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6. AUTHOR(S)
F. Alberto Grunbaum

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of California
Department of Mathematics
Berkeley, CA 94720

8. PERFORMING ORGANIZATION REPORT NUMBER
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During the period covered by the grant four areas were worked on: (1) Diffuse tomography, (2) Concentrating a signal in the physical and spectral domains, (3) New explicit solutions for the Kadomtsev-Petviashvili equation, and (4) Time and band limiting on the symmetric group.

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F. Alberto Grunbaum

UNIVERSITY OF CALIFORNIA

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During the period covered by this contract I have worked on a number of problems that can be loosely be catalogued in four areas

1. Diffuse tomography
2. Concentrating a signal in the physical and spectral domains.
3. New explicit solutions for the Kadomtsev-Petviashvili equation
4. Time and band limiting on the symmetric group

I have mentioned these four areas in the progress report submitted in 89, a copy of which is enclosed. In this final report I will go into more details in the first two areas. It is in these areas where I have put most of my energy in the last couple of years.

Diffuse tomography

A list of the papers that I have written during the last few years in this topic is given below. I will make some few remarks about the contents of each of them.

**Tomography with diffusion,Montpellier 1989

This paper is the first one to discuss a new approach to medical imaging with "low energy probes", like an infrared laser. It introduces a (two-step) Markov process and describes the problem of finding the "microscopic parameters" of this process from BOUNDARY measurements of the kind that one might do with a patient. It gives explicit solutions for VERY SMALL cases.

**Image reconstruction of the interior of bodies that diffuse radiation,Science vol 248 990-993 1990.

A discussion of the ideas of the previous paper for a (much) less mathematical audience.

**Imaging of media that diffuse and scatter radiation,Proc. IMA 1991.

This is a contribution to an IMA volume on new topics in Imaging. It describes the model considered in the papers above and discusses some of the open problems that will have to be addressed to make this approach practical. It deals in particular with the difficult issue of "brute force minimization" and the dangers of getting trapped in local minima.

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**Diffuse tomography: a refined model ,Oberwolfach 1990 editors A. Louis and F. Natterer

The main weakness of the models described elsewhere is taken care of in this paper, by allowing a richer discretization of velocities, while retaing the original discretization of positions.I show that the direct problem can be formulated exactly as before.

**Diffuse tomography, Proc. on Time Resolved Spectroscopy and Imaging of Tissues, SPIE vol 1431 pp-232-238

This paper, written for a "biosciences" audience, discusses the basic points of our inversion approach.It includes a description of the model and some numerical experiments.

**An inverse problem for a model of scattered and diffuse radiation, Australian Miniconference on Inverse Prob. in PDE 1991.

This paper reports on some numerical experiments design to study iterative methods for solving the inverse problem.They are based , as usual, on the analytical solution of the direct problem discussed in the previous papers.

**Backscattering comes to the rescue
consider the same problem ie Integral geometry and tomography, AMS 113 Contemporar

This paper was written an presented at an AMS meeting in June 89.Here I give for the first time explicit formulas for the "explicit solution of the inverse problem" under VERY RESTRICTIVE assumptions,i.e. no side scattering: only forward and backward.The point is to show that this extra freedom in the model allows one to get out of the well know nonuniqueness in the standard tomographic model with "too feww directions".

**Relating microscopic and macroscopic parameters for a 3 dimensional random walk,Comm. Math Phys 1990

Here I show the appropriate way to scale the scattering parameters at the microscopic scale if one is to obtain given macroscopic parameters in a HOMOGENEOUS medium.This result is an important tool in the modelling needed for the inverse problem, and does not by itself deal with the inversion problem.

**Renormalization of exit probabilities and a theorem of Poincare,Physica D 1990

Here I consider a simpler (i.e. two dimensional) version of the problem considered in the previous paper, and use some ideas of Poincare to study in detail the way in which the discretizations approach the continuos limit.

**Diffuse tomography:the isotropic case
submitted to Inverse Problems

Here I give (for the first time) an explicit method to solve the inverse problem.It requires (so far) the extra assumption that at each pixel the scattering parameters are all equal (i.e. if you do not die, then you migrate with equal probabilities in each of the adjacent pixels) Of course from pixel to pixel these probabilities change.

The algorithm works by solving a FINITE number of LINEAR problems.This should not be interpreted to mean that the inversion problem itself is linear.In fact this algorithm may not be "the way to go".

**Diffuse tomography:Computational aspects of the isotropic case.
submitted to Inverse Problems

Here I study the numerical performance of the algorithm introduced in the previous paper.The results indicate the need to look for more numerically stable strategies.This is being considered right now.

**An inverse problem in transport theory: diffuse tomography
Siam volume 1991 Conf. in honor of R. Kruger editor J. Coronnes

This paper written for the "transport theory community" tries to show how some of the ideas of "invariant embedding" can be extended to the higher dimensional case in the form of "addition formulas" , and then (and this is the novelty of the paper) used for the purposes of handling INVERSE problem.

**Uniqueness and nonuniqueness in diffuse tomography
Cape Cod ,1991.M. Bertero edit.

I present here three extreme situations in diffuse tomography:I exhibit an example showing that unless one imposses physical restrictions on the values of the scattering parameters one cannot expect uniqueness, I give an example showing that conditions like symmetry can give rise to a "smooth manifold of solutions" i.e. SEVERE nonuniqueness, and finally on the positive side I show that if one is willing to use "time of flight information" then invertibility becomes a rather easy task.

Concentrating a signal in the physical and spectral domains
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Once again I list the papers in this area and follow the list with comments on these papers.

**Three dimensional scattering and the Heisenberg inequality
Special Issue of J. of Computational and Applied Math.

I exhibit a family of examples of three dimensional potentials, essentially two delta shells centered around the origen.I show that for appropriate choice of the radii and strength of these shells one can concentrate the potential while the backscattering amplitude (as a function of the wave number k) does not spread out.This behaviour would take place within the Born approximation.
Much work remains to be done in this area.In particular it would be nice to test these predictions in an experimental setup.

**The scattering transform and the Heisenberg uncertainty principle
Nonlinear World voll (1989) pp 121-129   Kiev meeting 1989

I exhibit a family of potentials for the Schroedinger equation.The interesting feature here is that both the potentials in this family AND their scattering transforms can be concentrated at the origen by playing artfully with some parameters.

**Concentrating a potential and its scattering transform for a discrete version of the Schroedinger and Zhakarov-Shabat operators
Physica D 44 (1990) pp 92-98

l consider the question considered in the previous paper for properly chosen discrete analogs of the usual continuous problems.In both cases I manage to get examples showing that the "popular belief" about the scattering transform inhereting most prperties of the Fourier transform (to which it reduces in the limit of small potentials) is not necessarily true.

**The scattering transform for a piecewise linear potential in a Zakharov-Shabat system.
(this is a manuscript that is not quite finished)to be submitted

Here I manage to compute explicitely (in terms of the confluent hypergeometric function) the scattering matrix for such a system.Since every reasonably smooth potential can be well approximated by piecewise linear ones, one obtains a rather good ANALYTICAL tool for the approximate computation of these "reflection" and "transmission" coefficients.My ultimate goal (still not quite achieved) is to explore with these examples the space of potentials for good candidates to "violate" the Heisenberg principle.
Notice that in the case of the Schroedinger equation it was possible to "beat Heisenberg" with piecewise linear potentials.

Finally let me say that the paper

Some new explorations into the mystery of time and band limiting

will appear (sometime soon) in Advances in Appl. Math.
It deals with the fourth issue in the list given at the beginning of this report.