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BINOMIAL ANALYSIS OF RECOVERY AND MAINTENANCE SIMULATION RESULTS - AIRLAND BATTLE-FUTURE

FINAL REPORT

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#19 The Ordnance Center and School uses a simulation model to analyze recovery and maintenance operations on the future battlefield. It runs on a personal computer and is programmed with commericial SLAMSYSTEM software. It simulates an eight hour armored brigade battle in a European scenario. The model is used to evaluate the probable impact of improved recovery vehicles and maintenance vehicles on average repair time needed, recovery time required, and other parameters of interest. It is useful for answering typical "what if" questions.

After completing a run, the model provides data such as the number of tanks available at the end of the battle and at the end of the day. This is count data. The observed counts fall into just two categories, "operational" or "not operational". When this occurs, the data are called BINOMIAL data. The investigator's interest is in proportions - the percentage or number of events in one of the two classes. Statistical methods are needed to establish confidence limits on the proportions observed, and to demonstrate significant differences.

Mathematically exact methods for analyzing binomial data exist. However, the necessary computations are extremely demanding and time consuming. The use of published binomial tables presents practical difficulties that may lead to inaccuracies in the final results. Existing "short cut" approximation tests are frequently used. These tests often give good results, but occasionally they yield bad results. The user needs to exercise care in order to detect the occurrence of unacceptable results.

Our approach to binomial data analysis is the topic of this report.

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SIMULATION RESULTS - AIRLAND BATTLE-FUTURE

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BINOMIAL ANALYSIS OF RECOVERY AND MAINTENANCE SIMULATION RESULTS - AIRLAND BATTLE-FUTURE

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ABSTRACT

THE ORDNANCE CENTER AND SCHOOL USES A SIMULATION MODEL TO ANALYZE RECOVERY AND MAINTENANCE OPERATIONS ON THE FUTURE BATTLEFIELD. IT RUNS ON A PERSONAL COMPUTER AND IS PROGRAMMED WITH COMMERCIAL SLAMSYSTEM SOFTWARE. IT SIMULATES AN EIGHT HOUR ARMORED BRIGADE BATTLE IN A EUROPEAN SCENARIO. THE MODEL IS USED TO EVALUATE THE PROBABLE IMPACT OF IMPROVED RECOVERY VEHICLES AND MAINTENANCE VEHICLES ON AVERAGE REPAIR TIME NEEDED, RECOVERY TIME REQUIRED, AND OTHER PARAMETERS OF INTEREST. IT IS USEFUL FOR ANSWERING TYPICAL "WHAT IF" QUESTIONS.

AFTER COMPLETING A RUN, THE MODEL PROVIDES DATA SUCH AS THE NUMBER OF TANKS AVAILABLE AT THE END OF THE BATTLE AND AT THE END OF THE DAY. THIS IS COUNT DATA. THE OBSERVED COUNTS FALL INTO JUST TWO CATEGORIES, "OPERATIONAL" OR "NOT OPERATIONAL". WHEN THIS OCCURS, THE DATA ARE CALLED BINOMIAL DATA. THE INVESTIGATOR'S INTEREST IS IN PROPORTIONS - THE PERCENTAGE OR NUMBER OF EVENTS IN ONE OF THE TWO CLASSES. STATISTICAL METHODS ARE NEEDED TO ESTABLISH CONFIDENCE LIMITS ON THE PROPORTIONS OBSERVED, AND TO DEMONSTRATE SIGNIFICANT DIFFERENCES.

MATHEMATICALLY EXACT METHODS FOR ANALYZING BINOMIAL DATA EXIST. HOWEVER, THE NECESSARY COMPUTATIONS ARE EXTREMELY DEMANDING AND TIME CONSUMING. THE USE OF PUBLISHED BINOMIAL TABLES PRESENTS PRACTICAL DIFFICULTIES THAT MAY LEAD TO INACCURACIES IN THE FINAL RESULTS. EXISTING "SHORT CUT" APPROXIMATION TESTS ARE FREQUENTLY USED. THESE TESTS OFTEN GIVE GOOD RESULTS, BUT OCCASIONALLY THEY YIELD BAD RESULTS. THE USER NEEDS TO EXERCISE CARE IN ORDER TO DETECT THE OCCURRENCE OF UNACCEPTABLE RESULTS.

OUR APPROACH TO BINOMIAL DATA ANALYSIS IS THE TOPIC OF THIS REPORT.

BINOMIAL ANALYSIS OF RECOVERY AND MAINTENANCE
SIMULATION RESULTS - AIRLAND BATTLE-FUTURE

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INTRODUCTION

The Ordnance Center and School uses a simulation model to analyze recovery and maintenance operations on the future battlefield. It runs on a personal computer and is programmed with commercial SLAMSYSTEM software. It simulates an eight hour armored brigade battle in a European scenario.

One use of the model is to evaluate the probable impact of improved recovery vehicles and maintenance vehicles. We can make different assumptions about the average repair time needed, recovery time required, and other key parameters. After completing a run, the model provides data such as tank availability at the end of battle and at the end of the day, the number of tanks evacuated, and the number not recovered. We use the model to answer typical "what if" questions.

In order to generalize from this brigade size model, the battle simulation is repeated with different random number seeds to produce different results that might occur due to the laws of chance. Statistical methods are used to establish confidence limits and demonstrate significant differences.

THE STATISTICAL PROBLEMS

We needed typical statistical capabilities for analyzing quantitative data. Modest PC capabilities were available, and were subsequently enhanced through the acquisition of commercial software. We also needed to analyze a significant volume of count data. Our observed counts usually fall into just two categories, e.g., "operational" or "not operational", "hit" or "miss", "yes" or "no", etc. Our interest was in proportions - the percentage or number of events in one of the two classes. For example, a simulation run might show that 119 of the original 164 tanks are still operational after the battle. This is binomial data. Our approach to binomial data analysis is the topic of this paper.

THE APPROACH

Several options were available. We could treat our count data as if it were measurement data, and use statistical methods commonly applied to continuous data. We could use some of the "short cut" approximation tests that are available for count data. We could use binomial methods and rely on published binomial tables. We could obtain commercial software designed to handle binomial data. All of these options have shortcomings that will be discussed.

The option that we selected was to create our own binomial software for the PC. This software is fully operational. It consists of two programs written in FORTRAN, a language well equipped to handle the laborious calculations. The first program creates a table of the cumulative binomial when the user specifies a probability and any number of trials. The second program reads binomial sample data entered by the user. It computes the proportion observed, and several sets of confidence limits for this proportion. It enables the user to test for statistically significant differences at various significance levels. This software is available at no cost to the Department of Defense community.

Further discussion will include some background on the binomial distribution, the capabilities of the programs, some sample problems, hardware and software requirements, and a few interesting aspects of the binomial that may not be obvious.

SOME BACKGROUND ON THE BINOMIAL

The binomial distribution provides a method for handling count data for populations where the observations fall into just two categories. The investigator's interest is in proportions - the percentage or number of events in one of the two classes. Some of the statistical literature refers to this type of data as "quantal" data, or, "dichotomous" data, or "all-or-none" data. A two-class population has a very simple structure. It can be described by giving the proportion of the members of the population that fall in one class. In a random sample of size N, the probability of getting exactly 0, 1, 2, 3, ..., N successes can be worked out.

By definition, the binomial distribution is the probability distribution of the possible number of times that a particular event will occur in a sequence of trials. If the event has a given probability of occurring during any one trial, the binomial distribution states the probability of the event occurring any certain exact number of times in a sequence of trials. This is stated mathematically as, "The probability of the event occurring R times in N trials, or P(R). The program named BNTABLES generates tables that supply this information.

PROGRAM <u>BNTABLES</u>

We will introduce this program with a sample problem. If you flip an honest coin six times:

1. How many times would you expect to observe heads as the outcome? (Obviously the answer is three.)

2. What is the probability of observing heads <u>exactly</u> three times?

3. What is the probability of observing heads exactly one time?

4. What is the probability of observing heads exactly five times?

5. What is the probability of observing heads more than four times?

The user enters a probability, P. (This may be known, or assumed, depending upon how the the specific problem is structured.) He also enters the number of trials, N, (the sample size). Output can be directed to either the screen or the printer. In our example, the user would enter P = 0.5 and N = 6, and would obtain the following output:

CUMULATIVE BINOMIAL TABLE

P = .50000 N = 6

R	P(R)	SIGMA P(R) P(R OR LESS)	1 - SIGMA P(MORE THAN R)
0	.015625000	.015625000	.984375000
1	.093750000	.109375000	.890625000
2	.234375000	.343750000	.656250000
3	.312500000	.656250000	.343750000
4	.234375000	.890625000	.109375000
5	.093750000	.984375000	.015625000
6	.015625000	1.000000000	.000000000

The first column of the table contains all possible values for R (the number of heads that could occur). The second column contains P(R), the probability that the event (heads) will occur exactly R times in the N trials (the total number of coin flips).

What is usually of more interest and more practical value is the probability of an event occurring less than or more than some specified number of times. Accordingly, the third column contains the CUMULATIVE probability of the event occurring R times or less. The column heading is SIGMA P(R), since any number in the column is the sum of all of the P(R) values up to and including that row. The fourth column contains the probability of the event occurring more than R times. Since this is the complement of the third column, the heading is 1 - SIGMA.

Look at the second column of the table. It contains the probabilities of observing any exact number of heads in 6 trials. The largest probability appears in the row where R is 3 (three heads). It is 0.3125. You have answered questions #1 and #2. Look at the row where R is 1. The probability of observing heads exactly one time is 0.09375. You have answered question #3. Look at the row where R is 5. The probability of observing heads exactly five times is also 0.09375. You have answered question #4. Look at the row where R is 4. The probability of observing more than four heads appears in the right column. It is 0.109375. This completes the sample problem.

Take another look at the probabilities in column P(R) of the table. Note that if you were to plot the binomial distribution define by these probabilities, the resulting curve would be symmetric (Chart #1). It is interesting to note that the binomial distribution is symmetric only when P = 0.5. As P moves away from 0.5, the binomial becomes more and more skewed toward the center of the plot. However, if P is held constant at any value (even a value far removed from 0.5), the binomial becomes more symmetric as N increases. These fact will subsequently be examined in more detail, using Charts #2, #3, and #4.

ANOTHER EXAMPLE

Let's look at another small problem that is more practical. Assume that the Army has a contract with a manufacturer of spare parts for wheeled vehicles. Contract terms for one specific part state that the Army will accept the occurrence of random defects with probability P = 0.002 (an average of two defect per 1000 parts). Assume that the manufacturer is able to conform to these terms. An army unit orders 500 parts. You have been asked to determine several probabilities associated with the shipment.

What is the probability of no defects in the order? The probability of exactly one defect? The probability of two defects or less? The probability of four or more defects?

You would again run program BNTABLES, specifying P = .002 and N = 500, and would obtain the following output:

CUMULATIVE BINOMIAL TABLE

P = .00200 N = 500

R	P(R)	SIGMA P(R) P(R OR LESS)	1 - SIGMA P(MORE THAN R)
0	.367511255	.367511255	.632488745
1	.368247750	.735759005	.264240995
2	.184123875	.919882880	.080117120
3	.061251630	.981134510	.018865490
4	.015251533	.996386043	.003613957
5	.003031968	.999418011	.000581989
6	.000501277	.999919289	.000080711
7	.000070893	.999990182	.000009818
8	.000008755	.999998937	.000001063
9	.000000959	.999999896	.000000104

Look over the table. The first column, with heading R, contains values for possible numbers of defects in the sample of 500. The second column contains the probability of observing exactly R defects in 500 trials. The third column contains the cumulative probability of finding R or fewer defects.

The fourth column contains the cumulative probability of finding more than R defects. All of your answers can be read directly from the table:

The probability of zero defects = 0.3675 (1st figure in column 2).

The probability of one defect = 0.3682 (2nd figure in column 2).

The probability of two or less = 0.9199 (3rd figure in column 3).

Probability of four or more = 0.0189 (4th figure in column 4). This is the same as the probability of "more than three", which can be read directly from the table.

PROGRAM BNCONFLM

Suppose that instead of dealing with a binomial probability that is known or assumed, you have no information except for a random sample from a binomial population. You have no idea what the population probability (or proportion) might be. You can easily and quickly estimate this probability from the sample by computing the sample proportion:

P = Number of Successes / Total Number of Trials.

You would also like to obtain confidence limits for this proportion to get an idea of how good your estimate might be. An analogue to this would be computing confidence limits for a sample average obtained from a population with continuous data, when that population is known or assumed to be normally distributed. These computations are relatively easy. However, no simple method for direct computation of binomial confidence limits is available.

You might also wish to go a step farther and run a significance test, to infer whether the true proportion of your population is different from that of another population, or different from some "specified value" or "theoretical standard". If you have confidence limits available, you can easily determine statistical significance by using the method of overlapping confidence limits [Snedecor and Cochran, 1980, p. 66].

Program BNCONFLM provides binomial confidence limits quickly and conveniently. It reads data observed from a binomial sample and computes the proportion (or fraction) of the observations that are in the category of primary interest. It then computes several sets of confidence limits and confidence intervals (confidence ranges) for this proportion. These are computed for the 99%, 98%, 95%, 90%, and 80% confidence levels. With this information, the user can easily run significance tests at the various levels most commonly used in statistical work.

We will also demonstrate program BNCONFLM with a sample problem. In the responses to a questionnaire received from 250 soldiers at Fort Duncan we find that 33 soldiers (13.2%) show a negative attitude toward the quality of health care at the post. What is the maximum percent of the total soldier population at the post that we should expect to show this negative attitude assuming we have a representative sample? Use the 99% confidence level.

The user runs program BNCONFLM. He enters 33 as his count of observations in the category of primary interest, and 250 as the sample size (number of trials). He obtains the following output:

BINOMIAL CONFIDENCE LIMITS

COUNT IN CATEGORY OF INTEREST = 33 SAMPLE SIZE = 250

FRACTION OBSERVED = .13200

99% LIMITS =	.08233	AND	.19620	RANGE =	.11387
98% LIMITS =	.08641	AND	.18971	RANGE =	.10329
95% LIMITS =	.09263	AND	.18036	RANGE =	.08772
90% LIMITS =	.09820	AND	.17250	RANGE =	.07430
80% LIMITS =	.10487	AND	.16367	RANGE =	.05881

Note that the fraction observed is 0.132, and the 99% confidence limits are 0.082 and 0.196. We conclude that the best estimate of the negative attitude that we can make for the entire population (with the available data) is 13.2%, the fraction observed from the sample. We also conclude that we can be 99% sure that the true population attitude is contained within the limits of 8.2% and 19.6%. Therefore, the maximum negative response we should expect is 19.6%.

THE QUESTION OF SAMPLE SIZE

Binomial confidence limits can be used to obtain some insight about the effect of larger sample sizes on the confidence range for a proportion that is being estimated. Assume you have a sample of data where N = 10, and one of the 10 observations is a success. Running this through program BNCONFLM, you would obtain a confidence range of 0.44249 at the 95% confidence level. Assume that this confidence range is at least a partial indicator of the value obtained from your sample of 10 observations. You would assume that if you spend the time and money to obtain a larger sample, you could reduce the size of the confidence range, thereby improving your knowledge and enhancing the value of your information.

Assume further that you could obtain more sample observations and that the observed proportion would remain at 0.10. In the real world this would be most unlikely, but the purpose of this exercise is to demonstrate what would happen if nothing changed except the sample size. If you increase the sample size by adding observations in increments of 10, a summary of the results would appear as follows:

EFFECT OF SAMPLE SIZE CHANGES UPON 95% CONFIDENCE RANGES

PROPORTION OF SUCCESSES HELD CONSTANT AT 0.1

SAMPLE SIZE	CONFIDENCE RANGE	PERCENT REDUCTION FROM INITIAL RANGE	CHANGE FROM PREVIOUS PERCENT
10	0.44249	-	-
20	0.30463	31.2	31.2
30	0.24417	44.8	13.6
40	0.20871	52.8	8.0
50	0.18486	58.2	5.4
60	0.16747	62.2	3.4
70	0.15409	65.2	3.0
80	0.14339	67.6	2.4
90	0.13460	69.6	2.0

Note that when you add a second increment of 10 observations to your initial sample size, the confidence range is reduced by 31.2%. This appears to be a substantial reduction. The confidence limits (not shown) have moved closer together. You can now make a stronger statement about the population proportion. When the third increment of 10 observations is added, you obtain another 13.6% reduction - still very nice. As you continue to add more increments of 10, however, you will notice that you are buying less and less additional information each time. The same phenomenon exists with continuous data, however, the rate of change is not identical. At some point you will stop and ask yourself, "Is this next batch of data actually worth the additional time and money?". This is a practical question that often needs to be addressed.

MORE BACKGROUND ON THE BINOMIAL

We previously stated that the binomial distribution is symmetric only when P = 0.5. As P moves away from 0.5, the binomial becomes more and more skewed toward the center of the plot. A more general statement [James and James, 1976, p. 32] is, "When N is large, the binomial distribution can be approximated by a normal distribution with mean NP and variance NPQ (where Q = 1 - P). The binomial distribution can also be approximated by a Poisson distribution with mean NP if N is large".

Let's examine this statement by looking at a plot of the binomial where P = 0.01 and N = 210. We will then observe the impact of holding P constant while N is increased. The scales will be held constant so the changes to the curve will not be distorted. We will first double N, then increase it to five times its original value. When N = 210 the distribution is severely skewed to the right (Chart #2). When N = 420 the symmetry is much improved, although some skewness is still obvious (Chart #3). When N = 1050 the curve appears quite symmetric at first glance, but with careful observation some skewness can still be detected (Chart #4).

Because of facts such as those just discussed, statisticians have been able to derive several "shortcut" approximation methods for determining confidence limits and statistical significance that are widely used in analyzing binomial data. Frequently they give good results, but occasionally they give not-so-good results. The danger of obtaining bad results with approximation methods is greatest when N is small and when the probabilities approach extreme values (either zero or one). Statistical tests that are derived <u>directly</u> from the binomial distribution, however, are EXACT tests.

How can we make the preceding statement? The statistical literature seldom uses the term "EXACT test". However, the literature abounds with methodologies with names such as <u>Normal Approximation to the Binomial</u>, <u>Poisson Approximation to the Binomial</u>, and <u>Chi-square Approximation to the Binomial</u>. They imply that the binomial is a basic standard without stating it directly - that it is a fundamental mathematical truth so self-evident that the statement is unnecessary. To back up our EXACT test statement we offer the two following arguments:

1. The binomial distribution can be derived from the basic laws of mathematical probability [Snedecor, p. 107].

2. The binomial distribution exists in nature - in the real world - as surely as any law of physics. One need only identify a situation where observations fall into just two categories, and the probability of the event of interest occurring during any one trial is constant. This can be represented by a well constructed bead box containing beads of two colors, where the experimenter draws randomly with replacement. Or by an honest coin with honest flips. This may not sound impressive, until one thinks about the fact that many of the commonly used probability distributions <u>do not</u> exist in the real world. Their usefulness is in the fact that they give an excellent approximation of many things that do occur in the real world. And mathematicians have worked with them for many years and created convenient tables that will answer almost any conceivable question about them that might be asked.

The binomial distribution was first discovered by the Swiss mathematician Jacques Bernoulli (1654-1705). It was published in 1713, after his death. Some texts refer to the binomial as the Bernoulli distribution. Again, statistical tests that are derived directly from this distribution are EXACT tests.

WHY ISN'T THE BINOMIAL MORE WIDELY USED ?

One might ask the question, "If the binomial is so good, why isn't it more widely used? Why were so many short cut methods developed?" Unfortunately, the binomial requires a different table for each probability and sample size. Most general purpose statistical texts contain only a few. Several books of binomial tables have been published. One of these is called TABLES OF CUMULATIVE BINOMIAL PROBABILITIES, ORDNANCE CORPS, U.S. ARMY ORDP20-1, 1952. However, even books that are dedicated to binomial tables tend to not have the precise table that you really need for your practical application.

Direct computation of individual or cumulative probabilities is conceptually simple. However, it is usually impractical to do this manually unless the sample size is small. Large sample sizes require extensive calculations. Direct computation of confidence limits for a probability (or a proportion) that is estimated from a large sample requires unbelievably laborious calculations. As previously stated, both of the binomial programs were written in FORTRAN, a language well equipped to handle these computations.

It is not the goal of this paper to be critical of the normal distribution, or of the chi-square or the Poisson. These distributions all have important places in the statistician's tool kit. When they are used properly, they generally give very good results. But when you deal with binomial data, why not go back to the basics and use an exact test? Personal computers are available in most offices. The software is very "user friendly". And the user need not concern himself with questions such as whether populations are normally distributed, or chi-square expected frequencies are too small.

HARDWARE AND SOFTWARE SUMMARY:

HARDWARE REQUIRED: IBM PC or compatible, with 320 K RAM
SOFTWARE REQUIRED: MS-DOS 2.1 or higher (FORTRAN compiler not needed)
BINOMIAL TABLE LIMITATIONS: The user can request any probability greater than 0.0 and less than 1.0. He can request any integer value for N (number of trials) between 1 and 9,999,999.

AVAILABILITY OF THE PROGRAMS:

The programs are available from the software library of the Command and Control Microcomputer User's Group (C2MUG):

Associate Director, MCS CSE
ATTN: AMSEL-RD-SE-MCS (C2MUG)
Building 138
Fort Leavenworth, KS 66027-5600

The programs should be ordered through your C2MUG representative. If your organization does not have a representative, you can call C2MUG at DSN 552-7550 and ask how to order software. The binomial software is too new for inclusion in the 1991 C2MUG Software Catalog, but it will be included in the 1992 catalog. Individuals ordering before the new catalog is available will need to know the catalog number. It is 500-009.

QUESTIONS:

Questions, comments, and suggestions are encouraged. They should be directed to:

Commander, USAOC&S
ATTN: ATSL-CD-CS (Mr. Robert Dick)
Aberdeen Proving Ground, MD 21005-5201

Phone: DSN 298-2028
COM 301:278-2028

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Dixon, W. J. and Massey, F. J. 1969, Introduction to Statistical Analysis, Third Edition, McGraw-Hill.

James and James 1976, Mathematics Dictionary, Fourth Edition, Van Nostrand Reinhold Company.

Snedecor, G. W. and Cochran, W. G. 1980, Statistical Methods, Seventh Edition, The Iowa State University Press.

Volk, William 1958, Applied Statistics for Engineers, McGraw-Hill.

CHART 1

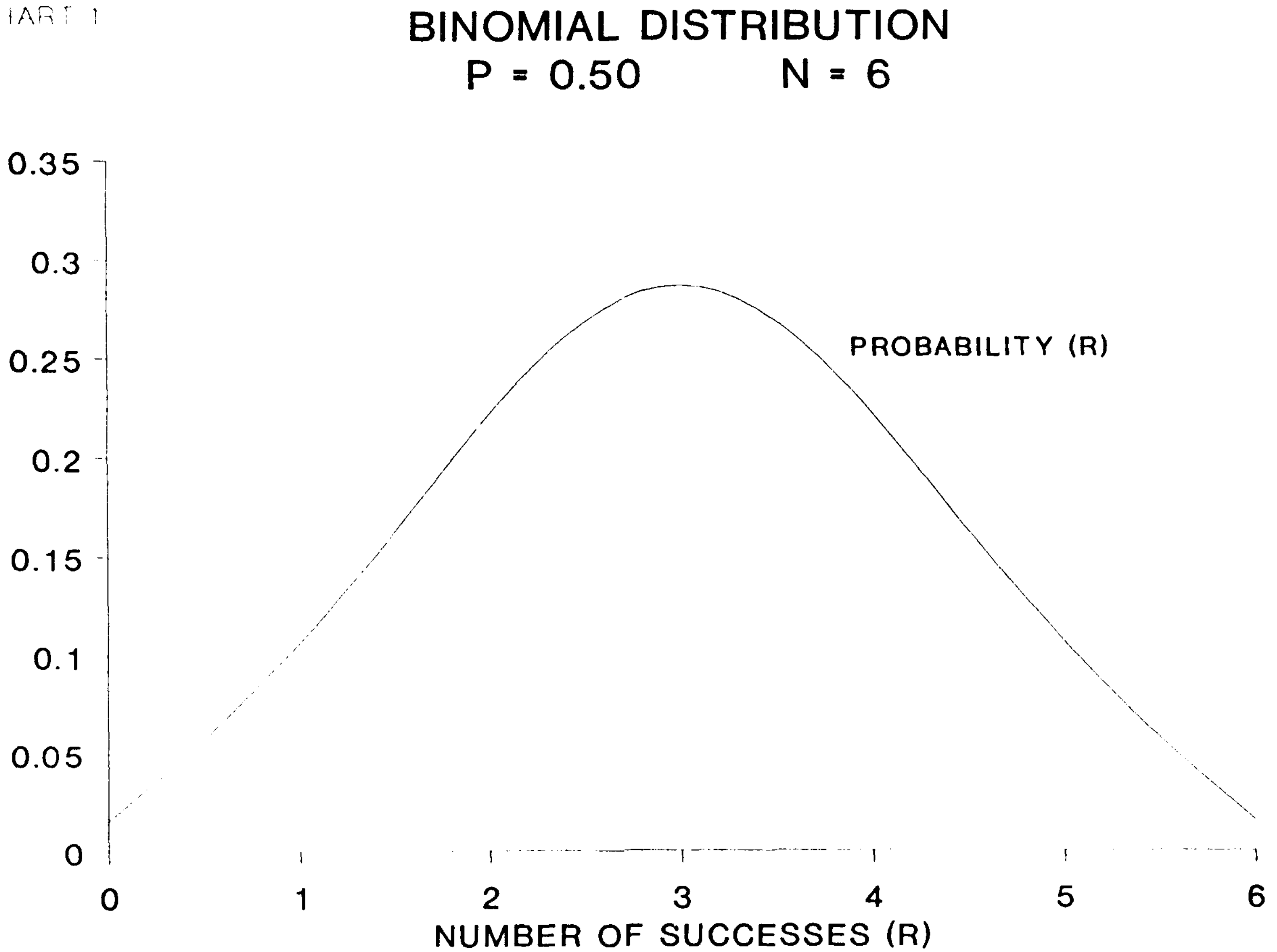
BINOMIAL DISTRIBUTION
P = 0.50 N = 6
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
PROBABILITY (R)
0
1
2
3
4
5
6
NUMBER OF SUCCESSES (R)

CHART 2

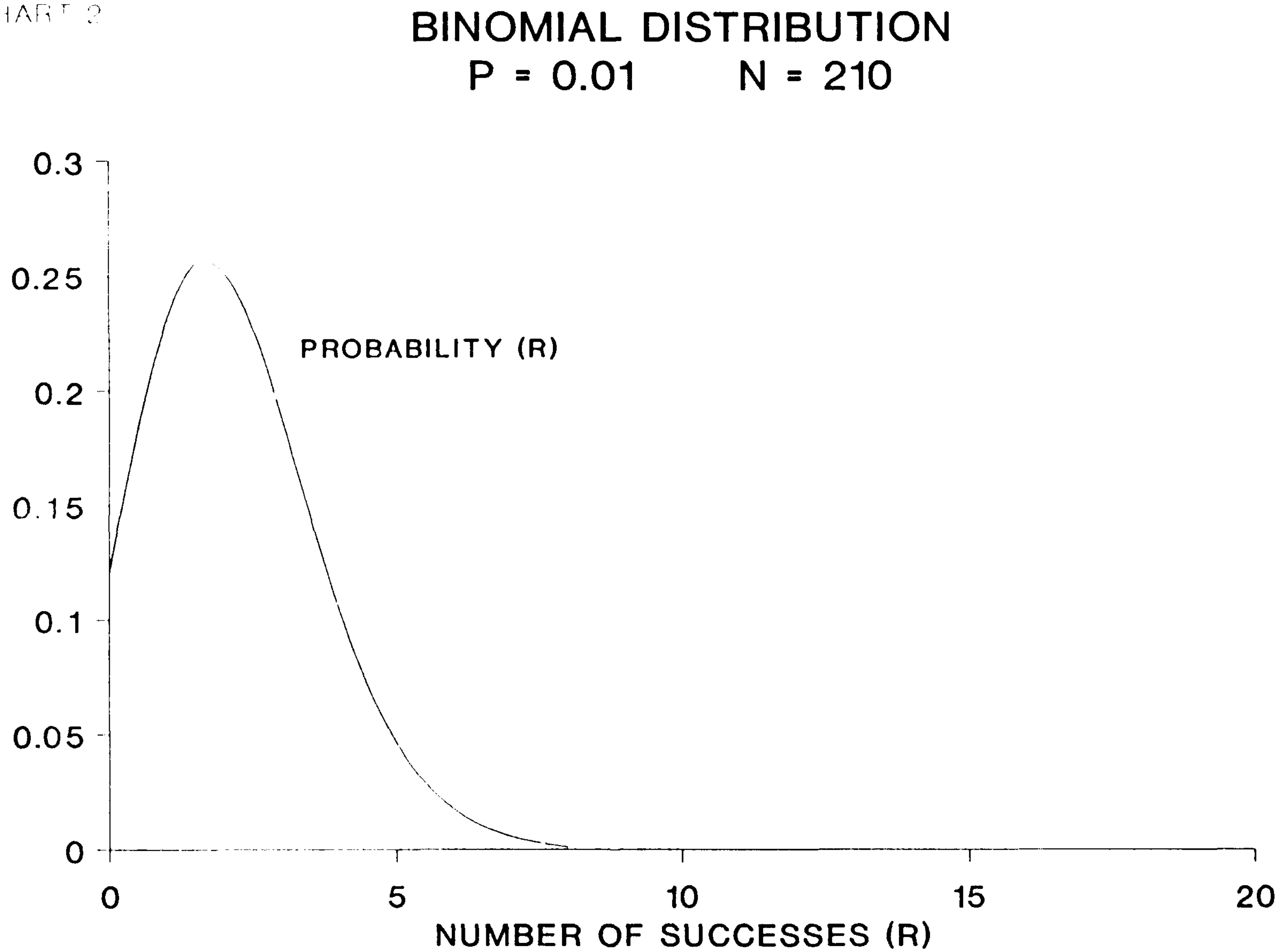
BINOMIAL DISTRIBUTION
P = 0.01 N = 210
PROBABILITY (R)
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
NUMBER OF SUCCESSES (R)

CHART 3

BINOMIAL DISTRIBUTION
P = 0.01 N = 420

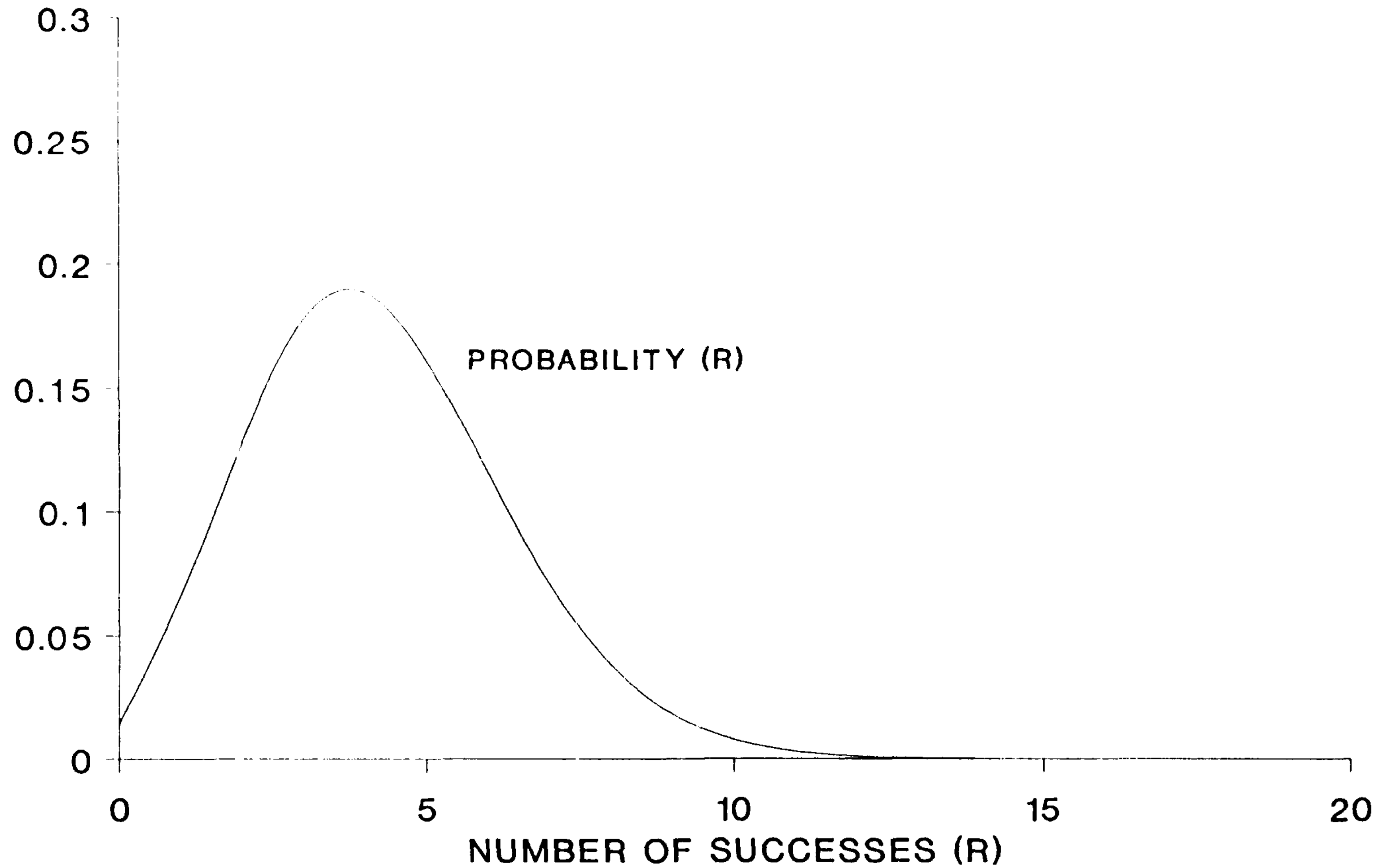

CHART 4

# BINOMIAL DISTRIBUTION
P = 0.01 N = 1050