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THESIS

ACOUSTIC IMPEDANCE OF MATERIALS FROM  
REVERBERATION TIME

by

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December 1991

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Acoustic Impedance of Materials from Reverberation Time

by

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Submitted in partial fulfillment of the  
requirements for the degree of

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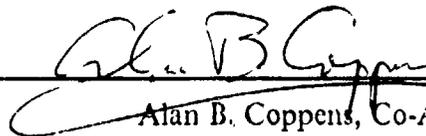
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## ABSTRACT

A theoretical model is derived to calculate the specific acoustic impedance of the absorptive material covering the walls of a cavity. This model will allow the experimental determination of the specific acoustic impedance from the measurement of the reverberation time in a water-filled cavity. The model assumes a wall of low absorption. It can not be used for rigid or pressure release walls and grazing incidence is excluded.

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## TABLE OF CONTENTS

I. INTRODUCTION .....	1
A. MOTIVATION .....	1
B. OBJECTIVE .....	1
C. METHOD .....	1
D. REMARKS .....	1
II. LITERATURE REVIEW .....	2
A. CHAMBER FOR REVERBERANT ACOUSTIC POWER MEASURE- MENTS IN AIR AND WATER [REF.1] .....	2
B. REVERBERATION TIME, ABSORPTION, AND IMPEDANCE [REF.2] ..	2
C. SABINE REVERBERATION EQUATION AND SOUND POWER CAL- CULATIONS [REF.3] .....	3
D. REVERBERATION TIME IN ENCLOSURES [REF.4] .....	4
E. OTHER LITERATURE .....	4
III. THEORETICAL MODEL .....	6
A. REVERBERATION IN CAVITIES .....	6
B. DAMPED NORMAL MODES .....	7
C. APPLICATION .....	9
1. SOLUTION (general) .....	11
2. RESULTS .....	14
3. CHECKS .....	16
a. Main assumption .....	16
b. Normal incidence .....	16
c. Power reflection coefficient .....	17
IV CONCLUSIONS .....	18
LIST OF REFERENCES .....	20
INITIAL DISTRIBUTION LIST .....	21

## LIST OF FIGURES

Figure 1. Coordinate system. ....	6
Figure 2. Value of "q". ....	13

## I. INTRODUCTION

### A. MOTIVATION

The design of coating materials that reduce the reflection of water borne sound is an important problem in underwater acoustics. Since it is difficult to theoretically predict the complex reflection coefficient of such a material, the reflection properties must be determined experimentally.

At high frequencies it is possible to measure the reflection coefficient of a slab of material by measuring the pressure after reflection of a narrow beam of sound incident at a given angle. However, this method becomes more difficult to use as the frequency of the sound is lowered.

Determining the acoustic impedance of a material by measuring the standing wave pattern in an "impedance tube", while the standard procedure in air borne sound, is very difficult for water born sound, because of the impossibility of approaching "rigid conditions" for the tube walls. This is because the " $\rho c$ " for water is approximately 5,000 times higher than that for air and is much closer to that of any wall material.

Therefore it is attractive to investigate the possibility of determining the acoustic impedance of a material from measurements of the reverberation time in a water-filled cavity with walls lined with the material.

### B. OBJECTIVE

To derive a theoretical model which will allow the experimental measurement of acoustical properties of absorption materials from measurements of reverberation time in a water filled cavity with wall lined with material.

### C. METHOD

The model is based on the normal mode theory in cavities.

It will be shown that the reverberation time in a cavity can be related to the temporal absorption coefficient of an acoustic field. From this fact, a relation between this coefficient and the complex specific acoustic impedance (resistance and reactance) will then obtained, under several assumptions and limitations.

### D. REMARKS

No references were found using the proposed approach. In addition most of the published articles in this field are for air as the medium in the cavity.

## II. LITERATURE REVIEW

This is a summary of published articles which are, in one way or the other, related to the subject. These summaries will give some idea about the accepted definitions and approximations relevant to this problem.

### A. CHAMBER FOR REVERBERANT ACOUSTIC POWER MEASUREMENTS IN AIR AND WATER [REF.1]

This was the only paper found in the Journal of the Acoustics Society of America where experiments made both in air and in water were reported, (in opinion corroborated by the authors).

The motivation for this work was the interest in obtaining sound-power measurements directly, and the idea that it might be possible with the help of a reverberant acoustic chamber. It is stated that if the sound field consists of many randomly excited modes (diffuse field), the radiation impedance of the tank is not affected by the fluid reaction, because the randomly phased fluid pressures cancel.

One of the major lessons to take out from this work is the fact that the reverberation time in the water, in a similar enclosure, is smaller than that of the air by a factor of ten. The author attributes this to the higher mean collision rate and higher sound speed in water. It is also noticeable that the air chamber gives a more uniform sound speed. This uniformity is important when the source radiation is concentrated in narrow frequency bands.

The fluid loading factor is given by  $\gamma = \rho_w c_w / \rho_p h 2\pi f$ , where  $\rho_w$  and  $c_w$  are the density and the sound speed in water,  $\rho_p$  and  $h$  the density and the thickness of the wall, and  $f$  the frequency. If  $\gamma \ll 1$ , the panels are very massive (hard walls) and specularly reflective; if  $\gamma \gg 1$ , the walls are very soft and specularly reflect sound as free surfaces. When  $\gamma \approx 1$  then the acoustic impedance of the wall is complex and the reflection is no longer specular, this the resistive and reactive parts of the wall impedance are function of the angle of incidence, this gives a more diffuse sound field.

### B. REVERBERATION TIME, ABSORPTION, AND IMPEDANCE [REF.2]

Dowell presents a rigorous theoretical model to calculate reverberation time in a room in terms of the impedance of absorption materials on the wall.

His approach is based on the coupling of the normal modes of a rigid wall room, through the damping due to wall absorption.

The sound pressure in the room is given by the summation of the contribution of each normal mode  $p = \rho_o c_o^2 \sum_n P_n F_n / M_n$ , where  $\rho_o$  and  $c_o$  are the medium density and speed of sound, respectively,  $P_n$  the coefficients in a normal mode expansion for pressure,  $F_n$  for a rectangular room is  $\cos(n_x \pi x / L_x) \cos(n_y \pi y / L_y) \cos(n_z \pi z / L_z)$  and  $M_n = \epsilon_x \epsilon_y \epsilon_z$  (for  $\epsilon_n = 1$  for  $n = 0$  and  $\epsilon_n = 1/2$  otherwise).  $P_n$  must satisfy the differential equation  $P_n + \omega_n^2 P_n + (A_A \rho_o c_o^2 / V) \sum_{r=1}^N P_r C_{nr} / M_r = 0$  with  $A_A$  being the area covered with absorption materials,  $\omega_n$  the natural frequency of the mode and  $C_n = \int (F_n F_r / z_a) dA_A / A_A$  and  $z_a$  the impedance of the material.

From the assumption that the sound field is diffuse (i.e., uniform all over the room), the initial conditions for  $p$ ,  $\dot{p}$ ,  $P_n$  and  $\dot{P}_n$  can be calculated. In this case if we assume large impedance on the walls ( $z_a / \rho_o c_o \gg 1$ ), all the coupling between modes are weak, and as  $P_n = \dot{P}_n = 0$  at  $t = 0$  for all modes except  $n = 0$ , the calculations can be made only considering this mode. Other approximation can be made if we consider the initial potential and kinetic energy on each mode as being the same, then we can find the initial conditions for, respectively,  $P_n$  and  $\dot{P}_n$ .

In the case of low absorption walls i.e., large impedance, the damping coupling coefficient is small ( $\zeta_n \ll 1$ ), Dowell found the modal reverberation time  $T_n = 6 \ln 10 V M_n \int (F_n / z_a) dA_A \rho_o c_o^2$ , except for  $T_o$ , which value is 1/2 of  $T_n$ . If a single mode dominates, the reverberation time is  $T_n$ .

Dowell claims that in the standard literature "the reverberation time is related to a random-absorption coefficient, the random absorption coefficient is related to the normal-absorption coefficient, and, finally, the normal-absorption coefficient is related to impedance" [Ref. 1: p. 183], it does not consider room geometry, and considers uniform absorption. He considers his model as a generalization of the previous methods, since it computes reverberation time directly from wall impedance and geometric factors.

### C. SABINE REVERBERATION EQUATION AND SOUND POWER CALCULATIONS [REF.3]

Various experiments show that the reverberation time changes with the amount and distribution of the absorption material in the cavity. This conflicts with the assumption commonly stated in the classical theory.

Young says the measurement of the decay time instead of the reverberation time is easier and more direct. It is also advantageous when several absorption materials are

tested on the same enclosure, because the results are obtained by comparison and the use of the decay time eliminates the eventual effects of any bulk absorption in the medium.

In order to obtain a diffuse sound field, required for the measurements, one must wait a while after the source is turned on, this is to give time for several reflections to occur.

#### **D. REVERBERATION TIME IN ENCLOSURES [REF.4]**

In this paper, the effect of the dependence of the sound absorption coefficient on the incident angle was tested.

They define  $\alpha_\theta$  as the dependence of the absorption coefficient  $\alpha$  on the angle of incidence and it is given by  $\alpha_\theta = 1 - |(Z/\rho c) \cos \theta - 1 / (Z/\rho c) \cos \theta + 1|^2$ , where  $Z$  and  $\rho c$  are the acoustic impedance of the material and the medium, respectively, and  $\theta$  the angle of incidence. Furthermore the average absorption coefficient is given by  $\alpha = \int_0^{\pi/2} \alpha_\theta \cos \theta d\theta$ .

This is a simulation of a two-dimensional enclosure, although its results permit some considerations about the physical problems connected to the reverberation phenomenon. It supports the previous authors, who states that the reverberation time depends on the distribution of absorption materials. Even in perfectly diffuse fields, the reverberation time varies as much as 20% with respect to the Sabine's formula.

Conclusion is that the angular-dependent sound absorption coefficient ( $\alpha_\theta$  increases as the incidence angle increases) has influence on the presence of specular reflections, what points to the believing that the sound absorption coefficient possesses the same property.

#### **E. OTHER LITERATURE**

All the other literature reviewed presented small or null interest for the problem. Most of them refer to work in air only and the approximation method do not consider normal mode theory and their application to experimental techniques is not convenient.

From this literature the major points to take is the fact of the reverberation time of cavities with walls covered by absorption material be related to the acoustical impedance of the walls. All the other physical properties of interest, reflection coefficient and absorption can be calculated from it. The article on the measurements of the reverberation time in air and water gives some feeling for the numerical results that one may expect from experiment

All the mathematics of the theories was intentionally omitted from this short review because it is of no interest for this work, all that we were looking for was the concepts and the mechanics of the problem.

### III. THEORETICAL MODEL

A model for calculating the complex impedance of a low-absorption wall from measured reverberation times will be developed. It is assumed that the reader is familiar with normal modes in rectangular cavities; otherwise, section 9.7 of KFCS [Ref. 2: pp.214-216] provides the necessary background to follow this development. All the subscripts ( $l, m, n$ ) are dropped from the equations for simplicity, but their constant presence must not be forgotten.

The geometry of the problem is presented in Fig. 1.

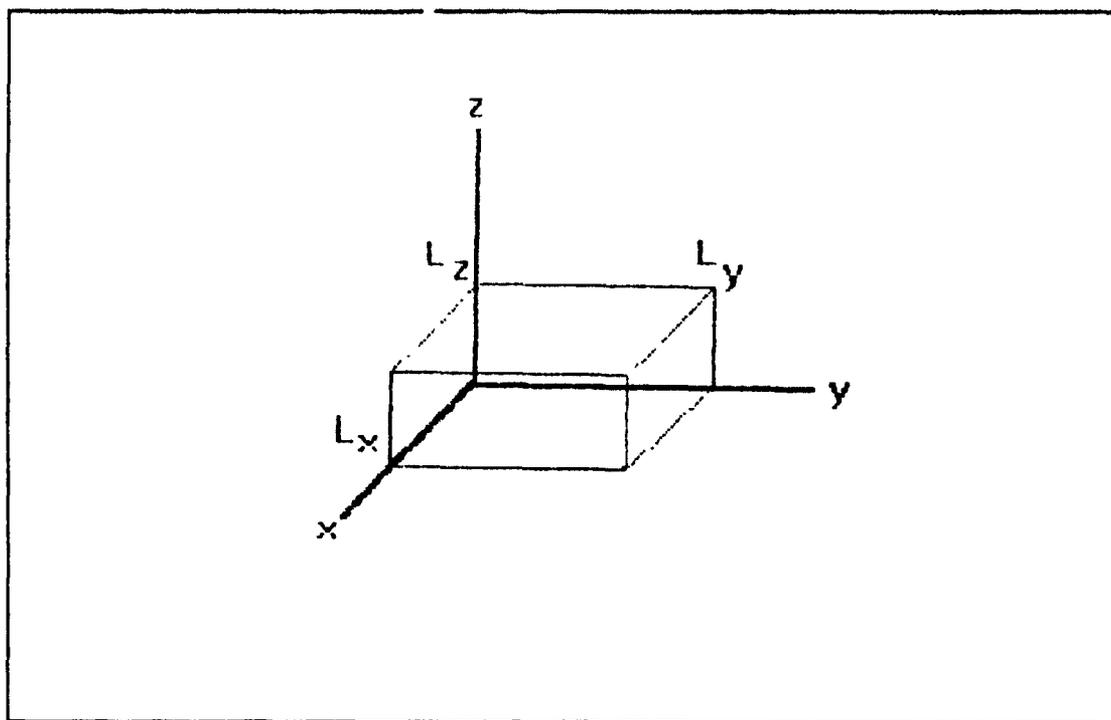


Figure 1. Coordinate system.

#### A. REVERBERATION IN CAVITIES

To measure the reverberation properties in a cavity, it needs to be completely filled with sound, i.e., the sound field must be diffuse; the more normal modes excited, the better, so that a standing wave pattern is not identifiable.

The number of normal modes  $N$  that are excited at a specific frequency  $f_c$  is given by Knudsen [Ref. 3: p.136]:

$$N = 4V \left( \frac{f_c}{c} \right)^3 \quad [1]$$

Here  $V$  is the volume of the cavity and  $c$  the speed of sound in the medium.

The equation governing the sound decay in this cavity with the sound turned off at  $t = 0$  is

$$p^2 = p^2(0) e^{-t/\tau} \quad [2]$$

where  $\tau$ , is the decay factor.

The reverberation time ( $T$ ) is defined as the time required for the sound pressure level to drop by 60 decibels (dB), so

$$20 \log \frac{P}{p(0)} = -60 \quad [3]$$

or

$$10 \log \frac{P^2}{P^2(0)} = 10 \log e^{-T/\tau} = -60 \quad [4]$$

which gives us, after some algebraic manipulation

$$T = 13.8\tau_e \quad [5]$$

## B. DAMPED NORMAL MODES

The lossy wave equation is

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad [6]$$

where  $p$  is the complex acoustic pressure and  $c$  the complex speed of sound in the medium, which turn out to be real since we will ignore losses on the medium (relaxation time negligible).

The Euler equation is

$$-\nabla p = \rho_o \frac{\partial \vec{u}}{\partial t} \quad [7]$$

where  $\rho_o$  is the density of the medium and  $\vec{u}$  the particle velocity.

As a solution to (6) try

$$p = P \cos(k_x x + \phi_x) \cos(k_y y + \phi_y) \cos(k_z z + \phi_z) e^{j\omega t} \quad [8]$$

where  $\underline{k} = \frac{\omega}{c}$  and the components of the complex wave number are:

$$\underline{k}_x = k_x + j\alpha_x \quad [9]$$

$$\underline{k}_y = k_y + j\alpha_y \quad [10]$$

$$\underline{k}_z = k_z + j\alpha_z \quad [11]$$

where  $k_x, k_y, k_z$  are the propagation terms and  $\alpha_x, \alpha_y, \alpha_z$  the absorption terms resulting from the wall expressed as a bulk absorption. The complex frequency is written

$$\underline{\omega} = \omega + j\beta \quad [12]$$

where  $\beta$  is the temporal absorption coefficient.

Substituting (8) into (6) gives

$$\underline{k}_x^2 + \underline{k}_y^2 + \underline{k}_z^2 = \frac{\underline{\omega}^2}{c^2} \quad [13]$$

or equating real parts

$$k_x^2 + k_y^2 + k_z^2 - (\alpha_x^2 + \alpha_y^2 + \alpha_z^2) = \left(\frac{\omega}{c}\right)^2 - \left(\frac{\beta}{c}\right)^2 \quad [14]$$

and imaginary parts

$$\alpha_x k_x + \alpha_y k_y + \alpha_z k_z = \frac{\omega \beta}{c^2} \quad [15]$$

The particle velocity can be found by substituting the value of  $p$ , from (8) into (7). Its components are:

$$u_x = j \frac{P}{\rho_0} \frac{k_x}{\underline{\omega}} \sin(k_x x + \phi_x) \cos(k_y y + \phi_y) \cos(k_z z + \phi_z) e^{j\omega t} \quad [16]$$

$$u_y = j \frac{P}{\rho_0} \frac{k_y}{\underline{\omega}} \cos(k_x x + \phi_x) \sin(k_y y + \phi_y) \cos(k_z z + \phi_z) e^{j\omega t} \quad [17]$$

$$\underline{u}_z = j \frac{P}{\rho_0} \frac{k_z}{\omega} \cos(k_x x + \phi_x) \cos(k_y y + \phi_y) \sin(k_z z + \phi_z) e^{j\omega t} \quad [18]$$

### C. APPLICATION

Assume that there is no bulk absorption and the wall at  $x = L_x$  is the only lossy and non-rigid wall. This implies  $\phi_x = 0$  and  $\alpha_y = \alpha_z = 0$ . Equation (15) simplifies to

$$k_x \alpha_x = \frac{\omega \beta}{c^2} \quad [19]$$

Assume that the physical property which determines the absorption characteristics of the wall is the normal specific acoustic impedance, (there are no shear waves on the walls) and let this be defined as

$$z_{wall} = (\gamma_x + j\eta_x) \rho_0 c \quad [20]$$

where  $\gamma_x$  and  $\eta_x$  are normal specific acoustic resistance and reactance coefficient.

The normal specific acoustic impedance at the wall (say at  $x = L_x$ ) is by definition, the ratio of the acoustic pressure ( $p$ ) to the normal component of the particle velocity vector ( $\underline{u}_x$ ), both evaluated at the wall.

$$\frac{p}{\underline{u}_x} = z \quad [21]$$

At the wall, the wave and the wall impedances must match. This gives us the desired boundary condition

$$\frac{p}{\underline{u}_x} \Big|_{x=L_x} = z_{wall} \quad [22]$$

or, substituting  $p$  (8),  $\underline{u}_x$  (16) and  $z_{wall}$  (20)

$$\rho_0 \frac{j\omega - \beta}{k_x - j\alpha_x} \cotn(k_x L_x) = (\gamma_x + j\eta_x) \rho_0 c \quad [23]$$

or rearranging,

$$\tan[(k_x + j\alpha_x)L_x] = j \frac{1}{\gamma_x + j\eta_x} \frac{\frac{\omega}{c} + j \frac{\beta}{c}}{k_x + j\alpha_x} \quad [24]$$

we have the fundamental relation for the physical quantities important to this problem.

The trigonometric identities

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \quad [25]$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \quad [26]$$

and defining  $\theta_o = k_x L_x$  and  $\delta = \alpha_x L_x$ , casts equation (22) into the form

$$\tan(\theta_o + j\delta) = \frac{\sin \theta_o \cos(j\delta) + \cos \theta_o \sin(j\delta)}{\cos \theta_o \cos(j\delta) - \sin \theta_o \sin(j\delta)} \quad [27]$$

If  $\delta \ll 1$  (i.e., low absorption) use of (19), and manipulation gives

$$j \frac{\omega}{k_x c (\gamma_x + j\eta_x)} \left[ 1 + j \left( \frac{\beta}{\omega} - \frac{\alpha_x}{k_x} \right) \right] = \frac{1 + j \delta \operatorname{ctn} \theta_o}{\operatorname{ctn} \theta_o - j \delta} \quad [28]$$

Working the right hand side separately, multiplying it by  $(\tan \theta_o / \tan \theta_o)$  and by the complex conjugate of the denominator

$$\frac{1 + j \delta \operatorname{ctn} \theta_o}{1 - j \delta \tan^2 \theta_o} = \frac{\tan \theta_o (1 - \delta^2)}{1 + \delta^2 \tan^2 \theta_o} + j \frac{\delta (1 + \tan^2 \theta_o)}{1 + \delta^2 \tan^2 \theta_o} \quad [29]$$

where, since  $\delta \ll 1$  the term  $(1 - \delta^2) \simeq 1$ ,

For the left hand side of (28) multiply by the complex conjugate of the denominator

$$j \frac{\omega}{k_x c (\gamma_x + j\eta_x)} \left[ 1 + j \left( \frac{\beta}{\omega} - \frac{\alpha_x}{k_x} \right) \right] = \frac{\omega}{k_x c (\gamma_x^2 + \eta_x^2)} \left\{ \eta_x - \gamma_x \left( \frac{\beta}{\omega} - \frac{\alpha_x}{k_x} \right) + j \left[ \gamma_x + \eta_x \left( \frac{\beta}{\omega} - \frac{\alpha_x}{k_x} \right) \right] \right\} \quad [30]$$

Now introduce the incidence angle (measured from the normal)  $\psi_i$ ,

$$\cos \psi_i = \frac{k_x}{k} \quad [31]$$

From equation (19) and knowing that  $\omega/c = k$

$$\alpha_x = \frac{k}{k_x} \frac{\beta}{c} \quad [32]$$

Substituting  $\alpha_x$  in  $\left(\frac{\beta}{\omega} - \frac{\alpha_x}{k_x}\right)$

$$\frac{\beta}{\omega} - \frac{\alpha_x}{k_x} = \frac{\beta}{\omega} \tan^2 \psi_i \quad [33]$$

So the right end side of (28) becomes (using the relation  $k_x = \frac{\omega}{c} \cos \psi_i$ )

$$\frac{\omega}{\cos \psi_i (\gamma_x^2 + \eta_x^2)} \left\{ \eta_x - \gamma_x \left( \frac{\beta}{\omega} \tan^2 \psi_i \right) + j \left[ \gamma_x + \eta_x \left( \frac{\beta}{\omega} \tan^2 \psi_i \right) \right] \right\} \quad [34]$$

and equation (29) becomes

$$\begin{aligned} \frac{\omega}{\cos \psi_i (\gamma_x^2 + \eta_x^2)} \left[ \eta_x - \gamma_x \left( \frac{\beta}{\omega} \tan^2 \psi_i \right) \right] + j \frac{\omega}{\cos \psi_i (\gamma_x^2 + \eta_x^2)} \left[ \gamma_x + \eta_x \left( \frac{\beta}{\omega} \tan^2 \psi_i \right) \right] = \\ \frac{\tan \theta_o (1 - \delta^2)}{1 + \delta^2 \tan^2 \theta_o} + j \frac{\delta (1 + \tan^2 \theta_o)}{1 + \delta^2 \tan^2 \theta_o} \end{aligned} \quad [35]$$

### 1. SOLUTION (general)

Equating real and imaginary parts of (35), considering  $\delta \ll 1$ , and (30), we get two equations for two unknowns  $\gamma_x$  and  $\eta_x$ :

$$\frac{j}{\cos \psi_i (\gamma_x^2 + \eta_x^2)} \left( \eta_x - \gamma_x \frac{\beta}{\omega} \tan^2 \psi_i \right) = \frac{\tan \theta_o}{1 + \delta^2 \tan^2 \theta_o} \quad [36]$$

and

$$\frac{1}{\cos \psi_i (\eta_x^2 + \gamma_x^2)} \left( \eta_x - \gamma_x \frac{\beta}{\omega} \tan^2 \psi_i \right) = \frac{\delta (1 + \tan^2 \theta_o)}{1 + \delta^2 \tan^2 \theta_o} \quad [37]$$

Equations (36) and (37) can be more useful if manipulated. First divide one by the other and cross multiply

$$\frac{\gamma_x}{\eta_x} = \left[ \frac{\delta (1 + \tan^2 \theta_o) - \frac{\beta}{\omega} \tan \theta_o \tan^2 \psi_i}{\tan \theta_o + \delta \frac{\beta}{\omega} \tan^2 \psi_i (1 + \tan^2 \theta_o)} \right] \quad [38]$$

The second equation can be obtained by the subtraction between (36) and (37):

$$(1 + \delta^2 \tan^2 \theta_o) \left[ \eta_x \left( 1 - \frac{\beta}{\omega} \tan^2 \psi_i \right) - \gamma_x \left( + \frac{\beta}{\omega} \tan^2 \psi_i \right) \right] = \quad [39]$$

$$\cos \psi_i (\gamma_x^2 + \eta_x^2) [ \tan \theta_o - \delta (1 + \tan^2 \theta_o) ]$$

Now define a couple of parameters in order to simplify the expressions (38) and (39) for further work (note that  $1 / \tan \theta_o = \text{ctn } \theta_o$ )

$$s = \frac{\beta}{\omega} \tan^2 \psi_i \quad [40]$$

$$q = \delta (\text{ctn } \theta_o + \tan \theta_o)$$

As one may notice, "q" is mainly a function of the rigidness of the wall through the  $\theta_o$ , and "s" a function of the incidence angle  $\psi_i$ .

Remember that, so far, the only assumption made is "low absorption" of the wall ( $\delta \ll 1$ ). Let us see what that say about  $\beta/\omega$ . Start with equations (19) and (31)

$$\beta = \frac{k_x}{k} \frac{\alpha_x}{c} = \frac{\alpha}{k} \cos \psi_i \quad [41]$$

which, dividing both sides by  $\omega$ , multiplying the right end side by  $L_x/L_x$ , and remembering the definition of  $\delta = \alpha_x L_x$ , becomes

$$\frac{\beta}{\omega} = \frac{\delta}{k L_x} \cos \psi_i \quad [42]$$

Since  $\cos \psi_i < 1$ ,  $k$  and  $L_x \gg \delta \ll 1$ , is clear that  $\beta/\omega \ll 1$

From this result we may induce that, if we avoid large values of  $\psi_i$ , than  $s \ll 1$ , i.e., the angle of incidence can not be close to grazing.

Fig. 2 shows that it is also acceptable to consider  $q$  as being of order  $\delta$  if we exclude extreme values of  $(\text{ctn } \theta_o + \tan \theta_o)$ , i.e.,  $\theta_o$  can not be close to 0 (quasi-rigid wall boundary condition) or to  $\pi$  (quasi-pressure release wall boundary condition).

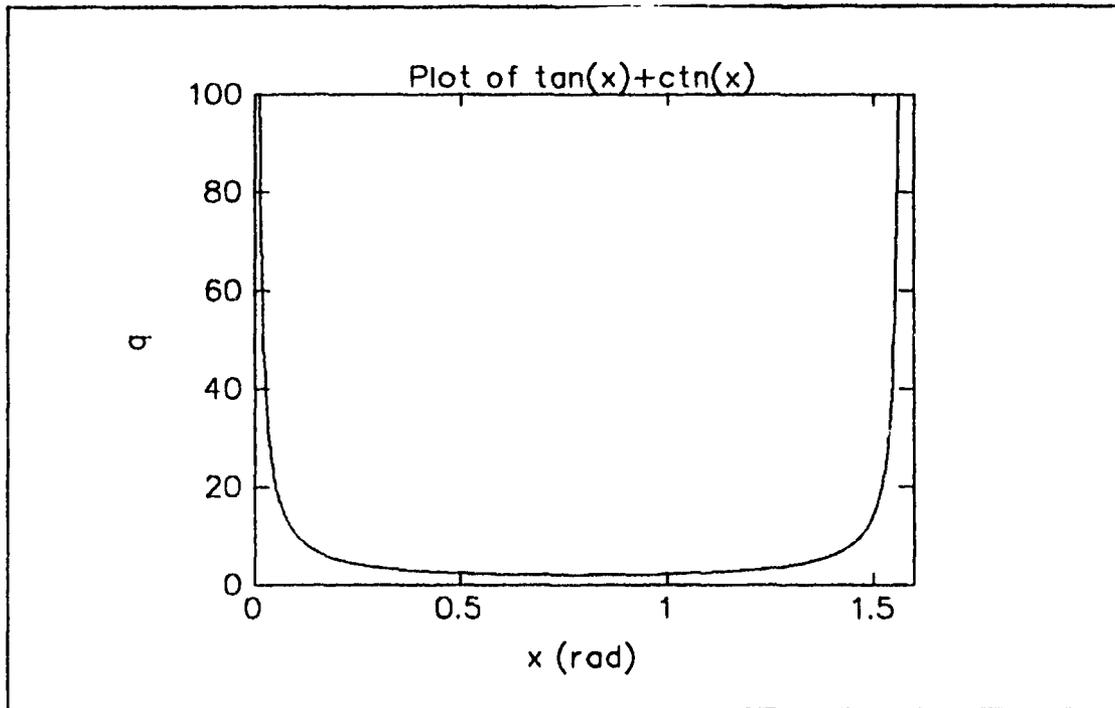


Figure 2. Value of "q".

It can also be shown that, with some exceptions,  $s < q$ , so that

$$\frac{\beta}{\omega} \tan^2 \psi_i < \delta (\text{ctn } \theta_o + \tan \theta_o)$$

Using (42) and substituting  $k$  by  $\omega/c$  gives

$$\frac{\beta}{\omega} = \frac{\delta c}{\omega L_x} \cos \psi_i$$

which when substituted in (43) and dividing both sides by  $\delta$  gives

$$\frac{c}{\omega L_x} < (\text{ctn } \theta_o + \tan \theta_o)$$

Assuming that  $kL_x \gg 1$  which excludes low frequencies (already excluded if we wish lots of normal modes excited in order to get a diffuse sound field) and since  $(\text{ctn } \theta_o + \tan \theta_o) \geq 2$  (see Figure 2), the relation is true if we avoid grazing incidence (already excluded).

The assumptions and restrictions, so far, are:

- low absorption at the wall ( $\delta \ll 1$ )
- exclude close to grazing incidence ( $\psi_i \neq \pi/2$ )
- avoid rigid or pressure release wall, ( $\theta_o \neq 0$  and  $\theta_o \neq \pi/2$ )
- $s < q \ll 1$ .

Note that the validity of the last relation can be extended when the dimensions of the cavity or the frequency are increased.

## 2. RESULTS

In this sub-section, expressions will be found for the normal resistance and reactance of the wall, using the normal incidence case as a check point. Insertion of the parameters  $s$  and  $q$  into (38) and (39) yields

$$\frac{\gamma_x}{\eta_x} = \frac{q-s}{1+sq} \quad [46]$$

$$\eta_x(1-s) - \gamma_x(1+s) = (\gamma_o^2 + \eta_o^2) \cos \psi_i \tan \theta_o (1-q) \quad [47]$$

Since  $\delta \ll 1$  and  $\tan \theta_o$  is not large, the term  $(1 + \delta^2 \tan^2 \theta_o) \approx 1$ .

Applying the approximations  $s < q \ll 1$ , which implies  $(q-s) \ll 1$  and  $sq \ll 1$ , it can be seen that the denominator of (46) goes to 1 and that  $(1+s) \approx (1-s) \approx (1-q) \approx 1$ . The ratio  $\gamma_o/\eta_o$  is much less than 1. It also tells us that both the resistance and the reactance have the same sign.

Equations (46) and (47) become:

$$\frac{\gamma_x}{\eta_x} \approx q-s \quad [48]$$

$$\eta_x - \gamma_o \approx (\gamma_o^2 + \eta_o^2) \cos \psi_i \tan \theta_o \quad [49]$$

Taking the value for  $\gamma_x$  from (48) and substituting on (49) yields:

$$\eta_x [1 - (q-s)] = [(q-s)^2 + 1] \eta_o^2 \cos \psi_i \tan \theta_o \quad [50]$$

Dividing both sides by  $\eta_x$  and noticing that  $(q-s) \ll 1$ , we have an expression for  $\eta_x$ :

$$\eta_x = \frac{1}{\cos \psi_i \tan \theta_o} \quad [51]$$

Now substitute  $\eta_x$  from (51) into (47) and get  $\gamma_x$

$$\gamma_x = \frac{q-s}{\cos \psi_i \tan \theta_o}$$

or, plugging back the values for the parameters  $s$  and  $q$  (39):

$$\gamma_x = \frac{\delta}{\cos \psi_i} (\operatorname{ctn}^2 \theta_o + 1) - \frac{\beta}{\omega} \frac{\tan^2 \psi_i}{\cos \psi_i \tan \theta_o} \quad [53]$$

The angle of incidence varies from 0 to  $\pi/2$ , so  $\cos \psi_i$  is always positive,  $\theta_o$  is a function of the mode number and the "rigidness" of the walls, so it can take any value, i.e.,  $\tan \theta_o$  can be either positive or negative. Under these arguments, looking to (51) and (52), and remembering that  $(q-s) > 0$  it is easy to verify that if  $\tan \theta_o > 0$ , then both  $\gamma_x$  and  $\eta_x$  are positive and they are negative if  $\tan \theta_o < 0$ . This is consistent with the expectations stated earlier in this section.

Applying the relation  $k_x = \omega/c \cos \psi_i$  to  $\theta_o$  yields

$$\theta_o = \frac{L_x \omega}{c} \cos \psi_i \quad [54]$$

and  $\alpha_x$  can be related to  $\beta$  by (31)

$$\alpha_x = \frac{\beta}{c} \frac{1}{\cos \psi_i} \quad [55]$$

The temporal absorption coefficient  $\beta$  can be obtained from the reverberation time  $T$  by direct comparison between equations (2) and (9) (on its maximum):

$$p_D^2 = p_D^2(0)e^{-\beta t} = p_D^2(0)e^{-t/\tau_e} \quad [56]$$

which leads to

$$\beta = \frac{1}{2\tau_e} \quad [57]$$

and with  $\tau_e$  and  $T$  related by equation (5), we have

$$\beta = \frac{13.8}{2T} \quad [58]$$

So all the variables in (53) and (51) are known or can be found from measurements, which permits the calculation of  $\gamma_x$  and  $\eta_x$  of the wall.

### 3. CHECKS

#### a. Main assumption

First we can check the assumption  $\delta \ll 1$ , to do so we may solve (51) and (53) for  $\delta$

$$\delta = \frac{\gamma_x / \eta_x}{(\text{ctn } \theta_o + \tan \theta_o) - \frac{c}{\omega L_x \tan^2 \psi_i}} \quad [59]$$

The denominator is always greater than 0 (except for grazing incidence), and it is larger than 1 because we assumed it when checking for  $s < q$ , since  $\frac{\gamma_x}{\eta_x} \ll 1$  our initial assumption is verified.

#### b. Normal incidence

We can start with (38) and (39), considering again that  $(1 + \delta^2 \tan^2 \theta_o) \approx 1$  and  $\psi_i = 0$ . Without any other assumptions that  $\delta \ll 1$  and  $\theta_o \neq (n+1)\pi/2$  or  $n\pi$ , equation (38) becomes

$$\frac{\gamma_x}{\eta_x} = \frac{\delta(1 + \tan^2 \theta_o)}{\tan \theta_o} = q \quad [60]$$

Substitution of  $\gamma_x$  in (39) yields

$$\eta_x(1-q) = \eta_x^2[1+(1-q)^2] \tan \theta_o \quad [61]$$

dividing both sides by  $\eta_x$  and noting that  $(1-q) \approx 1$  we get

$$\eta_x = \frac{1}{\tan \theta_o} \quad [62]$$

which, substituting in (60) gives

$$\gamma_x = \frac{q}{\tan \theta_o} = \delta(\text{ctn}^2 \theta_o + 1) \quad [63]$$

This results are the same as (52) and (53) if we set  $\psi_i = 0$ .

c. *Power reflection coefficient*

The power reflection coefficient ( $R_\pi$ ) is given in KFCS [Ref. 2: p. 139] by equation (6.45)

$$R_\pi = \frac{(\gamma_x \cos \psi_l - 1)^2 + \eta_x^2 \cos^2 \psi_l}{(\gamma_x \cos \psi_l + 1)^2 + \eta_x^2 \cos^2 \psi_l} \quad [64]$$

We want it to be close to one (low absorption), so, dividing everything by ( $\cos^2 \psi_l$ )

$$(\gamma_x - \sec \psi_l)^2 + \eta_x^2 \approx (\gamma_x + \sec \psi_l)^2 + \eta_x^2 \quad [65]$$

This tells us that  $\eta_x$  can take any value, and

$$\gamma_x^2 - 2\gamma_x \sec \psi_l + \sec^2 \psi_l \approx \gamma_x^2 + 2\gamma_x \sec \psi_l + \sec^2 \psi_l \quad [66]$$

or

$$-2\gamma_x \sec \psi_l \approx 4\gamma_x \sec \psi_l \quad [67]$$

which becomes

$$\gamma_x \ll \cos \psi_l \quad [68]$$

From this relation we find that  $\gamma_x$ , despite of being much smaller than  $\eta_x$ , has to be small. Recalling equation (46) one may see that

$$\gamma_x = \frac{q-s}{1+sq} \eta_x \ll \cos \psi_l \quad [69]$$

which, after some manipulation, and taking into account the approximations earlier made, tells us that  $\eta_x$  has to be also much smaller than ( $\cos \psi_l$ ). So we expect, for low attenuation at the walls, low resistance and low reactance.

#### IV. CONCLUSIONS

Using normal mode theory in cavities it is possible to calculate the normal acoustic resistance and reactance as functions of the reverberation time, the frequency of the acoustic field, the speed of sound in the medium, the dimensions of the cavity, and the incidence angle.

To measure the reverberation time, a diffuse field is required, i.e., a fairly uniform pressure is desired throughout the cavity. The reverberation time can be related to the temporal absorption coefficient and it is expected to increase with the increasing dimensions of the cavity, the decreasing absorption of the walls, and the decreasing  $\rho c$  of the medium.

Under the following limitations:

- low absorptive wall
- avoid rigid or pressure release walls
- exclude angles of incidence close to grazing

the results obtained as a function of determinable parameters are: from (53), the normal specific acoustic resistance is

$$\gamma_x = \frac{\beta}{c \cos^2 \psi_i} \left[ \operatorname{ctn}^2 \left( \frac{L_x \omega}{c} \cos \psi_i \right) + 1 \right] - \frac{\beta}{\omega} \frac{\tan^2 \psi_i}{\cos \psi_i \tan \left( \frac{L_x \omega}{c \cos \psi_i} \right)} \quad [70]$$

from (51), the normal specific acoustic reactance is

$$\eta_x = \frac{1}{\cos \psi_i \tan \left( \frac{L_x \omega}{c} \cos \psi_i \right)} \quad [71]$$

It is noticeable that, under the mentioned limitations, the resistive part has to be small and much smaller than the reactive part.

The exclusion of the "close to grazing" angles of incidence is assumed as not being very important because, for a diffuse field, there are many excited normal modes, so the modes excluded by this limitation are relatively few.

Despite the limitations on this model, it is a generalization of the theory presented in the textbooks [Refs. 5, 6].

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