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VISUAL DETERMINATION
OF INDUSTRIAL COLOR-DIFFERENCE TOLERANCES
USING PROBIT ANALYSIS

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by

Gregory D. Snyder

B.A. Southern Illinois University

(1979)

A thesis submitted in partial fulfillment
of the requirements for the degree of Master of Science
in t' e Center for Imaging Science
in the College of Graphic Arts and Photography
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VISUAL DETERMINATION
OF INDUSTRIAL COLOR-DIFFERENCE TOLERANCES
USING PROBIT ANALYSIS

by

Gregory D. Snyder

Submitted to the
Center for Imaging Science
in partial fulfillment of the requirements
for the Master of Science degree
at the Rochester Institute of Technology

ABSTRACT

A perceptibility study was conducted to visually determine the median tolerance values of 45 color-difference vectors in CIELAB color space using surface mode viewing of paint samples. Nine different color centers, each comprising five color vectors, were employed to collect a superthreshold dataset. Fifty color-normal observers made quantal judgements under simulated D_{65} illuminant regarding the magnitude of color-difference pairs based on comparisons to a near-neutral color-difference anchor pair. Probit analysis was applied to the response frequencies for each vector to estimate the parameters of the distribution and the median tolerance values. Results indicated the probit adequately models the response distributions of the human observer population.

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The late Dr. Franc Grum whose reputation and enthusiasm in the field of color studies inspired me to undertake this research effort.

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INTRODUCTION

A color tolerance is defined as a permissible color difference between a sample and a particular standard under specified conditions.[1] Color tolerance determinations can fall into two categories, acceptability and perceptibility, depending on the judgement basis and the intent of the measurements. In the former case, experiments are designed to measure preference of visual magnitudes of color differences, often being judged by experienced (biased and trained) personnel. The perceptibility case indicates an experiment designed to objectively measure visual magnitudes of color differences.[2] This latter situation is the class of color-difference tolerance intended for study in this experiment.

Numerous perceptibility experiments have been performed in the past to generate datasets for color-difference studies.[3,4,5,6,7,8,9] However, over the range of possible difference magnitudes these experiments have collected color data either at the threshold level, as in MacAdam's investigation of just-noticeable difference (jnd) ellipses, or at large difference levels typical of the Munsell Color System. This leaves a significant data gap in the super-threshold region representative of industrial color-difference tolerances. In order to develop color-difference equations for industrial applications, experimental datasets in the super-threshold range are necessary for the basis of such metrics.

The Colour Difference Subcommittee of CIE has made an appeal to the color research community to coordinate efforts in the establishment of perceptibility datasets over the entire range of color differences.[10] The intent of this proposal is to provide a structured approach to the complicated task of evaluating color difference issues and insure that data is collected for all conditions and stimuli of interest. Because past experiments have focused on threshold or large color differences, efforts will apparently be necessary to build on these past results and furnish the needed super-threshold datasets.

Many parameters can affect the results of a color-difference study, causing variability and even confusion between different experimental results. These factors as stated by Robertson[11] include,

a. Observer variability
b. Size of color difference
c. Sample separation
d. Adaptation
e. Mode of appearance
f. Acceptability vs. perceptibility
g. Weighting indices

Major differences regarding these factors in earlier color-difference studies from one experiment to another often makes comparison of results difficult at best. Because of the non-uniform nature of CIE tristimulus and the derived chromaticity color space, and the numerous related parameters, experiments must be designed that optimize the utility of the anticipated results. Consideration must be given to the augmentation of past experimental findings and to the prevention of duplicating efforts. Therefore, experimenters whose collected data is intended for community consumption must design their research in a manner that permits a broad range of applications and studies.

In establishing a perceptibility data base for modeling and/or validating color-difference equations a sound statistical analysis must be performed to demonstrate the merit of these results. Some a priori understanding of the data's statistical nature is helpful in the analysis of results. This information will assist in selecting a statistical method to perform the data analysis. For color-difference experiments it has been shown that the response data displays a normal distribution.[5,12] This is an important point to know in selecting a statistical model for analysis and a basic assumption in this experiment.

Probit Analysis

Certain types of research involve exposing subjects to a stimulus in varying intensities to determine their responses. In studies involving pass/fail (quantal) responses, such as the mortality of insects to different levels of insecticide, a cumulative response curve can be produced by plotting percent response versus stimulus intensity.[13] Finney[14] has shown that for data of this nature which has a normal distribution, the sigmoid curve of the cumulative distribution plot can be linearly transformed into probits and fit with a probit model by regression. This allows the researcher to make response predictions at an unlimited number of intensity levels (within the experimental range) from a minimal amount of data points.

Quantal experiments yield either an occurrence or non-occurrence dependent on the strength of the stimulus applied. The response to a specific stimulus for any one subject is dependent on that individual's tolerance level. For a given population the frequency of response as a function of 'dose' intensity can be expressed,

$$dP=f(x)dx \qquad (1)$$

where x is the dosage level and dP is the proportion of the population whose tolerance lies between x and $x+dx$. The proportion of subjects who would respond to a dose strength X or less is,

$$P=\int_{-\infty}^{X} f(x)dx \qquad (2).$$ [15]

When the population response falls in a normal distribution,

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (3)$$

where the mean, μ, and standard deviation, σ, represent the parameters that characterize the particular normal distribution curve.[16]

The shape of any normal distribution curve is defined by its parameters μ and σ. By Equation (3) each curve is symmetric about the mean, μ, and has two inflection points at $\pm\sigma$. A relative description of the normal probability curve can be specified by defining a z scale along the abscissa whose intervals are equal to 1σ (Figure 1). The symbol z is known as the standard normal deviate and is calculated by,

$$z=\frac{x-\mu}{\sigma} \qquad (4).^{17}$$

By combining Equations (2) and (3) we produce the cumulative distribution function for a normal probability density (Figure 2),

$$P=F(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{X}\exp^{-\frac{1}{2}(\frac{x-\mu}{\sigma})}dx \qquad (5)$$

where X is a particular dosage intensity. Substituting colorimetric variables the equation becomes,

$$P=F(dE)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{DE}\exp^{-\frac{1}{2}\frac{dE-\mu}{\sigma}}ddE \qquad (6)$$

where DE is a particular color-difference magnitude. Now by calculating P using the z scale Equation (5) becomes,

$$P=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{Z}\exp^{-\frac{z^2}{2}}dz \qquad (7)$$

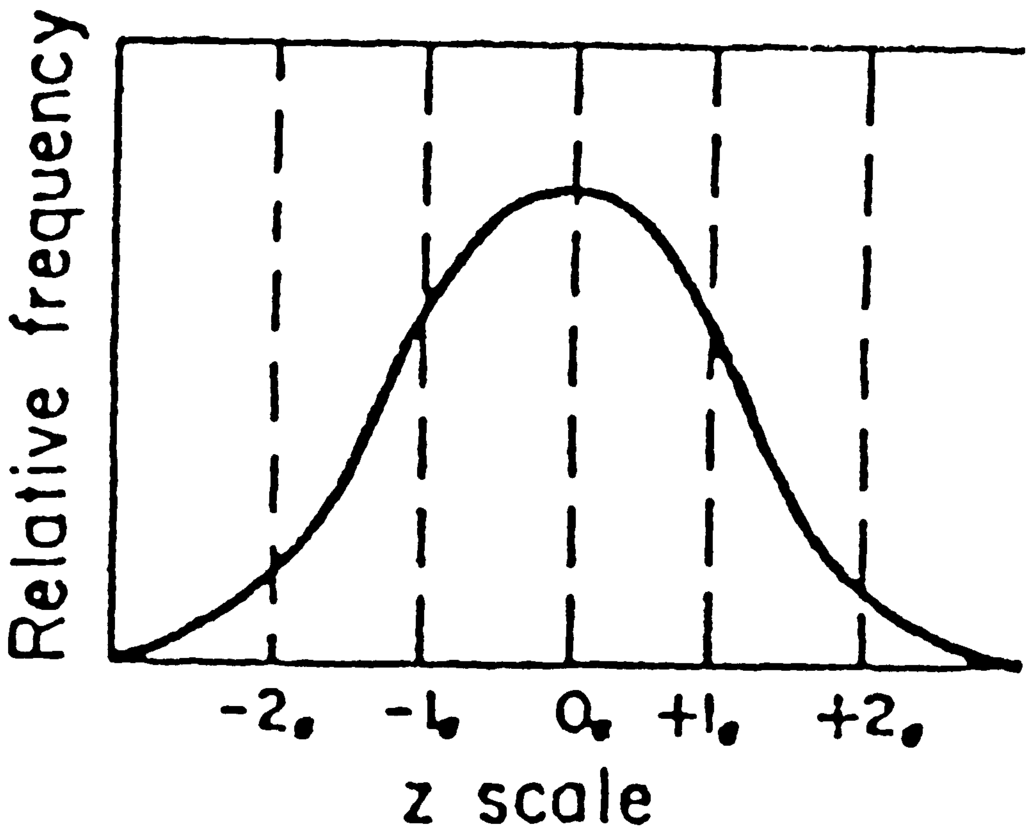


FIGURE 1. Normal Distribution Relation to Z-scale[18]

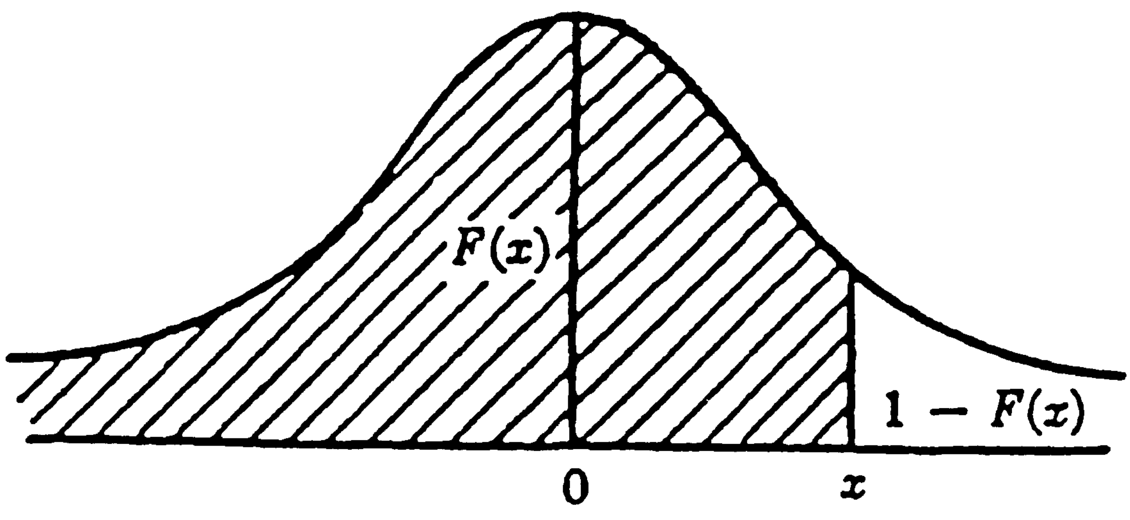


FIGURE 2. Cumulative Distribution Function for a Normal Probability Density[16]

where Z is a particular value on the z scale. From Equation (4) we show,

$$Z=\frac{X-\mu}{\sigma} \quad (8)$$

or,

$$Z=\frac{DE-\mu}{\sigma} \quad (9).$$

Therefore a linear relation exists between the dosage, x, or color-difference, dE, and the standard normal deviate, z, of the probability of response at that level of stimulus.

A probit (probability unit) is defined as simply the standard normal deviate increased by five, producing a relationship with dosage (color-difference) expressed as,

$$Z=5+\left(\frac{x+\mu}{\sigma}\right) \quad (10).$$ [19]

This provides a transformed scale along the abscissa of the normal probability curve similar to Figure 1 except that the mean corresponds to a z value of five instead of zero. The linear Equation (10) can be rewritten in the common form,

$$Z=\alpha+\beta x \quad (11)$$

where α, β replace μ, σ respectively as the new parameters.[20]

When the values of Equation (7) are plotted as cumulative percentage of response versus intensity of stimulus a normal sigmoid curve is formed. If Equation (7) is instead solved for probits and plotted as a function of stimulus, a linear curve is produced (Figure 3).[19] Notice

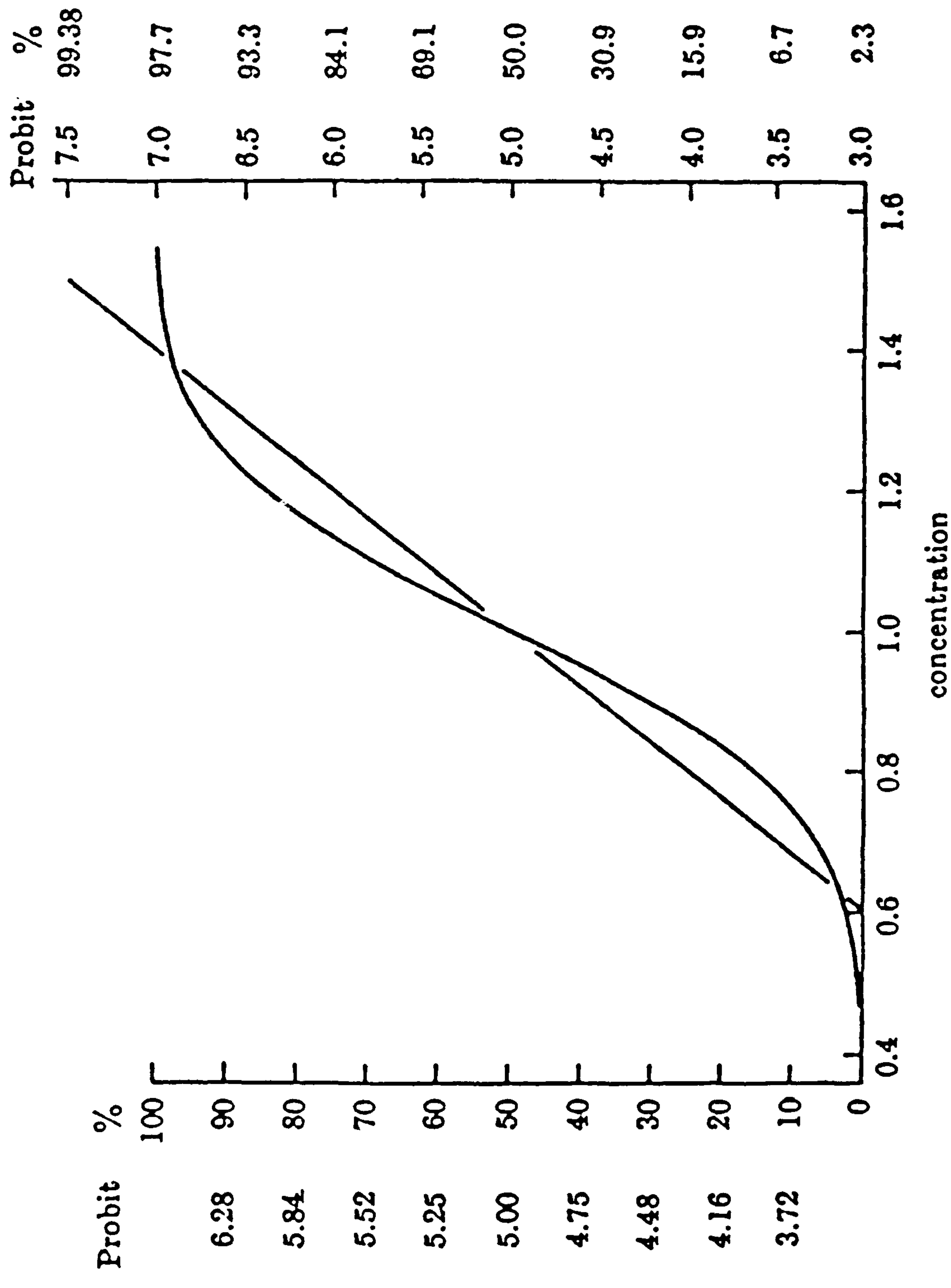


FIGURE 3. Sigmoidal Curve vs. Linear Probit[19]

that P = 0.5 and the probit value of five coincide. This is the dose at which half the population will respond and is commonly referred to as the median effective dose or MD50 in the biological community.[21] For this experiment the term T50 is substituted in place of MD50 to denote the median color-difference tolerance.

In order to solve Equation (7) for either P or Z the parameters μ and σ must be estimated. This is performed by fitting the linear regression of probits on stimulus values using either a graphical procedure or an arithmetical process known as maximum likelihood.[20] The graphical method is a rapid and sufficient means of accurately satisfying most problems; but, in cases where a more accurate assessment is required the maximum likelihood technique is employed. In both instances, experimental results relating stimulus to response frequencies are converted to probits and fitted to a straight line. From the linear parameters α , β of the fitted line the mean and standard deviation can be calculated for the probability distribution and substituted into Equation (5) or (6).

To determine the degree of conformance to a normal distribution, the chi-squared test is performed on the empirical data. A value of chi-squared within the boundaries of random variation signifies agreement between theoretical and empirical data. Chi-squared values larger than the limits of random variation are caused either by test subjects not responding independently or the experimental data not satisfactorily matching the linear curve relating stimuli to probit.[22]

Objective

There is an expressed need in the colorimetry community to develop perception datasets for color differences in the small super-threshold region.[10,23] The intended aim of this experiment is to develop such a data base under conditions appropriate to industrial practice. The experiment uses a quantal design for presenting color-difference stimuli to human observers. Probit analysis is applied to determine the frequency of response at which 50 percent of the observer population reject a given color-difference stimuli when compared to a near neutral standard anchor pair. This experiment will investigate the adequacy of probit analysis to estimate color-difference tolerance levels when using a limited number of sample stimuli.

EXPERIMENTAL

The experimental portion of this study is described in three parts: experimental design, materials and equipment, and the sample observations.

Experimental Design

In most scientific endeavors it is necessary through advanced planning to make the most efficient use of the research data collected to optimize the knowledge gained to the effort expended. This endeavor was no exception as it required many hours of valuable human participation and input. Needless to say then, the most critical part of this experiment was its initial design.

A major driver in the experimental design was the analysis method employed. In order to use probits the experiment was organized to permit such an analysis. Therefore, a series of tolerance judgements by human observers were designed as quantal responses to color-difference stimuli. Color-difference vectors were sampled at various magnitudes to determine the median tolerance, the point at which half the observer population responded to a level of stimuli. This is the level of tolerance at which $P = 0.5$ (Equation 5 or 6). In this experiment a response was defined as a rejection of a color-difference stimulus when compared to a near neutral anchor color-difference stimulus.

The experiment concentrated on nine color centers systematically distributed in CIELAB color space (Figure 4) to most effectively cover the range of chromatic variables. Such a distribution permitted the sampling of eight different hues and three levels of chroma. The color positions correspond to a gray center, four medium chroma hues (orange, yellow-green, cyan, purple), and four high chroma hues (red, blue, green, yellow). The lightness values varied to positions recommended by the CIE[24] for five of the experimental color centers (Figure 5). This produced a sampled lightness plane generally slanted downward from yellow (L^*=78) to blue (L^*=35).

Each color center was comprised of five color vectors oriented to sample the following color changes:

Vector	Color Change
A	$-L^*$ to L^*
B	$-a^*$ to a^*
C	$-b^*$ to b^*
D	$-a^*$ $-b^*$ to $+a^*$ $+b^*$
E	$-a^*$ $+b^*$ to $+a^*$ $-b^*$

This sampling design is illustrated in Figure 6. Along each vector nine sample positions were prepared at -2, -1, -0.5, -0.25, 0, 0.25, 0.5, 1, 2 CIELAB units from the center point. From these samples it was possible to construct color-difference pairs with no change (ΔE^*_{ab}=0) to pairs with relatively large color change (ΔE^*_{ab}=4). Pair

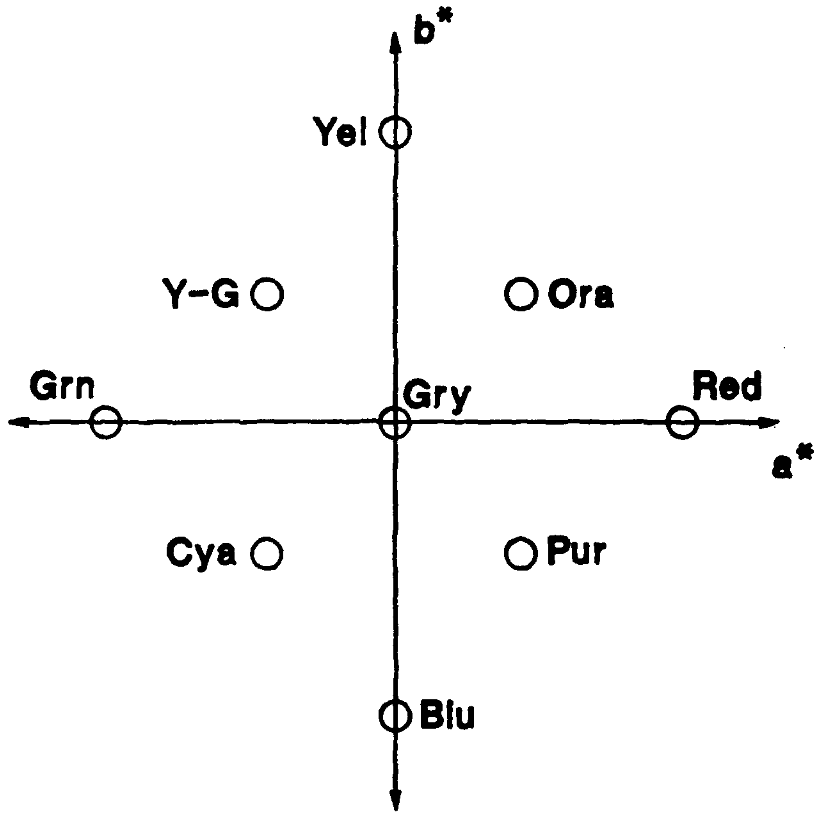


FIGURE 4. Distribution Design for Color Centers in CIELAB Color Space

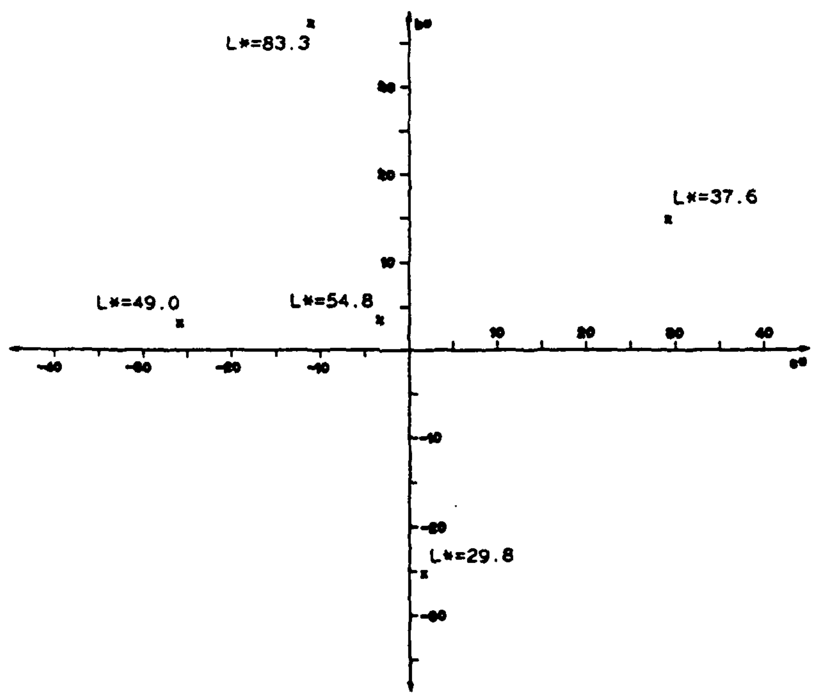


FIGURE 5. CIE Recommended Color Centers

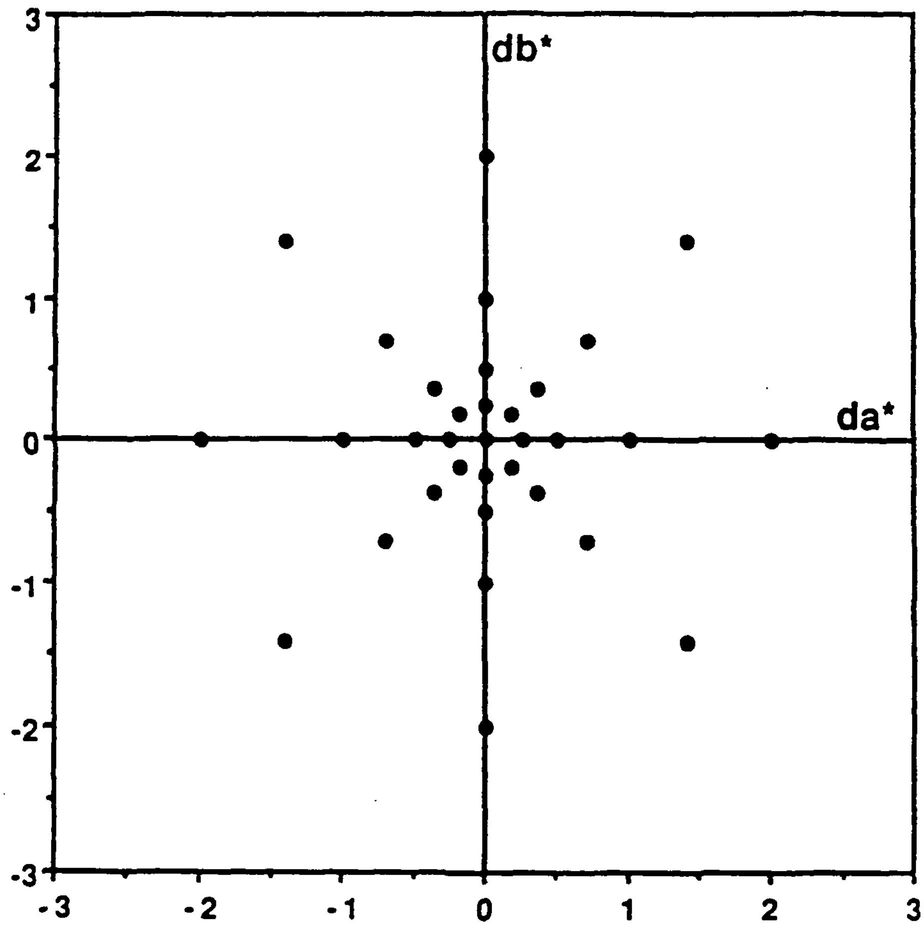


FIGURE 6. Color Sampling Design within a Color Center.

combinations were produced to provide adequate color-difference sampling around the ΔE^*_{ab}=1 magnitude. When producing color-difference pairs an effort was made to evenly distribute sampling along the range of -2 to 2 so as not to bias any segment of the sampled vector space.

For probit analysis to be applicable in this experiment, the samples selected along each color vector had to form a collinear set in each instance. To insure this criterion was met, eigenvectors were calculated for each of the 45 color vectors. Principal components analysis was applied for this determination to identify the fraction of variance associated with the principal direction of color-difference change. By confirming a coaxial sampling distribution the observed responses could suitably be described with a univariate distribution model.

In order to collect tolerance data typical of commercial color decisions (super-threshold), a standard or anchor color-difference magnitude pair of ΔE^*_{ab}=1 was employed throughout the experiment. Judgements regarding the acceptance or rejection of color-difference stimuli were based on a visual comparison between the stimulus pair and the anchor pair. By comparing different magnitudes of color differences to a known standard, we were able to determine the visual equivalent difference for each of the color vectors. The anchor pair selected for comparison was a near neutral gray varying in all color dimensions

(L^*,a^*,b^*) so as to excite all channels of the visual system. This type of variation in the anchor pair duplicated lightness and chromatic changes that occurred in the sample pairs.

Because the median tolerances of concern lie on the sloping segment of the sigmoidal response curve (Figure 3) it was critical that this region be the concentration of our sampling effort. In order to coarsely locate this area of the curve for each color vector a preliminary observation experiment was performed with a small population (relative to the subsequent main experiment) and limited color-difference sampling. By preceding the main observation experiment with a pilot test we tailor fit the final sampling for each color vector to insure that the region of the response curve undergoing change was adequately bracketed.

Materials and Equipment

The major tools employed in this experiment were the color sample materials and a light booth. Both required significant amounts of preparation and measurement prior to executing the observation experiments.

Color samples were produced and contributed to this effort by E.I. Du Pont De Nemours & Company (Inc.), Research & Development Division, Troy, MI courtesy of Dr. David H. Alman, Research Associate with the Color Operations Group. These materials were then used in the fabrication of color-difference samples used in the observation activities.

A light booth provided the proper and constant illumination environment used throughout the sample observations. This valuable piece of equipment was provided by the RIT Munsell Color Laboratory.

Color sample materials used in this activity were composed of glossy acrylic paints sprayed on an aluminum substrate or tile. This paint type provided a stable and durable color medium essential to providing an undeviating color-difference sample set through the duration of the experiment. For each of the nine color centers sampled, five vector sets were produced according to the intended design. Within each center the first vector (Set A) was perpendicular to the plane formed by the remaining vectors and incrementally sampled color exclusively in the L^* dimension. The other four vectors (Sets B,C,D,E) incrementally sampled chromatic differences along directions depicted in Figure 6 and described earlier in the experimental design. For a given color set, all of the samples were prepared from combinations of four colorants to produce simple, nonmetameric spectral variations within color-difference pairs.

A single gray vector set, different from the gray color center vectors, was used to fabricate the near neutral anchor color-difference pair employed as a judging standard. This anchor vector changed in all three color dimensions: decreasing L^*, b^* and increasing a^*. This vector was sampled in the same increments as the color center vectors. A pair of samples was selected from this anchor set to produce a color-difference of approximately $\Delta E^*_{ab}=1$.

An adequate number of replicates (2 to 3) for any given vector sample point were provided. This allowed room for mistakes or damage in cutting the aluminum tiles to the proper size. Also, since each replicate had a slightly different color coordinate, some variability was built in to allow the selection of the best sample point in tailoring color-difference pairs.

Color measurements were performed on all color tiles received prior to the observation experiments. Measurements were taken on a Hunterlab Labscan Spectrophotometer (Model LS-5100) containing a circular interference wedge monochromator and a silicon photodiode detector. All measurements took place using a single beam, 0/45 circumferential geometry. Illumination was provided by a filtered quartz-halogen, D_{65} simulator. Prior to measuring color tiles the spectrophotometer was allowed to warm-up for at least forty minutes. Initial standardization and calibration of the equipment was accomplished as directed in the instrument manual before sample measurements took place. During the measurement procedure, the instrument was zeroed and standardized again after approximately every thirty tiles. A five centimeter (cm) aperture was placed on the sample port of the instrument for all measurements.

Following color measurements, tiles were sized so that two sample tiles placed side-by-side subtended a ten degree angle from the eye of an average observer under experimental conditions. With a sample pair placed in the center of the light booth and the observer seated in such a manner to permit a viewing geometry of 45^{o}, the average distance, L, from eye to sample (Figure 7) was measured and the required sample size calculated to produce a 10^{o} pair. Sample tiles were calculated to be 6.5 cm on a side in order to subtend a 10^{o} Standard Observer viewing angle.

It was important when preparing the color-difference pairs that the two adjacent sample tiles provide an exact border to allow precise observer assessment without unnecessary distraction. Since the tiles to be used in sample pair preparation required cutting to proper dimensions, a precise instrument was utilized to minimize damage to the paint finish of the sample edges. This was accomplished using a hydraulic sheet metal cutter located at the RIT City Center Campus machine shop.

Once cut to size, the samples were paired to obtain desired color-difference values. Sample pairs were then mounted adjacent to one another on a gray aluminum tile

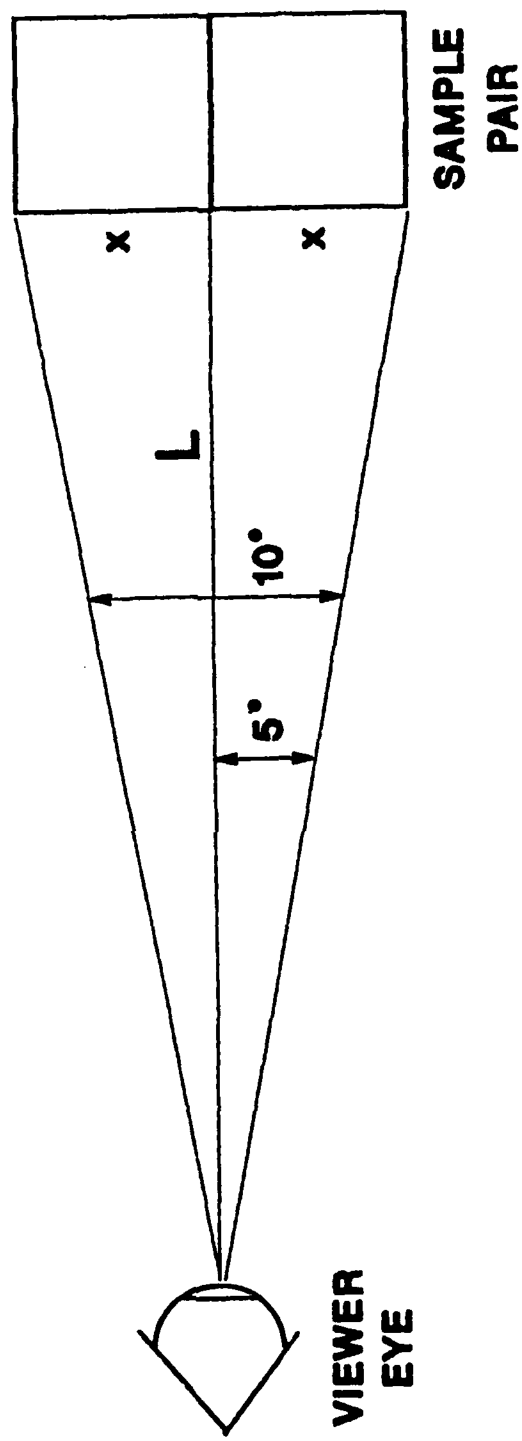


FIGURE 7. Configuration for Determination of 10° Viewing Dimensions

blank (the same as used in sample preparation). Double-sided tape was used to adhere the samples flat to the gray blanks and obtain a tight, clean adjoining interface between samples. The gray mounting tiles were measured with the Labscan Spectrophotometer to be,

$L^* = 38.26 \pm 0.22$
$a^* = -0.76 \pm 0.07$
$b^* = -0.71 \pm 0.17$

using 25 randomly selected tiles. The actual sample configuration is shown in Figure 8.

The tolerance judgements were all made under simulated D_{65} illuminant. This viewing environment was provided by a MacBeth Spectralight SPL-65 (Model PPM 01) booth using a filtered tungsten source. In order to provide a constant viewing background, gray matte board was placed in the bottom of the booth to match the gray tile mounts as closely as possible. Using the Labscan Spectrophotometer the matte board measured: $L^*=39.41$, $a^*=0.94$, $b^*=1.51$. To minimize reflection from the booth's back panel a piece of black felt cloth was used as a cover.

After configuring the light booth a spectral power distribution was performed to measure how well the booth simulated Illuminant D_{65} spectral characteristics.

Sample Observations

This section will be discussed in four parts: observer population, judgement presentation, pilot test,

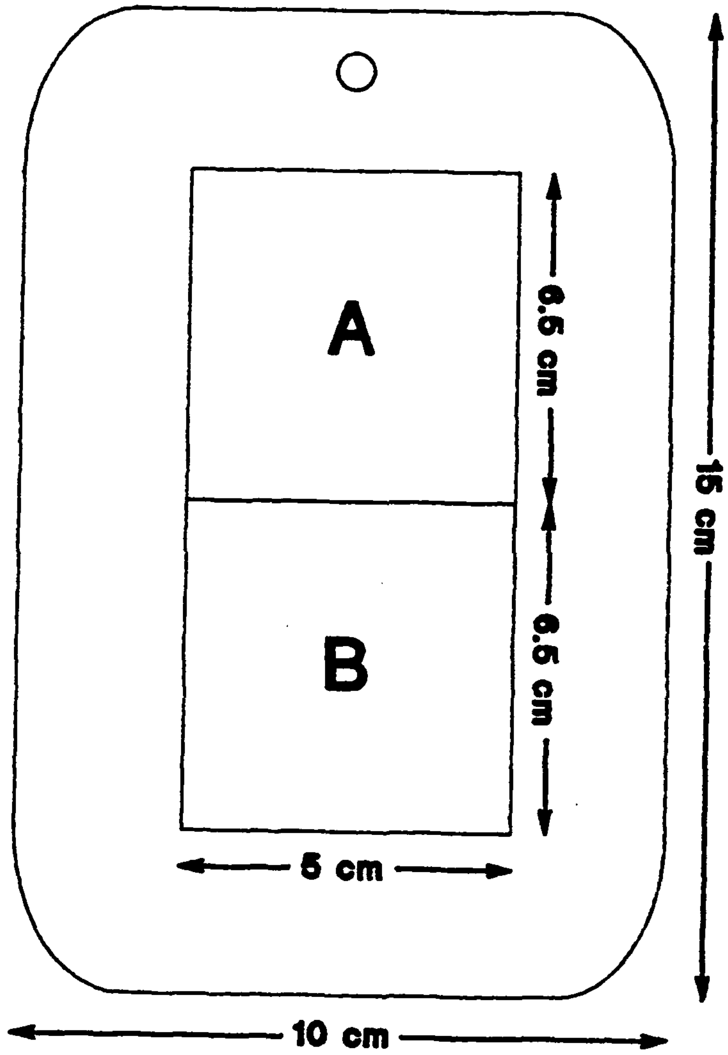


FIGURE 8. Color-Difference Sample Configuration

and main experiment.

Fifty observers participated in the experiment to obtain a statistically adequate population size. Two criterion were satisfied when selecting observers to judge: 1) they had no past experience in judging color differences or color matching, and 2) they were determined to be color-normal observers. Visual anomalies were tested for prior to the first sample viewing using a Dvorine (Ishihara) Test containing fifteen plates. The population was comprised of mostly college students, with no professional or experienced color matchers included in this group.

In order to acquire a consistent response dataset, observer preparation and instruction were uniformly administered to minimize variability of understanding and performance. In addition, the viewing sessions were accomplished in multiple, short segments of 15-20 minutes each to prevent observer fatigue and resultant poor decisions. A strong effort was made to treat each observer equally and expose each to identical conditions under which to perform.

All judgements were made using the same light booth described earlier under simulated D_{65}. Each observer was allowed at least three minutes to become adjusted to the illumination environment of the booth. During this visual adjustment, seating was arranged to permit comfort and to insure a 45^{o} viewing angle with the center of the light booth's bottom panel where comparisons would take place. Only after being properly acclimated to the viewing conditions were the observers allowed to start testing and making color-difference comparisons.

For first time observers a Dvorine Test and a D&H Test were administered and the results recorded along with the observer's age. Subsequent observation sessions did not repeat these visual testings. The Dvorine Test consisted of fifteen plates for which all candidate observers were required to obtain a perfect score. If any plates were missed a second Dvorine Test was given using fifteen different plates. Individuals not receiving a perfect score on the second test were dismissed as not color normal and omitted from further participation. The D&H Test was not used to determine color normalcy and was not used to exclude candidates from viewing.

In all viewing sessions, observers were provided with procedural ground rules prior to making comparisons. Observers were instructed to place the anchor pair flat in the center of the booth. Next, they placed the sample color-difference tile to be judged next to the anchor as shown in Figure 9. The observers were then directed to evaluate the visual color-difference magnitude of the sample pair with respect to the color-difference magnitude of the anchor. Observers were asked to judge the sample pair as acceptable if it appeared to have a smaller color-difference than the anchor or reject the sample pair if it appeared to have a larger color-difference than the anchor. The individual was asked to place the sample tile in one of two containers labeled either "ACCEPT" or "REJECT" depending on the decision and to continue the procedure with the next available sample tile. Comparisons of color-difference sample pairs with the anchor were accomplished one at a time and in the order presented. Observers were asked not to persist on a particular sample for more than about five seconds but to proceed with their initial interpretation.

Presentation of the sample tiles was ordered so as to minimize superfluous psychological contributions. Therefore, the experiment was arranged to diminish the biasing phenomenon associated with the prolonged viewing of a single color and its affect on the subsequent viewing of a different color. For this reason, color-difference sample tiles were presented to the observer in a random fashion so that the eye did not become "tuned" to any particular color. Samples were randomized using an International Mathematical and Statistical Libraries Inc. (IMSL) routine called GGPER.[25]

The observation experiment was executed in two phases, the first of which was a pilot test. The primary objective of the pilot test was to roughly approximate the median color-difference tolerances so the main experiment could

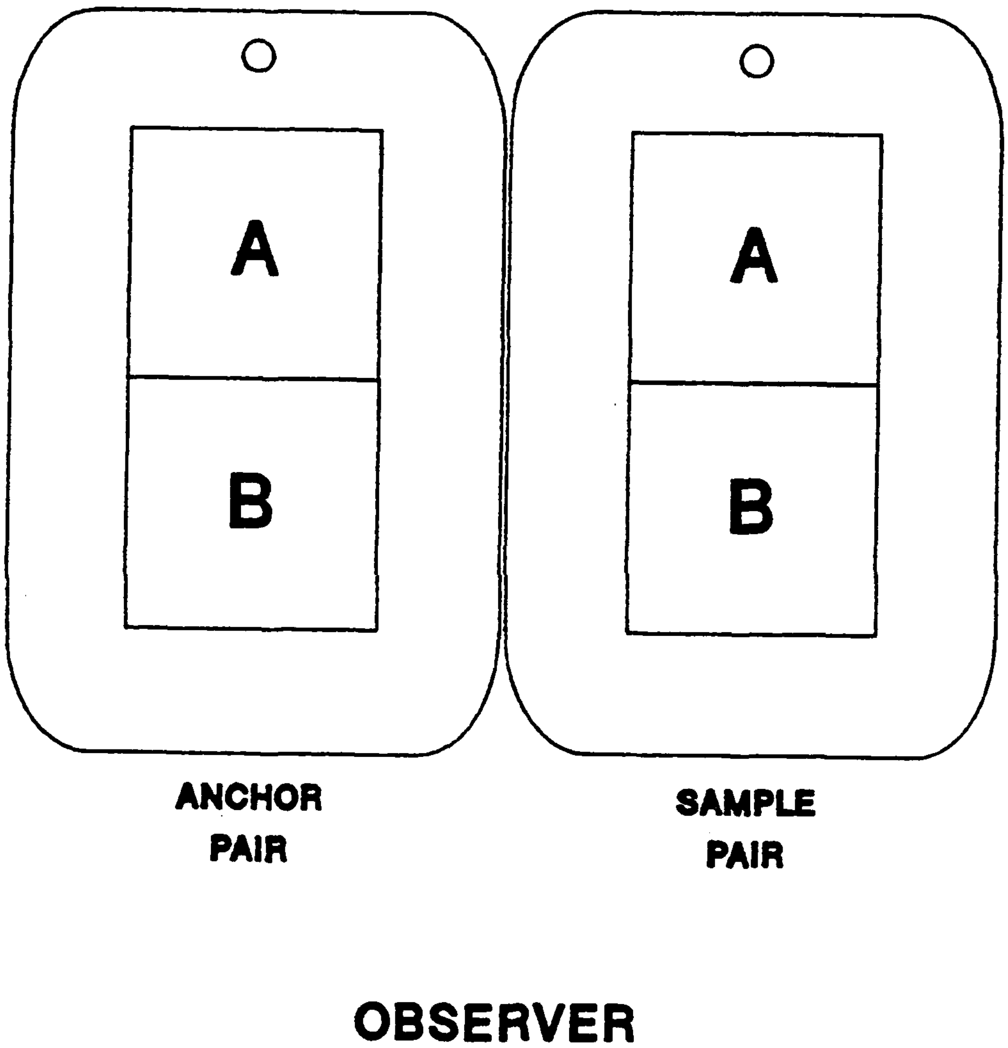


FIGURE 9. Color-Difference Anchor to Color-Difference Sample Viewing Arrangement

properly bracket the region of interest along the response curve. The secondary goal achieved by the pilot was to test the presentation/viewing procedure to detect any flaws or problems that were not anticipated.

For this initial testing of tolerance levels approximately the same color-difference samples were produced for each color vector. In order to evaluate a wide range of possible color differences the sampling increments were designed to include ΔE^*_{ab}s of 0.5, 1, 2, 3, 4 for all vectors. These color-difference pairs were prepared using the procedure described earlier. Ten observers were used to judge these 225 color-difference samples against the same anchor pair later used in the main experiment. Samples were presented in the same randomized order for all subjects. Observers judged samples in a variety of session numbers and lengths to determine the number of samples an observer could view in a single session before becoming fatigued.

Approximate color-difference tolerances were determined by inspection of the pilot test results. All but four vectors showed values of reasonable tolerance magnitudes. The four exceptions (Blue, E-Set; Green, B-Set; Purple, E-Set; Yellow, C-Set) displayed larger color-difference tolerances than expected. In double-checking the preparation, measuring and recording of samples used for these vectors no deviations were found and so was determined to further test these four color vectors using the same color-difference samples but increase the observer population to twenty. The results of this expanded pilot test provided more reasonable tolerances with which to prepare samples for the main experiment.

Sampling for the main experiment was tailored according to the findings of the pilot test. The number of color-difference samples prepared for each vector was kept to between six and nine based on the confidence with which the pilot tests estimated the location of the median tolerances. The main experiment consisted of 317 samples spread through all 45 color vectors. Fifty observers participated in this phase viewing the total number of samples within four separate sessions.

To avoid color biasing, sample pairs were randomly ordered for presentation to observers. A random list from 1 to 317 was generated using the IMSL routine mentioned earlier, and divided into four groupings of 79, 79, 79, 80 to make up the blocks of samples to be presented in the four sessions. The four blocks were then presented in random order to vary the experience level at which different observers viewed the same block of samples. By altering the order of presentation from one participant to the next, blocks were viewed at a similar mix of observer experience.

RESULTS

Color Sample Measurements

All color tiles used in this experiment were measured to determine CIELAB coordinates L^*, a^*, b^* prior to the fabrication and viewing of color-difference samples. Color measurements were performed using the spectrophotometer described earlier in the Experimental Section. Calibration and standardization of the instrument were accomplished with strict adherence to procedures stated in the equipment operating manual before and during the measurements. Tiles were handled with white cotton gloves at all times to avoid fingerprints and smudging that might alter the measurement results. During this procedure tiles were visually inspected for defects such as warping or blemishes caused in preparation that could affect accurate measurement. Tiles discovered to have such imperfections were properly noted and recorded. Of the approximately 1500 color tiles measured only five were noted to possess such physical anomalies and none of these were subsequently used in the preparation of color-difference samples.

The yellow color center required three attempts to produce a complete set of five satisfactory color vectors. In the first iteration vectors B ($-a^*$ to a^*), D ($-a^*$ $-b^*$ to $+a^*$ $+b^*$), and E ($-a^*$ $+b^*$ to $+a^*$ $-b^*$) were the only useful sets produced. Sets A ($-L^*$ to L^*) and C ($-b^*$ to b^*) were determined unsatisfactory due to excessive a^* change in both. In a second attempt to produce A and C, success was met only for the latter case. A third attempt was required to finally produce a useful yellow lightness vector.

In producing the near neutral anchor vector used to select a color-difference standard for comparison, success took two attempts. The first try did not produce enough a^* change along the color vector to effect an anchor pair changing in all three dimensions of CIELAB space.

Color-difference magnitudes used for the pilot test were systematically selected for each color vector. In all 45 color vectors, five samples were prepared for each having color-differences of 0.5, 1, 2, 3, and 4. The initial spectrophotometer measurements were used to calculate the exact ΔE^*_{ab} values for each sample pair. These values are found in Appendix A for each sample pair used in the pilot including the anchor pair.

Because of the problems in producing satisfactory A and C vectors for the yellow color center some concessions were made in producing color-difference pairs. For the yellow C color set, samples were prepared from the second attempt which had been successful. The yellow A set required three attempts to produce a usable vector set. However, to avoid delays in receiving samples from the third attempt it was decided to use the first attempt for the pilot. This still allowed for a rough determination of the threshold point along the vector even though an optimum vector direction was not employed.

The selection of color-difference pairs for the main experiment varied by vector set based on the rough tolerance levels determined in the pilot test. Exact measurements of the selected color-difference sample pairs and anchor pair used in this phase of the study are tabulated in Appendix B.

Principal Components Analysis

In order to employ probit analysis on the results of the main experiment, each of the sampled color vectors was required to be linear in nature. This requirement existed for the reason that the tolerance determinations be based on a single variation of stimulus in each vector set. To determine the degree of linearity in each vector, principal components was applied to determine the eigenvalue of the dominant vector direction. A principal components analysis routine provided by the SAS Institute, Inc.[26] was used to determine the eigenvector/eigenvalue of the first component within CIELAB color space. The results of this analysis are contained in Table 1.

Light Booth Measurements

Throughout this experiment all sample observations

TABLE 1. Principal Components Analysis Results

Color Vector		Eigenvector dL*	da*	db*	Eigenvalue
Blue	A	0.985	0.173	-0.013	0.9987
	B	-0.017	0.997	-0.078	0.9989
	C	-0.134	0.295	0.946	0.9990
	D	-0.018	0.720	0.694	0.9957
	E	-0.042	-0.339	0.940	0.9995
Cyan	A	0.998	0.032	-0.044	0.9997
	B	-0.076	0.996	-0.049	0.9997
	C	-0.078	0.351	0.933	0.9993
	D	-0.007	0.813	0.582	0.9982
	E	0.018	-0.486	0.874	0.9993
Gray	A	0.999	0.041	0.025	0.9995
	B	-0.084	0.990	-0.114	0.9984
	C	-0.029	0.099	0.995	0.9968
	D	-0.007	0.763	0.647	0.9981
	E	-0.007	-0.644	0.765	0.9968
Green	A	0.997	0.083	0.005	0.9993
	B	0.004	0.996	-0.090	0.9969
	C	-0.010	0.266	0.964	0.9960
	D	0.012	0.847	0.531	0.9925
	E	0.052	-0.666	0.744	0.9992
Orange	A	0.993	0.086	0.082	0.9987
	B	-0.049	0.993	0.105	0.9987
	C	0.068	0.138	0.988	0.9990
	D	-0.021	0.751	0.659	0.9996
	E	-0.069	0.747	-0.662	0.9979
Purple	A	0.995	-0.005	0.096	0.9994
	B	0.030	0.991	-0.134	0.9978
	C	0.058	0.087	0.995	0.9968
	D	0.064	0.815	0.576	0.9955
	E	-0.066	-0.609	0.790	0.9991
Red	A	0.999	-0.031	-0.014	0.9933
	B	0.014	1.000	0.002	0.9959
	C	-0.015	-0.004	1.000	0.9959
	D	0.008	0.689	0.725	0.9985
	E	-0.041	-0.616	0.787	0.9953
Yellow	A	1.000	0.015	0.022	0.9985
	B	0.058	0.985	0.162	0.9991
	C	0.130	0.079	0.988	0.9995
	D	0.160	0.637	0.754	0.9933
	E	-0.009	0.790	-0.613	0.9990
Yellow-	A	0.992	-0.117	0.038	0.9995
Green	B	-0.010	0.989	-0.149	0.9961
	C	-0.033	0.008	0.999	0.9996
	D	-0.163	0.729	0.664	0.9971
	E	0.084	-0.559	0.825	0.9997

were performed under simulated D_{65} illuminant. This was accomplished by use of the MacBeth light booth described in the Experimental Section. To insure the booth used for sample viewing accurately simulated the intended illuminant, a spectral power distribution was performed to characterize the illumination actually generated by the booth. Figures 10 and 11 portray the expected distribution for D_{65} and the measured distribution, respectively.

Pilot Test Response

A preliminary test response was conducted to coarsely determine the location of tolerance levels visually equivalent to the $\Delta E^*_{ab}=1$ near neutral anchor pair. An initial population of ten color-normal observers was employed to view 225 color-difference pairs.

Prior to making visual comparisons all observers were required to pass a Dvorine Test in order to establish color normalcy. In all ten cases a perfect response was scored by the observer on the test.

After allowing sufficient time to adjust to the viewing environment of the light booth (at least three minutes), and being properly instructed on the rules and criterion for judging, observers were presented color-difference samples in a random order of colors and magnitudes. Following the viewing session, results were recorded of the visual judgements for each sample pair.

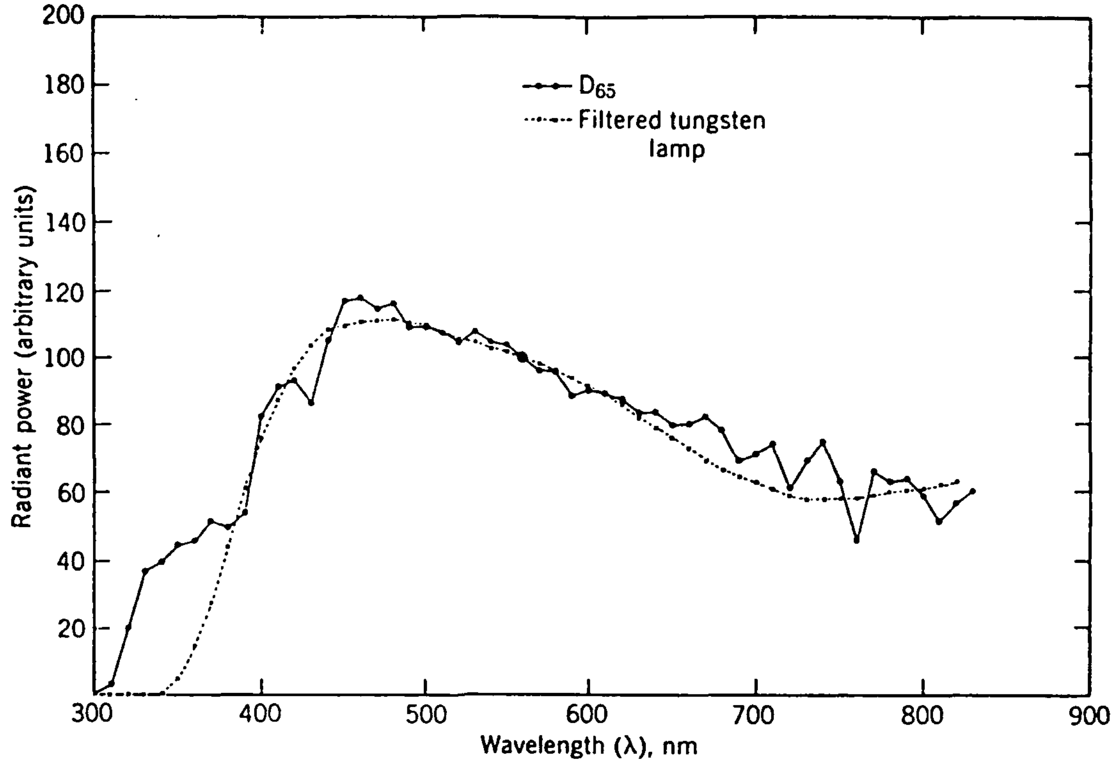


FIGURE 10. Spectral Distribution for CIE Illuminant D_{65}[27]

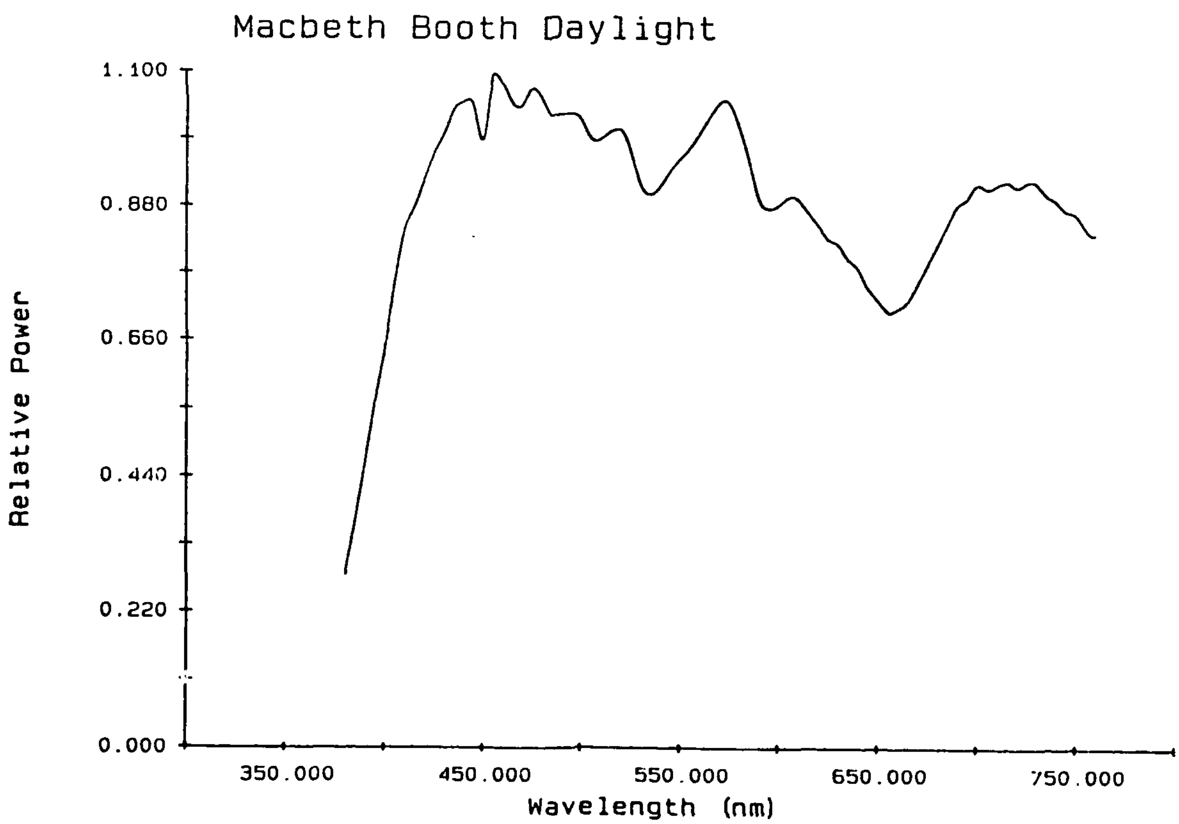


FIGURE 11. Measured Spectral Power Distribution for Light Booth Simulation of D_{65}

Appendix C contains the cumulative rejection response of the observer population for each color vector.

Approximate color-difference tolerance levels were identified based on inspection of the response frequencies collected from this preliminary test.

As a result of the pilot test four vector sets displayed unexpectedly high tolerances. Due to this outcome these vectors (Blue-E, Green-B, Purple-E, Yellow-C) were subjected to an expanded viewing population of twenty. The results recorded in Appendix C include this increased number of judgements for these four cases.

Main Experiment Response

The main experiment collected the visual responses of fifty color-normal observers for 317 color-difference stimuli judged against a near neutral anchor pair possessing a color-difference typical of industrial decisions. This required a total of 15,850 visual judgements just for this phase of the investigation. The results of these quantal responses were later used to estimate the color-difference tolerance perceived to be equivalent to a ΔE^*_{ab} approximately equal to one CIELAB unit.

As in the pilot test, all first time observers were administered a Dvorine Test to determine color-normalcy prior to the first viewing session. With one exception, all observers achieved perfect marks when given this test.

Observer #27 erred on one plate in the first testing attempt, but received a perfect score on a second test with different plates.

The color-difference samples used were first randomized for the experiment. This random ordering was then divided into four blocks of samples. So not to bias any particular block by always presenting it first, the four blocks were arbitrarily presented until each observer had viewed all four blocks.

The observer responses for the main experiment are tabulated in Appendix D. A response in this experiment (pilot and main) was considered to be any rejection judgement made by the observer. A sample pair that passed or was deemed smaller in ΔE^*_{ab} than the anchor pair was considered a non-response. Appendix D lists the cumulative responses (or rejections) for the observer population.

Probit Analysis

The frequency of response for the population of the main experiment was evaluated using probit analysis. This approach was used to estimate the level at which half the population responds to the color-difference stimulus and half the population does not respond.

A SAS[28] probit routine was used to calculate the T50 point for each of the 45 color vectors. The three variables necessary to run this statistical program were:

ΔE^*_{ab}, number of observers, and number of observer rejections. The routine then transforms the information to a probit scale and a straight line is iteratively fit to the data points by maximum likelihood estimation. The software then provides numerical information such as probabilities of rejection for associated stimuli levels, standard deviation, chi-squared values (See Appendix F). Graphical depictions are also output of the sigmoidal response curve (probability vs. ΔE^*_{ab}) and the linear probit line (probit vs. ΔE^*_{ab}) fit to the data points. The results of this analysis are listed in Table 2.

The center of each color vector was based on the zero sample used to prepare sample pairs. If more than one of the provided replicates was used for a vector, the average values were used to determine the center point of the vector. If none of the zero replicates were used, the center was determined by averaging all the replicates provided. The color vector center points are listed in Table 3. From this center point the color-difference tolerance value, T50, and the upper and lower fiducial limits were located along the eigenvector in both directions. Because this experiment measured color-difference magnitudes, the tolerance levels in both directions are equidistant from the center point on each vector.

Tolerance determinations assumed a normal distribution. Therefore, the estimation of median tolerance values

TABLE 2. Probit Analysis Results

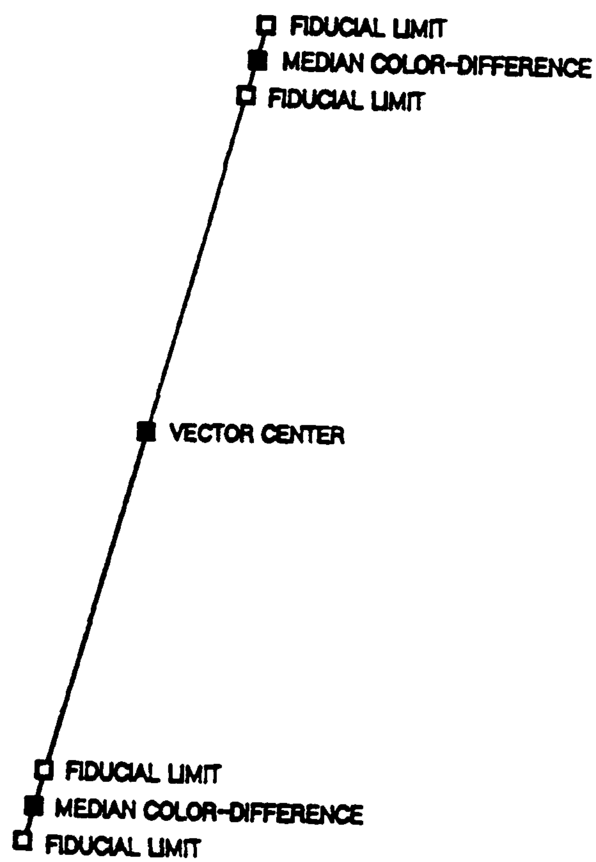
Color Vector		T50	(95%) Fiducial Limits Lower	Upper	Std. Dev.	d.f. (k-2)	(95%) Chi. Sq. (*=signif)
Blue	A	0.97	0.89	1.04	0.46	5	8.34
	B	1.36	1.28	1.44	0.47	5	7.96
	C	1.54	1.45	1.61	0.44	5	4.04
	D	1.12	0.97	1.28	0.38	5	13.94*
	E	2.81	2.58	3.03	0.78	6	12.28*
Cyan	A	0.80	0.66	0.93	0.35	5	12.07*
	B	1.62	1.54	1.69	0.42	5	2.07
	C	1.61	1.53	1.68	0.39	5	8.92
	D	1.79	1.59	1.98	0.59	5	10.86*
	E	1.51	1.40	1.62	0.58	5	5.72
Gray	A	0.94	0.88	1.00	0.27	5	3.74
	B	0.89	0.69	1.08	0.31	5	23.31*
	C	1.32	1.22	1.41	0.46	5	4.17
	D	0.91	0.86	0.96	0.25	5	1.19
	E	1.30	1.23	1.38	0.36	5	0.57
Green	A	0.95	0.89	1.01	0.31	5	4.83
	B	2.19	2.12	2.27	0.44	6	5.09
	C	1.30	1.24	1.37	0.37	5	4.53
	D	1.66	1.60	1.72	0.34	5	3.69
	E	1.77	1.71	1.84	0.40	5	1.17
Orange	A	0.90	0.84	0.96	0.32	5	7.76
	B	1.37	1.30	1.45	0.38	5	4.76
	C	1.46	1.30	1.61	0.39	4	8.60*
	D	1.60	1.51	1.68	0.47	5	3.60
	E	1.13	1.09	1.18	0.21	5	0.44
Purple	A	0.95	0.86	1.03	0.35	4	4.99
	B	1.48	1.41	1.54	0.34	5	7.69
	C	1.40	1.26	1.54	0.27	4	10.92*
	D	1.22	1.16	1.28	0.31	4	3.59
	E	2.84	2.74	2.94	0.54	7	4.69
Red	A	0.95	0.89	1.02	0.38	5	8.98
	B	1.94	1.88	2.00	0.34	6	7.71
	C	1.55	1.38	1.74	0.52	5	11.58*
	D	1.99	1.90	2.09	0.64	5	4.17
	E	1.33	1.25	1.40	0.39	4	2.53
Yellow	A	1.18	1.12	1.25	0.37	5	3.62
	B	1.45	1.38	1.52	0.41	5	2.09
	C	2.19	2.11	2.27	0.51	7	3.65
	D	1.61	1.55	1.68	0.35	4	3.14
	E	1.29	1.24	1.35	0.28	5	1.25
Yellow-Green	A	0.83	0.77	0.89	0.32	5	5.53
	B	1.14	1.08	1.20	0.32	5	5.92
	C	1.44	1.37	1.50	0.36	5	7.90
	D	1.21	1.15	1.26	0.28	5	1.98
	E	1.70	1.61	1.78	0.43	5	4.50

TABLE 3. Color Vector Center Positions

Color Vector		Center Position L*	a*	b*
Blue	A	36.11	- 1.36	-27.31
	B	35.83	- 1.40	-27.12
	C	35.86	- 1.53	-26.92
	D	35.96	- 1.89	-27.88
	E	36.26	- 1.97	-27.91
Cyan	A	50.61	-16.21	-11.22
	B	50.75	-16.17	-11.08
	C	50.64	-16.09	-11.00
	D	50.69	-16.19	-11.13
	E	50.82	-16.15	-11.10
Gray	A	59.76	- 1.00	1.28
	B	59.71	- 0.81	1.28
	C	59.50	- 1.04	1.21
	D	59.50	- 1.12	1.07
	E	59.45	- 0.99	1.24
Green	A	55.53	-27.11	2.82
	B	55.53	-27.13	2.80
	C	55.84	-27.20	2.76
	D	55.56	-27.23	2.78
	E	55.53	-27.06	2.77
Orange	A	63.55	12.35	20.89
	B	63.66	12.37	20.86
	C	63.67	12.18	20.78
	D	63.57	12.17	20.77
	E	63.71	12.18	20.75
Purple	A	46.46	12.74	-13.44
	B	46.29	12.73	-13.35
	C	46.25	12.55	-13.36
	D	46.45	12.70	-13.40
	E	46.51	12.67	-13.54
Red	A	42.29	36.02	20.43
	B	41.51	35.73	20.37
	C	41.20	35.85	20.66
	D	41.91	35.78	20.34
	E	41.89	35.91	20.42
Yellow	A	76.90	4.01	35.11
	B	78.34	1.36	37.23
	C	78.48	1.64	35.43
	D	78.60	1.17	34.27
	E	78.50	1.86	33.68
Yellow-Green	A	64.65	- 9.99	13.40
	B	64.80	-10.00	13.47
	C	64.72	-10.32	13.34
	D	64.91	-10.11	13.45
	E	64.82	- 9.94	13.35

did not use the logarithmic transformation to normalize the response distribution prior to applying probit analysis. The reason for this decision is examined in more detail in the Discussion section.

Using values estimated for each of the color vectors, graphical representations were produced to display the vectors' median color-difference tolerance, fiducial limits, and vector center in relation to each of the other four vectors within its color center. A sample vector is shown below. These plots are shown in Figures 12 to 20.



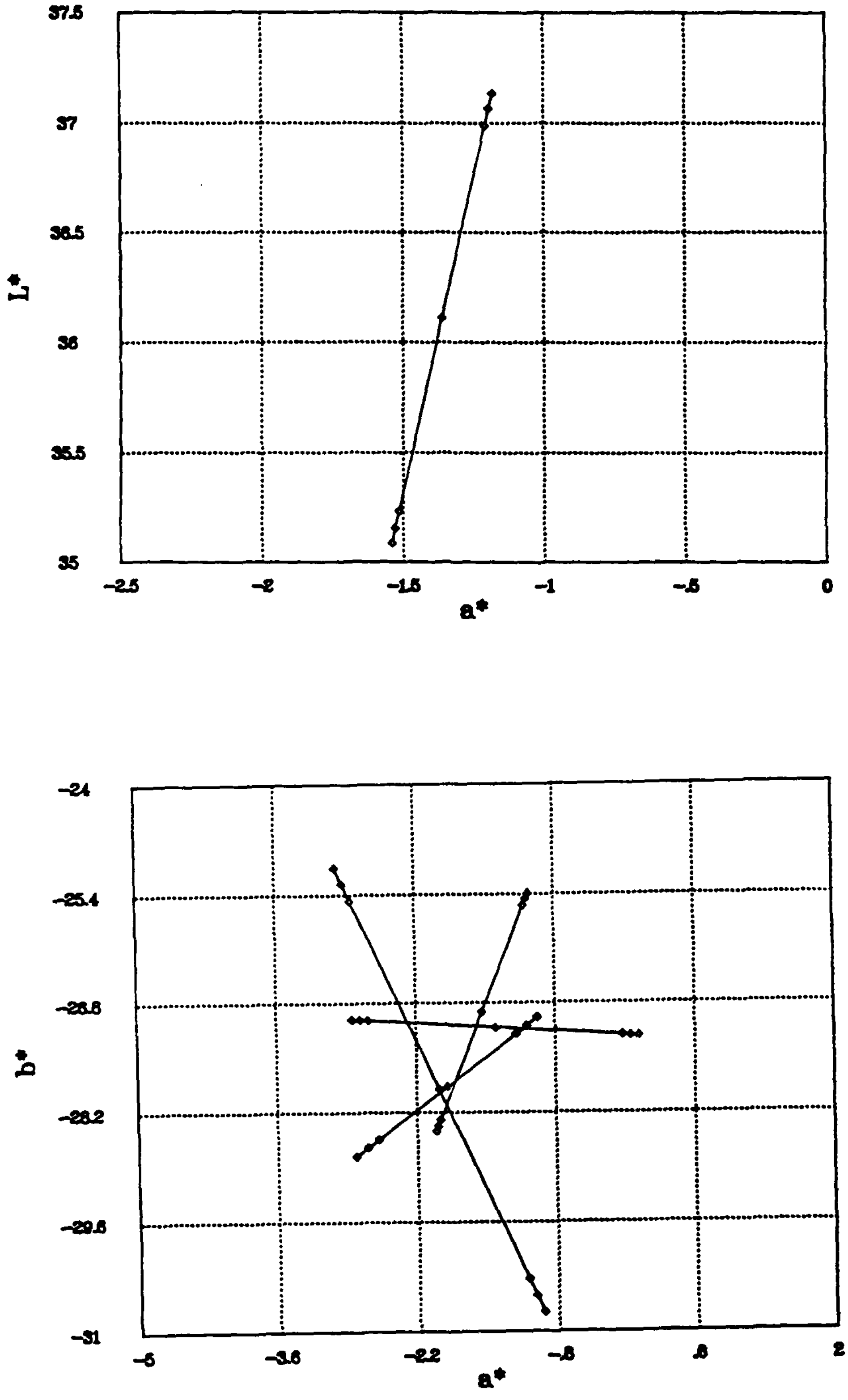


FIGURE 12. Blue Color Center Vector Tolerances

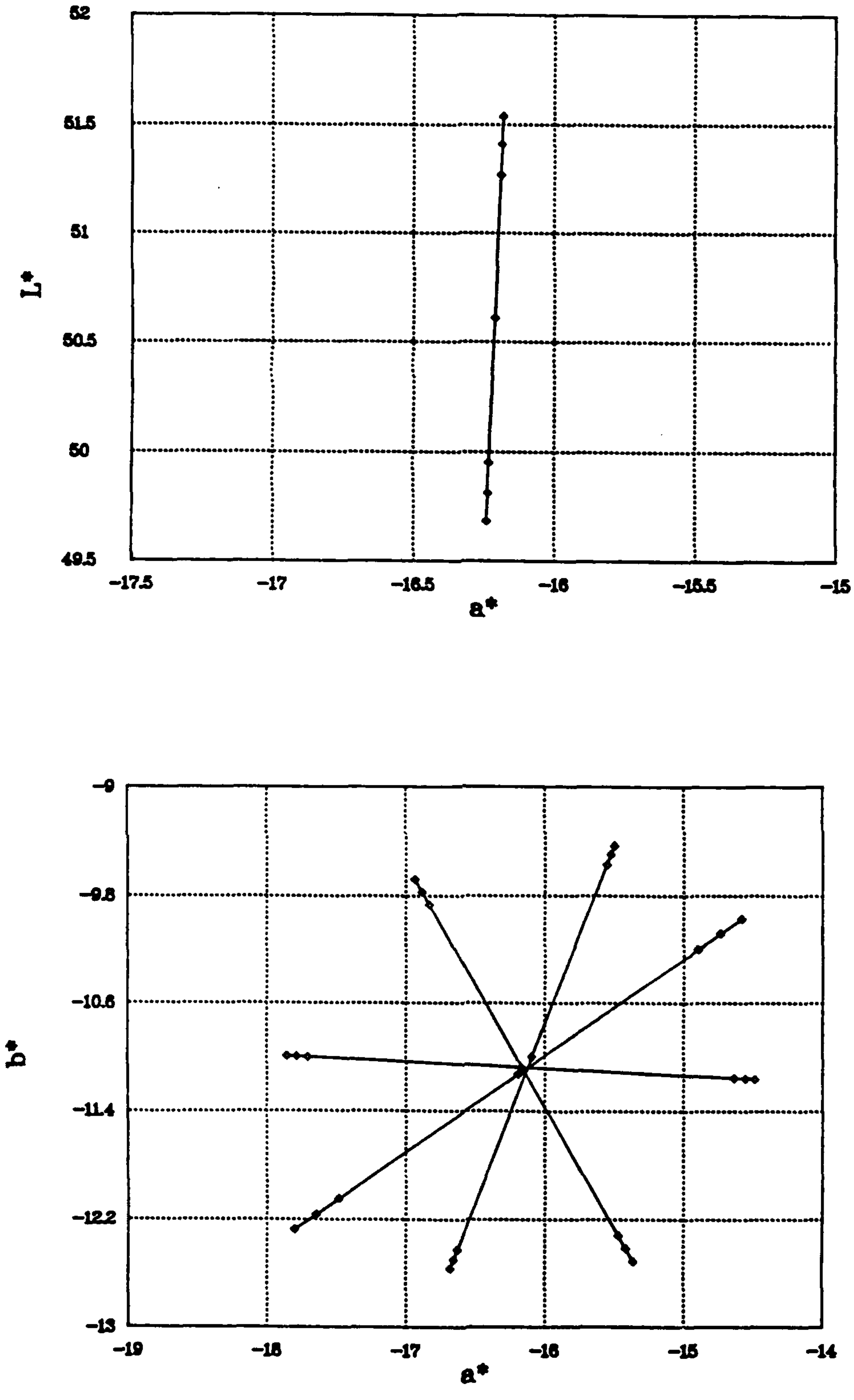


FIGURE 13. Cyan Color Center Vector Tolerances

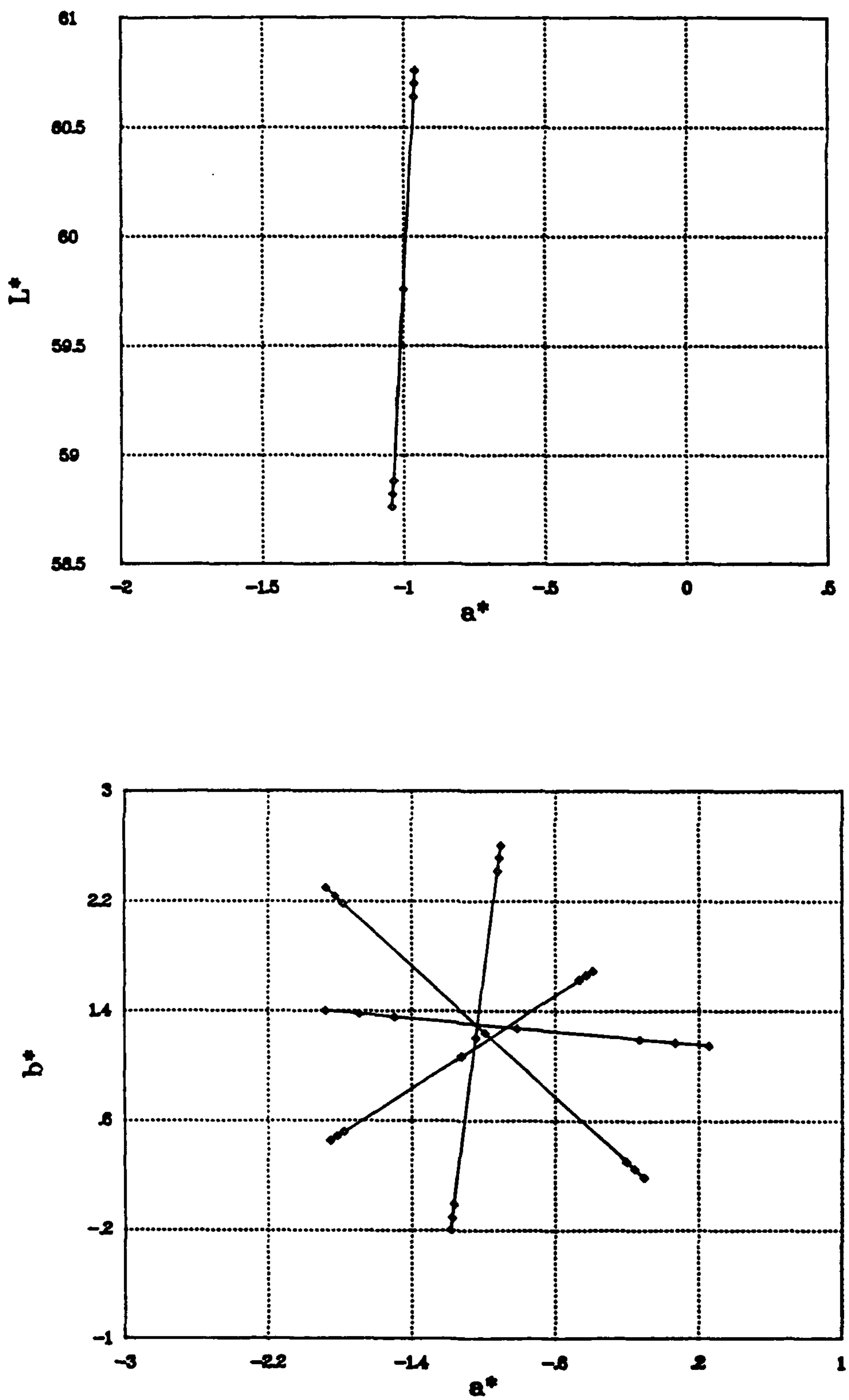


FIGURE 14. Gray Color Center Vector Tolerances

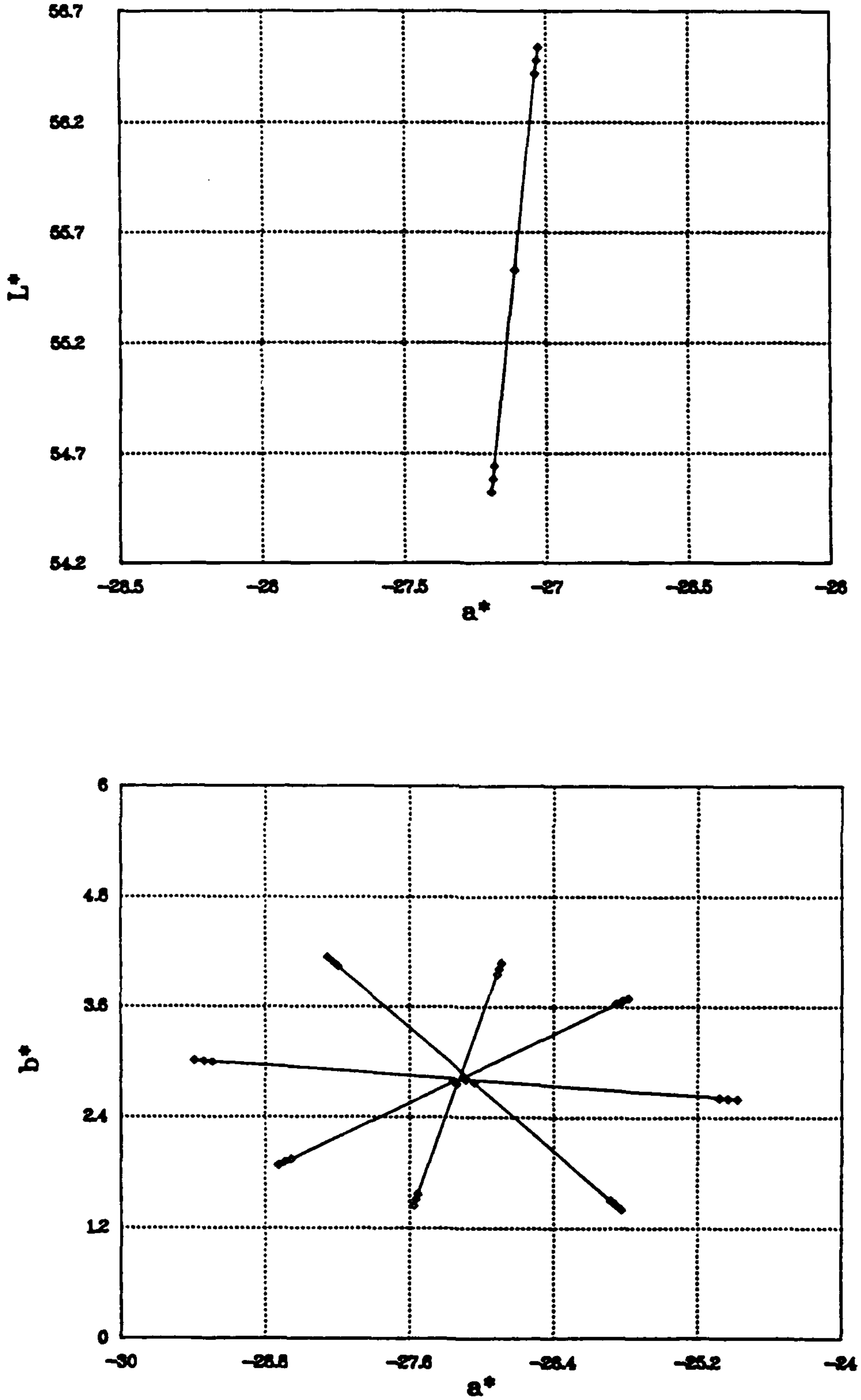


FIGURE 15. Green Color Center Vector Tolerances

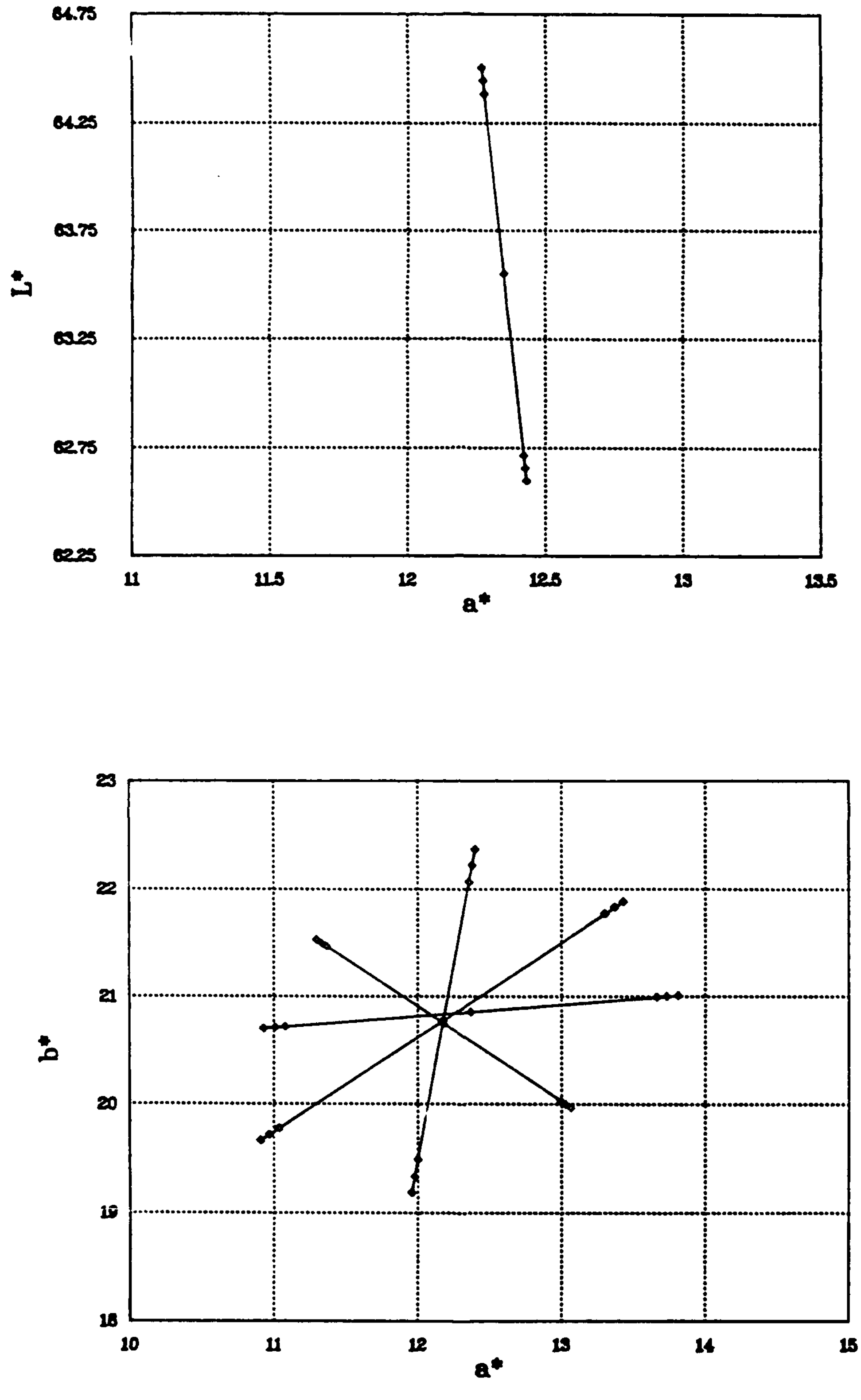


FIGURE 16. Orange Color Center Vector Tolerances

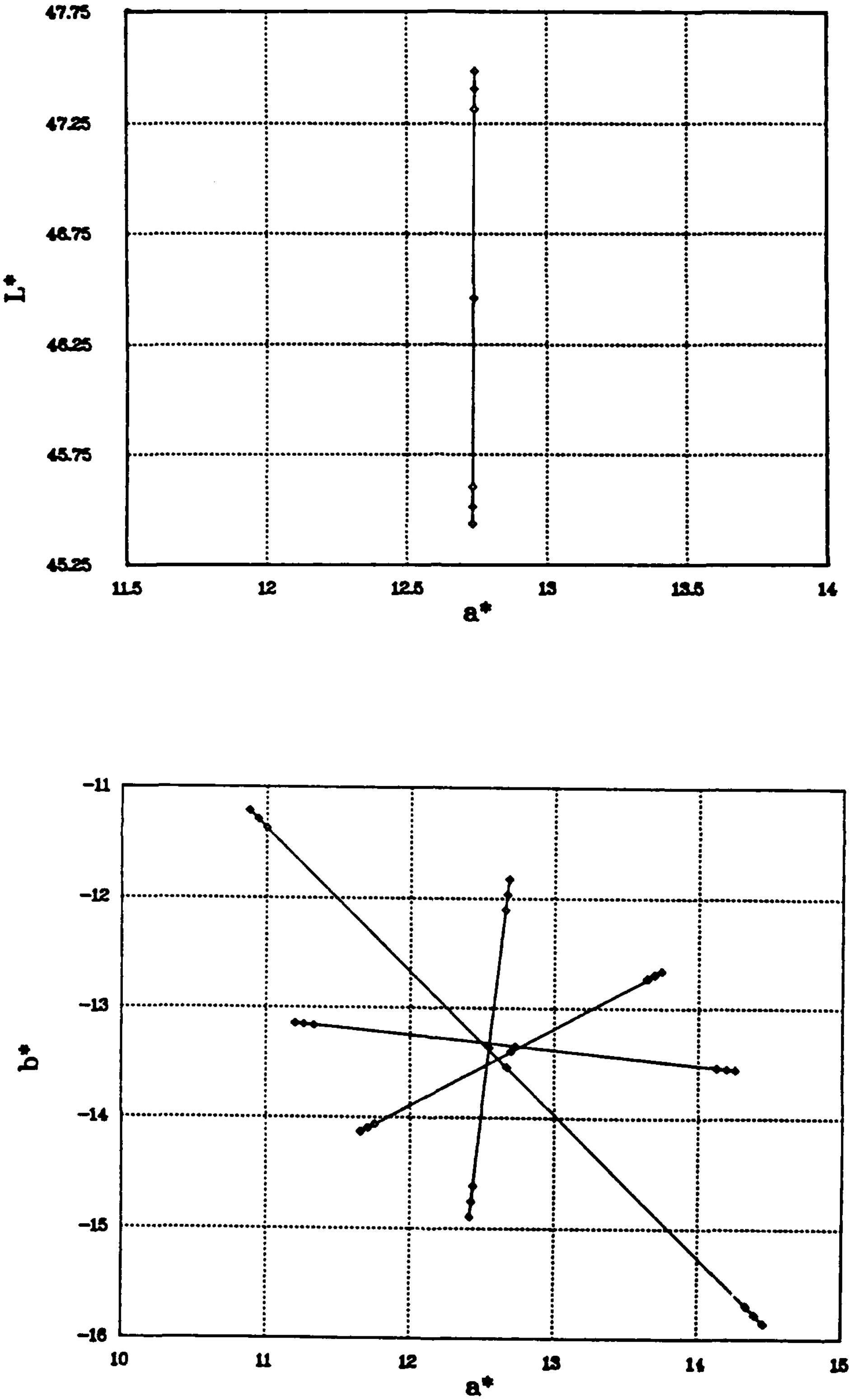


FIGURE 17. Purple Color Center Vector Tolerances

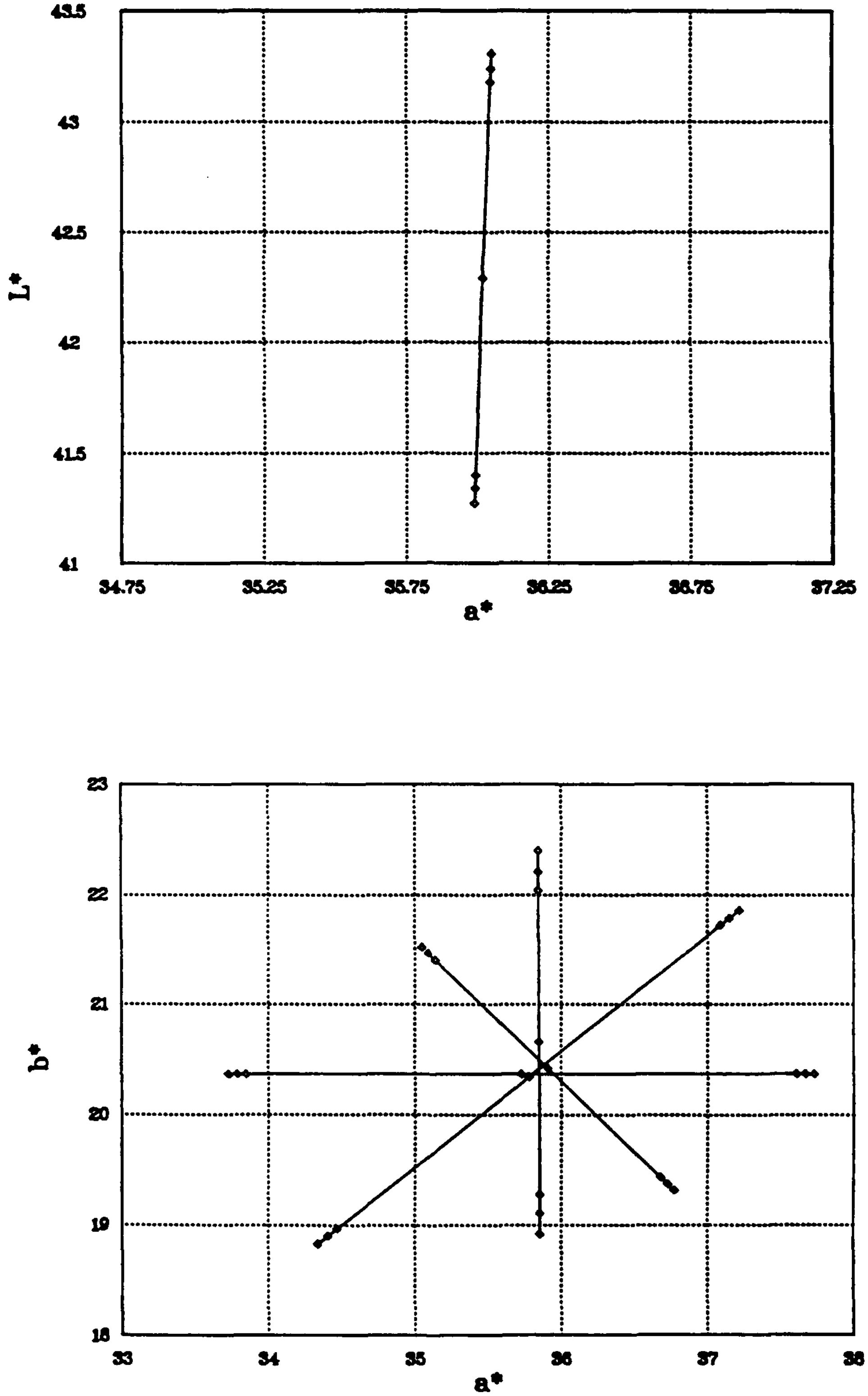
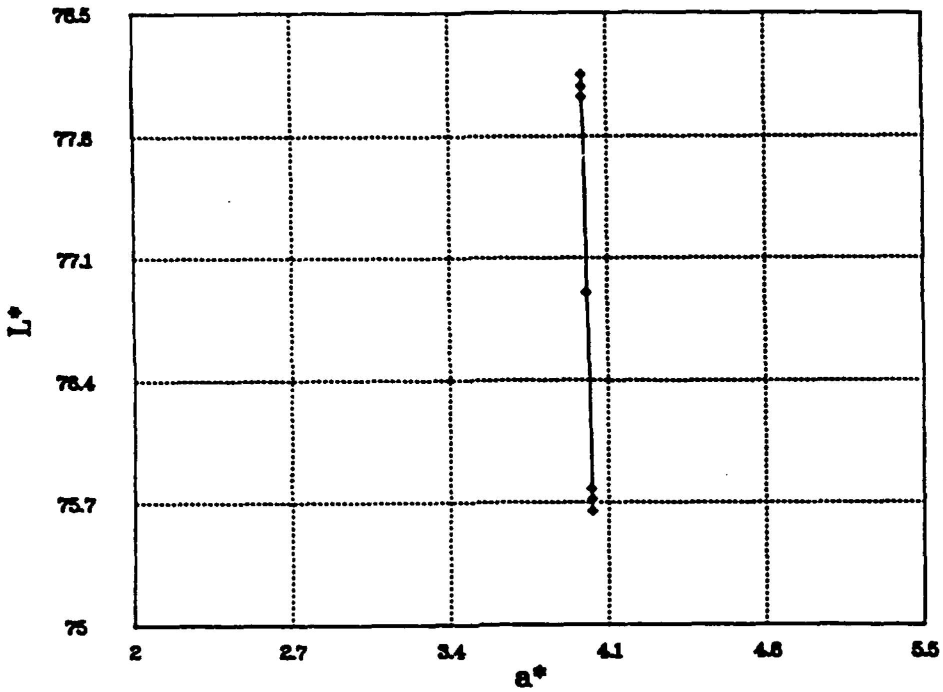


FIGURE 18. Red Color Center Vector Tolerances



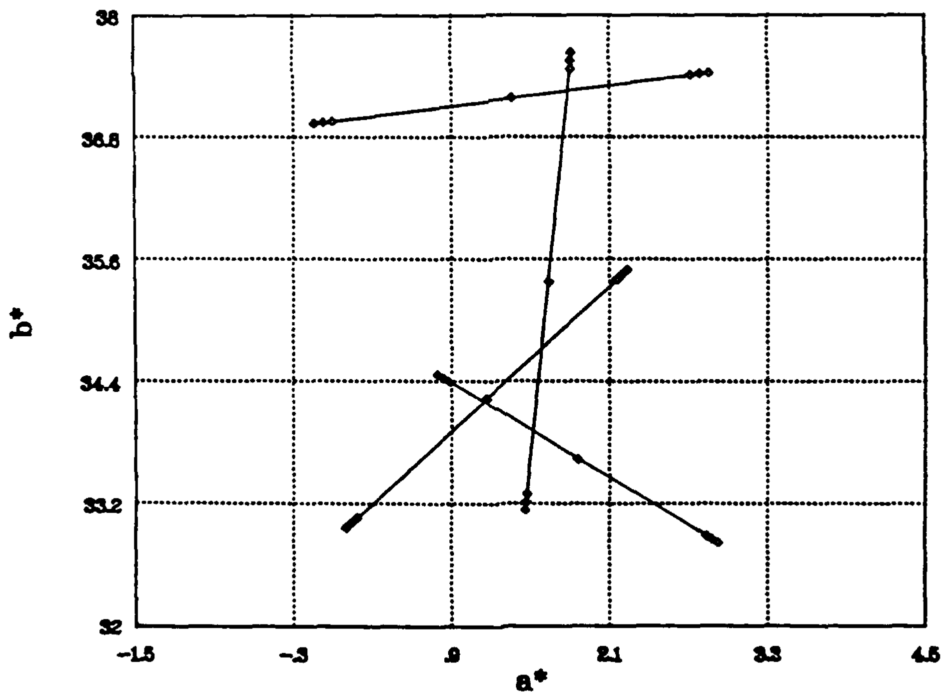
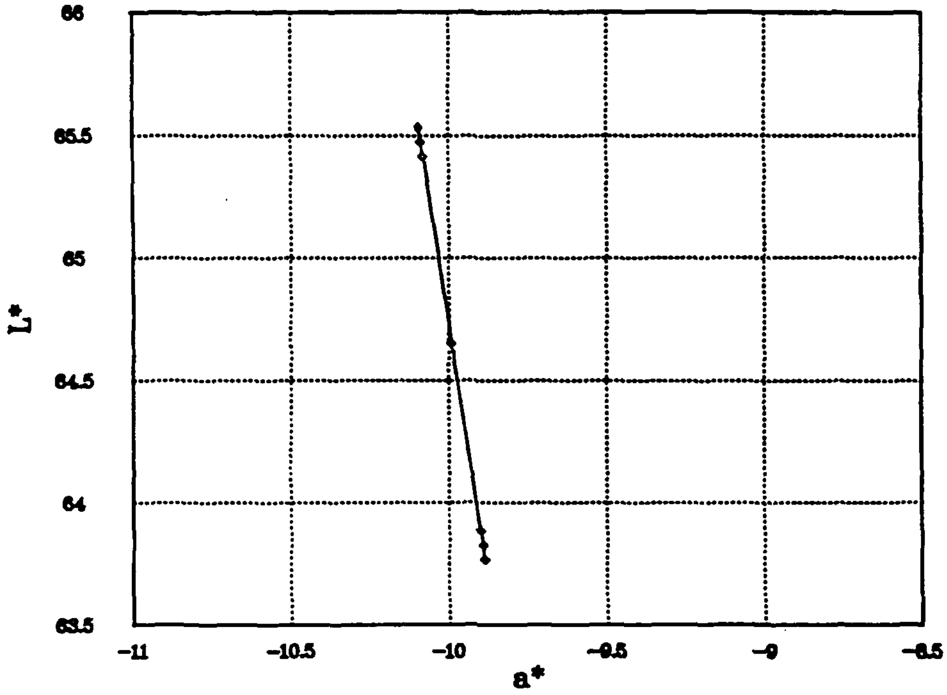


FIGURE 19. Yellow Color Center Vector Tolerances



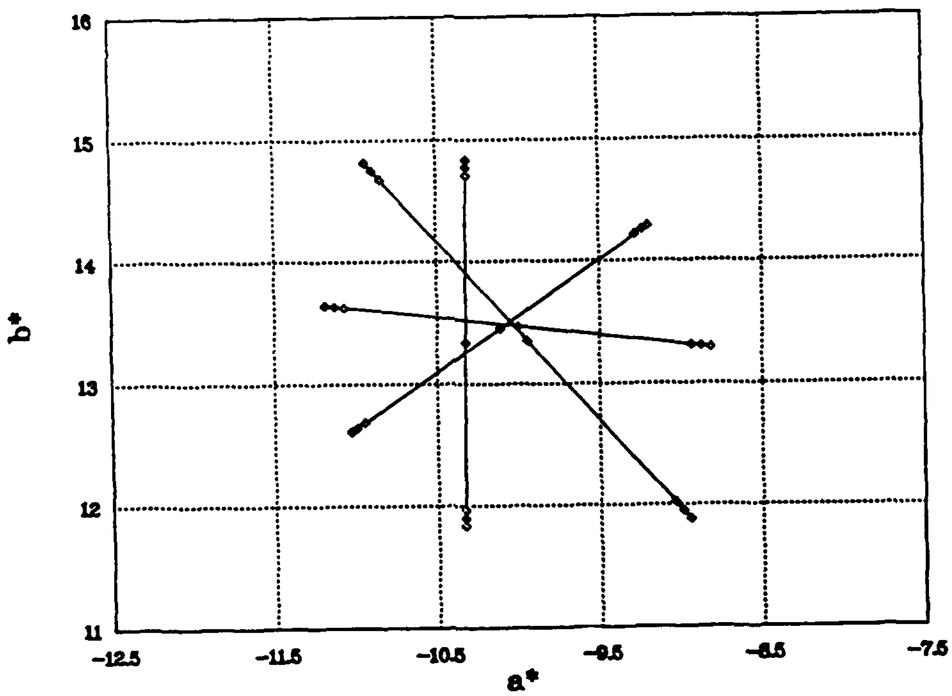


FIGURE 20. Yellow-Green Color Center Vector Tolerances

DISCUSSION

Experimental Design

A few critical assumptions were required in structuring this experiment to use probits in estimating color-difference tolerances. Most assumptions were based on knowledge gained from previous color investigations and were well founded.

One of these assumptions on which the experiment was based was that the perceptibility response of a color-normal population to color-difference magnitudes follows a normal distribution.[5,12] This is significant since the probit model is based on this same distribution and is a linear transformation of the normal distribution. This assumption is supported by the number of chi-squared values that indicate a good fit to the normal distribution. Only eight of the 45 vectors sampled did not exhibit a satisfactory chi-squared value. These eight vectors with significant chi-squared values are denoted with an asterisk (*) in Table 2.

The probit procedure provides a solution for altering distributions that do not conform to the normal distribution. This normalization is achieved by transforming the stimuli values to a logarithmic scale. This is a common technique in biological assay since tolerance distributions for given populations tend to possess a skewed profile.[27]

Because this experiment was performed on the premise that the population response would be distributed normally, a logarithmic transformation was not necessary. In fact, when the color-difference magnitudes were transformed to a logarithmic scale prior to applying the probit model, the resultant chi-squared values indicated poorer fits than without the transformation. The number of significant chi-squared tests increased two-fold (to sixteen color vectors) for the transformed case.

A model used to the same degree of success in biological assay is the logit[30] based on the logistic distribution. Compared to probit analysis, this method of tolerance determination yields similar estimations in all cases except for very small or very large probabilities.[31] For large subject populations probit and logit produce identical results. Besides the fundamental distribution, logit differs from probits in its method for fitting the regression line. Instead of performing a maximum likelihood estimation, logit minimizes the chi-squared value to estimate the unknown parameters. Since a basic assumption of this experiment is that the color-difference response is described by a normal distribution, the logit model was not applied to this particular dataset.

Another assumption considered during design involves the influence one stimulus has on a response for a subsequent stimulus. In the biological community probit analysis is often used to determine the lethal dose of a toxic substance to a particular population. Stimuli such as pesticides can have a lasting effect on a subject, in some cases as permanent as death. With this in mind Finney[32] states that probit analysis is applicable only in those cases where the subjects are tested once. In the context of biological assay this is a reasonable restriction.

For the purposes of this experiment the stimulus employed did not affect the observers' ability to make subsequent quantal decisions. A color-difference comparison was not considered debilitating with regard to subsequent visual comparisons. It was this rationale that led to the removal of the limitation expressed by Finney of only one test/judgement per participant.

A major interest of this effort was to collect perceptibility data typical of industrial color decisions. To achieve this goal the observer population was properly screened to avoid using experienced color matchers or individuals familiar with performing color-difference judgements. In general, the population of subjects used for making visual comparisons were college students and industrial workers. Placing this caveat on the selection of candidate observers precluded the chance of tainting the collected data with judgements based on an acceptability premise.

Isolation of the quantal responses to a single category of change in each case required designing the color-difference samples along precise vectors. The precision of these sample vector sets was determined using a statistical method termed principal component analysis. The resultant first component eigenvalues for all 45 color vectors exceeded 0.99 in all cases. Exact values can be found in Table 1. This confirmed that the probit model was fit to a univariate color change in each vector.

Probit Analysis

Probit analysis of the collected color-difference data was performed using the available SAS[28] routine. This program iteratively computes maximum-likelihood estimates for the parameters α (called INTERCEPT) and β (called SLOPE). In the same manner it also estimates the mean, μ, (called MU) and standard deviation, σ, (called SIGMA) of the stimulus tolerance. The estimated 'dose' along with the 95% fiducial limits for several probability levels are also listed in the printout. Two plots are produced: the empirical probit at each level of stimulus superimposed on the probit line, and the probability points at each level of stimulus superimposed on the normal sigmoid curve. Lastly, a summary of the input values are printed.

Several options are available to the program user to produce the desired results. The probability or p value used for the chi-square test can be set with the HPROB option. The default for p is 0.10 which was the value used in all experimental calculations. Stimulus data can be transformed using either a natural or base 10 logarithm. This option was not exercised for this effort. Additionally, an OPTC can be set to request that the estimation of the natural (threshold) response rate be optimized; however, this option was not utilized so the value defaulted to zero.

The chi-squared test was used to determine how well the empirical data agreed with the probit model. Since the HPROB option was not exercised the default probability 0.10 was used to determine significance of the test throughout the experiment. Eight vector sets exhibited significantly high chi-squared tests when compared to $\chi^2_{.90}$ for the respective degrees of freedom and are denoted in Table 2 by an asterisk. Because the parameters α , β were estimated the degrees of freedom were calculated for each vector as two less than the number of stimuli.

Significant chi-squared values indicate the data does not adequately fit the probit regression line. Such failures to conform to the assumed model are motivated by one of two types of heterogeneous behavior - random or systematic.[33] Random heterogeneity effects arise from ungoverned influences that may vary from stimuli to stimuli or from interaction between these. Data influenced by such random factors tends to display arbitrary dispersion about the regression line. Systematic heterogeneity on the other hand is experienced when the assumed mathematical model is incorrect. In this case, data takes on a curvilinear character which obviously deviates from the linear regression it is being fitted to.

Despite the frequent difficulty in distinguishing between random and systematic heterogeneity a strong case can be made for the former as the possible cause for departures in the eight color vectors. Because random deviations occur when the subjects do not respond wholly independent of one another, the rational for the initial concession regarding repeated testing of single observers may be flawed. The viewing procedure was designed with the allowance that subsequent comparisons should not be affected by those judgements made prior. This agreeably went against the normal convention of subject testing, but considering the nature of the stimuli employed seemed a trivial departure from customary practice.

In hindsight, the eight instances of high chi-squared tests can logically be attributed to unintended random factors designed into the experiment. Such influences could have been caused by changes in an observer's emotional state between sessions. Viewer fatique could have been a contributor even though sessions were shortened to prevent this occurrence. The integrity of decision making must change to some degree from the first test to the last test, and this certainly would not be a variable had subjects been restricted to a single test. Interaction between judgements may also have occurred if observers were prone to dwell on previous comparisons during subsequent judgements. The level of influence these factors had on the response frequencies attained is indeterminable, but nevertheless, is a conceivable source of deviation from the intended model.

Another possible explanation for the high chi-squared values could have been poor sampling around the 50% threshold point. Inadequate sampling in the center of the sloped region of the sigmoidal response curve precludes a thorough coverage of this critical area. Those color-difference vectors exhibiting significant chi-squared tests were, in most cases, sparsely sampled in this crucial sector of the curve.

Occurrences of random deviation can be calculated into the range of experimental error using the heterogeneity factor, h, which is determined by,

$$h=\frac{\chi^2}{k-2} \qquad (10).$$

Variances are then multiplied by h to increase the range of values within the experimental error. Use of the heterogeneity factor is not intended to accommodate those deviations of a systematic nature.

The heterogeneity factor was computed for all eight vectors exhibiting significant chi-squared values to adjust for the experimental error induced by random deviations. The SAS routine automatically performs this calculation for all cases exhibiting out of tolerance chi-squared tests whether caused by random or systematic contributions. For these cases the user is warned to check that high chi-squared values are not caused by systematic departures from the model and therefore left to determine the validity of using the *h* factor. It is conjectured that random deviation prompted this departure, consequently use of *h* is appropriate.

Fiducial limits are computed by the SAS routine for several probability levels with respect to *color-difference* magnitudes. If plotted the limits would produce two hyperbolic curves convex to the regression line approaching the line most closely at the 50% probability point. A horizontal line drawn anywhere along this plot would produce upper and lower limits asymmetric around the mean value. The limits at 50% probability show the most symmetry. As probabilities increase from the midpoint the lower limit digresses more rapidly than the upper limit and as probabilities decrease from the midpoint the opposite occurs.

For vectors with insignificant chi-squared values the fiducial limits are calculated using a *t* value of 1.96 for the 95% level of probability. In the eight cases where the chi-squared value was significant and a heterogeneity factor was in use, the *t* value is determined for the same level of probability (0.95) with the appropriate degrees of freedom. In either case a range of values is calculated within which the probit of the true response is certain (to 95%) lie in.

Fiducial limits differ from the conventional confidence limits in that the latter is typically symmetric about the mean. Fiducial limits used in probit analysis demonstrate that precision for estimating tolerance responses is best at the 50% level or Y=5.

CONCLUSIONS

The response frequency results of this analysis exhibit a reasonable fit to the probit model. Despite some significant chi-squared test values in eight of the 45 vector set cases the collected data displays undeniable agreement with the probit model.

The eight vector sets displaying high chi-squared tests can reasonably be explained by random deviations. Exposing observers to several color-difference stimuli may have caused interaction between decisions and contributed a heterogeneity of random nature to these vector sets. This method for testing subjects 'violates' the probit protocol customary to the bio-assay community, but for the determination of color-difference tolerances appears to be a reasonable modification to the normal convention.

Deviations from the normal distribution might also have been caused by insufficient sampling near the estimated T50 value. In the eight vector cases with high chi-squared values this crucial region of the response curve was not adequately tested.

The use of a near neutral anchor pair as a standard to base quantal decisions was effective and an easily understood criterion by all observers.

RECOMMENDATIONS

The nine color centers tested in this effort are a significant beginning to determine color-difference tolerance levels typical of industrial decisions. However, additional research is necessary to provide adequate data sets upon which to base color metric equation development. Future work should focus on validating the results of this experiment and continue the determination of tolerances in other regions of color space.

Specifically, lightness contributions to tolerance levels should be studied further. In this experiment the single vector within each color center to sample lightness effects was not found sufficient to conduct detailed analysis of this color variable. Future experiments could better characterize these effects by increased sampling with additional lightness vectors.

Further study should occur relevant to the eight vector sets that exhibited significant chi-squared values to demonstrate conformance to the probit model. The current results from this experiment can be used as a guide to optimize the sampling of sigmoid response curve for such a follow-up effort. Until this expanded evaluation is accomplished the tolerance levels estimated for the eight exceptions to the model should be used with discretion.

The color data collected in this and future experiments could be used to test applications to other statistical models such as the logit

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APPENDIX A

Pilot Test Sample Pair Measurements

Color Vector	Samples A&B (Paired)	L*	a*	b*	E
Anchor	STDA	49.53	- 0.08	5.65	
	STDB	48.89	0.17	4.90	1.02
Blue A	1A	35.95	- 1.39	-27.31	
	1B	36.28	- 1.30	-27.33	0.34
	2A	35.69	- 1.43	-27.32	
	2B	36.52	- 1.27	-27.31	0.85
	3A	35.28	- 1.52	-27.28	
	3B	37.10	- 1.15	-27.29	1.86
	4A	34.34	- 1.63	-27.24	
	4B	37.10	- 1.15	-27.29	2.80
	5A	34.34	- 1.63	-27.24	
	5B	37.88	- 0.89	-27.32	3.62
Blue B	1A	35.85	- 1.70	-27.09	
	1B	35.82	- 1.10	-27.16	0.60
	2A	35.79	- 1.93	-26.89	
	2B	35.86	- 0.86	-27.12	1.10
	3A	35.88	- 2.54	-27.03	
	3B	35.84	- 0.27	-27.19	2.28
	4A	35.92	- 3.71	-26.92	
	4B	35.84	- 0.27	-27.19	3.45
	5A	35.92	- 3.71	-26.92	
	5B	35.81	0.85	-27.26	4.57
Blue C	1A	35.91	- 1.58	-27.16	
	1B	35.84	- 1.45	-26.70	0.48
	2A	35.91	- 1.65	-27.40	
	2B	35.79	- 1.39	-26.49	0.95
	3A	35.98	- 1.78	-27.88	
	3B	35.75	- 1.21	-26.02	1.96
	4A	36.16	- 2.07	-28.80	
	4B	35.75	- 1.21	-26.02	2.94
	5A	36.16	- 2.07	-28.80	
	5B	35.59	- 0.84	-25.20	3.85
Blue D	1A	35.97	- 2.14	-28.11	
	1B	35.99	- 1.64	-27.54	0.76
	2A	35.99	- 2.40	-28.37	
	2B	35.99	- 1.40	-27.27	1.49
	3A	36.06	- 2.98	-28.92	

	3B	35.98	- 0.79	-26.72	3.11
	4A	36.23	- 4.07	-30.01	
	4B	35.98	- 0.79	-26.72	4.65
	5A	36.23	- 4.07	-30.01	
	5B	35.92	0.38	-25.74	6.18
Blue E	1A	36.25	- 2.08	-27.64	
	1B	36.33	- 1.86	-28.18	0.59
	2A	36.21	- 2.14	-27.42	
	2B	36.39	- 1.79	-28.41	1.07
	3A	36.22	- 2.38	-26.81	
	3B	36.33	- 1.54	-29.05	2.39
	4A	36.21	- 2.71	-25.76	
	4B	36.33	- 1.54	-29.05	3.49
	5A	36.21	- 2.71	-25.76	
	5B	36.42	- 1.10	-30.21	4.74
Cyan A	1A	50.37	-16.24	-11.22	
	1B	50.85	-16.21	-11.23	0.48
	2A	50.22	-16.23	-11.21	
	2B	51.05	-16.20	-11.24	0.83
	3A	49.73	-16.24	-11.18	
	3B	51.58	-16.16	-11.25	1.85
	4A	48.81	-16.28	-11.14	
	4B	51.58	-16.16	-11.25	2.77
	5A	48.81	-16.28	-11.14	
	5B	52.47	-16.10	-11.31	3.67
Cyan B	1A	50.79	-16.41	-11.09	
	1B	50.74	-15.99	-11.08	0.42
	2A	50.79	-16.63	-11.07	
	2B	50.72	-15.81	-11.05	0.82
	3A	50.83	-17.11	-11.05	
	3B	50.70	-15.29	-11.13	1.83
	4A	50.95	-18.02	-10.98	
	4B	50.70	-15.29	-11.13	2.75
	5A	50.95	-18.02	-10.98	
	5B	50.63	-14.38	-11.16	3.66
Cyan C	1A	50.67	-16.19	-11.21	
	1B	50.62	-16.01	-10.77	0.48
	2A	50.65	-16.26	-11.42	
	2B	50.62	-15.94	-10.55	0.93
	3A	50.75	-16.43	-11.91	
	3B	50.55	-15.74	-10.10	1.95
	4A	50.81	-16.76	-12.86	
	4B	50.55	-15.74	-10.10	2.95
	5A	50.81	-16.76	-12.86	
	5B	50.54	-15.36	- 9.16	3.97
Cyan D	1A	50.64	-16.40	-11.29	
	1B	50.71	-15.97	-11.00	0.52
	2A	50.73	-16.57	-11.44	
	2B	50.64	-15.81	-10.80	1.00
	3A	50.67	-17.06	-11.74	
	3B	50.73	-15.33	-10.52	2.12
	4A	50.74	-17.92	-12.36	

	4B	50.73	-15.33	-10.52	3.18
	5A	50.74	-17.92	-12.36	
	5B	50.79	-14.42	- 9.89	4.28
Cyan E	1A	50.80	-16.24	-10.96	
	1B	50.80	-16.07	-11.25	0.34
	2A	50.85	-16.34	-10.80	
	2B	50.78	-15.98	-11.43	0.73
	3A	50.85	-16.55	-10.41	
	3B	50.81	-15.77	-11.77	1.57
	4A	50.83	-16.89	- 9.74	
	4B	50.81	-15.77	-11.77	2.32
	5A	50.83	-16.89	- 9.74	
	5B	50.83	-15.37	-12.47	3.12
Gray A	1A	59.55	- 1.01	1.26	
	1B	60.00	- 0.99	1.24	0.45
	2A	59.34	- 1.01	1.25	
	2B	60.18	- 0.99	1.28	0.84
	3A	58.86	- 1.02	1.24	
	3B	60.85	- 0.93	1.30	1.99
	4A	57.95	- 1.08	1.21	
	4B	60.85	- 0.93	1.30	2.91
	5A	57.95	- 1.08	1.21	
	5B	61.72	- 0.93	1.30	3.77
Gray B	1A	59.70	- 1.05	1.30	
	1B	59.73	- 0.59	1.25	0.46
	2A	59.69	- 1.30	1.30	
	2B	59.76	- 0.37	1.23	0.94
	3A	59.70	- 1.77	1.37	
	3B	59.68	0.11	1.18	1.89
	4A	59.84	- 2.77	1.45	
	4B	59.68	0.11	1.18	2.90
	5A	59.84	- 2.77	1.45	
	5B	59.56	1.00	1.00	3.81
Gray C	1A	59.60	- 1.04	0.95	
	1B	59.61	- 1.01	1.46	0.51
	2A	59.69	- 1.08	0.69	
	2B	59.61	- 0.98	1.69	1.01
	3A	59.65	- 1.11	0.16	
	3B	59.61	- 0.93	2.24	2.09
	4A	59.79	- 1.17	- 0.94	
	4B	59.61	- 0.93	2.24	3.19
	5A	59.79	- 1.17	- 0.94	
	5B	59.61	- 0.78	3.23	4.19
Gray D	1A	59.53	- 1.30	0.92	
	1B	59.50	- 0.93	1.23	0.48
	2A	59.61	- 1.48	0.76	
	2B	59.60	- 0.75	1.38	0.96
	3A	59.57	- 1.84	0.46	
	3B	59.56	- 0.38	1.70	1.92
	4A	59.60	- 2.55	- 0.16	
	4B	59.56	- 0.38	1.70	2.86
	5A	59.60	- 2.55	- 0.16	

	5B	59.55	0.37	2.33	3.84
Gray E	1A	59.51	- 1.11	1.42	
	1B	59.49	- 0.82	1.07	0.45
	2A	59.48	- 1.26	1.58	
	2B	59.54	- 0.68	0.90	0.90
	3A	59.50	- 1.59	1.96	
	3B	59.51	- 0.33	0.52	1.91
	4A	59.61	- 2.17	2.64	
	4B	59.51	- 0.33	0.52	2.81
	5A	59.61	- 2.17	2.64	
	5B	59.67	0.25	- 0.32	3.82
Green A	1A	55.33	-27.11	2.82	
	1B	55.78	-27.08	2.81	0.45
	2A	55.15	-27.10	2.81	
	2B	55.99	-27.07	2.83	0.84
	3A	54.56	-27.18	2.83	
	3B	56.42	-27.01	2.84	1.87
	4A	53.69	-27.23	2.81	
	4B	56.42	-27.01	2.84	2.74
	5A	53.69	-27.23	2.81	
	5B	57.39	-26.92	2.84	3.71
Green B	1A	55.49	-27.37	2.83	
	1B	55.66	-26.86	2.74	0.55
	2A	55.61	-27.51	2.82	
	2B	55.58	-26.67	2.74	0.84
	3A	55.46	-28.10	2.90	
	3B	55.60	-26.16	2.67	1.96
	4A	55.59	-29.03	2.97	
	4B	55.60	-26.16	2.67	2.89
	5A	55.59	-29.03	2.97	
	5B	55.65	-25.29	2.59	3.76
Green C	1A	55.91	-27.28	2.54	
	1B	55.85	-27.12	2.94	0.43
	2A	55.88	-27.34	2.36	
	2B	55.90	-27.13	3.11	0.78
	3A	55.82	-27.38	2.00	
	3B	55.85	-27.00	3.55	1.60
	4A	55.93	-27.60	1.25	
	4B	55.85	-27.00	3.55	2.38
	5A	55.93	-27.60	1.25	
	5B	55.85	-26.71	4.43	3.30
Green D	1A	55.54	-27.45	2.67	
	1B	55.63	-27.10	2.88	0.42
	2A	55.87	-27.80	2.41	
	2B	55.70	-26.98	2.92	0.98
	3A	55.60	-28.03	2.27	
	3B	55.64	-26.50	3.24	1.81
	4A	55.56	-28.67	1.86	
	4B	55.64	-26.50	3.24	2.57
	5A	55.56	-28.67	1.86	
	5B	55.76	-25.67	3.73	3.54
Green E	1A	55.59	-27.18	2.94	

	1B	55.51	-26.92	2.64	0.40
	2A	55.59	-27.29	3.09	
	2B	55.50	-26.79	2.50	0.78
	3A	55.57	-27.62	3.43	
	3B	55.56	-26.52	2.20	1.65
	4A	55.66	-28.21	4.10	
	4B	55.56	-26.52	2.20	2.54
	5A	55.66	-28.21	4.10	
	5B	55.47	-26.03	1.64	3.29
Orange A	1A	63.26	12.36	20.92	
	1B	63.48	12.00	20.73	0.46
	2A	63.15	12.29	20.81	
	2B	63.99	12.40	20.93	0.86
	3A	62.60	12.29	20.82	
	3B	64.52	12.44	20.98	1.93
	4A	61.71	12.22	20.76	
	4B	64.52	12.44	20.98	2.83
	5A	61.71	12.22	20.76	
	5B	65.50	12.56	21.09	3.82
Orange B	1A	63.66	12.13	20.84	
	1B	63.65	12.60	20.87	0.47
	2A	63.71	11.83	20.73	
	2B	63.65	12.82	20.87	1.00
	3A	63.68	11.35	20.79	
	3B	63.65	13.37	20.95	2.03
	4A	63.79	10.30	20.66	
	4B	63.65	13.37	20.95	3.09
	5A	63.79	10.30	20.66	
	5B	63.56	14.42	21.12	4.15
Orange C	1A	63.69	12.14	20.55	
	1B	63.65	12.27	21.07	0.54
	2A	63.60	12.14	20.37	
	2B	63.74	12.30	21.28	0.93
	3A	63.55	12.11	19.94	
	3B	63.79	12.38	21.81	1.90
	4A	63.53	11.99	18.97	
	4B	63.79	12.38	21.81	2.88
	5A	63.53	11.99	18.97	
	5B	63.78	12.51	22.73	3.80
Orange D	1A	63.61	11.98	20.59	
	1B	63.63	12.32	20.87	0.44
	2A	63.61	11.79	20.42	
	2B	63.58	12.51	21.05	0.96
	3A	63.63	11.40	20.07	
	3B	63.62	12.89	21.39	1.99
	4A	63.69	10.67	19.43	
	4B	63.62	12.89	21.39	2.96
	5A	63.69	10.67	19.43	
	5B	63.58	13.68	22.07	4.01
Orange E	1A	63.73	12.06	20.87	
	1B	63.74	12.35	20.60	0.40
	2A	63.78	11.87	21.00	

	2B	63.71	12.49	20.46	0.83
	3A	63.81	11.53	21.30	
	3B	63.72	12.80	20.16	1.71
	4A	63.91	10.81	21.88	
	4B	63.72	12.80	20.16	2.64
	5A	63.91	10.81	21.88	
	5B	63.69	13.36	19.56	3.45
Purple A	1A	46.25	12.77	-13.47	
	1B	46.81	12.70	-13.38	0.57
	2A	46.09	12.73	-13.47	
	2B	46.93	12.73	-13.39	0.84
	3A	45.56	12.77	-13.54	
	3B	47.47	12.75	-13.34	1.92
	4A	44.60	12.75	-13.58	
	4B	47.47	12.75	-13.34	2.88
	5A	44.60	12.75	-13.58	
	5B	48.44	12.75	-13.23	3.86
Purple B	1A	46.30	12.45	-13.30	
	1B	46.24	12.98	-13.38	0.54
	2A	46.29	12.17	-13.22	
	2B	46.36	13.18	-13.40	1.03
	3A	46.25	11.68	-13.18	
	3B	46.34	13.72	-13.45	2.06
	4A	46.27	10.74	-13.08	
	4B	46.34	13.72	-13.45	3.00
	5A	46.27	10.74	-13.08	
	5B	46.44	14.77	-13.63	4.07
Purple C	1A	46.14	12.48	-13.55	
	1B	46.24	12.59	-13.14	0.44
	2A	46.16	12.45	-13.77	
	2B	46.24	12.57	-12.87	0.91
	3A	46.16	12.44	-14.30	
	3B	46.33	12.69	-12.46	1.86
	4A	46.09	12.42	-15.34	
	4B	46.33	12.69	-12.46	2.90
	5A	46.09	12.42	-15.34	
	5B	46.32	12.77	-11.55	3.81
Purple D	1A	46.44	12.53	-13.57	
	1B	46.53	12.84	-13.27	0.44
	2A	46.48	12.33	-13.68	
	2B	46.52	13.02	-13.12	0.89
	3A	46.45	11.94	-13.95	
	3B	46.58	13.42	-12.87	1.84
	4A	46.45	11.94	-13.95	
	4B	46.58	13.42	-12.87	2.77
	5A	46.45	11.94	-13.95	
	5B	46.71	14.10	-12.34	3.64
Purple E	1A	46.46	12.52	-13.36	
	1B	46.57	12.79	-13.74	0.48
	2A	46.48	12.40	-13.19	
	2B	46.62	12.90	-13.90	0.88
	3A	46.54	12.01	-12.75	

	3B	46.64	13.26	-14.34	2.02
	4A	46.43	11.44	-12.00	
	4B	46.64	13.26	-14.34	2.97
	5A	46.43	11.44	-12.00	
	5B	46.77	13.81	-15.09	3.91
Red A	1A	41.95	36.06	20.50	
	1B	42.62	35.93	20.38	0.69
	2A	41.75	35.98	20.42	
	2B	42.73	35.85	20.30	1.00
	3A	41.26	35.94	20.38	
	3B	43.27	35.91	20.39	2.01
	4A	40.29	35.89	20.36	
	4B	43.27	35.91	20.39	2.98
	5A	40.29	35.89	20.36	
	5B	44.21	35.92	20.46	3.92
Red B	1A	41.52	35.50	20.35	
	1B	41.51	36.01	20.43	0.52
	2A	41.58	35.26	20.37	
	2B	41.43	36.35	20.54	1.11
	3A	41.40	34.86	20.48	
	3B	41.51	36.79	20.45	1.93
	4A	41.38	33.86	20.49	
	4B	41.51	36.79	20.45	2.93
	5A	41.38	33.86	20.49	
	5B	41.55	37.78	20.43	3.92
Red C	1A	41.20	35.84	20.40	
	1B	41.22	35.81	20.90	0.50
	2A	41.22	35.95	20.26	
	2B	41.21	35.80	21.13	0.88
	3A	41.18	35.99	19.73	
	3B	41.21	35.92	21.80	2.07
	4A	41.27	35.87	18.40	
	4B	41.21	35.92	21.80	3.40
	5A	41.27	35.87	18.40	
	5B	41.14	36.03	22.93	4.53
Red D	1A	41.82	35.78	20.36	
	1B	41.89	35.98	20.54	0.28
	2A	41.88	35.57	20.13	
	2B	41.82	36.31	20.90	1.07
	3A	41.83	35.34	19.86	
	3B	41.92	36.56	21.17	1.79
	4A	41.90	34.46	18.87	
	4B	41.92	36.56	21.17	3.11
	5A	41.90	34.46	18.87	
	5B	41.91	37.34	21.95	4.22
Red E	1A	41.94	35.84	20.67	
	1B	41.88	36.14	20.33	0.46
	2A	41.87	35.66	20.85	
	2B	42.01	36.14	19.96	1.02
	3A	41.84	35.34	21.26	
	3B	41.95	36.57	19.63	2.04
	4A	41.79	34.73	22.04	

	4B	41.95	36.57	19.63	3.04
	5A	41.79	34,73	22.04	
	5B	41.90	37.30	18.89	4.07
Yellow A	1A	77.99	1.39	37.05	
	1B	78.54	1.50	37.11	0.56
	2A	77.73	1.28	37.02	
	2B	78.81	1.59	37.17	1.13
	3A	77.24	1.18	36.90	
	3B	79.26	1.82	37.30	2.16
	4A	76.16	0.92	36.79	
	4B	79.26	1.82	37.30	3.27
	5A	76.16	0.92	36.79	
	5B	80.33	2.15	37.61	4.42
Yellow B	1A	78.32	1.04	37.12	
	1B	78.32	1.64	37.23	0.61
	2A	78.27	0.74	37.12	
	2B	78.30	1.95	37.30	1.22
	3A	78.23	0.16	37.02	
	3B	78.35	2.51	37.43	2.39
	4A	78.16	- 0.98	36.87	
	4B	78.35	2.51	37.43	3.54
	5A	78.16	- 0.98	36.87	
	5B	78.47	3.71	37.73	4.78
Yellow C	1A	78.43	1.56	35.15	
	1B	78.47	1.65	35.70	0.56
	2A	78.38	1.55	34.90	
	2B	78.51	1.64	35.94	1.05
	3A	78.31	1.51	34.38	
	3B	78.64	1.69	36.49	2.14
	4A	78.21	1.50	33.36	
	4B	78.64	1.69	36.49	3.17
	5A	78.21	1.50	33.36	
	5B	78.78	1.82	37.69	4.38
Yellow D	1A	78.69	0.99	34.08	
	1B	78.72	1.28	34.48	0.49
	2A	78.70	0.84	33.91	
	2B	78.73	1.47	34.61	0.94
	3A	78.37	0.54	33.47	
	3B	78.75	1.85	35.04	2.08
	4A	78.30	- 0.07	32.78	
	4B	78.75	1.85	35.04	3.00
	5A	78.30	- 0.07	32.78	
	5B	78.93	2.55	35.88	4.11
Yellow E	1A	78.49	1.68	33.78	
	1B	78.55	2.00	33.48	0.44
	2A	78.55	1.47	33.90	
	2B	78.53	2.17	33.37	0.88
	3A	78.48	1.12	34.19	
	3B	78.50	2.53	33.09	1.79
	4A	78.55	0.38	34.78	
	4B	78.50	2.53	33.09	2.74
	5A	78.55	0.38	34.78	

	5B	78.50	3.21	32.58	3.58
Yellow-Green A	1A	64.37	- 9.97	13.40	
	1B	64.84	-10.01	13.41	0.47
	2A	64.14	- 9.96	13.34	
	2B	65.08	-10.07	13.41	0.95
	3A	63.63	- 9.90	13.38	
	3B	65.62	-10.13	13.45	2.00
	4A	62.64	- 9.80	13.32	
	4B	65.62	-10.13	13.45	3.00
	5A	62.64	- 9.80	13.32	
	5B	66.63	-10.28	13.48	4.02
Yellow-Green B	1A	64.65	-10.15	13.51	
	1B	64.78	- 9.83	13.45	0.35
	2A	64.77	-10.28	13.47	
	2B	64.68	- 9.65	13.44	0.64
	3A	64.69	-10.66	13.60	
	3B	64.76	- 9.37	13.38	1.31
	4A	64.74	-11.31	13.69	
	4B	64.76	- 9.37	13.38	1.96
	5A	64.74	-11.31	13.69	
	5B	64.64	- 8.73	13.33	2.61
Yellow-Green C	1A	64.76	-10.34	13.11	
	1B	64.77	-10.33	13.50	0.39
	2A	64.76	-10.33	12.89	
	2B	64.76	-10.33	13.72	0.83
	3A	64.80	-10.34	12.49	
	3B	64.70	-10.31	14.16	1.67
	4A	64.80	-10.33	11.74	
	4B	64.70	-10.31	14.16	2.42
	5A	64.80	-10.33	11.74	
	5B	64.71	-10.30	15.04	3.30
Yellow-Green D	1A	64.92	-10.24	13.32	
	1B	64.78	- 9.94	13.54	0.40
	2A	64.96	-10.36	13.17	
	2B	64.72	- 9.81	13.66	0.77
	3A	64.98	-10.63	12.97	
	3B	64.70	- 9.52	13.96	1.51
	4A	65.01	-11.10	12.50	
	4B	64.70	- 9.52	13.96	2.17
	5A	65.01	-11.10	12.50	
	5B	64.58	- 8.95	14.46	2.94
Yellow-Green E	1A	64.86	-10.06	13.51	
	1B	64.86	- 9.83	13.21	0.38
	2A	64.88	-10.16	13.67	
	2B	64.76	- 9.72	13.04	0.78
	3A	64.97	-10.39	14.00	
	3B	64.80	- 9.47	12.68	1.62
	4A	65.02	-10.90	14.76	
	4B	64.80	- 9.47	12.68	2.53
	5A	65.02	-10.90	14.76	
	5B	64.75	- 9.07	12.05	3.28

APPENDIX B

Main Experiment Sample Pair Measurements

Color Vector	Samples A&B (Paired)	L*	a*	b*	E
Anchor	STDA	49.53	- 0.08	5.65	
	STDA	48.89	0.17	4.90	1.02
Blue A	001A	35.69	- 1.43	-27.32	
	001B	35.95	- 1.39	-27.31	0.26
	002A	35.26	- 1.52	-27.28	
	002B	35.76	- 1.46	-27.26	0.50
	003A	36.33	- 1.32	-27.32	
	003B	37.06	- 1.15	-27.30	0.75
	004A	36.11	- 1.36	-27.31	
	004B	37.10	- 1.15	-27.29	1.01
	005A	35.90	- 1.39	-27.31	
	005B	37.10	- 1.15	-27.29	1.22
	006A	34.28	- 1.63	-27.28	
	006B	35.95	- 1.39	-27.39	1.69
	007A	34.34	- 1.63	-27.24	
	007B	36.30	- 1.31	-27.31	1.99
Blue B	008A	35.86	- 1.39	-27.13	
	008B	35.80	- 0.84	-27.16	0.53
	009A	35.82	- 1.10	-27.16	
	009B	35.84	- 0.28	-27.18	0.82
	010A	35.80	- 1.41	-27.10	
	010B	35.84	- 0.27	-27.19	1.14
	011A	35.83	- 1.68	-27.11	
	011B	35.84	- 0.28	-27.18	1.40
	012A	35.90	- 3.73	-26.93	
	012B	35.85	- 1.98	-27.03	1.75
	013A	35.83	- 1.12	-27.16	
	013B	35.84	0.84	-27.24	1.96
	014A	35.90	- 3.73	-26.93	
	014B	35.86	- 1.39	-27.13	2.35
Blue C	015A	35.91	- 1.58	-27.16	
	015B	35.84	- 1.45	-26.70	0.48
	016A	35.90	- 1.61	-27.14	
	016B	35.68	- 1.20	-26.05	1.19
	017A	35.89	- 1.64	-27.42	
	017B	35.75	- 1.21	-26.02	1.47

	018A	36.12	- 2.09	-28.80	
	018B	35.90	- 1.61	-27.14	1.74
	019A	35.98	- 1.78	-27.88	
	019B	35.75	- 1.21	-26.02	1.96
	020A	36.12	- 2.09	-28.80	
	020B	35.84	- 1.45	-26.70	2.21
	021A	36.12	- 2.09	-26.70	
	021B	35.76	- 1.41	-26.47	2.45
Blue D	022A	35.99	- 2.40	-28.37	
	022B	35.95	- 2.16	-28.13	0.34
	023A	35.96	- 1.89	-27.88	
	023B	35.99	- 1.64	-27.54	0.42
	024A	35.97	- 2.14	-28.11	
	024B	35.99	- 1.64	-27.54	0.76
	025A	35.99	- 2.40	-28.37	
	025B	35.96	- 1.64	-27.59	1.09
	026A	35.96	- 1.64	-27.59	
	026B	35.98	- 0.79	-26.72	1.22
	027A	35.99	- 2.40	-28.37	
	027B	35.99	- 1.40	-27.27	1.49
	028A	36.06	- 2.98	-28.92	
	028B	35.96	- 1.64	-27.59	1.89
Blue E	029A	36.20	- 2.73	-25.75	
	029B	36.35	- 1.88	-28.16	1.47
	030A	36.21	- 2.71	-25.76	
	030B	36.23	- 2.08	-27.65	1.99
	031A	36.20	- 2.73	-25.75	
	031B	36.35	- 1.88	-28.16	2.56
	032A	36.20	- 2.73	-25.75	
	032B	36.36	- 1.80	-28.42	2.83
	033A	36.25	- 2.18	-27.37	
	033B	36.36	- 1.11	-30.23	3.06
	034A	36.21	- 2.73	-25.75	
	034B	36.35	- 1.57	-29.03	3.48
	035A	36.22	- 2.39	-26.79	
	035B	36.36	- 1.11	-30.23	3.67
	036A	36.21	- 2.73	-25.75	
	036B	36.46	- 1.12	-30.19	4.73
Cyan A	037A	50.83	-16.22	-11.25	
	037B	51.08	-16.19	-11.24	0.25
	038A	49.72	-16.24	-11.18	
	038B	50.22	-16.23	-11.21	0.50
	039A	50.83	-16.22	-11.25	
	039B	51.58	-16.17	-11.26	0.75
	040A	50.61	-16.21	-11.22	
	040B	51.58	-16.17	-11.26	0.97
	041A	50.37	-16.24	-11.22	
	041B	51.58	-16.17	-11.26	1.21
	042A	48.81	-16.26	-11.14	
	042B	50.37	-16.24	-11.22	1.56
	043A	48.81	-16.26	-11.14	
	043B	50.61	-16.21	-11.22	1.80

Cyan B	044A	50.83	-17.13	-11.03	
	044B	50.79	-16.41	-11.09	0.72
	045A	50.83	-17.13	-11.03	
	045B	50.75	-16.17	-11.08	0.96
	046A	50.91	-18.05	-10.98	
	046B	50.79	-16.63	-11.07	1.43
	047A	50.75	-16.17	-11.08	
	047B	50.64	-14.38	-11.16	1.80
	048A	50.95	-18.03	-10.99	
	048B	50.74	-15.99	-11.08	2.05
	049A	50.79	-16.63	-11.07	
	049B	50.64	-14.38	-11.16	2.26
	050A	50.91	-18.05	-10.98	
	050B	50.77	-15.73	-11.12	2.33
Cyan C	051A	50.60	-16.01	-10.79	
	051B	50.58	-15.74	-10.08	0.76
	052A	50.64	-16.18	-11.21	
	052B	50.58	-15.74	-10.08	1.21
	053A	50.75	-16.41	-11.92	
	053B	50.61	-15.93	-10.54	1.47
	054A	50.60	-16.01	-10.79	
	054B	50.53	-15.37	- 9.16	1.75
	055A	50.75	-16.43	-11.91	
	055B	50.55	-15.74	-10.10	1.95
	056A	50.73	-16.24	-11.43	
	056B	50.53	-15.37	- 9.16	2.44
	057A	50.81	-16.76	-12.86	
	057B	50.55	-15.74	-10.10	2.95
Cyan D	058A	50.73	-16.57	-11.44	
	058B	50.64	-15.81	-10.80	1.00
	059A	50.64	-16.40	-11.29	
	059B	50.77	-15.30	-10.52	1.35
	060A	50.67	-17.06	-11.74	
	060B	50.66	-15.80	-10.82	1.56
	061A	50.75	-17.91	-12.36	
	061B	50.64	-16.40	-11.29	1.85
	062A	50.67	-17.06	-11.74	
	062B	50.73	-15.33	-10.52	2.12
	063A	50.74	-17.92	-12.36	
	063B	50.66	-15.80	-10.82	2.62
	064A	50.74	-17.92	-12.36	
	064B	50.73	-15.33	-10.52	3.18
Cyan E	065A	50.85	-16.34	-10.80	
	065B	50.78	-15.98	-11.43	0.73
	066A	50.87	-16.54	-10.41	
	066B	50.76	-16.08	-11.27	0.98
	067A	50.85	-16.55	-10.41	
	067B	50.81	-15.77	-11.77	1.57
	068A	50.84	-16.89	- 9.75	
	068B	50.79	-15.96	-11.44	1.93
	069A	50.83	-16.89	- 9.74	
	069B	50.81	-15.77	-11.77	2.32

	070A	50.87	-16.87	- 9.77	
	070B	50.78	-15.38	-12.44	3.06
	071A	50.83	-16.89	- 9.74	
	071B	50.83	-15.37	-12.47	3.12
Gray A	072A	59.76	- 1.00	1.28	
	072B	60.00	- 0.99	1.24	0.24
	073A	58.86	- 1.02	1.24	
	073B	59.34	- 1.01	1.25	0.48
	074A	59.55	- 1.01	1.26	
	074B	60.27	- 0.97	1.22	0.72
	075A	57.93	- 1.08	1.20	
	075B	58.89	- 1.03	1.22	0.96
	076A	59.55	- 1.01	1.26	
	076B	60.84	- 0.94	1.29	1.29
	077A	60.00	- 0.99	1.24	
	077B	61.72	- 0.93	1.30	1.72
	078A	58.86	- 1.02	1.24	
	078B	60.85	- 0.93	1.30	1.99
Gray B	079A	59.72	- 1.05	1.29	
	079B	59.71	- 0.81	1.28	0.24
	080A	59.79	- 1.81	1.34	
	080B	56.69	- 1.30	1.30	0.52
	081A	59.79	- 1.81	1.34	
	081B	59.72	- 1.05	1.29	0.76
	082A	59.79	- 1.81	1.34	
	082B	59.71	- 0.81	1.28	1.00
	083A	59.75	- 1.07	1.29	
	083B	59.63	0.13	1.19	1.21
	084A	59.73	- 0.59	1.25	
	084B	59.54	0.99	1.01	1.61
	085A	59.71	- 0.81	1.28	
	085B	59.54	0.99	1.01	1.83
Gray C	086A	59.60	- 1.04	0.95	
	086B	59.61	- 0.98	1.69	0.74
	087A	59.69	- 1.08	0.69	
	087B	59.61	- 0.98	1.69	1.01
	088A	59.69	- 1.08	0.69	
	088B	59.58	- 0.93	2.24	1.56
	089A	59.65	- 1.11	0.16	
	089B	59.61	- 0.93	2.24	2.09
	090A	59.79	- 1.17	- 0.93	
	090B	59.50	- 1.04	1.22	2.17
	091A	59.69	- 1.07	0.70	
	091B	59.64	- 0.77	3.22	2.54
	092A	59.65	- 1.11	0.16	
	092B	59.64	- 0.77	3.22	3.08
Gray D	093A	59.53	- 1.30	0.92	
	093B	59.50	- 0.93	1.23	0.48
	094A	59.50	- 0.93	1.23	
	094B	59.57	- 0.39	1.69	0.71
	095A	59.61	- 1.48	0.76	
	095B	59.60	- 0.75	1.38	0.96

	096A	59.55	- 1.85	0.46	
	096B	59.50	- 0.93	1.23	1.20
	097A	59.57	- 1.84	0.46	
	097B	59.60	- 0.75	1.38	1.43
	098A	59.59	- 2.56	- 0.16	
	098B	59.53	- 1.30	0.92	1.66
	099A	59.53	- 1.30	0.92	
	099B	59.55	0.37	2.33	2.19
Gray E	100A	59.48	- 1.26	1.58	
	100B	59.54	- 0.68	0.90	0.90
	101A	59.50	- 1.59	1.96	
	101B	59.49	- 0.82	1.07	1.18
	102A	59.48	- 1.26	1.58	
	102B	59.52	- 0.34	0.52	1.40
	103A	59.50	- 1.59	1.96	
	103B	59.51	- 0.33	0.52	1.91
	104A	59.58	- 2.17	2.64	
	104B	59.54	- 0.68	0.90	2.29
	105A	59.48	- 1.26	1.58	
	105B	59.67	0.25	- 0.32	2.43
	106A	59.61	- 2.17	2.64	
	106B	59.51	- 0.33	0.52	2.81
Green A	107A	55.52	-27.11	2.82	
	107B	55.82	-27.10	2.81	0.30
	108A	55.25	-27.12	2.84	
	108B	55.78	-27.08	2.81	0.53
	109A	55.25	-27.12	2.84	
	109B	55.99	-27.07	2.83	0.74
	110A	54.56	-27.18	2.83	
	110B	55.54	-27.11	2.82	0.98
	111A	54.56	-27.18	2.84	
	111B	55.82	-27.10	2.81	1.26
	112A	53.69	-27.23	2.81	
	112B	55.15	-27.10	2.81	1.47
	113A	55.33	-27.11	2.82	
	113B	57.35	-26.91	2.85	2.03
Green B	114A	55.48	-27.63	2.87	
	114B	55.66	-26.84	2.74	0.82
	115A	55.48	-27.63	2.87	
	115B	55.65	-26.16	2.66	1.49
	116A	55.66	-26.86	2.74	
	116B	55.54	-25.18	2.66	1.69
	117A	55.53	-27.13	2.80	
	117B	55.54	-25.18	2.66	1.96
	118A	55.59	-29.03	2.97	
	118B	55.66	-26.84	2.74	2.20
	119A	55.59	-29.03	2.97	
	119B	55.56	-26.63	2.75	2.41
	120A	55.46	-28.10	2.90	
	120B	55.54	-25.18	2.66	2.93
	121A	55.64	-28.95	2.97	
	121B	55.65	-25.29	2.59	3.68

Green C	122A	55.82	-27.38	2.00	
	122B	55.91	-27.28	2.54	0.56
	123A	55.88	-27.34	2.36	
	123B	55.90	-27.13	3.11	0.78
	124A	55.91	-27.28	2.54	
	124B	55.86	-26.99	3.54	1.04
	125A	55.88	-27.34	2.36	
	125B	55.85	-27.00	3.55	1.24
	126A	55.85	-27.12	2.94	
	126B	55.91	-26.76	4.41	1.51
	127A	56.02	-27.66	1.22	
	127B	55.85	-27.12	2.94	1.81
	128A	55.81	-27.27	2.39	
	128B	55.91	-26.76	4.41	2.09
Green D	129A	55.61	-28.06	2.25	
	129B	55.58	-27.58	2.55	0.57
	130A	55.87	-27.80	2.41	
	130B	55.70	-26.98	2.92	0.98
	131A	55.71	-28.15	2.21	
	131B	55.63	-27.10	2.88	1.25
	132A	55.50	-26.86	3.00	
	132B	55.67	-25.63	3.77	1.46
	133A	55.56	-28.67	1.86	
	133B	55.56	-27.23	2.78	1.71
	134A	55.56	-28.67	1.86	
	134B	55.70	-26.98	2.92	2.00
	135A	55.54	-27.45	2.67	
	135B	55.67	-25.63	3.77	2.13
Green E	136A	55.57	-27.62	3.43	
	136B	55.51	-26.92	2.64	1.06
	137A	55.57	-27.34	3.10	
	137B	55.56	-26.52	2.20	1.22
	138A	55.53	-27.06	2.77	
	138B	55.47	-26.03	1.64	1.53
	139A	55.59	-27.18	2.94	
	139B	55.47	-26.01	1.65	1.75
	140A	55.59	-27.29	3.09	
	140B	55.47	-26.01	1.65	1.93
	141A	55.66	-28.21	4.10	
	141B	55.50	-26.79	2.50	2.15
	142A	55.66	-28.21	4.10	
	142B	55.56	-26.52	2.20	2.54
Orange A	143A	63.26	12.36	20.92	
	143B	63.53	12.36	20.89	0.27
	144A	63.07	12.32	20.88	
	144B	63.56	12.34	20.88	0.49
	145A	63.26	12.36	20.92	
	145B	63.99	12.40	20.93	0.73
	146A	62.60	12.29	20.82	
	146B	63.56	12.34	20.88	0.96
	147A	63.26	12.36	20.92	
	147B	64.52	12.44	20.98	1.26

	148A	63.99	12.40	20.93	
	148B	65.50	12.56	21.09	1.53
	149A	62.60	12.29	20.82	
	149B	64.52	12.44	20.98	1.93
Orange B	150A	63.66	12.13	20.84	
	150B	63.65	12.60	20.87	0.47
	151A	63.71	11.83	20.73	
	151B	63.65	12.60	20.87	0.78
	152A	63.66	12.13	20.84	
	152B	63.65	13.37	20.95	1.24
	153A	63.79	10.30	20.66	
	153B	63.71	11.83	20.73	1.53
	154A	63.79	10.30	20.66	
	154B	63.67	12.12	20.84	1.83
	155A	63.68	11.35	20.79	
	155B	63.65	13.37	20.95	2.03
	156A	63.66	12.13	20.84	
	156B	63.56	14.42	21.12	2.31
Orange C	157A	63.69	12.14	20.55	
	157B	63.65	12.27	21.07	0.54
	158A	63.53	11.99	18.97	
	158B	63.55	12.11	19.94	0.98
	159A	63.55	12.11	19.94	
	159B	63.70	12.28	21.25	1.33
	160A	63.70	12.28	21.25	
	160B	63.78	12.51	22.73	1.50
	161A	63.53	11.99	18.97	
	161B	63.67	12.18	20.78	1.83
	162A	63.67	12.18	20.78	
	162B	63.76	12.52	22.76	2.01
Orange D	163A	63.61	11.98	20.59	
	163B	63.58	12.51	21.05	0.70
	164A	63.61	11.79	20.42	
	164B	63.58	12.51	21.05	0.96
	165A	63.63	11.40	20.07	
	165B	63.63	12.32	20.87	1.22
	166A	63.69	10.67	19.43	
	166B	63.61	11.98	20.59	1.75
	167A	63.63	11.40	20.07	
	167B	63.62	12.89	21.39	1.99
	168A	63.61	11.98	20.59	
	168B	63.58	13.68	22.07	2.25
	169A	63.63	11.40	20.07	
	169B	63.58	13.68	22.07	3.03
Orange E	170A	63.78	11.87	21.00	
	170B	63.73	12.06	20.87	0.24
	171A	63.81	11.53	21.30	
	171B	63.71	12.18	20.75	0.86
	172A	63.81	11.53	21.30	
	172B	63.74	12.35	20.60	1.08
	173A	63.78	11.87	21.00	
	173B	63.72	12.80	20.16	1.25

	174A	63.74	12.35	20.60	
	174B	63.69	13.36	19.56	1.45
	175A	63.91	10.81	21.88	
	175B	63.74	12.35	20.60	2.01
	176A	63.91	10.81	21.88	
	176B	63.71	12.49	20.46	2.21
Purple A	177A	46.01	12.80	-13.49	
	177B	46.25	12.77	-13.47	0.24
	178A	46.01	12.80	-13.49	
	178B	46.75	12.74	-13.42	0.75
	179A	46.25	12.77	-13.47	
	179B	47.47	12.75	-13.34	1.23
	180A	44.60	12.75	-13.58	
	180B	46.09	12.73	-13.47	1.49
	181A	46.75	12.74	-13.42	
	181B	48.44	12.75	-13.23	1.70
	182A	46.46	12.74	-13.44	
	182B	48.44	12.75	-13.23	1.99
Purple B	183A	46.30	12.45	-13.30	
	183B	46.24	12.98	-13.38	0.54
	184A	46.29	12.17	-13.22	
	184B	46.36	13.18	-13.40	1.03
	185A	46.30	12.45	-13.30	
	185B	46.34	13.72	-13.45	1.28
	186A	46.27	10.74	-13.08	
	186B	46.38	12.21	-13.25	1.48
	187A	46.24	12.98	-13.38	
	187B	46.44	14.77	-13.63	1.82
	188A	46.25	11.68	-13.18	
	188B	46.34	13.72	-13.45	2.06
	189A	46.27	10.74	-13.08	
	189B	46.24	12.98	-13.38	2.26
Purple C	190A	46.05	12.53	-13.58	
	190B	46.24	12.57	-12.87	0.74
	191A	46.09	12.42	-15.34	
	191B	46.16	12.44	-14.30	1.04
	192A	46.11	12.44	-14.31	
	192B	46.16	12.56	-13.11	1.21
	193A	46.16	12.44	-14.30	
	193B	46.24	12.57	-12.87	1.44
	194A	46.25	12.55	-13.36	
	194B	46.32	12.77	-11.55	1.82
	195A	46.09	12.42	-15.34	
	195B	46.25	12.55	-13.36	1.99
Purple D	196A	46.45	11.94	-13.95	
	196B	46.44	12.53	-13.57	0.70
	197A	46.49	11.12	-14.42	
	197B	46.45	11.94	-13.95	0.95
	198A	46.44	12.53	-13.57	
	198B	46.58	13.42	-12.87	1.14
	199A	46.49	11.12	-14.42	
	199B	46.48	12.33	-13.68	1.42

	200A	46.45	12.70	-13.40	
	200B	46.71	14.10	-12.34	1.78
	201A	46.44	12.53	-13.57	
	201B	46.71	14.10	-12.34	2.01
Purple E	202A	46.54	12.00	-12.74	
	202B	46.60	12.76	-13.73	1.25
	203A	46.54	12.34	-13.15	
	203B	46.64	13.25	-14.33	1.49
	204A	46.60	12.76	-13.73	
	204B	46.77	13.81	-15.09	1.73
	205A	46.54	12.00	-12.74	
	205B	46.64	13.25	-14.33	2.02
	206A	46.47	11.43	-11.97	
	206B	46.57	12.79	-13.74	2.23
	207A	46.47	11.43	-11.97	
	207B	46.61	12.90	-13.91	2.44
	208A	46.48	11.44	-12.00	
	208B	46.73	13.15	-14.27	2.85
	209A	46.51	12.03	-12.75	
	209B	46.68	13.91	-15.13	3.04
	210A	46.48	11.44	-12.00	
	210B	46.77	13.81	-15.09	3.91
Red A	211A	42.29	36.02	20.43	
	211B	42.62	35.93	20.38	0.35
	212A	41.26	35.94	20.38	
	212B	41.75	35.98	20.42	0.49
	213A	41.83	36.01	20.45	
	213B	42.59	35.97	20.42	0.76
	214A	41.75	35.98	20.42	
	214B	42.73	35.85	20.30	1.00
	215A	41.26	35.94	20.38	
	215B	42.52	35.88	20.35	1.26
	216A	41.75	35.98	20.42	
	216B	43.27	35.91	20.39	1.52
	217A	41.26	35.94	20.38	
	217B	43.27	35.91	20.39	2.01
Red B	218A	41.51	36.01	20.43	
	218B	41.42	36.75	20.43	0.75
	219A	41.52	35.50	20.35	
	219B	41.42	36.75	20.43	1.26
	220A	41.58	35.26	20.37	
	220B	41.42	36.75	20.43	1.50
	221A	41.45	36.00	20.41	
	221B	41.52	37.75	20.41	1.75
	222A	41.51	35.73	20.37	
	222B	41.52	37.75	20.41	2.02
	223A	41.38	33.86	20.49	
	223B	41.51	36.01	20.43	2.15
	224A	41.58	35.26	20.37	
	224B	41.52	37.75	20.41	2.49
	225A	41.40	34.83	20.46	
	225B	41.55	37.80	20.48	2.97

Red C	226A	41.20	35.84	20.40	
	226B	41.22	35.81	20.90	0.50
	227A	41.19	35.94	20.22	
	227B	41.18	35.88	21.21	0.99
	228A	41.18	35.99	19.73	
	228B	41.19	35.88	20.97	1.24
	229A	41.15	35.94	19.65	
	229B	41.21	35.80	21.13	1.49
	230A	41.25	35.91	18.42	
	230B	41.19	35.94	20.22	1.80
	231A	41.27	35.87	18.40	
	231B	41.20	35.84	20.40	2.00
	232A	41.15	36.01	20.32	
	232B	41.17	35.97	22.86	2.54
Red D	233A	41.82	35.78	20.36	
	233B	41.87	36.57	21.18	1.14
	234A	41.82	36.31	20.90	
	234B	41.91	37.34	21.95	1.47
	235A	41.83	35.34	19.86	
	235B	41.92	36.56	21.17	1.79
	236A	41.90	34.46	18.87	
	236B	41.91	35.78	20.34	1.98
	237A	41.82	35.78	20.36	
	237B	41.91	37.34	21.95	2.23
	238A	41.88	35.57	20.13	
	238B	41.91	37.34	21.95	2.54
	239A	41.78	35.38	19.90	
	239B	41.81	37.43	22.04	2.96
Red E	240A	41.94	35.84	20.67	
	240B	41.89	35.91	20.42	0.26
	241A	41.87	35.66	20.85	
	241B	42.01	36.14	19.96	1.02
	242A	41.84	35.34	21.26	
	242B	41.88	36.14	20.33	1.23
	243A	41.79	34.73	22.04	
	243B	41.87	35.66	20.85	1.51
	244A	41.79	34.73	22.04	
	244B	41.94	35.84	20.67	1.77
	245A	41.84	35.34	21.26	
	245B	41.95	36.57	19.63	2.04
	246A	41.87	35.66	20.85	
	246B	41.90	37.30	18.89	2.56
Yellow A	247A	76.90	4.01	35.11	
	247B	77.36	4.02	35.09	0.46
	248A	75.92	4.02	35.16	
	248B	76.66	4.02	35.15	0.74
	249A	75.92	4.02	35.16	
	249B	76.90	4.01	35.11	0.98
	250A	76.66	4.02	35.15	
	250B	77.91	4.02	35.13	1.25
	251A	76.43	4.02	35.13	
	251B	77.91	4.02	35.13	1.48

	252A	77.16	4.02	35.14	
	252B	78.94	4.07	35.23	1.78
	253A	76.90	4.01	35.11	
	253B	78.98	4.07	35.23	2.08
Yellow B	254A	78.27	0.16	37.02	
	254B	78.29	0.79	37.10	0.64
	255A	78.27	0.16	37.02	
	255B	78.31	1.06	37.18	0.91
	256A	78.27	0.74	37.12	
	256B	78.30	1.95	37.30	1.22
	257A	78.31	1.06	37.18	
	257B	78.36	2.56	37.44	1.52
	258A	78.16	- 0.98	36.87	
	258B	78.27	0.74	37.12	1.74
	259A	78.11	- 0.93	36.85	
	259B	78.31	1.06	37.18	2.03
	260A	78.23	0.16	37.02	
	260B	78.35	2.51	37.43	2.39
Yellow C	261A	78.33	1.54	34.40	
	261B	78.43	1.56	35.15	0.76
	262A	78.33	1.54	34.40	
	262B	78.50	1.64	35.41	1.03
	263A	78.64	1.69	36.49	
	263B	78.78	1.82	37.69	1.22
	264A	78.51	1.64	35.94	
	264B	78.76	1.85	37.66	1.75
	265A	78.21	1.50	33.36	
	265B	78.50	1.64	35.41	2.08
	266A	78.46	1.63	35.45	
	266B	78.75	1.83	37.70	2.28
	267A	78.17	1.47	33.33	
	267B	78.46	1.63	35.45	2.50
	268A	78.38	1.55	34.90	
	268B	78.75	1.83	37.70	2.84
	269A	78.21	1.50	33.36	
	269B	78.64	1.69	36.49	3.17
Yellow D	270A	78.37	0.54	33.47	
	270B	78.69	0.99	34.08	0.82
	271A	78.60	1.17	34.27	
	271B	78.75	1.85	35.04	1.04
	272A	78.69	0.99	34.08	
	272B	78.75	1.85	35.02	1.28
	273A	78.30	- 0.07	32.78	
	273B	78.70	0.84	33.91	1.50
	274A	78.30	- 0.07	32.78	
	274B	78.69	0.99	34.08	1.72
	275A	78.37	0.54	33.47	
	275B	78.75	1.85	35.04	2.08
Yellow E	276A	78.58	1.46	33.93	
	276B	78.53	2.01	33.47	0.72
	277A	78.55	0.38	34.78	
	277B	78.48	1.12	34.19	0.95

	278A	78.51	1.11	34.21	
	278B	78.53	2.01	33.47	1.17
	279A	78.55	0.35	34.80	
	279B	78.55	1.47	33.90	1.44
	280A	78.50	1.86	33.68	
	280B	78.50	3.21	32.58	1.74
	281A	78.49	1.68	33.78	
	281B	78.50	3.21	32.58	1.94
	282A	78.48	1.12	34.19	
	282B	78.50	3.21	32.58	2.64
Yellow-	283A	64.37	- 9.97	13.40	
Green A	283B	64.65	- 9.99	13.40	0.28
	284A	64.37	- 9.97	13.40	
	284B	64.84	-10.01	13.41	0.47
	285A	63.63	- 9.90	13.38	
	285B	64.37	- 9.97	13.40	0.74
	286A	62.64	- 9.80	13.32	
	286B	63.63	- 9.90	13.38	1.00
	287A	64.37	- 9.97	13.40	
	287B	62.62	-10.13	13.45	1.26
	288A	65.08	-10.07	13.41	
	288B	66.58	-10.27	13.49	1.52
	289A	63.63	- 9.90	13.38	
	289B	62.62	-10.13	13.45	2.00
Yellow-	290A	64.78	- 9.83	13.45	
Green B	290B	64.74	- 9.34	13.39	0.50
	291A	64.65	-10.15	13.51	
	291B	64.76	- 9.37	13.38	0.80
	292A	64.69	-10.66	13.60	
	292B	64.68	- 9.65	13.44	1.02
	293A	64.80	-10.00	13.47	
	293B	64.64	- 8.73	13.33	1.29
	294A	64.74	-11.31	13.69	
	294B	64.78	- 9.83	13.45	1.50
	295A	64.74	-11.31	13.69	
	295B	64.68	- 9.65	13.44	1.68
	296A	64.74	-11.31	13.69	
	296B	64.76	- 9.37	13.38	1.96
Yellow-	297A	64.77	-10.33	13.50	
Green C	297B	64.70	-10.31	14.16	0.66
	298A	64.75	-10.33	13.13	
	298B	64.71	-10.32	14.15	1.02
	299A	64.76	-10.33	12.89	
	299B	64.70	-10.31	14.16	1.27
	300A	64.77	-10.33	13.50	
	300B	64.71	-10.30	15.04	1.54
	301A	64.80	-10.33	11.74	
	301B	64.77	-10.33	13.50	1.76
	302A	64.80	-10.33	11.74	
	302B	64.76	-10.33	13.72	1.98
	303A	64.80	-10.33	11.74	
	303B	64.70	-10.31	14.16	2.42

Yellow-Green D	304A	64.98	-10.63	12.97	
	304B	64.91	-10.11	13.45	0.71
	305A	64.92	-10.24	13.32	
	305B	64.70	- 9.52	13.96	0.99
	306A	64.96	-10.36	13.17	
	306B	64.70	- 9.52	13.96	1.18
	307A	64.98	-10.63	12.97	
	307B	64.70	- 9.52	13.96	1.51
	308A	65.01	-11.10	12.50	
	308B	64.72	- 9.81	13.66	1.76
	309A	64.96	-10.36	13.17	
	309B	64.58	- 8.95	14.46	1.95
	310A	65.01	-11.10	12.50	
	310B	64.70	- 9.52	13.96	2.17
Yellow-Green E	311A	64.82	- 9.94	13.35	
	311B	64.80	- 9.47	12.68	0.82
	312A	65.02	-10.90	14.76	
	312B	64.86	-10.06	13.51	1.51
	313A	64.86	-10.06	13.51	
	313B	64.75	- 9.07	12.05	1.77
	314A	64.88	-10.16	13.67	
	314B	64.75	- 9.07	12.05	1.96
	315A	64.97	-10.39	14.00	
	315B	64.75	- 9.07	12.05	2.37
	316A	65.02	-10.90	14.76	
	316B	64.80	- 9.47	12.68	2.53
	317A	65.04	-10.89	14.74	
	317B	64.76	- 9.08	12.05	3.25

APPENDIX C

Rejection Frequency of Pilot Test

Color Center	Approx. E_{ab}	Vector Response A	B	C	D	E
Blue	0.5	3	0	1	2	0
	1.0	5	3	1	9	1
	2.0	10	9	7	9	5
	3.0	10	10	10	10	14
	4.0	10	9	10	10	19
Cyan	0.5	3	0	0	0	0
	1.0	7	0	2	0	0
	2.0	10	6	7	7	4
	3.0	10	10	10	10	7
	4.0	10	10	10	9	7
Gray	0.5	0	0	0	0	0
	1.0	5	8	2	2	1
	2.0	10	10	4	10	6
	3.0	10	10	10	10	10
	4.0	10	10	10	10	10
Green	0.5	0	1	0	0	0
	1.0	8	0	0	0	0
	2.0	9	8	8	7	2
	3.0	10	12	10	10	9
	4.0	10	18	10	10	10
Orange	0.5	0	0	0	0	0
	1.0	6	3	1	1	3
	2.0	10	8	10	6	9
	3.0	10	10	10	8	9
	4.0	10	10	10	9	10
Purple	0.5	2	0	0	0	0
	1.0	6	0	0	1	0
	2.0	9	9	10	10	1
	3.0	10	10	9	10	12
	4.0	10	10	10	10	15

Red	0.5	2	1	1	0	1
	1.0	8	1	1	0	3
	2.0	10	5	7	3	8
	3.0	10	9	10	10	10
	4.0	10	10	10	10	10
Yellow	0.5	0	0	0	0	0
	1.0	6	2	2	0	0
	2.0	10	9	10	10	7
	3.0	10	10	20	9	9
	4.0	10	10	20	10	10
Yellow-Green	0.5	0	0	0	0	0
	1.0	6	0	1	0	0
	2.0	10	6	6	6	3
	3.0	9	9	10	10	8
	4.0	10	10	10	9	10

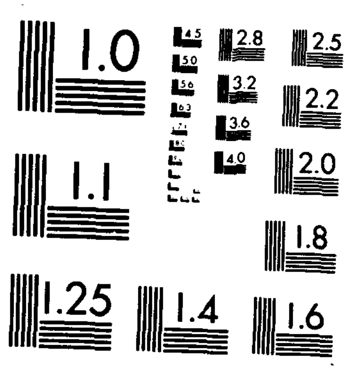
NOTES:

- Actual E_{ab} values are listed in Appendix A.
- Sample sizes for each color-difference pair were N = 10 except for samples in the Blue/E, Green/B, Purple/E, and Yellow/C vectors which were N = 20.

AD-A243 545 VISUAL DETERMINATION OF INDUSTRIAL COLOR-DIFFERENCE TOLERANCES USING PROBIT ANALYSIS(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH G D SNYDER JUN 91 2/2

UNCLASSIFIED AFIT/CI/CIA-91-083 XF-AFIT NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

APPENDIX D

Rejection Frequency of Main Experiment

Color Center	Vector Set	Sample #	E_{ab}	Freq. of Rejectn.
Blue	A	001	0.26	0
		002	0.50	10
		003	0.75	16
		004	1.01	27
		005	1.22	39
		006	1.69	47
		007	1.99	48
	B	008	0.53	1
		009	0.82	5
		010	1.14	16
		011	1.40	31
		012	1.75	41
		013	1.96	46
		014	2.35	47
	C	015	0.48	0
		016	1.19	14
		017	1.47	22
		018	1.74	30
		019	1.96	41
		020	2.21	47
		021	2.45	50
	D	022	0.34	0
		023	0.42	3
		024	0.76	5
		025	1.09	22
		026	1.22	39
		027	1.49	41
		028	1.89	47
	E	029	1.47	1
		030	1.99	8
		031	2.56	14
		032	2.83	29
		033	3.06	33

		034	3.48	41
		035	3.67	45
		036	4.73	48
Cyan	A	037	0.25	3
		038	0.50	6
		039	0.75	25
		040	0.97	36
		041	1.21	47
		042	1.56	47
		043	1.80	50
	B	044	0.72	1
		045	0.96	3
		046	1.43	14
		047	1.80	37
		048	2.05	41
		049	2.26	47
		050	2.33	48
	C	051	0.76	0
		052	1.21	10
		053	1.47	17
		054	1.75	28
		055	1.95	46
		056	2.44	48
		057	2.95	50
	D	058	1.00	3
		059	1.35	13
		060	1.56	17
		061	1.85	23
		062	2.12	44
		063	2.62	44
		064	3.18	49
	E	065	0.73	3
		066	0.98	8
		067	1.57	31
		068	1.93	38
		069	2.32	46
		070	3.06	49
		071	3.12	50
Gray	A	072	0.24	1
		073	0.48	2
		074	0.72	8
		075	0.96	27
		076	1.29	46
		077	1.72	50
		078	1.99	50

	B	079	0.24	0
		080	0.52	5
		081	0.76	15
		082	1.00	38
		083	1.21	44
		084	1.61	49
		085	1.83	49
	C	086	0.74	3
		087	1.01	17
		088	1.56	33
		089	2.09	47
		090	2.17	49
		091	2.54	50
		092	3.08	50
	D	093	0.48	1
		094	0.71	12
		095	0.96	30
		096	1.20	43
		097	1.43	49
		098	1.66	50
		099	2.19	50
	E	100	0.90	6
		101	1.18	19
		102	1.40	31
		103	1.91	47
		104	2.29	50
		105	2.43	50
		106	2.81	50
Green	A	107	0.30	2
		108	0.53	4
		109	0.74	10
		110	0.98	30
		111	1.26	39
		112	1.47	49
		113	2.03	50
	B	114	0.82	0
		115	1.49	3
		116	1.69	4
		117	1.96	17
		118	2.20	23
		119	2.41	39
		120	2.93	46
		121	3.68	50
	C	122	0.56	0
		123	0.78	4
		124	1.04	14

		125	1.24	23
		126	1.51	36
		127	1.81	43
		128	2.09	50
	D	129	0.57	0
		130	0.98	1
		131	1.25	5
		132	1.46	17
		133	1.71	25
		134	2.00	40
		135	2.13	48
	E	136	1.06	2
		137	1.22	4
		138	1.53	13
		139	1.75	22
		140	1.93	35
		141	2.15	42
		142	2.54	48
Orange	A	143	0.27	2
		144	0.49	3
		145	0.73	18
		146	0.96	23
		147	1.26	47
		148	1.53	48
		149	1.93	50
	B	150	0.47	1
		151	0.78	1
		152	1.24	18
		153	1.53	37
		154	1.83	43
		155	2.03	47
		156	2.31	50
	C	157	0.54	2
		158	0.98	5
		159	1.33	14
		160	1.50	26
		161	1.83	45
		162	2.01	46
	D	163	0.70	1
		164	0.96	2
		165	1.22	14
		166	1.75	33
		167	1.99	39
		168	2.25	45
		169	3.03	50

	E	170	0.24	0
		171	0.86	5
		172	1.08	19
		173	1.25	37
		174	1.45	46
		175	2.01	50
		176	2.21	50
Purple	A	177	0.24	0
		178	0.75	15
		179	1.23	43
		180	1.49	46
		181	1.70	48
		182	1.99	50
	B	183	0.54	0
		184	1.03	2
		185	1.28	16
		186	1.48	31
		187	1.82	40
		188	2.06	46
		189	2.26	50
	C	190	0.74	0
		191	1.04	3
		192	1.21	10
		193	1.44	37
		194	1.82	46
		195	1.99	48
	D	196	0.70	2
		197	0.95	8
		198	1.14	23
		199	1.42	37
		200	1.78	49
		201	2.01	49
	E	202	1.25	0
		203	1.49	0
		204	1.73	2
		205	2.02	2
		206	2.23	4
		207	2.44	14
		208	2.85	26
		209	3.04	34
		210	3.91	48
Red	A	211	0.35	2
		212	0.49	4
		213	0.76	19
		214	1.00	25
		215	1.26	43

		216	1.52	46
		217	2.01	49
	B	218	0.75	0
		219	1.26	1
		220	1.50	5
		221	1.75	15
		222	2.02	24
		223	2.15	43
		224	2.49	46
		225	2.97	50
	C	226	0.50	0
		227	0.99	4
		228	1.24	20
		229	1.49	26
		230	1.80	28
		231	2.00	42
		232	2.54	48
	D	233	1.14	4
		234	1.47	9
		235	1.79	17
		236	1.98	27
		237	2.23	37
		238	2.54	39
		239	2.96	45
	E	240	0.26	1
		241	1.02	12
		242	1.23	16
		243	1.51	37
		244	1.77	43
		245	2.04	48
		246	2.56	50
Yellow	A	247	0.46	2
		248	0.74	6
		249	0.98	11
		250	1.25	29
		251	1.48	42
		252	1.78	46
		253	2.08	50
	B	254	0.64	2
		255	0.91	4
		256	1.22	13
		257	1.52	27
		258	1.74	40
		259	2.03	47
		260	2.39	49

	C	261	0.76	0
		262	1.03	0
		263	1.22	1
		264	1.75	12
		265	2.08	24
		266	2.28	25
		267	2.50	37
		268	2.84	45
		269	3.17	48
	D	270	0.82	1
		271	1.04	4
		272	1.28	6
		273	1.50	16
		274	1.72	33
		275	2.08	46
	E	276	0.72	1
		277	0.95	6
		278	1.17	16
		279	1.44	36
		280	1.74	46
		281	1.94	50
		282	2.64	50
Yellow-Green	A	283	0.28	2
		284	0.47	3
		285	0.74	25
		286	1.00	36
		287	1.26	44
		288	1.52	49
		289	2.00	50
	B	290	0.50	2
		291	0.80	4
		292	1.02	19
		293	1.29	36
		294	1.50	42
		295	1.68	49
		296	1.96	49
	C	297	0.66	1
		298	1.02	9
		299	1.27	14
		300	1.54	26
		301	1.76	39
		302	1.98	50
		303	2.42	50
	D	304	0.71	1
		305	0.99	13
		306	1.18	22

	307	1.51	44
	308	1.76	48
	309	1.95	50
	310	2.17	50
E	311	0.82	1
	312	1.51	13
	313	1.77	31
	314	1.96	38
	315	2.37	48
	316	2.53	47
	317	3.25	50

NOTE: - Sample sizes for each color-difference pair were N = 50.

APPENDIX E

Software Listings

PRINCIPAL COMPONENTS FORTRAN PROGRAM:

```
      CHARACTER*50 FILENAME
      CHARACTER*5 STUFF
      DIMENSION PRIN1(20), MEAN(3), EIGEN(3)
      REAL MEAN, NEWLCENTER, NEWACENTER, NEWBCENTER

      OPEN (UNIT=7, FILE='PLOT.OUT', STATUS='NEW')
      OPEN (UNIT=8, FILE='RECORD.OUT', STATUS='NEW')

1000  TYPE*, ' ENTER FILENAME'
      ACCEPT 99, FILENAME
99    FORMAT(A)
      OPEN(UNIT=3, FILE=FILENAME, STATUS='OLD')

      WRITE(7, 300) FILENAME
      WRITE(8, 300) FILENAME
300   FORMAT(1X,A)

      DO 100 I=1,4
100   READ(3,*)
      READ(3,110)N
110   FORMAT(7X,I2)
      DO 120 I=1,8
120   READ(3,*)
      READ(3,130) (MEAN(J),J=1,3)
130   FORMAT(23X,3F14.5)

C     WRITE(8,140)  (MEAN(J),J=1,3)
C140  FORMAT(1X,'MEAN L* a* b*:', 3F10.2)
      DO 150 I=1,19
150   READ(3,*)
      READ(3,160) VALUE
160   FORMAT(68X,F7.5)
      WRITE(8,165) VALUE
165   FORMAT(1X,'EIGENVALUE 1ST COMPONENT:',F8.5)
      DO 170 I=1,9
170   READ(3,*)
      DO 180 J=1,3
180   READ(3,190) EIGEN(J)
190   FORMAT(23X,F14.5)
```

```
      WRITE(8,200) (EIGEN(J),J=1,3)
200   FORMAT(1X,'EIGENVECTORS 1ST COMPONENT:',3F10.5)
      DO 210 I=1,4
210   READ(3,*)
      DO 220 J=1,N
220   READ(3,*)NO, PRIN1(J)

           NEWLCENTER=MEAN(1)+PRIN1(N)*EIGEN(1)
           NEWACENTER=MEAN(2)+PRIN1(N)*EIGEN(2)
           NEWBCENTER=MEAN(3)+PRIN1(N)*EIGEN(3)

      WRITE(8,230)NEWLCENTER,NEWACENTER,NEWBCENTER
230   FORMAT(1X,'COLOR CENTER L* a* b* on vector:',3F10.2)

      TYPE*, ' ENTER DELTAE'
      ACCEPT*,AIMDE
      WRITE(8,240) AIMDE
240   FORMAT(1X,'DELTA E AT LD50:', F8.5)
      TYPE*, ' ENTER LOWER CONFIDENCE LIMIT'
      ACCEPT*, CFL
      WRITE(8,250) CFL
250   FORMAT(1X,'LOWER FIDUCIAL LIMIT:',F8.5)
      TYPE*, ' ENTER UPPER CONFIDENCE LIMIT'
      ACCEPT*, CFH
      WRITE(8,260) CFH
260   FORMAT(1X,'UPPER FIDUCIAL LIMIT:',F8.5)

      E = 0.
270   E = E + 0.01

      TOLPL = NEWLCENTER + PRIN1(N) * EIGEN(1) * E
      TOLPA = NEWACENTER + PRIN1(N) * EIGEN(2) * E
      TOLPB = NEWBCENTER + PRIN1(N) * EIGEN(3) * E

           DL=NEWLCENTER - TOLPL
           DA=NEWACENTER - TOLPA
           DB=NEWBCENTER - TOLPB
           D=SQRT(DL**2.+DA**2.+DB**2.)

      IF (D. LT. AIMDE) GO TO 270
      TOLPL = NEWLCENTER + PRIN1(N) * EIGEN(1) * E * 5.
      TOLPA = NEWACENTER + PRIN1(N) * EIGEN(2) * E * 5.
      TOLPB = NEWBCENTER + PRIN1(N) * EIGEN(3) * E * 5.
      PCSTAR = SQRT(TOLPA**2. + TOLPB**2.)
      WRITE(7,280) TOLPL, TOLPA, TOLPB, PCSTAR
280   FORMAT(1X,4F8.2)

      E = 0.
290   E = E + 0.01

      TOLPL = NEWLCENTER - PRIN1(N) * EIGEN(1) * E
      TOLPA = NEWACENTER - PRIN1(N) * EIGEN(2) * E
```

```
     TOLPB = NEWBCENTER - PRIN1(N) * EIGEN(3) * E

          DL=NEWLCENTER - TOLPL
          DA=NEWACENTER - TOLPA
          DB=NEWBCENTER - TOLPB
          D=SQRT(DL**2.+DA**2.+DB**2.)

     IF (D. LT. AIMDE) GO TO 290
     TOLPL = NEWLCENTER + PRIN1(N) * EIGEN(1) * E * 5.
     TOLPA = NEWACENTER + PRIN1(N) * EIGEN(2) * E * 5.
     TOLPB = NEWBCENTER + PRIN1(N) * EIGEN(3) * E * 5.
     PCSTAR = SQRT(TOLPA**2. + TOLPB**2.)
     WRITE(7,280) TOLPL, TOLPA, TOLPB, PCSTAR

     CLOSE(UNIT=3)
     TYPE*, ' MORE TO DO? 1 FOR YES'
     ACCEPT*, MORE
     IF (MORE .EQ. 1) GO TO 1000

     STOP
     END
```

RANDOMIZE SAMPLES FORTRAN INTERFACE PROGRAM TO IMSL GGPER:

```
     INTEGER IPER(10),K
     DOUBLE PRECISION DSEED
     K=10
     DSEED=123457.0D0

     CALL GGPER(DSEED,K,IPER)
     WRITE(6,500)IPER
500  FORMAT(1X,I3)

     STOP
     END
```

PROBIT ANALYSIS SAS PROGRAM:

```
     OPTIONS NODATE LINESIZE=80;
     DATA COLPROB;
          FILENAME COLPROB 'BLA.DAT';
          INFILE COLPROB;
          INPUT DELTAE TOTALOBS OBSFAIL;
     PROC PROBIT;
          VAR DELTAE TOTALOBS OBSFAIL;
          RUN;
     PROC PRINT;
```

APPENDIX F

SAS Probit Analysis Output Listing Example

ORANGE-R 1

PROBIT ANALYSIS ON DELTAE

ITERATION	INTERCEPT	SLOPE	MU	SIGMA
0	1.45116577	2.55145775	1.39090456	0.39193281
1	1.34890710	2.65750645	1.37387922	0.37629260
2	1.34084416	2.66422118	1.37344297	0.37534421
3	1.34081862	2.66424076	1.37344246	0.37534145

COVARIANCE MATRIX

	INTERCEPT	SLOPE
INTERCEPT	0.12835598	-0.08184247
SLOPE	-0.08184247	0.05633865

COVARIANCE MATRIX

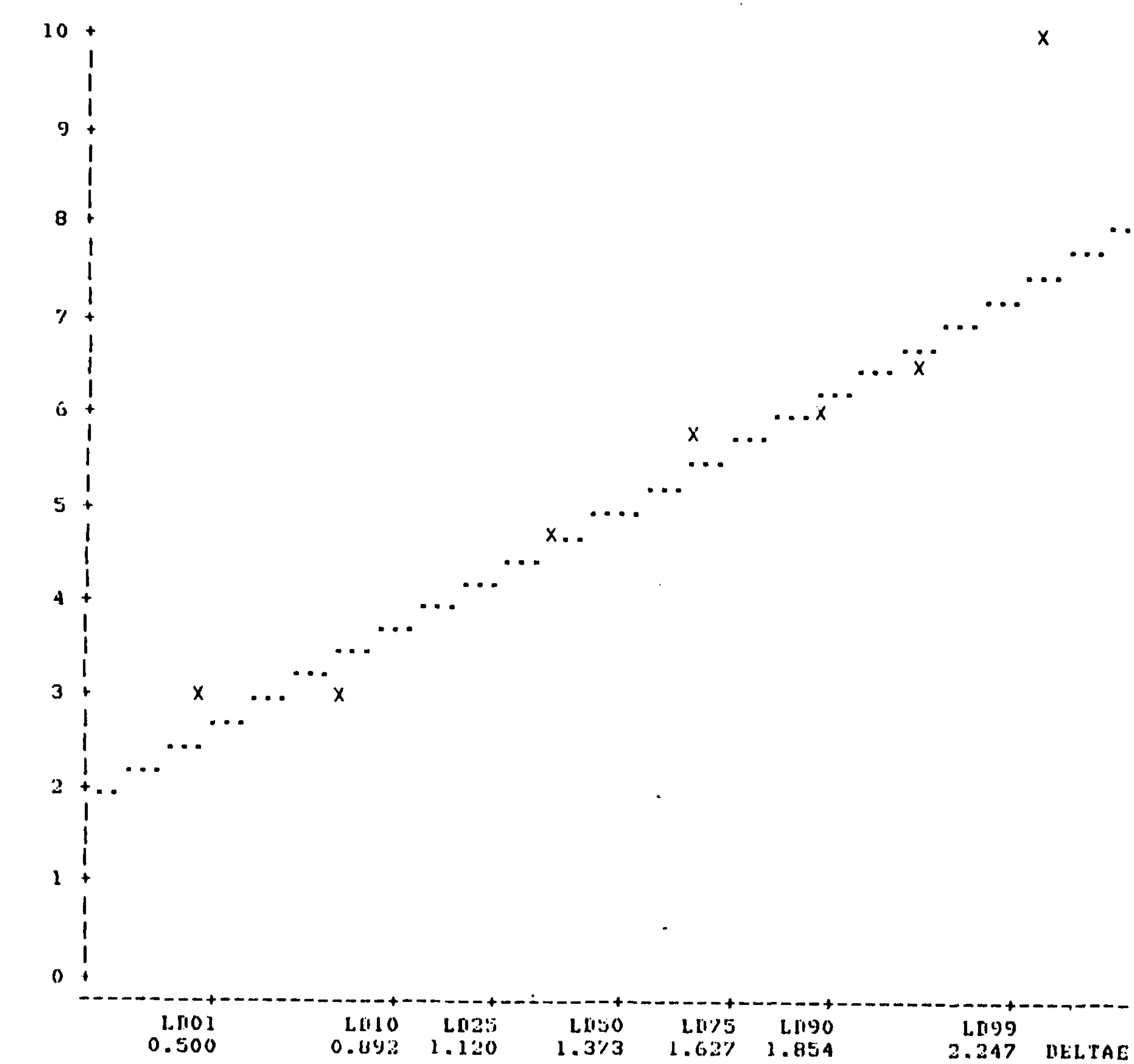
	MU	SIGMA
MU	0.00138321	-0.00023608
SIGMA	-0.00023608	0.00111818

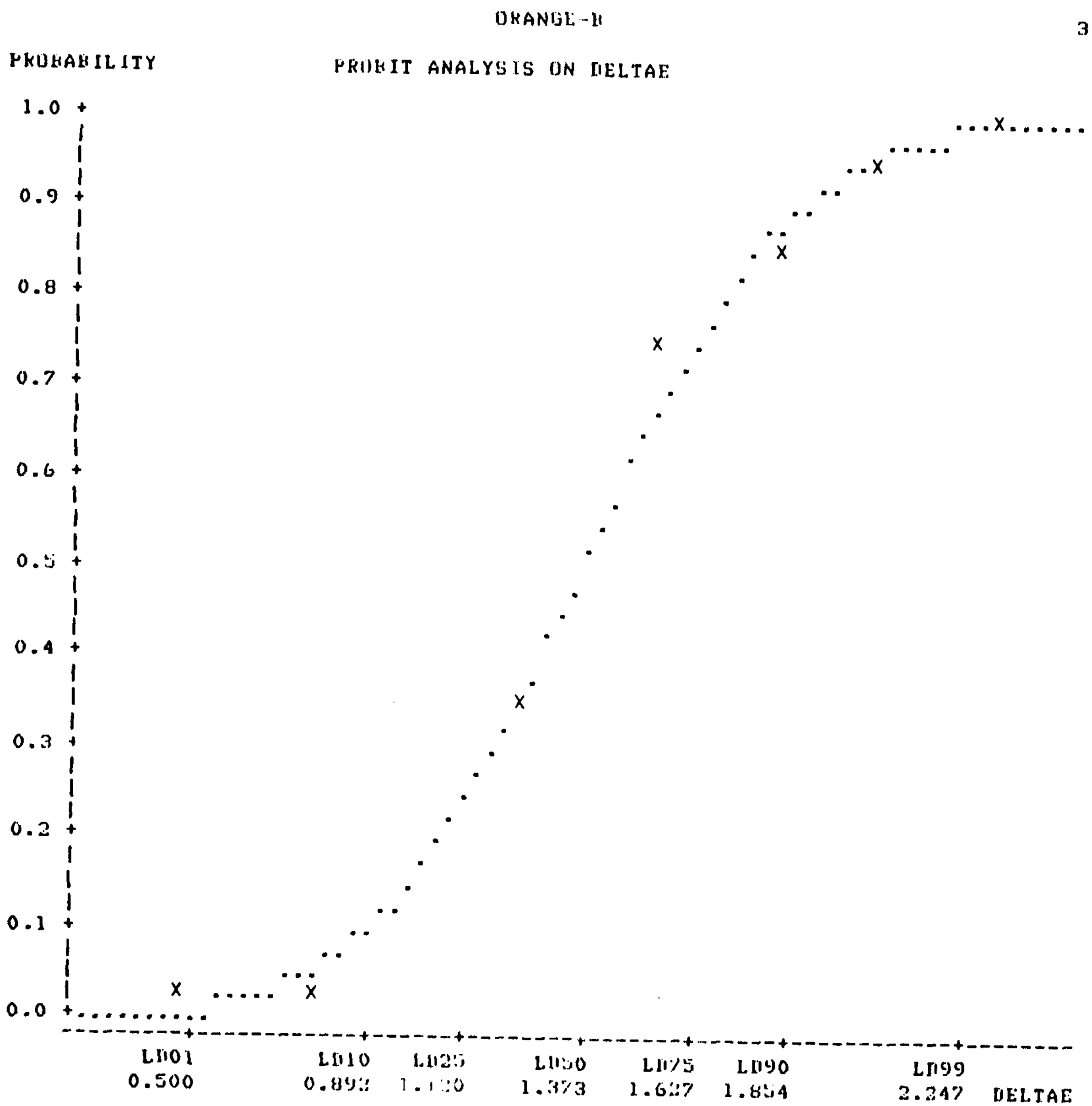
CHI-SQ = 4.7621 WITH 5 DF PROB > CHI-SQ = 0.4456

NOTE: SINCE THE CHI-SQUARE IS SMALL (P > 0.10), FIDUCIAL
LIMITS WILL BE COMPUTED USING A T VALUE OF 1.96 .

ORANGE-B 2

PROBIT ANALYSIS ON DELTAE




ORANGE-B
3
PROBABILITY
PROBIT ANALYSIS ON DELTAE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
LD01
0.500
LD10
LD25
LD50
1.373
LD75
LD90
LD99
2.347
DELTAE

ORANGE-B 4

PROBIT ANALYSIS ON DELTAE

PROBABILITY	DELTAE	95 PERCENT FIDUCIAL LIMITS LOWER	UPPER
0.01	0.50026767	0.28401083	0.65661740
0.02	0.60258536	0.40636168	0.74533770
0.03	0.66750265	0.48380389	0.80181337
0.04	0.71633740	0.54194377	0.84441469
0.05	0.75606071	0.58914959	0.87915407
0.06	0.78987148	0.62925962	0.90879228
0.07	0.81951691	0.66436942	0.93483803
0.08	0.84606086	0.69575452	0.95821044
0.09	0.87020152	0.72425171	0.97951301
0.10	0.89242303	0.75044102	0.99916446
0.15	0.98442604	0.85836097	1.08103752
0.20	1.05754712	0.94339367	1.14684628
0.25	1.12027850	1.01565569	1.20399280
0.30	1.17661321	1.07986396	1.25599740
0.35	1.22881571	1.13865373	1.30489618
0.40	1.27835079	1.19368919	1.35204662
0.45	1.32627655	1.24613200	1.39846985
0.50	1.37344246	1.29687559	1.44502481
0.55	1.42060837	1.34668364	1.49251532
0.60	1.46853413	1.39628992	1.54177508
0.65	1.51806920	1.44649118	1.59375972
0.70	1.57027171	1.49826081	1.64967864
0.75	1.62660642	1.55292728	1.71122504
0.80	1.68933780	1.61251875	1.78104211
0.85	1.76245887	1.68057076	1.86383156
0.90	1.85446189	1.76453193	1.96966339
0.91	1.87668340	1.78459043	1.99544565
0.92	1.90082406	1.80629957	2.02353628
0.93	1.92736801	1.83008015	2.05451320
0.94	1.95701344	1.85653861	2.08931030
0.95	1.99082421	1.88659830	2.12889885
0.96	2.03054751	1.92177454	2.17566780
0.97	2.07938227	1.96483961	2.23334394
0.98	2.14429956	2.02182933	2.31037209
0.99	2.24661725	2.11117915	2.43199343

ORANGE-B

OBS	DELTAE	TOTALOBS	OBSFAIL
1	0.47	50	1
2	0.78	50	1
3	1.24	50	18
4	1.53	50	37
5	1.83	50	43
6	2.03	50	47
7	2.31	50	50