is traversing the arc. The aim of the single evader is, knowing the interdictor's strategy, to pick a path which maximizes the probability of successful traversal through the network. For a small network, this problem can be analyzed in terms of a two person zero-sum game which can be easily solved by linear programming. For a large network, the game matrix becomes too large to handle. Wollmer therefore proposes a solution technique which is based on marginal analysis or the steepest ascent approach. Initially, all values of π_{ij} are set to zero and are incremented in such a way that the ratio of the increase in π_{ij} and the decrease in K is minimized. Without any formal proof, he claims that the solution is exact for a scenario of a single interceptor versus a single evader and yields an approximately optimal answer for multiple interceptors. The main weakness of the model is its inability to handle multiple infiltrators traversing the network simultaneously. The optimal strategy of the evader is also not given in the model.

Danskin 1962 develops a game theory model of convoy routing. The analysis solves a two-person zero-sum game in which one player, the convoy command, directs two routes; the first represented by a vector **x** for the merchant vessels and the second by a vector **y** for escorts. The other player, the enemy submarine command, controls one vector **z**. The payoff P, in terms of tonnage delivered / dollar cost, is defined as the ratio of T the total tonnage delivered on a set of routes and C the total route cost including the cost of losses. The convoy commander seeks to maximize P which is a function of (**x**, **y**, **z**) whilst the submarine commander seeks to minimize P. The model determines a critical turnaround time a_0 using a Lagrangian method. If a route i has its turnaround time $a_i > a_0$, it is not used. If $a_i < a_0$, it is used. The main difficulty of this model, when applied to network interdiction problem, is that the determination of a_0 and other threshold values would require the solving of equations involving the sum of non-linear functions of T_i and C_i over all paths through the network.

D. MODEL ASSUMPTIONS

The assumptions made for the development of the various model throughout this paper are as follows:

a. Damage function. It is assumed that there would be no partial destruction of a target. The capacity of an arc will be reduced to zero upon interdiction, which is a reasonable assumption with the advent of "smart" weapons. Other types of damage functions may approximated by expanding arcs into sets of independent parallel arcs.

b. Time frame. We are assuming that once the link is destroyed, the arc capacity is reduced instantaneously to zero. The enemy could employ other support means, such as constructing pontoon bridges next to the destroyed bridges, but this would take some time.

c. Independent Arc. It is assumed that the destruction of one arc has no impact on the performance of other arcs in the network.

e. Node Capacities. We assume that each node will have infinite capacity. If interdiction of a node, say a road intersection, is a possibility, this can be accomplished by splitting the node i into two nodes i_1 and i_2 and connecting a capacitated arc between these two nodes. In the case of a directed network, all arcs originally entering node i now enter node i_1 and all arcs leaving node i now leave i_2. The model can also be generalized to undirected networks.

II. DETERMINISTIC MODELS - SINGLE COMMODITY NETWORKS

In this chapter, integer programs are formulated for the interdiction of a single commodity network. The aims of the attacker are to reduce to the maximum flow of materials, minimize the resource used to cut off all supplies to the enemy and minimize the maximum flow of supplies to the units, prioritized according to their tactical values. Most of the models discussed here are generalizations of those described in Wood 1991.

A. NETWORK FLOW BASICS

Let $G = (N,A)$ be a directed graph with node set N and arc set A. An arc is an ordered pair (i,j) where $i,j \in N$. Associated with each arc (i,j), there is a flow capacity u_{ij}. The lower bound of arc capacity is assumed to be zero. Let f be the amount of flow in the network from the source node s to sink node t. The maximum flow problem can be stated as:

$$\text{Maximize} \quad f \tag{1.1}$$

$$\text{Subject to} \quad \sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} f & \text{if } i = s \\ 0 & \text{if } i \neq s \text{ or } t \\ -f & \text{if } i = t \end{cases}$$

$$x_{ij} \le u_{ij} \quad \forall\ (i,j)$$

$$x_{ij} \ge 0 \quad \forall\ (i,j)$$

where the sums and inequalities are taken over existing arcs in the network.

Let S be any set of nodes in the network such that S contains node s but not t. let $T = N - S$. Then $(S,T) \equiv \{(i,j) : i \in S, j \in T\}$ is a cut-set separating node s from node t.

The dual of the maximum flow problem is as follows:

$$\text{Minimize} \quad \sum_i \sum_j u_{ij}\beta_{ij} \tag{1.2}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + \beta_{ij} \ge 0 \quad \forall\ (i,j)$$

$$\beta_{ij} \ge 0 \quad \forall\ (i,j)$$

$$\alpha_i \quad \text{unrestricted} \quad \forall\ i$$

where α is the dual variable corresponding to the conservation equations and β is the dual variable corresponding to $\mathbf{x} \le \mathbf{u}$.

In the single commodity flow problem, it is well known from the Maximum-flow/minimum-cut theorem by Ford and Fulkerson that, for a given maximum flow f, a cut (S,T) is minimal if, and only if all the arcs of (S,T) are saturated while the arcs in the form (T,S) are flowless with respect to f. Hence, we are guaranteed an integer optimal solution if the arc capacities are integer. This also arises from the fact that the constraint matrix of the maximum flow problem is unimodular. We shall now develop a constructive proof for the following lemma:

LEMMA 1 The dual of the maximum flow problem has a solution in which all dual variables are 0 or 1.

Assume that G is connected and that we have solved the maximum flow problem using a bounded-variable simplex algorithm as in Bradley, Brown and Graves 1977. However, instead of starting with a full artificial basis, assume that full rank has been achieved by discarding the last flow-balance constraint in the maximum flow problem. This implies that $\alpha_{|N|} \equiv 0$. Now, any basis which is associated with the flow balance constraints from a network flow problem can be put into the upper triangular form. Let B be the optimal basis in triangulated form and assume that variables have been reflected about their upper bounds as necessary to put all +1s on the diagonal of B. Reflection does not change the dual solution. Let α denote the dual row vector of length $|N| - 1$ and let c_B denote the row vector of length $|N| - 1$ containing the costs associated with the optimal basis. If $c_B = 0$ it follows that $\alpha = 0$. If not, $c_B = (000...010...0)$ where the 1 is in position l say, and α can be computed from

$$\alpha B = c_B$$

Now $B_{ii} = 1$, and we let $p(i) < i$ denote the row of the off-diagonal element in column i of B. If there is no off-diagonal element $p(i) = 0$ and we define $\alpha_0 \equiv 0$. The above equation can now be solved by substitution:

$$\alpha_1 = 0$$

$$\alpha_2 - \alpha_{p(2)} = 0 \Rightarrow \alpha_2 = 0 \text{ since } \alpha_0 = \alpha_1 = 0 \text{ and } p(2) = 0 \text{ or } 1$$

$$\vdots$$

$$\alpha_{l-1} - \alpha_{p(l-1)} = 0 \Rightarrow \alpha_{l-1} = 0 \text{ since } \alpha_0 = \ldots = \alpha_{l-2} = 0 \text{ and } p(l-1) \le l - 2$$

$$\alpha_l - \alpha_{p(l)} = 1 \Rightarrow \alpha_l = 1 \text{ since } \alpha_0 = \ldots = \alpha_{l-1} = 0 \text{ and } p(l) \le l - 1$$

$$\alpha_{l+1} - \alpha_{p(l+1)} = 0 \Rightarrow \alpha_{l+1} = 1 \text{ if } \alpha_{p(l+1)} = 1 \text{ or } \Rightarrow \alpha_{l+1} = 0 \text{ if } \alpha_{p(l+1)} = 0$$

$$\vdots$$

$$\alpha_{N+1} - \alpha_{p(N+1)} = 0 \Rightarrow \alpha_{N+1} = 1 \text{ if } \alpha_{p(N+1)} = 1 \text{ or } \Rightarrow \alpha_{N+1} = 0 \text{ if } \alpha_{p(N+1)} = 0$$

Thus, $\alpha_i = 0$ or $\alpha_i = 1$ for all $i \in N$. Next, for a specified arc (i,j) note that (a) $\beta_{ij} \geq 0$, (b) β_{ij} appears in only one inequality in the dual formulation (1.2), and (c) the objective coefficient u_{ij} associated with β_{ij} is positive. It therefore follows that

1. if $\alpha_i = 0$ and $\alpha_j = 0$, then $\beta_{ij} = 0$,
2. if $\alpha_i = 1$ and $\alpha_j = 0$, then $\beta_{ij} = 0$,
3. if $\alpha_i = 0$ and $\alpha_j = 1$, then $\beta_{ij} = 1$, and
4. if $\alpha_i = 1$ and $\alpha_j = 1$, then $\beta_{ij} = 0$.

Thus, the dual of the maximum flow problem has an optimum solution in which all dual variables are 0 or 1. *QED*

B. MINIMIZE MAXIMUM FLOW

1. Problem Statement

The logistics planner of an enemy wishes to force through as much of a single commodity from a source node to a sink node in a directed and capacitated network. Given insufficient resources to completely cut the supply network, the problem of the interdictor will be to minimize the maximum amount of flow that the enemy can force through the unbroken arcs. The strike planner can allocate any one type of resource available to interdict any one arc. Simultaneous strikes by multiple resources on a single target are not necessary. (This model can be generalized to include a co-ordinated attack of multiple strike resources against a single target.) Destruction of an arc is always assumed to be total.

2. Model 1 - Network with Directed Arcs

For an interdicted network model, the indices used are:

i,j : nodes,

l : the type of strike resources available, e.g., cruise missiles, aircraft, special forces, etc.

The data of the model are:

e_{ijl} : amount resource of type l required to destroy arc (i,j)

E_l : the total strike assets available for resource type l,

u_{ij} : the capacity of arc (i,j).

The decision variables are:

γ_{ijl} : the binary variable which is 1 if arc (i,j) is destroyed by resource type l, and is otherwise 0,

x_{ij} : flow through arc (i,j),

f : total flow through the network.

The Min-Max problem that solves the interdictor's problem is as follows:

$$\underset{\gamma \in \Gamma}{\text{Minimize}} \quad \text{Maximize} \quad f \tag{2.1}$$

Subject to

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} f & \text{if } i = s \\ 0 & \text{if } i \neq s \text{ or } t \\ -f & \text{if } i = t \end{cases}$$

$$x_{ij} \leq u_{ij}(1 - \gamma_{ijl}) \qquad \forall\ (i,j)\ ,\ l$$

$$x_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

where $\Gamma = \{\gamma_{ijl} \mid \gamma_{ijl} \in (0,1)\ \forall\ (i,j)\ ,\ l \quad and \ \sum_{(i,j)} e_{ijl}\gamma_{ijl} \leq E_l\ \forall\ l\}$.

The second constraint of (2.1) ensures that the capacity of arc (i,j) is reduced to zero once γ_{ijl} is equal to one. The solution of the Min-Max problem will yield an optimum solution corresponding to the maximum flow given that the total resources **E** are insufficient to reduce the capacity of any cut-set (S,T) in the network to zero.

To solve the above Min-Max problem, we can take the dual of the minimization problem and convert the Min-Max problem into a non-linear program as follows:

$$\underset{\gamma \in \Gamma}{\text{Minimize}} \quad \text{Minimize} \quad \sum_{l} \sum_{(i,j)} u_{ij}(1 - \gamma_{ijl})\beta_{ijl} \tag{2.2}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\ (i,j)$$

$$\beta_{ijl} \geq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\alpha_i \quad \text{unrestricted} \qquad \forall\ i$$

$$\Gamma = \{\ \gamma_{ijl} \mid \gamma_{ijl} \in (0,1)\ \forall\ (i,j)\ ,\ l\ , \quad \text{and} \sum_{(i,j)} e_{ijl}\ \gamma_{ijl} \leq E_l\ \forall\ l\ \}$$

where α is the dual variable associated with the flow conservation constraint and β is the dual variable associated with the second constraint of (2.1).

Now, let

$$h_{ijl} = (1 - \gamma_{ijl})\beta_{ijl}$$

Substituting the above into the second constraint of the (2.2), we have,

$$\alpha_i - \alpha_j + \sum_{i} h_{ijl} + \sum_{l} \gamma_{ijl}\beta_{ijl} \geq 0$$

The second constraint of (2.2) is now non-linear. However, we can prove that;

$$\gamma_{ijl}\beta_{ijl} = \gamma_{ijl}$$

or ,

$$\gamma_{ijl}(\beta_{ijl} - 1) = 0 \qquad (2.3)$$

From Lemma 1, we know that for fixed values of γ_{ijl} the optimum solution of the inner minimization problem has β_{ijl} equal to 0 or 1. This is true since, for fixed γ_{ijl}, problem (2.1) is just a maximum flow problem with multiple upper bounds on each arc and such a problem can be solved by throwing out all bounds but the smallest. (Throwing out a bound sets some β_{ijl} to 0.) Let p be the event $\{ \gamma_{ijl} = 0\}$, and q be the event $\{ \beta_{ijl} = 1\}$. Equation (2.3) is true if p or q is true. It is false, if and only if both p and q are false, i.e., only when $\{ \gamma_{ijl} = 1\}$ and $\{ \beta_{ijl} = 0\}$. Looking back at the objective function of (2.2), if $\{ \beta_{ijl} = 0\}$ we may assume that $\{ \gamma_{ijl} = 0\}$ because setting $\gamma_{ijl} = 1$ would not affect the value of the objective function but it would unnecessarily use up e_{ijl} units of resources. Therefore, we may linearize the second constraint of (2.2) by replacing the non-linear term $\gamma_{ijl}\beta_{ijl}$ with γ_{ijl} and the constraint becomes:

$$\alpha_i - \alpha_j + \sum_l h_{ijl} + \sum_l \gamma_{ijl} \geq 0$$

We now have the following mixed integer program (MIP):

$$\text{Minimize} \quad \sum_{l} \sum_{(i,j)} u_{ij} h_{ijl} \tag{2.4}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + \sum_{l} h_{ijl} + \sum_{l} \gamma_{ijl} \geq 0 \qquad \forall\ (i,j)$$

$$\sum_{(i,j)} e_{ijl} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$h_{ijl} \geq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

$$\alpha_i \quad \text{unrestricted} \qquad \forall\ i$$

Since, for this model, we will select at the most one type of resource to interdict one particular arc, we have for each arc (i,j). $\Sigma_l \gamma_{ijl} \leq 1$. In an arc (i,j) in a cut-set, there may be at most 1 of γ_{ijl} terms taking on the value of 1. Hence, $\Sigma_l h_{ijl}$ can be replaced by a single term h_{ij}.

The final formulation becomes:

$$\text{Minimize} \quad \sum_{(i,j)} u_{ij} h_{ij} \tag{2.5}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + h_{ij} + \sum_l \gamma_{ijl} \geq 0 \qquad \forall\ (i,j)$$

$$\sum_{(i,j)} e_{ijl} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$h_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

$$\alpha_i \ \text{unrestricted} \qquad \forall\ i$$

We can interpret the above model as being based on a modified dual of the maximum flow linear programming formulation. Essentially, an (S,T) cut has been identified with all $\alpha_i = 1$ for $i \in T$ and $\alpha_i = 0$ for $i \in S$. The value of $\Sigma_l \gamma_{ijl} = 1$ if (i,j) is a forward arc across the cut and that arc is to be broken; h_{ij} is 1 if (i,j) is a forward arc across the cut but that arc is not to be broken; and all other h_{ij} and $\Sigma_l \gamma_{ijl}$ are 0. Thus, we see that a cut is identified and arcs are broken in that cut so as to leave as little remaining capacity as possible.

3. Model 2 - Network with Undirected Arcs

If the network $G' = (N, A')$ is undirected, we would have to convert G' into an appropriate directed graph in order to use Model 1. In the maximum flow problem, no flow need enter the source nodes and no flow need leave the sink nodes so that any arc incident to a source node can be represented by a single arc directed out from the source node and any arc incident to a sink node can be represented by a single arc directed into the sink node. Other undirected arcs in G' can be replaced by two independent directed arcs in anti-parallel, each directed arc with the capacity u_{ij} and resource e_{ij} taken from the original network G. If one of the two directed arcs in the anti-parallel pair is destroyed, there is no possibility of the other arc being interdicted simultaneously requiring a total of $2e_{ij}$ units of resource to destroy the undirected arc. Specifically, if one directed arc (i,j) is destroyed, we have $\alpha_i = 0$ and $\alpha_j = 1$. This means that, for the other directed arc (j,i), the term $\{h_{ij} + \Sigma_l \gamma_{ijl}\}$ will be zero without violating the second constraint of (2.5).

Model 1 can solve the problem of undirected graph using the above transformation, but introduces a large number of variables as it does not take advantage of the fact that each pair of directed arcs share the same values of capacity and destruction effort. Denote an undirected arc as an unordered pair of nodes $[i,j]$

where $i,j \in G'$. The modified model becomes:

$$\text{Minimize} \quad \sum_{[i,j]} u_{ij} h_{ij} \tag{2.6}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + h_{ij} + \sum_l \gamma_{ijl} \geq 0 \qquad \forall\ [i,j]$$

$$\alpha_j - \alpha_i + h_{ij} + \sum_l \gamma_{ijl} \geq 0 \qquad \forall\ [i,j]$$

$$\sum_{[i,j]} e_{ijl} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$h_{ij} \geq 0 \qquad \forall\ [i,j]$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ [i,j]\ ,\ l$$

$$\alpha_i \ \text{unrestricted} \qquad \forall\ i$$

C. MINIMIZE INTERDICTION EFFORT

1. Problem Statement

The attacker has sufficient resources **E** to stop completely the flow of material in the supply network. The time taken for force projection to arc (i,j) and recovery of resource type l back to base depends on a factor w_{ijl}. The aim of the interdictor is to destroy the set of arcs so as to cut the network into two disjoint subsets utilizing minimum resource and assigning the missions in such a way which minimizes the maximum mission turnaround time.

2. Model 3 - Network with Directed Arcs

When the interdictor has only one type of resource, the minimum resource required to reducing the network flow to zero and the associated cut-set can be determined by solving a maximum flow problem with the arc capacities equal to the interdiction efforts. The minimal capacity cut-set of the maximum flow problem yields the results of the total minimum resource and the set of arcs to be interdicted.

To solve the problem of multiple resources, we need to introduce the following modifications to Model 1:

a. Delete all terms with h_{ij}, as there is now zero network flow,

b. Introduce a conversion factor w_{ijl} for each resource type l; the conversion factor being the measure of the mission turnaround time and of how the strike planners value the opportunity cost of a particular resource availability. The formulation becomes:

$$\text{Minimize} \quad \sum_{l} \sum_{(i,j)} w_{ijl}\, e_{ijl} \gamma_{ijl} \tag{2.7}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + \sum_{l} \gamma_{ijl} \geq 0 \qquad \forall\ (i,j)$$

$$\sum_{(i,j)} e_{ij} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

$$\alpha_i \ \text{unrestricted} \qquad \forall\ i$$

3. Model 4 - Network with Undirected Arcs

For an undirected network, let an undirected arc be represented by a pair of nodes $[i,j]$, the formulation is as follows:

$$\text{Minimize} \quad \sum_{l} \sum_{[i,j]} w_{ijl}\, e_{ijl} \gamma_{ijl} \tag{2.8}$$

$$\text{Subject to} \quad \alpha_t - \alpha_s = 1$$

$$\alpha_i - \alpha_j + \sum_{l} \gamma_{ijl} \geq 0 \qquad \forall\ [i,j]$$

$$\alpha_j - \alpha_i + \sum_{l} \gamma_{ijl} \geq 0 \qquad \forall\ [i,j]$$

$$\sum_{[i,j]} e_{ij} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ [i,j]\ ,\ l$$

$$\alpha_i \ \text{unrestricted} \qquad \forall\ i$$

D. MAXIMIZE SHORTFALL IN PRIORITIZED DEMAND

1. Problem Statement

The enemy wants to push through a road network sufficient flow of materials from a set of depots I_s to meet the demands of the field units, especially of those tactically crucial units I_d which are in critical shortage of supplies. The problem of the attacker is to select a set of arcs with a fixed resource **E** in order to maximize the prioritized shortfall of the demands measured according to tactical values r_i of the units.

2. Model 5 - Network with Directed Arcs

In this model the additional notation used is:

I_s : the set of supply nodes,

I_d : the set of demand nodes,

I_o : the set of transhipment nodes.

The additional data required is:

s_i : the amount of supply at node i,

d_i : the amount of demand at node i,

r_i : the tactical values of units; the higher the value of the unit the greater is its tactical importance,

The additional variables introduced here are:

y_i^+ : the amount of unused supply at supply node i,

y_i^- : the amount of shortfall at demand node i.

The Max-Min problem formulation is as follows:

$$\underset{\gamma \in \Gamma}{\text{Maximize}} \quad \text{Minimize} \quad \sum_{i \in I_d} r_i \, y_i^- \tag{2.9}$$

$$\begin{aligned}
\text{Subject to} \quad & \sum_j x_{ij} - \sum_j x_{ji} + y_i^+ = s_i && \forall \, (i \in I_s) \\
& \sum_j x_{ij} - \sum_j x_{ji} = 0 && \forall \, (i \in I_o) \\
& \sum_j x_{ij} - \sum_j x_{ji} - y_i^- = -d_i && \forall \, (i \in I_d) \\
& x_{ij} \leq u_{ij}(1 - \gamma_{ijl}) && \forall \, (i,j) \, , \, l \\
& x_{ij} \geq 0 && \forall \, (i,j) \\
& y_i^+ \geq 0 && \forall \, (i \in I_s) \\
& y_i^- \geq 0 && \forall \, (i \in I_d)
\end{aligned}$$

where $\Gamma = \{\gamma_{ijl} \mid \gamma_{ijl} \in (0,1) \; \forall \, (i,j) \, , \, l \quad \text{,and} \sum_{(i,j)} e_{ijl} \, \gamma_{ijl} \leq E_l \; \forall \, l \, \}$

Let α_i be the dual variable associated with the flow conservation constraint and β_{ijl}, the capacity constraint. Converting the minimization LP into its dual, we have:

$$\max_{\gamma \in \Gamma} \ \text{Max} \quad \sum_{i \in I_s} s_i \alpha_i - \sum_{i \in I_d} d_i \alpha_i + \sum_{l} \sum_{(i,j)} u_{ij}(1 - \gamma_{ijl}) \beta_{ijl} \qquad (2.10)$$

$$\begin{array}{lrl} \text{Subject to} & \alpha_i - \alpha_j + \sum_l \beta_{ijl} \le 0 & \forall\ (i,j) \\ & \alpha_i \le 0 & \forall\ (i \in I_s) \\ & -\alpha_i \le r_i & \forall\ (i \in I_d) \\ & \beta_{ijl} \le 0 & \forall\ (i,j)\ ,\ l \\ & \alpha_i \quad \text{unrestricted} & \forall\ \ i \notin (I_s\ ,\ I_d) \end{array}$$

$$\Gamma = \{\ \gamma_{ijl}\ |\ \gamma_{ijl} \in (0,1)\ \forall\ (i,j)\ ,\ l\ ,\quad \sum_{(i,j)} e_{ijl}\gamma_{ijl} \le E_l\ \forall\ l\ \}$$

The model can be linearized by replacing $(1-\gamma_{ijl})\beta_{ijl}$ with h_{ijl} and adding the constraint $h_{ijl} \le \beta_{ijl} - \gamma_{ijl}\underline{\beta}_{ijl}$ where $\underline{\beta}_{ijl}$ is a lower bound on β_{ijl}. Looking back at (2.8), since β_{ijl} is the dual variable for constraint $x_{ij} \le u_{ij}(1-\gamma_{ijl})$, it can be interpreted as the rate of change of the objective function value if an arc is interdicted. Since β_{ijl} is negative and the maximum rate of change in the objective function value of (2.8) is $\max_i r_i$, it follows that $\underline{\beta}_{ijl} \ge -\max_i r_i$.

The formulation for the MIP is thus:

$$\text{Maximize} \quad \sum_{i \in I_s} s_i \alpha_i - \sum_{i \in I_d} d_i \alpha_i + \sum_{l} \sum_{(i,j)} h_{ijl} u_{ij} \tag{2.11}$$

Subject to

$$\alpha_i - \alpha_j + \sum_l \beta_{ijl} \leq 0 \qquad \forall\ (i,j)$$

$$h_{ijl} - \beta_{ijl} + \underline{\beta}_{ijl} \gamma_{ijl} \leq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\sum_{(i,j)} e_{ijl} \gamma_{ijl} \leq E_l \qquad \forall\ \ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

$$\alpha_i \leq 0 \qquad \forall\ (i \in I_s)$$

$$-\alpha_i \leq r_i \qquad \forall\ (i \in I_d)$$

$$\alpha_i \quad \text{unrestricted} \qquad \forall\ \ i \notin (I_s\ ,\ I_d)$$

$$\beta_{ijl} \leq 0 \qquad \forall\ \ (i,j)\ ,\ l$$

$$h_{ijl} \leq 0 \qquad \forall\ \ (i,j)\ ,\ l$$

3. Model 6 - Network with Undirected Arcs

For undirected graph, we can apply a transformation similar to the previous sections to the network, and the formulation is as follows:

$$\text{Maximize} \quad \sum_{i \in I_s} s_i \alpha_i - \sum_{i \in I_d} d_i \alpha_i + \sum_{l} \sum_{[i,j]} h_{ijl} u_{ij} \qquad (2.12)$$

Subject to

$$\alpha_i - \alpha_j + \sum_{l} \beta_{ijl} \leq 0 \qquad \forall\ [i,j]$$

$$\alpha_j - \alpha_i + \sum_{l} \beta_{ijl} \leq 0 \qquad \forall\ [i,j]$$

$$h_{ijl} - \beta_{ijl} + \underline{\beta}_{ijl} \gamma_{ijl} \leq 0 \qquad \forall\ [i,j]\ ,\ l$$

$$\sum_{[i,j]} e_{ijl} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ [i,j]\ ,\ l$$

$$\alpha_i \leq 0 \qquad \forall\ [i \in I_s]$$

$$-\alpha_i \leq r_i \qquad \forall\ [i \in I_d]$$

$$\alpha_i \quad \text{unrestricted} \qquad \forall\ i \notin [I_s\ ,\ I_d]$$

$$\beta_{ijl} \leq 0 \qquad \forall\ [i,j]\ ,\ l$$

$$h_{ijl} \leq 0 \qquad \forall\ [i,j]\ ,\ l$$

III. DETERMINISTIC MODELS - MULTICOMMODITY NETWORKS

A military logistics supply problem usually involves the expeditious shipment of certain distinct commodities, such as ammunition, fuel, spare parts, food and etc, from the supply depots and ammo dumps to their respective destinations along the arcs of an underlying transportation network. This scenario results in the well-known multicommodity flow problem. It also occurs in communication systems, urban traffic systems and railway problems, as well as in many others. The commodities interact when flowing on the same arc either by competing for arc capacity, or by causing congestion.

A. MINIMIZE MAXIMUM FLOW

1. Problem Statement

In this operational setting, the problem of the strategic strike planners can be stated as follows:

> The enemy has a policy of forcing through the unbroken arcs of the network the flow of a range of commodities. Commodity k flows from source node s_k to a sink node t_k. The aim of the interdictor is to reduce the sum of the maximum flow of the multiple commodities utilizing a fixed amount of resources and, if given sufficient resources, to stem the flow of materials with minimum resource and mission turnaround time.

2. Model 7 - Network with Directed Arcs.

For the multicommodity network model, the following notation is used:

i,j : the index sets for the nodes of the network,

k : the index set ıor the commodity,

l : the index set for the types of strike resources.

The given data for the model are:

u_{ij} : the combined capacity per unit time in a standard unit for the arc (i,j).

The variables in the models are:

f_k : the amount of flow for commodity k through the network,

x_{ijk} : the amount of flow for commodity k in arc (i,j);

γ_{ijl} : the binary variable which is 1 if arc (i,j) is destroyed by strike resource type l, and is otherwise 0.

The formulation of the interdiction problem from the attacker's viewpoint is as follows:

$$\underset{\gamma \in \Gamma}{\text{Minimize}} \quad \text{Maximize} \quad \sum_k f_k \tag{3.1}$$

Subject to

$$\sum_j x_{ijk} - \sum_j x_{jik} = \begin{cases} f_k & \text{if } i = s_k \\ 0 & \text{if } i \neq s_k \text{ , } s_k \\ -f_k & \text{if } i = t_k \end{cases} \qquad \forall\, k$$

$$\sum_k x_{ijk} \leq u_{ij}(1 - \gamma_{ijl}) \qquad \forall\, (i,j) \text{ , } l$$

$$x_{ijk} \geq 0 \qquad \forall\, (i,j) \text{ , } k$$

$$\Gamma = \{ \gamma_{ijl} \mid \gamma_{ijl} \in (0,1)\ \forall\, (i,j) \text{ , } l \text{ , } \quad \text{and} \sum_{(i,j)} e_{ijl}\gamma_{ijl} \leq E_l,\ \forall\, l \}$$

Let α_{ik} be the dual variable associated with the flow conservation equations, and let β_{ijl} be the dual variable associated with the combined capacity constraints. By taking the dual of the maximization problem the Min-Max problem becomes:

$$\underset{\gamma \in \Gamma}{\text{Minimize}} \quad \text{Minimize} \quad \sum_{l} \sum_{(i,j)} u_{ij} \beta_{ijl} (1 - \gamma_{ijl}) \tag{3.2}$$

Subject to

$$-\alpha_{sk} + \alpha_{tk} = 1 \qquad \forall\, k$$

$$\alpha_{ik} - \alpha_{jk} + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\, (i,j)\,,\, k$$

$$\beta_{ijl} \geq 0 \qquad \forall\, (i,j)\,,\, l$$

$$\alpha_{ik} \ \text{unrestricted} \qquad \forall\, i\,,\, k$$

$$\Gamma = \{\gamma_{ijl} \mid \gamma_{ijl}\ \forall\, (i,j)\,,\, l\,, \quad \text{and} \sum_{(i,j)} e_{ijl}\, \gamma_{ijl} \leq E_l\ \forall\, l\}$$

The above model can be linearized by replacing $\beta_{ijl}(1 - \gamma_{ijl})$ with h_{ijl} and introducing an additional constraint:

$$h_{ijl} \geq \beta_{ijl} - \bar{\beta}_{ijl} \gamma_{ijl} \qquad \forall\, (i,j)\,,\, l$$

where $\bar{\beta}_{ijl}$ is a known upper bound on β_{ijl}; and a suitable value is $\bar{\beta}_{ijl} = 1$. In the multicommodity flow problem, not all dual variables of the maximum flow problem need to be 0 or 1. There is no guarantee that the optimum solution will be integer (unless we have a *completely planar* network, Sakarovitch 1966).

The full model is:

$$\text{Minimize} \quad \sum_{l} \sum_{(i,j)} u_{ij} h_{ijl} \tag{3.3}$$

Subject to

$$
\begin{array}{rl}
-\alpha_{sk} + \alpha_{tk} = 1 & \forall\ k \\
\alpha_{ik} - \alpha_{jk} + \sum_{l} \beta_{ijl} \geq 0 & \forall\ (i,j)\ ,\ k \\
h_{ijl} - \beta_{ijl} + \bar{\beta}_{ijl} \gamma_{ijl} \geq 0 & \forall\ (i,j)\ ,\ l \\
\sum_{(i,j)} e_{ijl} \gamma_{ijl} \leq E_l & \forall\ l \\
\gamma_{ijl} \in (0,1) & \forall\ (i,j)\ ,\ l \\
\beta_{ijl} \geq 0 & \forall\ (i,j)\ ,\ l \\
h_{ijl} \geq 0 & \forall\ (i,j)\ ,\ l \\
\alpha_{ik} \ \text{unrestricted} & \forall\ i\ ,\ k
\end{array}
$$

3. Model 8 - Network with Undirected Arcs.

In the undirected-arc multicommodity flow problem, the material can flow in either direction of an arc so that the total flow is limited by the sum of flows of different commodities in both directions. An undirected arc in this type of network can be modeled by two anti-parallel directed arcs with a joint capacity constraint. Flows of same commodity in opposite direction will cancel, while flows of different commodities are cumulative regardless of direction. The arc capacity constraint for undirected arc *[i,j]* is then:

$$\sum_k (x_{ijk} + x_{jik}) \leq u_{ij} \qquad \forall\ k$$

The Min-Max problem from the attacker's viewpoint is:

$$\underset{\gamma \in \Gamma}{\text{Minimize}} \quad \text{Maximize} \quad \sum_k f_k \tag{3.4}$$

Subject to

$$\sum_j x_{ijk} - \sum_j x_{jik} = \begin{cases} f_k & \text{if } i = s_k \\ 0 & \text{if } i \neq s_k, t_k \\ -f_k & \text{if } i = t_k \end{cases} \qquad \forall\, k$$

$$\sum_k \left(x_{ijk} + x_{jik} \right) \leq u_{ij}(1 - \gamma_{ijl}) \qquad \forall\, [i,j]\,,\, l$$

$$x_{ijk} \geq 0 \qquad \forall\, [i,j]\,,\, k$$

$$\Gamma = \{ \gamma_{ijl} \mid \gamma_{ijl} \in (0,1)\ \forall\, [i,j]\,,\, l\,, \quad \text{and} \sum_{[i,j]} e_{ijl}\gamma_{ijl} \leq E_l,\ \forall\, l \}$$

Let an undirected arc be denoted by $[i,j]$. Following the same methodology for the development of Model 6, the formulation for the undirected-arc multicommodity network problem becomes:

$$\text{Minimize} \quad \sum_{l} \sum_{[i,j]} u_{ij} h_{ijl} \qquad (3.5)$$

Subject to

$$-\alpha_{sk} + \alpha_{tk} = 1 \qquad \forall\ k$$

$$\alpha_{ik} - \alpha_{jk} + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\ [i,j]\ ,\ k$$

$$\alpha_{jk} - \alpha_{ik} + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\ [i,j]\ ,\ k$$

$$h_{ijl} - \beta_{ijl} + \bar{\beta}_{ijl} \gamma_{ijl} \geq 0 \qquad \forall\ [i,j]\ ,\ l$$

$$\sum_{[i,j]} e_{ijl} \gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ [i,j]\ ,\ l$$

$$\beta_{ijl} \geq 0 \qquad \forall\ [i,j]\ ,\ l$$

$$h_{ijl} \geq 0 \qquad \forall\ [i,j], l$$

$$\alpha_{ik} \text{ unrestricted} \qquad \forall\ i\ ,\ k$$

B. MINIMIZE INTERDICTION EFFORT

1. Problem Statement

The attacker has sufficient resources to completely stop the flow of multiple commodities through the supply network. The aim of the attacker is to create with the minimum resource and minimum mission turnaround time a disconnecting set which is defined as a set of arcs whose removal from the network destroys all paths from each source to its respective sink .

2. Model 9 - Network with Directed Arcs

To solve the above problem, we need to introduce the following modifications to Model 6 in (3.3):

a. since there are now sufficient resources and we assume that we have solved Model 6, all h_{ijl} terms in the objective function will be zero indicating zero flow in the network. All h_{ijl} terms may then be deleted from the model,

b. the objective function of the new model should now be the amount of total resources weighted by the factor w_{ijl} which is the measure of the turnaround time for mission of type l on arc (i,j).

The model is as follows:

$$\text{Minimize} \quad \sum_{l} \sum_{(i,j)} w_{ijl}\, e_{ijl}\, \gamma_{ijl} \tag{3.6}$$

Subject to

$$-\alpha_{sk} + \alpha_{tk} = 1 \qquad \forall\ k$$

$$\alpha_{ik} - \alpha_{jk} + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\ (i,j)\ ,\ k$$

$$-\beta_{ijl} + \bar{\beta}_{ijl}\gamma_{ijl} \geq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\sum_{(i,j)} e_{ijl}\gamma_{ijl} \leq E_l \qquad \forall\ l$$

$$\beta_{ijl} \geq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

$$\alpha_{ik} \ \text{unrestricted} \qquad \forall\ i\ ,\ k$$

3. Model 10 - Network with Undirected Arcs

For the undirected graph, the formulation becomes:

$$\text{Minimize} \quad \sum_{l} \sum_{[i,j]} w_{ijl}\, e_{ijl}\, \gamma_{ijl} \tag{3.7}$$

Subject to

$$-\alpha_{sk} + \alpha_{tk} = 1 \qquad \forall\, k$$

$$\alpha_{ik} - \alpha_{jk} + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\, [i,j]\,,\, k$$

$$\alpha_{jk} - \alpha_{ik} + \sum_{l} \beta_{ijl} \geq 0 \qquad \forall\, [i,j]\,,k$$

$$-\beta_{ijl} + \bar{\beta}_{ijl}\gamma_{ijl} \geq 0 \qquad \forall\, [i,j]\,,\, l$$

$$\sum_{(i,j)} e_{ijl}\gamma_{ijl} \leq E_l \qquad \forall\, l$$

$$\beta_{ijl} \geq 0 \qquad \forall\, [i,j]\,,\, l$$

$$\gamma_{ijl} \in (0,1) \qquad \forall\, [i,j]\,,\, l$$

$$\alpha_{ik} \;\; \text{unrestricted} \qquad \forall\, i\,,\, k$$

IV. PROBABILISTIC MODELS

In this chapter, we will present probabilistic models for network interdiction. The models to be presented are applicable to infiltration and counterinsurgency situation.

A. ONE EVADER VERSUS ONE INTERDICTOR

1. Problem Statement

An evader attempts to travel from the source node s of a supply network to the sink node t. An interdictor would like to stop his opponent by positioning with probability π_{ij} on arc (i,j) so that the probability of successful infiltration by the evader is minimized. The probability that the interdictor detects the evader given that the interdictor positions himself on arc (i,j) and the evader traverses on arc (i,j) is p_{ij}. (If detection is certain, p_{ij} could, instead, represent a reward for catching the evader on arc (i,j).) Knowing the interdictor's strategy, the evader will have to develop an optimum strategy by selecting path k with probability δ_k so as to minimize the probability of interception.

2. Model

For this model, the indices used are:

i,j : nodes,

The data of the model are:

p_{ij} : probability of successful interception given that the evader has chosen to cross the arc and the interdictor has placed himself there.

The decision variables are:

π_{ij} : the probability of the interdictor deploying at arc (i,j),

ϕ_{ij} : the probability of the evader traversing arc (i,j).

The Min-Max formulation is as follows:

$$\underset{\phi}{\text{Minimize}} \quad \underset{\pi}{\text{Maximize}} \quad \sum_{(i,j)} p_{ij}\phi_{ij}\pi_{ij} \tag{4.1}$$

Subject To

$$\sum_{(i,j)} \pi_{ij} = 1$$

$$\sum_j \phi_{ij} - \sum_j \phi_{ji} = \begin{cases} 1 & \text{if } i = s \\ 0 & \text{if } i \neq s \text{ or } t \\ -1 & \text{if } i = t \end{cases}$$

$$\pi_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$\phi_{ij} \geq 0 \qquad \forall\ (i,j)$$

The aim of the interdictor is to maximize the total reward while the aim of the evader is to minimize the total reward. The first constraint of (4.1) ensures that the probability of deployment by the interdictor is one. The second set of constraints ensure that only one evader is traversing the network. Let v be the dual variable associated with the first constraint of (4.1) and taking the dual of the maximization problem, we have:

$$\text{Minimize} \quad v \qquad (4.2)$$

Subject To

$$v - p_{ij}\phi_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$\sum_j \phi_{ij} - \sum_j \phi_{ji} = \begin{cases} 1 & \text{if } i = s \\ 0 & \text{if } i \neq s \text{ or } t \\ -1 & \text{if } i = t \end{cases}$$

$$\pi_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$\phi_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$v \quad \text{unrestricted}$$

The optimal solution will occur for that value of v such that the minimum cut capacity (with arc capacities at v/p_{ij}) is 1. The dual prices of the first set of constraints in the optimum solution will be the optimum values of π_{ij} and are positive on the minimum cut-set. The optimal strategy of the evader can be recovered as follows:

(1) Let

$$a_{(i,j).k} = \begin{cases} 1 & \text{if path } k \text{ contains arc } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

and δ_k be the probability that the evader chooses path k.

(2) Having known the optimal values of ϕ_{ij} by solving (4.2), δ_k can be determined by solving the following set of equations:

$$\sum_k a_{(i,j),k}\, \delta_k = \phi_{ij} \qquad \forall\ (i,j)$$

As Lawler 1976 points out, there exists a solution of such a system with at most $|E|$ positive δ_k. In fact, a set of $|E|$ or fewer arcs which will satisfy this system can be derived as follows: Start with the optimal "flow of probability" in the system from (4.2). Discard any arcs with zero flow. Find a directed path k_1 from s to t and suppose the minimum flow arc on that arc has flow x_1. Then, associated with this path k_1, $\delta_{k_1} = x_1$. Subtract x_1 units of flow from arcs along the path and delete any arcs with no flow remaining. Repeat this process at most $|E|$ - 1 more times until all flow has been allocated to at most $|E|$ paths.

B. ONE EVADER VERSUS MULTIPLE INTERDICTORS

1. Problem Statement

A team of n interdictors is deployed in the network in an attempt to stop an evader from traversing from the source node to the sink node. The evader has the probability ϕ_{ij} of choosing to travel on arc (i,j). At arc (i,j), at most one interceptor l may be deployed, with the probability of π_{ijl}. The reward to an interdictor of deploying at arc (i,j) given that the evader has chosen to traverse that arc is p_{ij}. The objective of the interdictor team is to maximize the total reward while that of the evader is to minimize it.

2. Model

In this model, the additional index used is:

l : interdictor number which is $1...n$,

The additional decision variables are:

π_{ijl} : the probability of interdictor l deploying at arc (i,j),

δ_{ijl} : the binary variable which is 1 if interdictor l is deployed at arc (i,j) and is otherwise 0.

The Max-Min formulation is as follows:

$$\underset{\pi,\,\delta}{\text{Maximize}} \quad \underset{\phi}{\text{Minimize}} \quad \sum_{(i,j)} p_{ij}\phi_{ij}\left(\sum_{l}\pi_{ijl}\right) \tag{4.3}$$

Subject To

$$\sum_j \phi_{ij} - \sum_j \phi_{ji} = \begin{cases} 1 & \text{if } i = s \\ 0 & \text{if } i \neq s \text{ or } t \\ -1 & \text{if } i = t \end{cases}$$

$$\pi_{ijl} - \delta_{ijl} \leq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\sum_l \delta_{ijl} \leq 1 \qquad \forall\ (i,j)$$

$$\sum_{(i,j)} \pi_{ijl} \leq 1 \qquad \forall\ l$$

$$\pi_{ijl} \geq 0 \qquad \forall\ (i,j)\ ,\ l$$

$$\phi_{ij} \geq 0 \qquad \forall\ (i,j)$$

$$\delta_{ijl} \in (0,1) \qquad \forall\ (i,j)\ ,\ l$$

The objective of the interdictor is to manipulate π_{ijl} and δ_{ijl} so that the total reward would be maximized. The evader aims to minimize this reward, manipulating ϕ_{ij}. The objective function is correct since, at any one time, only one interdictor would be present at an arc. This is so due to the second and third constraints of (4.3). This formulation does restrict the interdictors in that if interdictor l ever interdicts arc (i,j), no other interdictor $l' = l$ can ever interdict that arc.

Taking the dual of the minimization LP of the above Max-Min problem and letting α_i be the dual variable associated with the first constraint of (4.3), we have:

$$\text{Maximize} \quad \alpha_s - \alpha_t \tag{4.4}$$

Subject To

$$\begin{aligned}
\alpha_i - \alpha_j - p_{ij} \sum_l \pi_{ijl} &\leq 0 && \forall\, i \\
\pi_{ijl} - \delta_{ijl} &\leq 0 && \forall\, (i,j)\,,\, l \\
\sum_l \delta_{ijl} &\leq 1 && \forall\, (i,j) \\
\sum_{(i,j)} \pi_{ijl} &\leq 1 && \forall\, l \\
\pi_{ijl} &\geq 0 && \forall\, (i,j)\,,\, l \\
\phi_{ij} &\geq 0 && \forall\, (i,j) \\
\delta_{ijl} &\in (0,1) && \forall\, (i,j)\,,\, l
\end{aligned}$$

V. COMPUTATIONAL RESULTS

In this chapter, a network interdictor's problem which originally motivated the writing of this thesis is solved. GAMS (General Algebraic Modeling System) programs are developed and ıun on an Amdahl 5990 mainframe computer. The integer programs are solved using ZOOM (Zero/One Optimization Methods) which is available in GAMS. The network to be solved is an undirected network, and the relevant models tested are:

a. Model 2 - minimize the maximum flow (single commodity),

b. Model 4 - minimize the interdiction effort (single commodity),

c. Model 8 - minimize the maximum flow (multicommodity),

d. Model 10 - minimize the interdiction effort (multicommodity).

A. SCENARIO

The road transportation network in question is as shown in Figure 1. It is a land transportation network system found in Southeast Asia and consists of 112 nodes and 176 undirected arcs. The capacities of the arcs in term of standard truck-loads per day as shown in Table 1 which is in Appendix A. The land transportation network in question presents itself as a realistic scenario for a strategic strike planner to solve. The locations of enemy front-line units, its supply depots and the orientation of the network are as shown in Figure 1.

1. Opposing Forces

The network interdictor has at his disposal the following types of assets:

a. Ground Attack Fighter Squadrons. Fighter aircraft laden with weapons such as laser-guided bombs and other precision guided munitions which are suitable for destructions of bridges.

b. Naval Gunfire Support. Naval combatants can be deployed along the coasts to provide gun fire support.

c. Special Forces. This consists of highly specialized and trained personnel operating autonomously in small teams deep in enemy's territories to carry out clandestine sabotage missions.

2. Weapons Efforts

Against a particular target, the suitability of a weapon system and the required efforts depend on:

a. the types of defenses which the attacker is expected to encounter,

b. the effective range of the weapon,

c. in the case of aerial attacks, the degree of difficulty in detecting and identifying targets, and,

d. the size of target.

The resources required to destroy each target is given in Table 1. The data presented represent a realistic range of possibilities but are hypothetical and only meant for the purpose of testing the model. Accurate resource data could be obtained from detailed mission planning for each target with the aid of the Joint Munitions Effectiveness Manual (JMEM), 1991.

B. MINIMUM CUT-SET - SINGLE COMMODITY NETWORK

In this scenario, the enemy wishes to supply his front-line troops at nodes [108] through [112] by forcing the flow of a single commodity from the supply depots at nodes [1] through [7] through the network. The maximum flow of the un-interdicted network is 190 units and the minimum cut-set is as shown in Figure 2.

C. MODEL 2 - MINIMIZING MAXIMUM FLOW - SINGLE COMMODITY NETWORK

For this model, the GAMS program that solves the problem is in the Appendix. Given that the interdictor has 4 aircraft, 4 naval gunfire support (NGFS) units and 4 teams of special forces with which to interdict the network, the maximum flow is reduced to 20 units. The arcs to be interdicted and the force allocation are as follows:

	Arcs	Weapons Allocation
1.	(84,85)	2 Aircraft
2.	(83,86)	4 NGFS
3.	(54,87)	2 Aircraft

	Arcs	Weapons Allocation
4.	(53,52)	2 Special Forces
5.	(51,94)	2 Special forces

The cut-set is as shown in Figure 3. In this cut-set, arc (50,52) is the only arc not interdicted, and allows a flow of 20 units. The model contains 356 single equations, 817 real variables and 528 discrete variables. The generation time and execution times are 1.58 seconds and 1.66 seconds respectively. The generation time is the time spent preparing the model for solution, while the execution time is the time used after the syntactic check is finished, including the time spent generating the model.

D. MODEL 4 - MINIMIZE INTERDICTION EFFORT - SINGLE COMMODITY NETWORK

Given that the interdictor has sufficient resources to stop completely the flow of material in the supply network, and, for simplicity, assuming equal turnaround times, the minimum effort required and the force allocation are as follows:

	Arcs	Weapons Allocation
1.	(84,85)	2 Aircraft
2.	(83,86)	2 Aircraft
3.	(54,87)	2 Aircraft
4.	(52,82)	4 Special Forces
5.	(51,94)	2 Special Forces

Figure 4 shows the location of strikes and the force allocation. The model has 356 equations, 641 real variables and 628 discrete variables. It takes 1.59 seconds to generate and 1.66 seconds to execute the model.

E. MODEL 8 - MINIMIZE MAXIMUM FLOW - MULTICOMMODITY NETWORK

For the multicommodity flow scenario, we assume that the enemy wishes to supply:

1. the unit at node [112] with water from the water "source" at node [90],
2. the unit at node [108] with ammunition from the ammunition dump at node [6],
3. the unit at node [112] with fuel from the fuel depot at node [5].

The maximum multicommodity flow is 260 units which is the sum of flow of the three commodities in terms of standard truckloads per day. With the available assets of 4 aircraft, 4 NGFS units and 4 teams of special forces, the maximum flow is reduced to 100 units. Figure 5 shows the locations of attack and the following force allocation:

	Arcs	Weapons Allocation
1.	(51,94)	2 Special Forces
2.	(54,55)	2 Aircraft
3.	(54,83)	2 Aircraft
4.	(83,86)	4 NGFS
5.	(84,85)	2 Special Forces

Note that the unit at node [112] will receive 80 units of water supply and 20 units of fuel and ammunition will flow through the network. The model has 1588 equations, 1921 real variables and 528 discrete variables. It takes 6.01 seconds to generate and 6.11 second to execute the model.

F. MODEL 10 - MINIMIZE INTERDICTION EFFORT - MULTICOMMODITY NETWORK

To disconnect all the units concerned from their respective sources of supply, the minimum interdiction effort is 14 units and the force allocation is as follows:

	Arcs	Weapons Allocation
1.	(5,12)	4 Aircraft
2.	(6,9)	2 Aircraft
3.	(97,112)	8 NGFS

As shown in Figure 6, the interdiction efforts are channelled to the arcs incident to the fuel depot, ammunition dumps and the tactical unit requiring water supply. If the enemy augments the defenses of these arcs by increasing their destruction efforts e_{ijl} to an extent that it is no longer possible for the attacker to strike these arcs (in this example, we make $e_{ijl} > E_l$ for these three arcs), we will see a distinct shift in the attacker's strategy. Figure 7 shows the new strategy adopted by the attackers. The reallocation of the strike resources is as follows:

	Arcs	Weapons Allocation
1.	(84,85)	2 Aircraft
2.	(83,86)	2 Aircraft
3.	(54,87)	2 Aircraft
4.	(52,94)	4 Special Forces
5.	(51,94)	2 Special Forces
6.	(90,89)	4 NGFS
7.	(90,91)	4 Aircraft

The model has 1588 single equations, 1393 real variables and 528 discrete variables. It takes 5.72 seconds to generate and 5.82 seconds to execute the model.

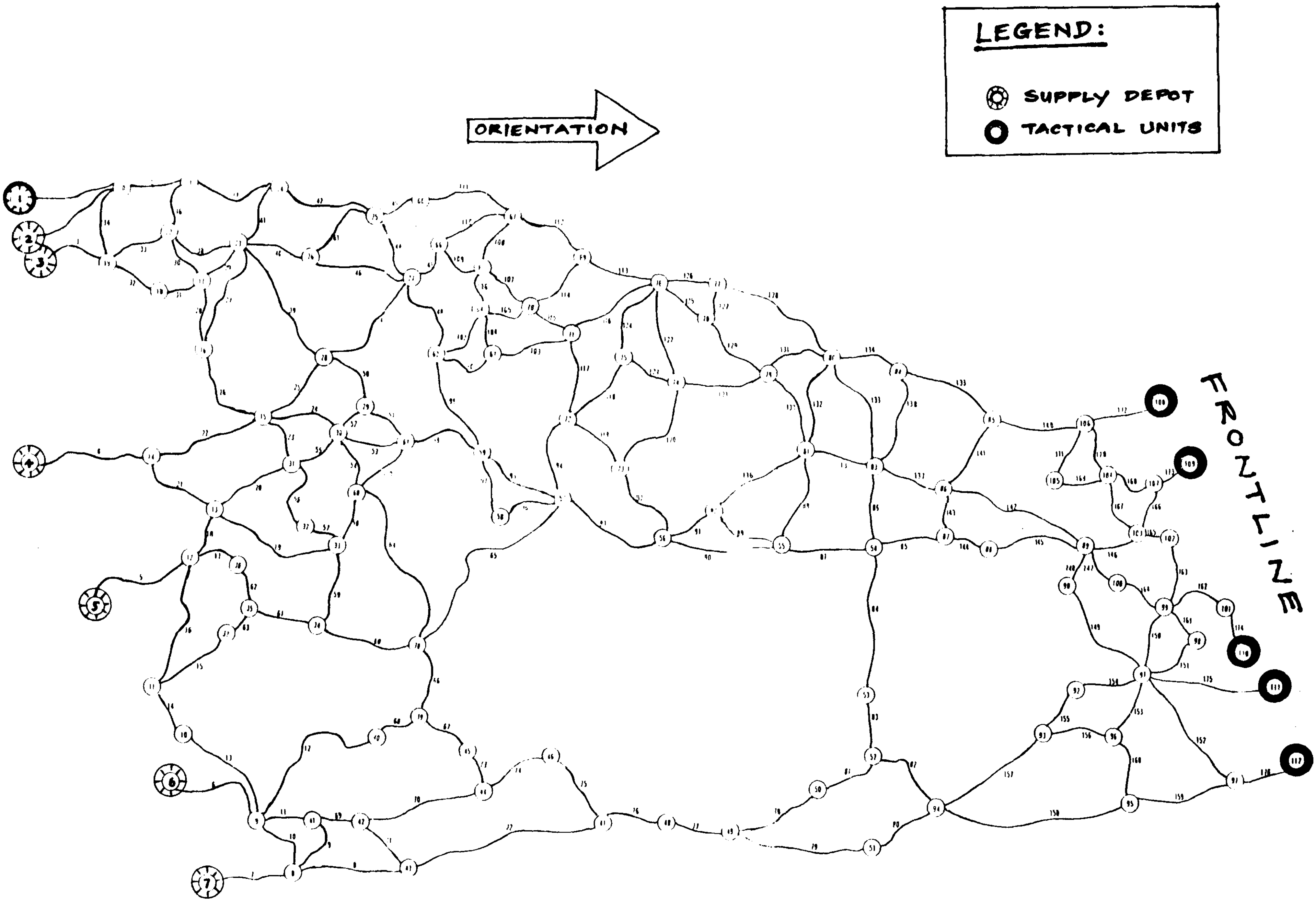

FIGURE 1 : ROAD TRANSPORTATION NETWORK FOR LOGISTIC SUPPLY

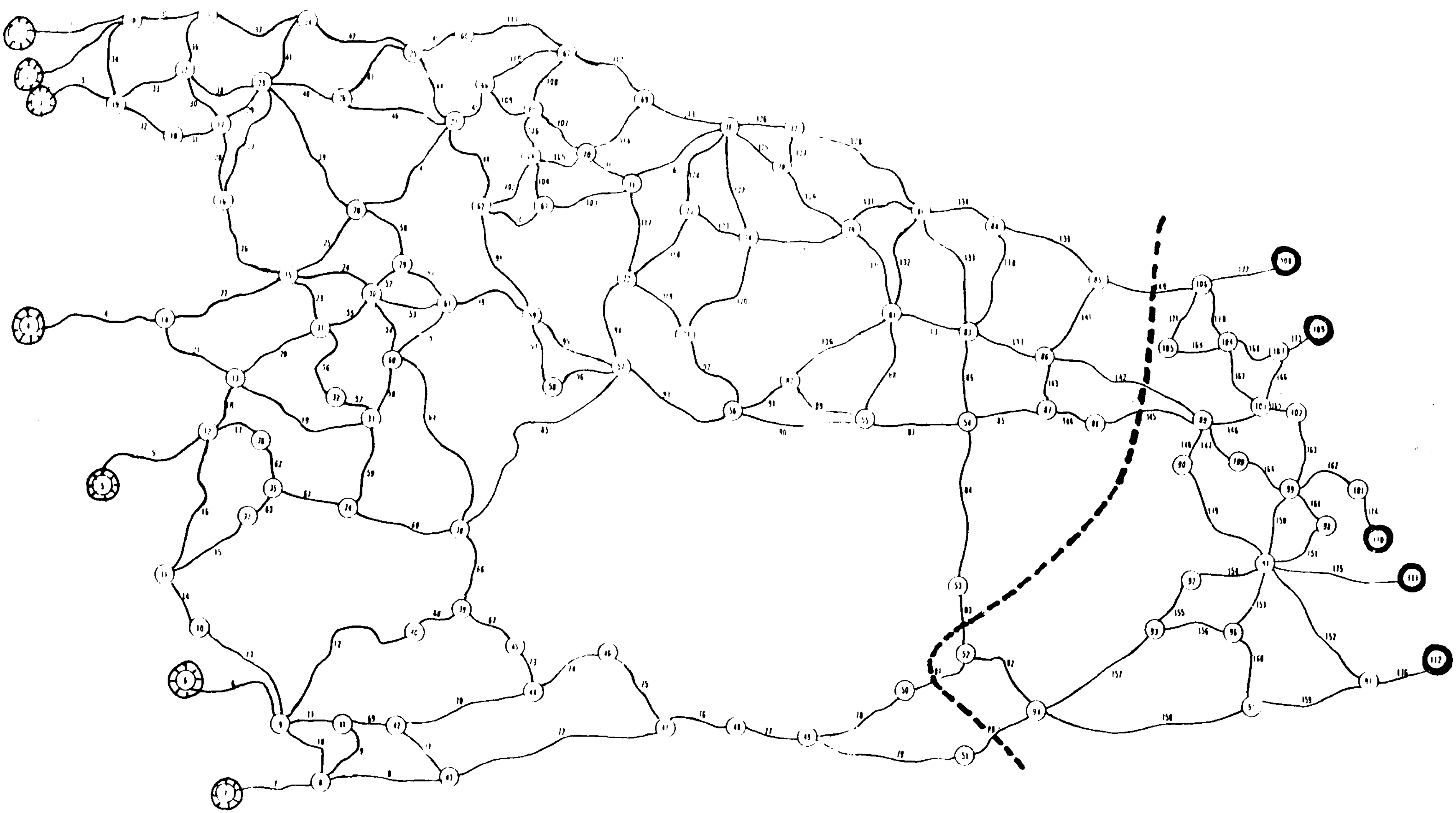

FIGURE 2 - MINIMUM CUT - SET (SINGLE COMMODITY)

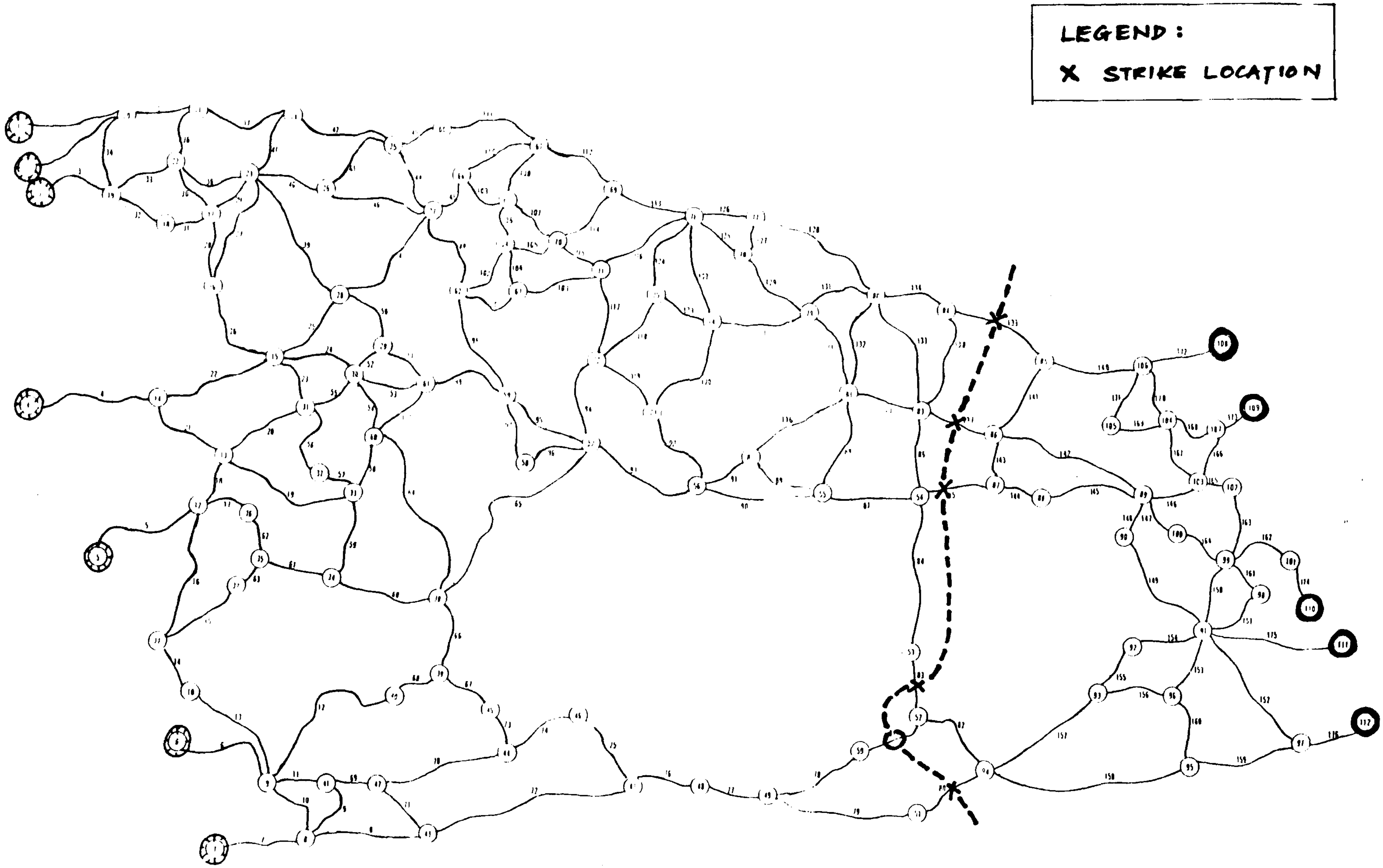

FIGURE 3 – MINIMIZE THE MAXIMUM FLOW (SINGLE COMMODITY)

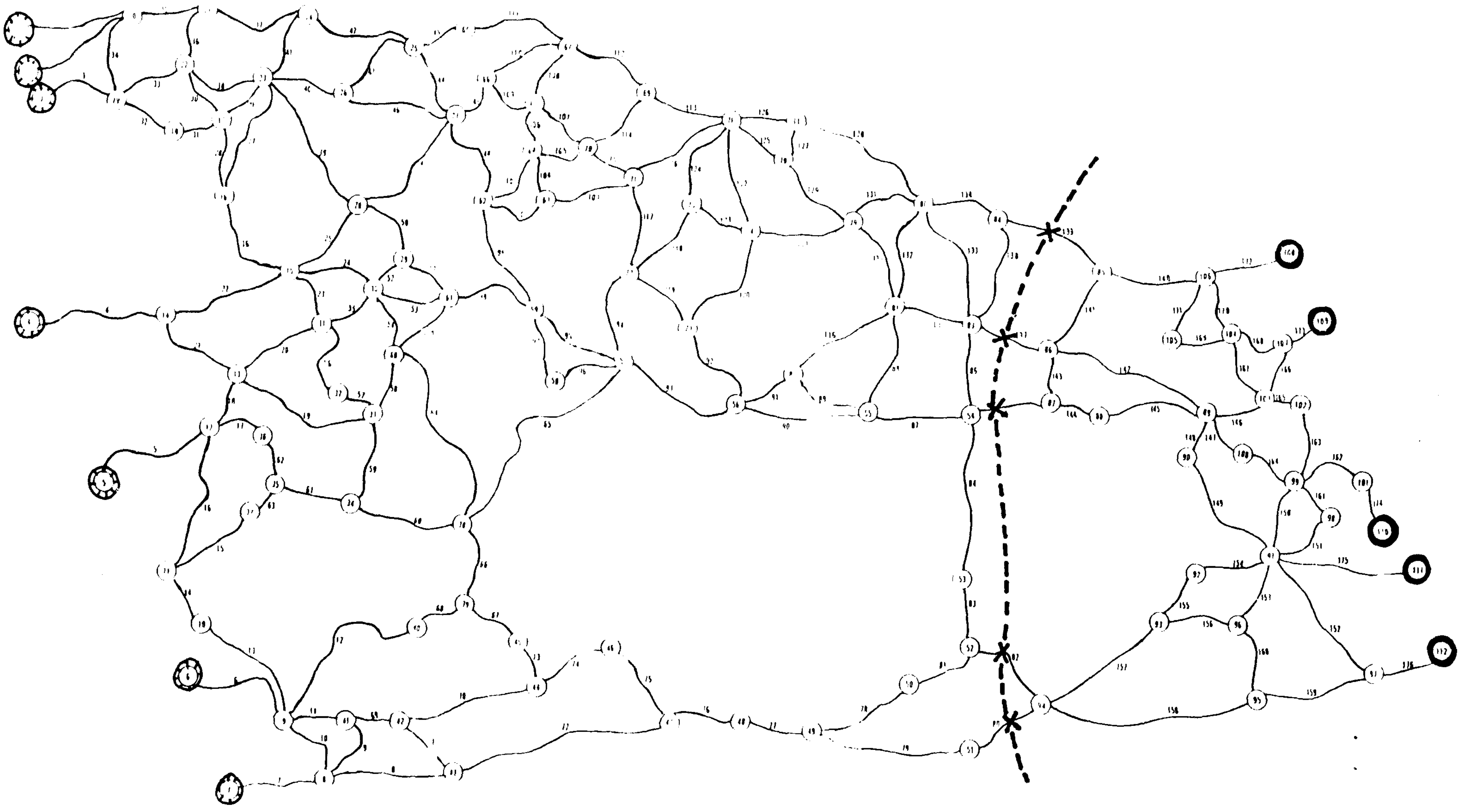

FIGURE 4: MINIMIZE INTERDICTION EFFORT (SINGLE COMMODITY)

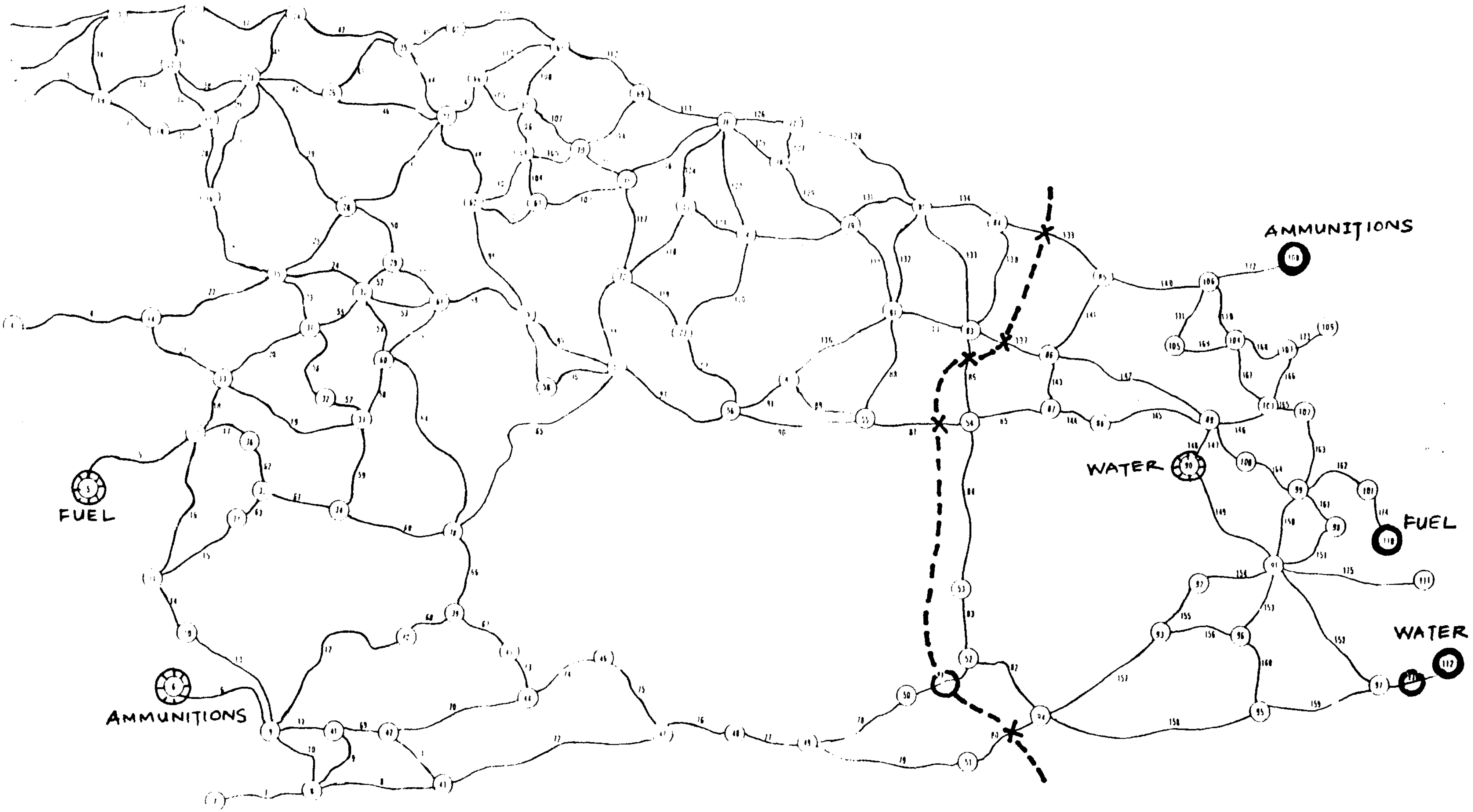

FIGURE 5 : MINIMIZE THE MAXIMUM FLOW (MULTIPLE COMMODITIES)

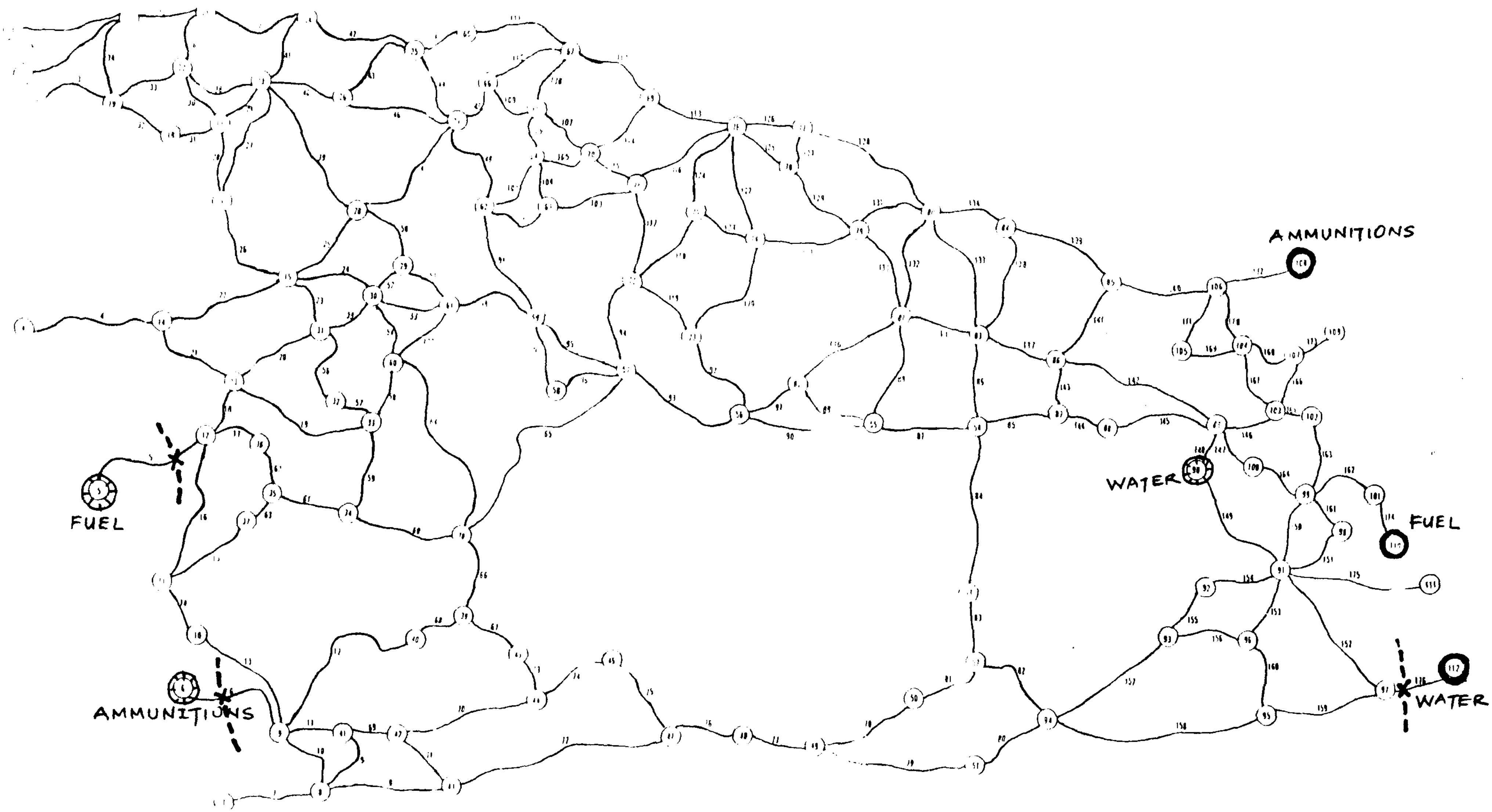

FIGURE 6 : MINIMIZE INTERDICTION EFFORT (MULTIPLE COMMODITIES)

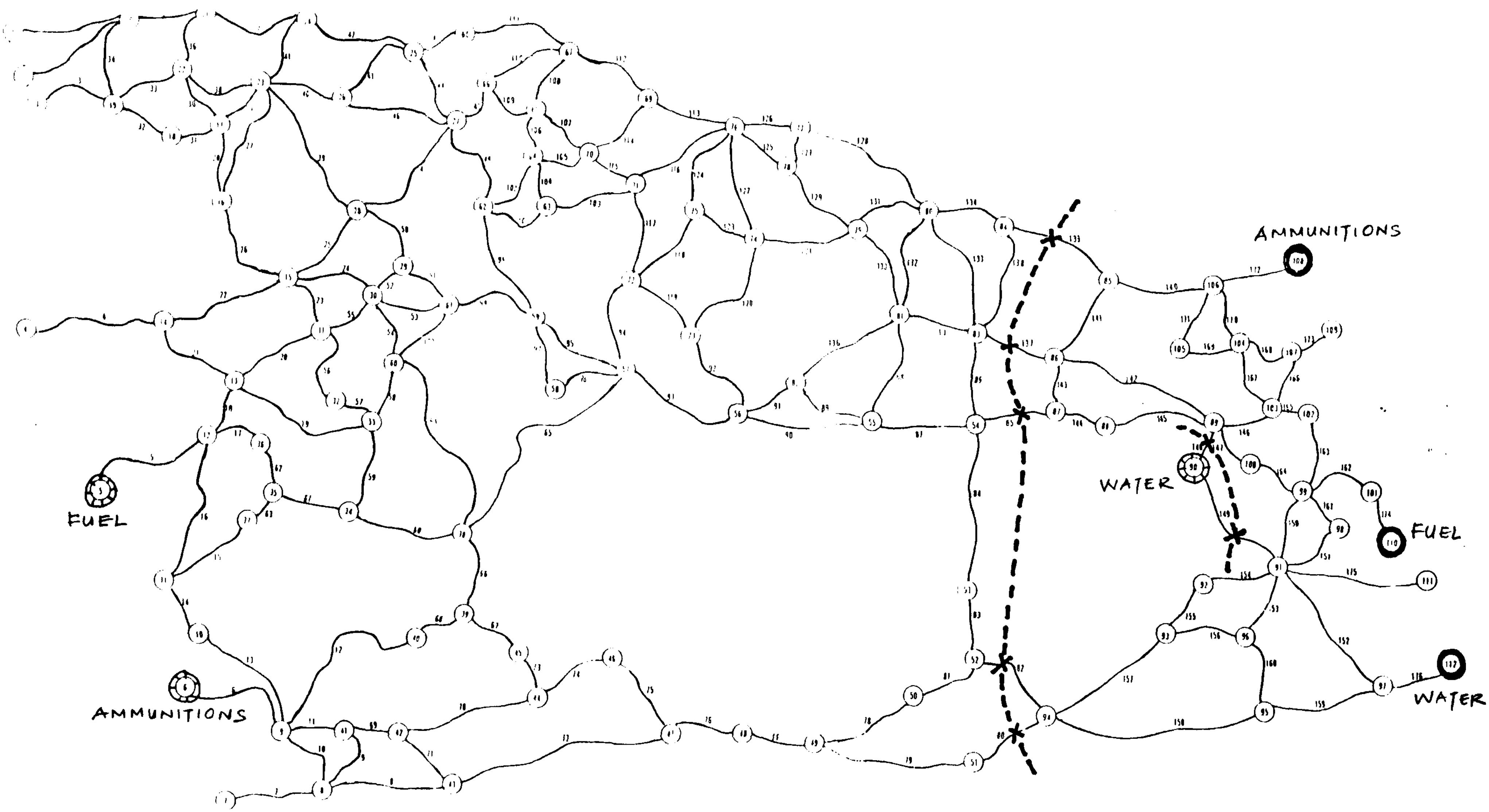

FIGURE 7: MINIMIZE INTERDICTION EFFORT - AUGMENTED DEFENSE (MULTIPLE COMMODITIES)

VI. SUMMARY AND EXTENSIONS

A. SUMMARY

In this report, we present two main types of model, vis., the deterministic and probabilistic models, for the analysis of strategic strike against a land communication network.

In the deterministic models, the technique of mixed integer programming is used and has been shown to be quite versatile in that generalizations to include, among other features, multiple strike resources, are easy. Under this category, the main models we have developed are:

a. Single Commodity Network. The aim of the network interdictor is to minimize the maximum throughput of material supply by allocating a fixed amount of multiple resources to interdict the most "profitable" arcs of the underlying transportation network. If there are sufficient resources to stop completely the flow of materials, the problem becomes one of allocating, in an optimal manner, the minimum effort over the multiple strike resources. Another way of degrading the performance of the supply network is to maximize the prioritized shortfalls of the demands at the tactical units.

b. Multicommodity Network. In this situation, the enemy wishes to send a few types of commodities from their supply depots to the respective tactical units requiring specialized support. The attacker's problem is either to minimize the sum of the maximum flow of the multiple commodities or to utilize minimum resources to achieve a disconnecting set, which severs the paths connecting all sources to their respective sinks.

Another type of model presented is a probabilistic model, which is applicable to the interdiction of a lightly travelled network in which the arc capacity is not a factor. The objective of a single interdictor or a team of interdictors is to minimize the probability of infiltration by a single evader through a network while the objective of the evader is just the opposite.

B. EXTENSIONS

There are a few useful model extensions which warrant further research.

1. Deterministic Models

a. Partial Arc Destruction. One possible model extension would be to allow for partial arc destruction, by assuming a linear or piece-wise linear damage function.

b. Co-ordinated Strikes. Certain missions may require the co-ordinated execution of a few types of strike resources. Examples include the employment of ground laser designator to illuminate targets for fighter bombers, and target spotting for naval gun fire support. The model presented can be modified to incorporate this feature.

2. Probabilistic Models

a. Multiple Evaders Versus Multiple Interceptors. It will be useful to extend the model to include the situation where multiple interceptors are available to defend the network against multiple evaders.

b. Resource Constraint. The issue of resource constraint over time can be addressed to improve model realism.

APPENDIX A. NETWORK DATA

	Arc	Capacity	Weapons Efforts		
			Aircraft	NGFS	Sp Forces
1.	(1,20)	100	12	10	15
2.	(2,20)	40	14	8	15
3.	(3,19)	100	14	10	15
4.	(4,14)	110	6	12	15
5.	(5,12)	80	4	9	15
6.	(6,9)	130	2	15	15
7.	(7,8)	30	12	6	15
8.	(8,43)	50	10	6	15
9.	(8,41)	160	14	18	15
10.	(8,9)	140	12	15	15
11.	(9,41)	100	16	15	15
12.	(9,40)	100	18	15	15
13.	(9,10)	80	12	9	15
14.	(10,11)	90	16	10	15
15.	(11,37)	100	14	12	15
16.	(11,12)	50	16	28	15
17.	(12,36)	50	4	29	2
18.	(12,13)	40	6	24	4
19.	(13,33)	80	4	28	4
20.	(13,31)	100	8	22	5
21.	(13,14)	20	4	23	15
22.	(14,15)	40	4	25	15
23.	(15,31)	50	4	28	15
24.	(15,30)	50	2	28	15
25.	(15,28)	60	4	6	15
26.	(15,16)	70	6	9	2
27.	(16,23)	130	4	14	5
28.	(16,17)	120	6	15	4

	Arc	Capacity	Weapons Efforts		
			Aircraft	NGFS	Sp Forces
29.	(17,23)	120	2	18	4
30.	(17,22)	110	6	18	4
31.	(17,18)	120	16	18	4
32.	(18,19)	120	10	14	4
33.	(19,22)	30	10	4	2
34.	(19,20)	50	18	8	2
35.	(20,21)	130	14	18	4
36.	(21,22)	40	12	8	2
37.	(21,24)	50	14	8	2
38.	(22,23)	40	12	8	2
39.	(23,28)	80	6	8	2
40.	(23,26)	70	4	8	2
41.	(23,24)	90	6	12	4
42.	(24,25)	50	6	8	2
43.	(25,26)	60	6	8	2
44.	(25,27)	130	6	18	5
45.	(25,68)	20	6	4	2
46.	(26,27)	50	6	4	2
47.	(27,28)	60	6	8	2
48.	(27,62)	40	6	6	2
49.	(27,66)	90	4	10	6
50.	(28,29)	50	4	5	15
51.	(29,61)	70	4	27	15
52.	(29,30)	50	6	25	15
53.	(30,61)	60	4	26	15
54.	(30,60)	70	8	28	15
55.	(30,31)	100	4	22	15
56.	(31,32)	120	4	24	15
57.	(32,33)	150	4	26	15
58.	(33,60)	140	4	24	15
59.	(33,34)	60	4	29	6
60.	(34,38)	20	8	23	2

	Arc	Capacity	Weapons Efforts		
			Aircraft	NGFS	Sp Forces
61.	(34,35)	140	4	28	8
62.	(35,36)	160	2	20	8
63.	(35,37)	30	4	26	2
64.	(38,60)	60	6	27	2
65.	(38,57)	40	4	28	2
66.	(38,39)	20	2	4	2
67.	(39,45)	80	2	9	4
68.	(39,40)	90	2	10	6
69.	(41,42)	40	2	6	2
70.	(42,44)	90	2	10	5
71.	(42,43)	60	2	6	2
72.	(43,47)	30	2	4	2
73.	(44,45)	140	12	13	15
74.	(44,46)	160	10	15	15
75.	(46,47)	180	16	16	15
76.	(47,48)	130	10	14	15
77.	(48,49)	140	14	15	15
78.	(49,50)	30	16	4	15
79.	(49,51)	150	18	20	15
80.	(51,94)	50	18	8	2
81.	(50,52)	20	18	4	2
82.	(52,94)	90	12	10	4
83.	(52,53)	30	12	25	2
84.	(53,54)	50	12	26	2
85.	(54,87)	30	2	24	2
86.	(54,83)	70	2	28	4
87.	(54,55)	30	2	24	2
88.	(55,81)	150	4	25	8
89.	(55,82)	140	4	25	6
90.	(55,56)	30	2	24	2
91.	(56,82)	70	4	27	2
92.	(56,73)	80	2	20	4

	Arc	Capacity	Weapons Efforts		
			Aircraft	NGFS	Sp Forces
93.	(56,57)	90	6	22	5
94.	(57,72)	40	6	26	2
95.	(57,59)	30	6	24	2
96.	(57,58)	60	6	26	2
97.	(58,59)	50	4	26	2
98.	(59,61)	150	2	20	15
99.	(59,62)	140	6	28	15
100.	(60,61)	20	4	24	15
101.	(62,63)	70	18	8	5
102.	(62,64)	90	14	10	5
103.	(63,71)	40	14	6	5
104.	(63,64)	30	14	5	2
105.	(64,70)	80	10	8	5
106.	(64,65)	80	10	8	5
107.	(65,70)	50	10	6	2
108.	(65,67)	40	14	4	2
109.	(65,66)	10	18	4	2
110.	(66,67)	120	14	10	8
111.	(67,68)	130	14	10	15
112.	(67,69)	60	14	6	15
113.	(69,76)	70	18	8	15
114.	(69,70)	40	16	5	15
115.	(70,71)	60	14	8	15
116.	(71,76)	70	18	8	15
117.	(71,72)	80	16	8	15
118.	(72,75)	40	14	4	15
119.	(72,73)	140	18	18	15
120.	(73,74)	150	14	18	15
121.	(74,79)	120	16	18	15
122.	(74,76)	60	16	8	15
123.	(74,75)	30	16	8	15
124.	(75,76)	20	14	4	15

	Arc	Capacity	Weapons Efforts		
			Aircraft	NGFS	Sp Forces
125.	(76,78)	50	14	5	15
126.	(76,77)	30	14	4	15
127.	(77,78)	80	12	8	15
128.	(77,80)	50	12	6	15
129.	(78,79)	30	12	5	15
130.	(79,81)	140	14	18	15
131.	(79,80)	90	12	10	5
132.	(80,81)	30	14	4	2
133.	(80,83)	40	10	4	3
134.	(80,84)	50	12	8	2
135.	(81,83)	50	2	8	2
136.	(81,82)	60	2	8	5
137.	(83,86)	30	2	4	2
138.	(83,84)	90	2	10	6
139.	(84,85)	80	2	8	2
140.	(85,106)	40	4	8	15
141.	(85,86)	70	4	8	15
142.	(86,89)	10	8	4	15
143.	(86,87)	160	8	20	15
144.	(87,88)	180	6	24	15
145.	(88,89)	40	6	4	15
146.	(89,103)	30	4	4	15
147.	(89,100)	90	2	13	15
148.	(89,90)	40	6	4	15
149.	(90,91)	60	4	5	15
150.	(91,99)	90	18	11	15
151.	(91,98)	60	16	5	15
152.	(91,97)	40	14	4	15
153.	(91,96)	140	14	12	15
154.	(91,92)	150	10	12	15
155.	(92,93)	150	10	12	15
156.	(93,96)	30	14	4	15

	Arc	Capacity	Weapons Efforts		
			Aircraft	NGFS	Sp Forces
157.	(93,94)	40	12	4	15
158.	(94,95)	80	12	8	15
159.	(95,97)	90	12	10	15
160.	(95,96)	90	12	10	15
161.	(98,99)	80	14	8	15
162.	(99,101)	90	12	10	15
163.	(99,102)	80	12	8	15
164.	(99,100)	60	12	6	15
165.	(102,103)	50	12	6	15
166.	(103,107)	90	12	10	15
167.	(103,104)	150	10	14	15
168.	(104,107)	120	10	12	15
169.	(104,105)	130	16	12	16
170.	(104,106)	50	10	8	14
171.	(105,106)	80	10	8	14
172.	(106,108)	100	14	10	15
173.	(107,109)	90	14	8	14
174.	(101,110)	100	12	10	16
175.	(91,111)	100	12	10	16
176.	(97,112)	80	12	8	14

APPENDIX B. COMPUTER PROGRAM

GAMS COMPUTER LISTING

```
                              08/04/91 10:31:48  PAGE     1
                                                               GAMS
2.19 IBM CMS


OPTIONS
  LIMCOL =  0, LIMROW  = 0, SOLPRINT = OFF , DECIMALS = 2
  RESLIM =10000, ITERLIM = 90000, OPTCR    = 0.05;

*---------DEFINITIONS AND DATA--------------------------

SET
  I nodes in the network /1*112/
  L types of asset        /AIR,NGS,SF/;

ALIAS  (I,J);

PARAMETERS  CAP(I,J)    arc capacity

      /1  .20      100
        2 .20       40
        3 .19      100
        4 .14      110
        5 .12       80
        6 .9       130
        7 .8        30
        8 .43       50
        8 .41      160
        8 .9       140
        9 .41      100
        9 .40      100
        9 .10       80
       10 .11       90
       11 .37      100
       11 .12       50
```

39	12 .36	50
40	12 .13	40
41	13 .33	80
42	13 .31	100
43	13 .14	20
44	14 .15	40
45	15 .31	50
46	15 .30	50
47	15 .28	60
48	15 .16	70
49	16 .23	130
50	16 .17	120
51	17 .23	120
52	17 .22	110
53	17 .18	120
54	18 .19	120
55	19 .22	30
56	19 .20	50
57	20 .21	130
58	21 .22	40
59	21 .24	50
60	22 .23	40

61	23 .28	80
62	23 .26	70
63	23 .24	90
64	24 .25	50
65	25 .26	60
66	25 .27	130
67	25 .68	20
68	26 .27	50
69	27 .28	60
70	27 .62	40
71	27 .66	90
72	28 .29	50
73	29 .61	70
74	29 .30	50
75	30 .61	60

76	30 .60	70
77	30 .31	100
78	31 .32	120
79	32 .33	150
80	33 .60	140
81	33 .34	60
82	34 .38	20
83	34 .35	140
84	35 .36	160
85	35 .37	30
86	38 .60	60
87	38 .57	40
88	38 .39	20
89	39 .45	80
90	39 .40	90
91	41 .42	40
92	42 .44	90
93	42 .43	60
94	43 .47	30
95	44 .45	140
96	44 .46	160
97	46 .47	180
98	47 .48	130
99	48 .49	140
100	49 .50	30
101	49 .51	150
102	51 .94	50
103	50 .52	20
104	52 .94	90
105	52 .53	30
106	53 .54	50
107	54 .87	30
108	54 .83	70
109	54 .55	30
110	55 .81	150
111	55 .82	140
112	55 .56	30
113	56 .82	70
1.4	56 .73	80
115	56 .57	90
116	57 .72	40

08/04/91 10:31:48 PAGE 3

117	57 .59	30
118	57 .58	60
119	58 .59	50
120	59 .61	150
121	59 .62	140
122	60 .61	20
123	62 .63	70
124	62 .64	90
125	63 .71	40
126	63 .64	30
127	64 .70	80
128	64 .65	80
129	65 .70	50
130	65 .67	40
131	65 .66	10
132	66 .67	120
133	67 .68	130
134	67 .69	60
135	69 .76	70
136	69 .70	40
137	70 .71	60
138	71 .76	70
139	71 .72	80
140	72 .75	40
141	72 .73	140
142	73 .74	150
143	74 .79	120
144	74 .76	60
145	74 .75	30
146	75 .76	20
147	76 .78	50
148	76 .77	30
149	77 .78	80
150	77 .80	50
151	78 .79	30
152	79 .81	140
153	79 .80	90
154	80 .81	30
155	80 .83	40

156	80 .84	50
157	81 .83	50
158	81 .82	60
159	83 .86	30
160	83 .84	90
161	84 .85	80
162	85 .106	40
163	85 .86	70
164	86 .89	10
165	86 .87	160
166	87 .88	180
167	88 .89	40
168	89 .103	30
169	89 .100	90
170	89 .90	40
171	90 .91	60
172	91 .99	90

173	91 .98	60
174	91 .97	40
175	91 .96	140
176	91 .92	150
177	92 .93	150
178	93 .96	30
179	93 .94	40
180	94 .95	80
181	95 .97	90
182	95 .96	90
183	98 .99	80
184	99 .101	90
185	99 .102	80
186	99 .100	60
187	102.103	50
188	103.107	90
189	103.104	150
190	104.107	120
191	104.105	130
192	104.106	50

```
        105.106    80
        106.108   100
        107.109    90
        101.110   100
        91 .111   100
        97 .112    80 /


             EFFORT(I,J,L)
/ 1  .20 .AIR   12,  1  .20 .NGS   10,  1  .20 .SF   15
  2  .20 .AIR   14,  2  .20 .NGS    8,  2  .20 .SF   15
  3  .19 .AIR   14,  3  .19 .NGS   10,  3  .19 .SF   15
  4  .14 .AIR    6,  4  .14 .NGS   12,  4  .14 .SF   15
  5  .12 .AIR    4,  5  .12 .NGS    9,  5  .12 .SF   15
  6  .9  .AIR    2,  6  .9  .NGS   15,  6  .9  .SF   15
  7  .8  .AIR   12,  7  .8  .NGS    6,  7  .8  .SF   15
  8  .43 .AIR   10,  8  .43 .NGS    6,  8  .43 .SF   15
  8  .41 .AIR   14,  8  .41 .NGS   18,  8  .41 .SF   15
  8  .9  .AIR   12,  8  .9  .NGS   15,  8  .9  .SF   15
  9  .41 .AIR   16,  9  .41 .NGS   15,  9  .41 .SF   15
  9  .40 .AIR   18,  9  .40 .NGS   15,  9  .40 .SF   15
  9  .10 .AIR   12,  9  .10 .NGS    9,  9  .10 .SF   15
  10 .11 .AIR   16,  10 .11 .NGS   10,  10 .11 .SF   15
  11 .37 .AIR   14,  11 .37 .NGS   12,  11 .37 .SF   15
  11 .12 .AIR   16,  11 .12 .NGS   28,  11 .12 .SF   15
  12 .36 .AIR    4,  12 .36 .NGS   29,  12 .36 .SF    2
  12 .13 .AIR    6,  12 .13 .NGS   24,  12 .13 .SF    4
  13 .33 .AIR    4,  13 .33 .NGS   28,  13 .33 .SF    4
  13 .31 .AIR    8,  13 .31 .NGS   22,  13 .31 .SF    5
  13 .14 .AIR    4,  13 .14 .NGS   23,  13 .14 .SF   15
  14 .15 .AIR    4,  14 .15 .NGS   25,  14 .15 .SF   15
  15 .31 .AIR    4,  15 .31 .NGS   28,  15 .31 .SF   15
  15 .30 .AIR    2,  15 .30 .NGS   28,  15 .30 .SF   15
  15 .28 .AIR    4,  15 .28 .NGS    6,  15 .28 .SF   15
  15 .16 .AIR    6,  15 .16 .NGS    9,  15 .16 .SF    2
  16 .23 .AIR    4,  16 .23 .NGS   14,  16 .23 .SF    5
```

08/04/91 10:31:48 PAGE 5 GAMS

2.19 IBM CMS

```
  16 .17 .AIR    6,  16 .17 .NGS   15,  16 .17 .SF    4
```

230	17 .23 .AIR	2,	17 .23 .NGS	18,	17 .23 .SF	4
231	17 .22 .AIR	6,	17 .22 .NGS	18,	17 .22 .SF	4
232	17 .18 .AIR	16,	17 .18 .NGS	18,	17 .18 .SF	4
233	18 .19 .AIR	10,	18 .19 .NGS	14,	18 .19 .SF	4
234	19 .22 .AIR	10,	19 .22 .NGS	4,	19 .22 .SF	2
235	19 .20 .AIR	18,	19 .20 .NGS	8,	19 .20 .SF	2
236	20 .21 .AIR	14,	20 .21 .NGS	18,	20 .21 .SF	4
237	21 .22 .AIR	12,	21 .22 .NGS	8,	21 .22 .SF	2
238	21 .24 .AIR	14,	21 .24 .NGS	8,	21 .24 .SF	2
239	22 .23 .AIR	12,	22 .23 .NGS	8,	22 .23 .SF	2
240	23 .28 .AIR	6,	23 .28 .NGS	8,	23 .28 .SF	2
241	23 .26 .AIR	4,	23 .26 .NGS	8,	23 .26 .SF	2
242	23 .24 .AIR	6,	23 .24 .NGS	12,	23 .24 .SF	4
243	24 .25 .AIR	6,	24 .25 .NGS	8,	24 .25 .SF	2
244	25 .26 .AIR	6,	25 .26 .NGS	8,	25 .26 .SF	2
245	25 .27 .AIR	6.	25 .27 .NGS	18,	25 .27 .SF	5
246	25 .68 .AIR	6,	25 .68 .NGS	4,	25 .68 .SF	2
247	26 .27 .AIR	6,	26 .27 .NGS	4,	26 .27 .SF	2
248	27 .28 .AIR	6,	27 .28 .NGS	8,	27 .28 .SF	2
249	27 .62 .AIR	6,	27 .62 .NGS	6,	27 .62 .SF	2
250	27 .66 .AIR	4,	27 .66 .NGS	10,	27 .66 .SF	6
251	28 .29 .AIR	4,	28 .29 .NGS	5,	28 .29 .SF	15
252	29 .61 .AIR	4,	29 .61 .NGS	27,	29 .61 .SF	15
253	29 .30 .AIR	6,	29 .30 .NGS	25,	29 .30 .SF	15
254	30 .61 .AIR	4,	30 .61 .NGS	26,	30 .61 .SF	15
255	30 .60 .AIR	8,	30 .60 .NGS	28,	30 .60 .SF	15
256	30 .31 .AIR	4,	30 .31 .NGS	22,	30 .31 .SF	15
257	31 .32 .AIR	4,	31 .32 .NGS	24,	31 .32 .SF	15
258	32 .33 .AIR	4,	32 .33 .NGS	26,	32 .33 .SF	15
259	33 .60 .AIR	4,	33 .60 .NGS	24,	33 .60 .SF	15
260	33 .34 .AIR	4,	33 .34 .NGS	29,	33 .34 .SF	6
261	34 .38 .AIR	8,	34 .38 .NGS	23,	34 .38 .SF	2
262	34 .35 .AIR	4,	34 .35 .NGS	28,	34 .35 .SF	8
263	35 .36 .AIR	2,	35 .36 .NGS	20,	35 .36 .SF	8
264	35 .37 .AIR	4,	35 .37 .NGS	26,	35 .37 .SF	2
265	38 .60 .AIR	6,	38 .60 .NGS	27,	38 .60 .SF	2
266	38 .57 .AIR	4,	38 .57 .NGS	28,	38 .57 .SF	2
267	38 .39 .AIR	2,	38 .39 .NGS	4,	38 .39 .SF	2
268	39 .45 .AIR	2,	39 .45 .NGS	9,	39 .45 .SF	4
269	39 .40 .AIR	2,	39 .40 .NGS	10,	39 .40 .SF	6
270	41 .42 .AIR	2,	41 .42 .NGS	6,	41 .42 .SF	2
271	42 .44 .AIR	2,	42 .44 .NGS	10,	42 .44 .SF	5
272	42 .43 .AIR	2,	42 .43 .NGS	6,	42 .43 .SF	2

273	43 .47 .AIR	2,	43 .47 .NGS	4,	43 .47 .SF	2
274	44 .45 .AIR	12,	44 .45 .NGS	13,	44 .45 .SF	15
275	44 .46 .AIR	10,	44 .46 .NGS	15,	44 .46 .SF	15
276	46 .47 .AIR	16,	46 .47 .NGS	16,	46 .47 .SF	15
277	47 .48 .AIR	10,	47 .48 .NGS	14,	47 .48 .SF	15
278	48 .49 .AIR	14,	48 .49 .NGS	15,	48 .49 .SF	15
279	49 .50 .AIR	16,	49 .50 .NGS	4,	49 .50 .SF	15
280	49 .51 .AIR	18,	49 .51 .NGS	20,	49 .51 .SF	15
281	51 .94 .AIR	18,	51 .94 .NGS	8,	51 .94 .SF	2
282	50 .52 .AIR	18,	50 .52 .NGS	4,	50 .52 .SF	2
283	52 .94 .AIR	12,	52 .94 .NGS	10,	52 .94 .SF	4
284	52 .53 .AIR	12,	52 .53 .NGS	25,	52 .53 .SF	2

285	53 .54 .AIR	12,	53 .54 .NGS	26,	53 .54 .SF	2
286	54 .87 .AIR	2,	54 .87 .NGS	24,	54 .87 .SF	2
287	54 .83 .AIR	2,	54 .83 .NGS	28,	54 .83 .SF	4
288	54 .55 .AIR	2,	54 .55 .NGS	24,	54 .55 .SF	2
289	55 .81 .AIR	4,	55 .81 .NGS	25,	55 .81 .SF	8
290	55 .82 .AIR	4,	55 .82 .NGS	25,	55 .82 .SF	6
291	55 .56 .AIR	2,	55 .56 .NGS	24,	55 .56 .SF	2
292	56 .82 .AIR	4,	56 .82 .NGS	27,	56 .82 .SF	2
293	56 .73 .AIR	2,	56 .73 .NGS	20,	56 .73 .SF	4
294	56 .57 .AIR	6,	56 .57 .NGS	22,	56 .57 .SF	5
295	57 .72 .AIR	6,	57 .72 .NGS	26,	57 .72 .SF	2
296	57 .59 .AIR	6,	57 .59 .NGS	24,	57 .59 .SF	2
297	57 .58 .AIR	6,	57 .58 .NGS	26,	57 .58 .SF	2
298	58 .59 .AIR	4,	58 .59 .NGS	26,	58 .59 .SF	2
299	59 .61 .AIR	2,	59 .61 .NGS	20,	59 .61 .SF	15
300	59 .62 .AIR	6,	59 .62 .NGS	28,	59 .62 .SF	15
301	60 .61 .AIR	4,	60 .61 .NGS	24,	60 .61 .SF	15
302	62 .63 .AIR	18,	62 .63 .NGS	8,	62 .63 .SF	5
303	62 .64 .AIR	14,	62 .64 .NGS	10,	62 .64 .SF	5
304	63 .71 .AIR	14,	63 .71 .NGS	6,	63 .71 .SF	5
305	63 .64 .AIR	14,	63 .64 .NGS	5,	63 .64 .SF	2
306	64 .70 .AIR	10,	64 .70 .NGS	8,	64 .70 .SF	5
307	64 .65 .AIR	10,	64 .65 .NGS	8,	64 .65 .SF	5
308	65 .70 .AIR	10,	65 .70 .NGS	6,	65 .70 .SF	2
309	65 .67 .AIR	14,	65 .67 .NGS	4,	65 .67 .SF	2

310	65 .66 .AIR	18,	65 .66 .NGS	4,	65 .66 .SF	2
311	66 .67 .AIR	14,	66 .67 .NGS	10,	66 .67 .SF	8
312	67 .68 .AIR	14,	67 .68 .NGS	10,	67 .68 .SF	15
313	67 .69 .AIR	14,	67 .69 .NGS	6,	67 .69 .SF	15
314	69 .76 .AIR	18,	69 .76 .NGS	8,	69 .76 .SF	15
315	69 .70 .AIR	16,	69 .70 .NGS	5,	69 .70 .SF	15
316	70 .71 .AIR	14,	70 .71 .NGS	8,	70 .71 .SF	15
317	71 .76 .AIR	18,	71 .76 .NGS	8,	71 .76 .SF	15
318	71 .72 .AIR	16,	71 .72 .NGS	8,	71 .72 .SF	15
319	72 .75 .AIR	14,	72 .75 .NGS	4,	72 .75 .SF	15
320	72 .73 .AIR	18,	72 .73 .NGS	18,	72 .73 .SF	15
321	73 .74 .AIR	14,	73 .74 .NGS	18,	73 .74 .SF	15
322	74 .79 .AIR	16,	74 .79 .NGS	18,	74 .79 .SF	15
323	74 .76 .AIR	16,	74 .76 .NGS	8,	74 .76 .SF	15
324	74 .75 .AIR	16,	74 .75 .NGS	8,	74 .75 .SF	15
325	75 .76 .AIR	14,	75 .76 .NGS	4,	75 .76 .SF	15
326	76 .78 .AIR	14,	76 .78 .NGS	5,	76 .78 .SF	15
327	76 .77 .AIR	14,	76 .77 .NGS	4,	76 .77 .SF	15
328	77 .78 .AIR	12,	77 .78 .NGS	8,	77 .78 .SF	15
329	77 .80 .AIR	12,	77 .80 .NGS	6,	77 .80 .SF	15
330	78 .79 .AIR	12,	78 .79 .NGS	5,	78 .79 .SF	15
331	79 .81 .AIR	14,	79 .81 .NGS	18,	79 .81 .SF	15
332	79 .80 .AIR	12,	79 .80 .NGS	10,	79 .80 .SF	5
333	80 .81 .AIR	14,	80 .81 .NGS	4,	80 .81 .SF	2
334	80 .83 .AIR	10,	80 .83 .NGS	4,	80 .83 .SF	3
335	80 .84 .AIR	12,	80 .84 .NGS	8,	80 .84 .SF	2
336	81 .83 .AIR	2,	81 .83 .NGS	8,	81 .83 .SF	2
337	81 .82 .AIR	2,	81 .82 .NGS	8,	81 .82 .SF	5
338	83 .86 .AIR	2,	83 .86 .NGS	4,	83 .86 .SF	2
339	83 .84 .AIR	2,	83 .84 .NGS	10,	83 .84 .SF	6
340	84 .85 .AIR	2,	84 .85 .NGS	8,	84 .85 .SF	2

341	85 .106.AIR	4,	85 .106.NGS	8,	85 .106.SF	15
342	85 .86 .AIR	4,	85 .86 .NGS	8,	85 .86 .SF	15
343	86 .89 .AIR	8,	86 .89 .NGS	4,	86 .89 .SF	15
344	86 .87 .AIR	8,	86 .87 .NGS	20,	86 .87 .SF	15
345	87 .88 .AIR	6,	87 .88 .NGS	24,	87 .88 .SF	15
346	88 .89 .AIR	6,	88 .89 .NGS	4,	88 .89 .SF	15
347	89 .103.AIR	4,	89 .103.NGS	4,	89 .103.SF	15

```
89 .100.AIR    2,  89 .100.NGS   13,  89 .100.SF   15
89 .90 .AIR    6,  89 .90 .NGS    4,  89 .90 .SF   15
90 .91 .AIR    4,  90 .91 .NGS    5,  90 .91 .SF   15
91 .99 .AIR   18,  91 .99 .NGS   11,  91 .99 .SF   15
91 .98 .AIR   16,  91 .98 .NGS    5,  91 .98 .SF   15
91 .97 .AIR   14,  91 .97 .NGS    4,  91 .97 .SF   15
91 .96 .AIR   14,  91 .96 .NGS   12,  91 .96 .SF   15
91 .92 .AIR   10,  91 .92 .NGS   12,  91 .92 .SF   15
92 .93 .AIR   10,  92 .93 .NGS   12,  92 .93 .SF   15
93 .96 .AIR   14,  93 .96 .NGS    4,  93 .96 .SF   15
93 .94 .AIR   12,  93 .94 .NGS    4,  93 .94 .SF   15
94 .95 .AIR   12,  94 .95 .NGS    8,  94 .95 .SF   15
95 .97 .AIR   12,  95 .97 .NGS   10,  95 .97 .SF   15
95 .96 .AIR   12,  95 .96 .NGS   10,  95 .96 .SF   15
98 .99 .AIR   14,  98 .99 .NGS    8,  98 .99 .SF   15
99 .101.AIR   12,  99 .101.NGS   10,  99 .101.SF   15
99 .102.AIR   12,  99 .102.NGS    8,  99 .102.SF   15
99 .100.AIR   12,  99 .100.NGS    6,  99 .100.SF   15
102.103.AIR   12,  102.103.NGS    6,  102.103.SF   15
103.107.AIR   12,  103.107.NGS   10,  103.107.SF   15
103.104.AIR   10,  103.104.NGS   14,  103.104.SF   15
104.107.AIR   10,  104.107.NGS   12,  104.107.SF   15
104.105.AIR   16,  104.105.NGS   12,  104.105.SF   16
104.106.AIR   10,  104.106.NGS    8,  104.106.SF   14
105.106.AIR   10,  105.106.NGS    8,  105.106.SF   14
106.108.AIR   14,  106.108.NGS   10,  106.108.SF   15
107.109.AIR   14,  107.109.NGS    8,  107.109.SF   14
101.110.AIR   12,  101.110.NGS   10,  101.110.SF   16
91 .111.AIR   12,  91 .111.NGS   10,  91 .111.SF   16
97 .112.AIR   12,  97 .112.NGS    8,  97 .112.SF   14/

          ASSET (L)
            / AIR  4
              NGS  4
              SF   4 / ;

POSITIVE VARIABLE
          H(I,J);

VARIABLE
          A(I)
          MAXCAP;

```

```
BINARY VARIABLES
          G(I,J,L);

          A.FX('1') = 0;
          A.FX('2') = 0;
          A.FX('3') = 0;
```

08/04/91 10:31:48 PAGE 8 GAMS

2.19 IBM CMS

```
          A.FX('4') = 0;
          A.FX('5') = 0;
          A.FX('6') = 0;
          A.FX('7') = 0;
          A.FX('108') = 1;
          A.FX('109') = 1;
          A.FX('110') = 1;
          A.FX('111') = 1;
          A.FX('112') = 1;

EQUATIONS   OBJ          define objective function
            ARC1(I,J)    equation for forward arc
            ARC2(I,J)    equation for backward arc
            WPN(L)       weapon expenditure for each arc;

* > > > minimize < < <
OBJ..
     MAXCAP =E= SUM ( (I,J) $ ( CAP(I,J) GT 0 ) , CAP(I,J) *
H(I,J));

* > > > subject to < < <


ARC1(I,J) $ ( CAP(I,J) GT 0)..
        A(I) - A(J) + H(I,J) + SUM (L, G(I,J,L))  =G= 0;


ARC2(I,J) $ ( CAP(I,J) GT 0)..
        A(J) - A(I) + H(I,J) + SUM (L, G(I,J,L))  =G= 0;

WPN(L)..
```

```
428 SUM( (I,J) $ ( CAP(I,J) GT 0 ),EFFORT(I,J,L) * G(I,J,L)) =L=
429 ASSET(L) ;
430
431 MODEL NETINT  /ALL/;
432 SOLVE NETINT USING MIP MINIMIZING MAXCAP;
433 DISPLAY MAXCAP.L;
434 DISPLAY H.L;
435 OPTION G:0:2:1; DISPLAY G.L;


COMPILATION TIME     =       0.380 SECONDS

08/04/91 10:31:48  PAGE     9
MODEL STATISTICS    SOLVE NETINT USING MIP FROM LINE 432
                    GAMS 2.19 IBM CMS



MODEL STATISTICS

BLOCKS OF EQUATIONS     4    SINGLE EQUATIONS    356
BLOCKS OF VARIABLES     4    SINGLE VARIABLES    817
NON ZERO ELEMENTS    2817    DISCRETE VARIABLES  528

GENERATION TIME      =       1.550 SECONDS


EXECUTION TIME       =       1.630 SECONDS
1
08/04/91 10:31:56  PAGE    10
SOLUTION REPORT     SOLVE NETINT USING MIP FROM LINE 432
                    GAMS 2.19 IBM CMS


          S O L V E   S U M A R Y

     MODEL   NETINT            OBJECTIVE  MAXCAP
     TYPE    MIP               DIRECTION  MINIMIZE
     SOLVER  ZOOM              FROM LINE  432

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      1 OPTIMAL
```

```
**** OBJECTIVE VALUE              20.0000

RESOURCE USAGE, LIMIT          1.162    10000.000
ITERATION COUNT, LIMIT       303        90000

Z O M / X M P   ---   Version 2.1 Jun 1988

Courtesy of Dr Roy E. Marsten,
        Department of Management Information Systems,
        University of Arizona,
        Tucson    Arizona  85721,  U.S.A.


PROBLEM SPECIFICATIONS
---------------------
BEGIN
*
* SPECS FILE, VERSION 2.1  JUN 1987
*
* AMOUNT OF PRINTOUT IN STATUS FILE
*
PRINT CONTINUOUS 0
PRINT BRANCH     0
PRINT HEURISTIC  0
PRINT TOUR       0
*
* PARAMETERS CONTROLLING LP
*
*
* PARAMETERS CONTROLLING THE HEURISTIC
*
HEURISTIC YES
*HEURISTIC NO
*
* PARAMETERS CONTROLLING THE BRANCH AND BOUND.
*
BRANCH    YES
QUIT      NO
DIVE      YES
INCUMBANT=300.0
INVERT =  50
*
MAX SAVE  5
```

EXPAND 3
SELECT 3
*

08/04/91

10:31:56 PAGE 11
SOLUTION REPORT SOLVE NETINT USING MIP FROM LINE 432
GAMS 2.19 IBM CMS

END

Work space needed (estimate) -- 55974 words.
Work space available -- 55974 words.
Maximum obtainable -- 295165 words.

The LU factors occupied 1319 slots (estimate 7388).

```
Iterations: Initial LP        303, Time:   0.83
            Heuristic           0,         0.00
            Branch and bound    0,         0.00
            Final LP            0,         0.00
```

```
**** REPORT SUMMARY :     0    NONOPT
                          0 INFEASIBLE
                          0 UNBOUNDED
```

08/04/91

10:31:56 PAGE 12
E X E C U T I N G
GAMS 2.19 IBM CMS

```
----   433 VARIABLE  MAXCAP.L        =        20.00


----   434 VARIABLE  H.L

            52

50        1.00


----   435 VARIABLE  G.L
```

	AIR	NGS	SF
51 .94			1
53 .54			1
54 .87	1		
83 .86		1	
84 .85	1		

**** FILE SUMMARY FOR USER 8847P

INPUT SC2 GAMS A
OUTPUT SC2 LISTING A

EXECUTION TIME = 0.270 SECONDS

REFERENCES

1. Bradley, G., Brown, G., and Graves, G., "Implementation of Large-Scale Primal Transshipment Algorithms," *Management Science*, **24**, pp. 1-34,1977.

2. Capps, L.R., "A Dynamic Model for the Allocation of Airstrikes Against a Lines-of-Communication Network", M.S. Thesis, Naval Postgraduate School, Monterey, California, Sep 1970.

3. Danskin, J.M., "A Game Theory Model of Convoy Routing", *Operations Research*, Volume 10, pp. 774-785, Nov-Dec 1962.

4. Ghare, P.M., Montgomery, D.C., and Turner, W.C., "Optimal Interdiction Policy for a Flow Network", *Naval Research Logistics Quarterly*, 18, No. 4, December 1971.

5. Helmbold, R.L., "A Countercapaciity Network Interdiction Model", The Rand Corporation, R-611-PR, March 1971.

6. Joint Technical Co-ordinating Group For Munitions Effectiveness (JTCG/ME), *"Joint Munitions Effectiveness Manuals"*, Aberdeen Proving Ground, Maryland 21005-5071, May 1991.

7. Lawler, E.L., *Combinatorial Optimization, Networks and Matroids*, Holt, Rinehart and Winston, New York, 1976.

8. Mustin, T. M. , "Optimum Allocation of Air Strikes for Interdiction of a Transportation Network", M.S. Thesis, Naval Postgraduate School, Monterey, California, June 1967.

9. Nugent, R.O., "Optimal Allocation of Air Strikes against a Transportation Network for an Exponential Damage Function", M.S. Thesis, Naval Postgraduate School, Monterey, California, June 1969.

10. Preston, C.P., "Interdiction of a Transportation Network", M.S. Thesis, Naval Postgraduate School, Monterey, California, March 1972.

11. Sakarovitch, M., "The Multicommodity Maximal Flow Problems," ORC 66-25, University of California, Berkeley, 1966.

12. Wollmer, R.D., "Algorithms for Targeting strikes in a Lines-of-Communication Network", *Operations Research*, Vol 18, pp 497-515, May-June 1970.

13. Wollmer, R.D., "An Interdiction Model for Sparsely Travelled Networks", The Rand Corporation, RM-5539-PR, April 1968.

14. Wood, R.K. "Deterministic Network Interdiction", working paper, Naval Postgraduate School, Monterey, California, July 1991.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22304-6145	2
2.	Library, Code 52 Naval Postgraduate School Monterey, California 93942-5002	2
3.	Dr. R. Kevin Wood, Code OR/Wd Department of Operations Research Naval Postgraduate School Monterey, California 93943	1
4.	Dr. Gerald Brown, Code OR/Bn Department of Operations Research Naval Postgraduate School Monterey, California 93943	1
5.	LTC Kok-Hua Loh 2 Merryn Road Singapore 1129 Republic of Singapore	2