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The report summarizes the main results obtained, with emphasis on the energy approach introduced first for a single degree of freedom model, and then developed for a two degree of freedom beam model. The elastic strain energy function is plotted as a surface over the displacement coordinate plane, whose topography depends on the plastic strains. It provides a guide to the various types of beam response, which may be quasiperiodic or chaotic, and enables their dependence on parameters such as impulse magnitude and damping to be clearly understood.

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Dynamic Plastic Instabilities in Nonlinear Inelastic Response to Pulse Loading

Final Report
ARO Proposal Number 24362-EG, Contract/Grant No. DAALO3-87-K-0038
(15 March 1987 - 14 September 1991)

Abstract. The report summarizes the main results obtained, with emphasis on the energy approach introduced first for a single degree of freedom beam model, and then developed for one of two degrees of freedom. The elastic strain energy function is plotted as a surface over the displacement coordinate plane, whose topography depends on the plastic strains. It provides a guide to the various types of beam response, quasiperiodic and chaotic, and enables their dependence on parameters such as impulse magnitude and damping ratio to be clearly understood.

1. Statement of problem

The subject of the research project was defined as the investigation of anomalous inelastic responses of certain structural elements (beams and plates) to short pulse transverse loading. These "anomalous" behaviors are observed in computed final (permanent) deflections that exhibit extreme sensitivities to parameters. In certain circumstances, because of these not even the *sign* of the major deflection can be reliably predicted, either for a physical laboratory specimen or "on paper" for a mathematically well defined problem.

These phenomena indicate the presence of multiple equilibrium states, stable and unstable, in the new geometry of the structure created by plastic deformations resulting from the pulse loading. The structures concerned are beams and plates fixed at their boundaries, so that transverse loading which causes small plastic deformations effectively converts the beam/plate to a shallow arch/shell. The dynamic instabilities are thus related to those involved in snap-buckling. The fundamental difference is that there the structure and loading are known and specified in advance, while in the present class of problems the relevant structure and loading cannot be specified except by means of *calculated* response quantities: plastic deformations and available energies, in particular.

2. Summary of main results

Following the accidental discovery of these anomalous behaviors in 1984, preliminary research received support from an ARO "Feasibility Study" grant (Proposal No. 23225-EG). Papers [1 - 4] in the list of publications present results obtained prior to the present grant.

These preliminary results further displayed the sensitivity of the numerical solutions for discretized uniform beams to the parameters of the problem and the numerical treatment. The 1DoF model studies with damping showed that the previous calculations for an undamped model are incapable of predicting the sign of the final rest deflection of the damped specimen. The "characteristic diagram" for the damped model, in which the final deflection is plotted against the load pulse parameter, showed the final deflection alternating abruptly between positive and negative values. The discontinuous transitions correspond to singular points in the phase diagram. The pattern of final displacements has some regularity, but the dependence on damping strength and other parameters appears to be quite complicated [4].

The main result of the research supported by the present grant is a new approach, introduced in [5], to displaying and interpreting the response features, simpler and more direct than the usual stability studies in terms of phase space geometry. The discontinuous alternation of the final state between positive and negative values, and all other features of the characteristic diagram of this model, become immediately understandable from energy diagrams, constructed by plotting the elastic strain energy and the total energy against the displacement, as furnished by the numerical solution of the equations of dynamics and plasticity. The approach is based simply on the energy balance equation, i.e. the first integral of the equations of motion. The sum of the current kinetic and elastic strain energies is equal to the total available energy, which can be written as a reference value less the energies subsequently dissipated in damping and in plastic work, if any. Applied to the 1DoF model, the elastic strain energy function is a quartic polynomial function of the displacement, with constant coefficients as soon as the plastic strains have become constant. Typically it then exhibits a local maximum between two local minima, i.e. a potential hill between two wells. The final rest state is always at one or the other of the local minima.

An early application of the insights provided by these diagrams was to explain the origin of erroneous results in a contemporary paper making use of the same model. In this paper the authors considered loading specified by initial conditions (as in our work), as well as periodic loading. In the former case they obtained a diagram in which the coordinates are the initial velocity and displacement, and a dot is entered if the final displacement is negative (opposite to the initial impulse). Their diagram is extremely complex, with data points suggestive of fractal boundaries between attracting basins. The actual response has no such complications; no fractal geometry or other signs of chaotic behavior. It is obvious from the energy plots that the numerical solution is everywhere calculable without difficulty, contrary to the authors' statement that attempted numerical solutions are "meaningless". The basic error of the published paper was explained in [6] with further development in [7]. Numerical difficulties are encountered only if the damping coefficient is taken excessively small.

Publications [8, 9] provide further examples of finite element solutions for pin-ended and fully clamped beams, illustrating unexpected (apparently chaotic) features of time histories, contrasting with the comparative simplicity of the 1DoF model. The energy approach was reviewed in these papers for the 1DoF model, and in [9] was applied to cases of uniform beams, taking data from the finite element solutions, and plotting the two energies as functions of the midpoint displacement. These diagrams illustrate the expected main features of damped elastic-plastic responses, but are irregular and uninstructive since the strain energy is a function not only of the midpoint displacement, but of all the kinematic variables of the discretized beam.

Since September 1989 the work has concentrated on studying a two degree of freedom (2DoF) beam model. We again took this to be of Shanley type, with deformable sandwich beam elements at the two quarterpoints and midpoint. Assuming symmetrical deformations, the two transverse displacements at the midpoint and quarterpoint specify the configuration. The phase space (displacements and velocities) now having four dimensions, a much richer variety of behaviors, including chaotic vibrations, is exhibited. The use of a 2DoF model is advantageous especially for displaying the energy approach. The strain energy is a function of the two displacements, with the four plastic strains appearing as parameters. When they become constant, the function can be represented as a surface over the displacement coordinate plane. The important typical features of this surface created by plastic straining are a local maximum, two saddle points, and two local minima. The character of the response, after the completion of plastic deformation, depends on the position of the total energy plane relative to these five singular points. The total energy monotonically decreases, rapidly when plastic flow occurs and more gradually due to damping losses. Thus the corresponding plane steadily descends, with intersection curves of various shapes and steadily decreasing area. The actual state point must lie in the interior of the current intersection curve. If the total energy plane is initially above the top of the potential hill (local maximum), the response may be either quasiperiodic or weakly chaotic until it reaches the vicinity of the maximum and saddle points. Then it typically becomes strongly chaotic, with a highly irregular mixture of large amplitude vibrations and small amplitude positive and negative vibrations. Eventually there is an abrupt transition to a small amplitude vibration, which retains its sign and becomes smoother (quasiperiodic) as the amplitude decreases to zero. This transition indicates that the total energy has fallen below the energy at the lower saddle point, so that the solution path is trapped in one or the other of the two potential wells. The final state, of course, is at one of the two local minima. The particular one chosen cannot be predicted, since in the preceding chaotic interval the solution path wanders irregularly over the permitted region, and selects the positive or negative well in an apparently random manner.

Publications [11-14] present the main results for the 2DoF model. They should be compared with those of a uniform beam on the one hand, and the 1DoF model on the other. Paper [11] includes illustrations of uniform beam time histories for comparison with those of the 2DoF model, for which the quasiperiodic and chaotic behaviors are identified by characteristic phase plane diagrams and plots of power spectral density and Lyapunov characteristic exponents. The use of the energy approach is detailed in publications [12] and [13], with pictures of strain energy surfaces for several cases, and comparisons between the characteristic diagrams (final deflection plotted as function of the load pulse parameter) of the single and two degree of freedom models. The orderly pattern in the former case contrasts with chaotic irregularity in the latter [12]. A major contribution of [13] is a pair of diagrams showing the correlation between the sign of the largest Lyapunov exponent and the position of the total energy plane relative to the energies at the five stationary points of the strain energy surface. Also briefly discussed in [13] is an alternative problem definition, in which the four plastic strains are regarded as independently chosen parameters, and the reponse behavior is studied for various initial displacement and velocity conditions. Preliminary calculations show the sensitivity of the character of the response, which may be either quasiperiodic or chaotic at a fixed energy level, to the orientation of the initial displacement vector.

Laboratory experiments were a fundamental part of the investigation. A summary of our experimental results is given in [15]. Anomalous final deflections were obtained in tests on specimens of aluminum alloy 6061-T6 with fully fixed ends. These were somewhat more difficult to obtain than anticipated. This was probably because effects of damping were stronger than expected; calculations [9] had shown that anomalous behaviors tended to be suppressed by energy losses in damping. Some progress was made toward finding a method of estimating the circumstances in which anomalous behavior may be expected.

3. Publications

In the following list, publications [1-4] are included for reference; the work they describe was supported in part by the preceding ARO "Feasibility Study" grant, Proposal No. 23225-EG.

1. Symonds, P.S. and Yu, T.X. "Counterintuitive Behavior in a Problem of Elastic-Plastic Beam Dynamics", *ASME J. Applied Mechanics*, Vol. 52(3), pp. 517-522, 1985.

2. Symonds, P.S., McNamara, J.F., and Genna, F. "Vibrations and Permanent Displacements of a Pin-Ended Beam Deformed Plastically by Short Pulse Excitation", *Int. J. Impact Engineering*, Vol. 4(2), pp. 73-82, 1986; also published in *Material Nonlinearity in Vibration Problems, ed.* M. Sathyamoorthy, ASME AMD-Vol. 71, pp. 69-78, 1985.

3. Genna, F. and Symonds, P.S. "Induced Vibrations and Dynamic Plastic Instabilities of a Nonlinear Structural Model due to Pulse Loading", *Meccanica*, Vol. 22, pp. 144-149, 1987.

4. Genna, F. and Symonds, P.S. "Dynamic Plastic Instabilities in Response to Short Pulse Excitation", *Proc. Royal Society of London*, Vol. A417, pp. 31-44, 1988.

5. Borino, G., Perego, U., and Symonds, P.S. "An Energy Approach to Anomalous Damped Elastic-Plastic Response to Short Pulse Loading", *ASME J. Applied Mechanics*, Vol. 56(2), pp. 430-438, 1989.

6. Symonds, P.S., Borino, G., and Perego, U. "Chaotic Motion of an Elastic-Plastic Beam", *ASME J. Applied Mechanics*, Vol. 55(3), pp. 745-746, 1988. Discussion of paper of same title by Poddar, B., Moon, F.C., and Mukherjee, S., *ASME J. Applied Mechanics*, Vol. 55(1), pp.185-189, 1988.

7. Perego, U., Borino, G., and Symonds, P.S. "The Role of Damping in Anomalous Response to Short Pulse Loading", *Proc. ASCE, J. Engineering Mechanics*, Vol. 115(2), pp. 2782-2788, 1989.

8. Symonds, P.S., Perego, U., Borino, G., and Genna, F. "Anomalous Elastic-Plastic Response to Short Pulse Loading - An Outline of Recent Progress" in *Omaggio a Giulio Ceradini - Note Scientifiche in Occasione del 70^o Compleano*, ed. U. Andreus, et al., Dept. of Structural and Geotechnical Engineering, University of Rome "La Sapienza", pp. 641-668, 1988.

9. Symonds, P.S., and Lee, J.-Y. "Anomalous and Unpredictable Response to Short Pulse Loading", in *Recent Advances in Impact Dynamics of Engineering Structures*, ed. D. Hui and N. Jones, ASME AMD-Vol. 105, Book No. H00542, pp.31-38, 1989.

10. Symonds, P.S., Genna, F., and Ciullini, A. "Special Cases in Study of Anomalous Elastic-Plastic Response of Beams by a Simple Model", *Int. J. Solids & Structures*, Vol. 27(3), pp. 299-314, 1991.

11. Lee, J.-Y., Symonds, P.S., and Borino, G. "Chaotic Responses of a Fixed Ended Elastic-Plastic Beam Model to Short Pulse Loading" Part I "Response Characteristics, Sensitivity to Parameters", submitted to *ASME J. Applied Mechanics*, March, 1991.

12. Lee, J.-Y., Symonds, P.S., and Borino, G. "Chaotic Responses of a Fixed Ended Elastic-Plastic Beam Model to Short Pulse Loading" Part II "Energy Approach; Role of Damping and Plastic Strains", submitted to *ASME J. Applied Mechanics*, March, 1991.

13. Lee, J.-Y. and Symonds, P.S. "Extended Energy Approach to Chaotic Elastic-Plastic Response to Impulsive Loading", *Int. J. Mechanical Sciences* (in press).

14. Symonds, P.S. and Lee, J.-Y. "Unpredictable and Chaotic Response to Impulsive Loading: and Energy Approach to Nonlinear Inelastic Behavior", *Proc. 62nd Shock & Vibration Symposium*, ed. H.D. Kohn (SAVIAC, Booz-Allen & Hamilton Inc., Arlington, VA) Vol. 1, pp. 82-91, 1991.

15. Kolsky, H., Rush, P., and Symonds, P.S. "Some Experimental Observations of Anomalous Response of Fully Clamped Beams", *Int. J. Impact Engineering* (in press).

Since 1988, the results obtained under the present grant have been presented in seven talks at scheduled conferences or symposia, and twelve invited seminar lectures.

4. Participating Personnel

P.S. Symonds, Professor of Engineering (Research)
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Mr. Jae-Yeong Lee was awarded the degree M.Sc. in Applied Mathematics in May 1990. He defended his thesis for the Ph.D. degree in Engineering in August, 1991, and will receive the degree in May 1992.