A Computer Simulation of an Adaptive Noise Canceler with a Single Input

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NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The citation of trade names and names of manufacturers in this report is not to be construed as official Government endorsement or approval of commercial products or services referenced herein.
A description of an adaptive noise canceler using Widrow's LMS algorithm is presented. A computer simulation of canceler performance (adaptive convergence time and frequency transfer function) was written (for use as a design tool). The simulations, assumptions, and input parameters are described in detail. The simulation is used in a design example to predict the performance of an adaptive noise canceler in the simultaneous presence of both strong and weak narrow-band signals (a coiled frequency hopping radio scenario).

On the basis of the simulation results, it is concluded that the simulation is suitable for use as an adaptive noise canceler design tool; i.e., it can be used to evaluate the effect of design parameter changes on canceler performance.
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Convergence Parameter vs Number of Iterations
Necessary for a 30 db Reduction in Interference Power
This report presents the results of a computer simulation of the operation of an adaptive noise canceler in the simultaneous presence of both strong and weak narrow-band signals. The simulation is intended to be used as a tool for designing such a canceler. For a given scenario, it will identify the design parameter values needed to cause the canceler to attenuate the strong signal to a user specified level and pass the weak signal.

This study was motivated by the cosite interference problem encountered by frequency hopping radios. When two or more such radios and their antennas are independently operated in close proximity, i.e., in a jeep or communication shelter, a cosite interference problem can develop. In this type of situation, the radio may not be able to meet its specified bit error rate. A degraded bit error rate means that the receiver sensitivity will be degraded and, as a result, communication range will be decreased.

This type of interference problem is caused by the transmitter's strong signal being too close to the frequency the receiver is tuned to. The difference in power levels between the interfering transmitter signal at the receiver input and the minimum signal the receiver is capable of detecting could be in excess of 130 dB. The receiver may not be able to provide the entire 130 dB of interference rejection filtering needed at the transmitter frequency. Therefore, an external applique capable
of supplying the additional filtering may be required. An adap-
tive noise canceler with a single input is one possible way of
providing this required additional filtering. The next two
sections describe the adaptive noise canceling concept.
ADAPTIVE NOISE CANCELLING

An Adaptive Noise Canceler is shown in Figure 1. It works as follows.

"A signal is transmitted over a channel to a sensor that receives the signal plus an uncorrelated noise \(N_o\). The combined signal and noise \(S+N_o\) form the 'primary input' to the canceler. A second sensor receives a noise \(N_1\) which is uncorrelated with the signal but correlated in some unknown way with the noise \(N_o\). This sensor provides the 'reference input' to the canceler. The noise \(N_1\) is filtered to produce an output \(Y\) that is a close replica of \(N_o\). This output is subtracted from the primary input \(S+N_o\) to produce the system output \(N_o - Y\).\[1\]

The system output \(E = S + N_o - Y\) of the canceler is used to modify, via an adaptive algorithm, the frequency response of the adaptive filter.

The adaptive filter will usually be implemented as a programmable transversal filter (PTF) (see Figure 2). A transversal filter is the preferred implementation because:

1. It is one of the simplest filter structures. The filter output is simply the sum of delayed and scaled inputs.

2. There is no feedback from the taps to the input.

3. It is stable. Since there is no feedback, a finite
Figure 1. Adaptive Noise-Canceling Concept (from Ref. 9).
Figure 2. Programmable Transversal Filter (PTF).
filter input produces a finite filter output. If feedback were present, then an inappropriate choice of filter tap weights could cause the filter output to become unbounded, i.e., the output would oscillate. A transversal filter cannot oscillate.

4. It has a linear phase characteristic, i.e., it produces a phase shift that is linearly proportional to frequency. It can be shown\(^\text{14}\) that if a signal is to be passed through a linear system without any resultant distortion, the overall system frequency response must have a constant amplitude gain characteristic over the frequency spectrum of the input signal and its phase shift must be linear over the same frequency spectrum. Filtering without distortion is important for adaptive noise canceling because the adaptive filter must pass the interferer without distortion so that it can be subtracted (at the summer) from the unfiltered interferer. If the adaptive filter introduces distortion, then the summer is no longer subtracting two identical interferers. As a result, the error power and error amplitude with adaptive filter distortion will be higher than without distortion. This is not desirable.

5. There is a simple and analytically tractable relationship between the transfer function of a transversal filter and its parameters (see Equation 2 of the simulation description section). The complicated nonlinear relationship between parameters and transfer function
for most other filter structures makes the analysis and
calculation of adaptive algorithms much more difficult
than for transversal filters.

6. Widrow's algorithm, one of the most widely used adaptive
algorithms, assumes a transversal filter structure.

A PTF forms a weighted sum of delayed versions of the input
signal. It is programmable in that the weights can be changed.
Changing the weights changes the frequency transfer function of
the PTF. A PTF is identical in structure to a programmable
finite impulses response (FIR) digital filter. The specific
technology used to implement a PTF will depend on the frequency
range of interest. For VHF and UHF applications, Surface Acous-
tic Wave (SAW) devices are an appropriate technology. At VHF and
UHF frequencies, SAW technology can give the appropriate sam-
pling rates (intertap delay) and total delay times necessary to
implement transversal filters with the required bandwidth and
frequency resolution needed for cosite interference reduction.
Before an adaptive noise canceler can be implemented, a reference signal correlated with the interfering signal but not the intended signal must be generated. When the interfering signal $N_o$ is much stronger than the intended signal $S$, the reference signal can be generated by modifying the adaptive noise canceler of Figure 1 to give the circuit shown in Figure 3 (an adaptive noise canceler with a single input). In Figure 3, the primary and reference inputs are connected together. In effect, Figure 3 assumes that the reference input is equal to the primary input. This may at first appear contradictory. The reference input $N_1$ (see Figure 1) has to be correlated to the interference $N_o$, not the signal $S$. But since the signal $S$ is part of the primary input, it will be part of reference input if reference input = primary input as per Figure 3. Hence the reference input appears to be correlated to the signal, also.

When the interfering signal $N_o$ is much larger than the intended signal ($N_o \gg S$) the apparent contradiction is resolved. In this case the reference input $N_1$ ($N_1 = S + N_o$ = primary input) is highly correlated with and "looks" like the interfering signal (i.e., $N_1 \approx N_o$). While $S$ is a component of $N_1$, and therefore, will correlate to a certain extent with $N_1$, $N_o$ is so much larger than $S$ that $N_1$ will be much more highly correlated to $N_o$ than $S$. So to a very good approximation, the reference input $N_1$ is correlated to the interference $N_o$, not the signal $S$. (For a rigorous proof, see Reference 2.)
Figure 3. Adaptive Noise Canceler with a Single Input.
It will now be shown why the reference input must be correlated to the interference and not the signal. The adaptive filter within the canceler must filter the reference input $N_1$ to produce an output $Y$ that is a close replica of $N_o$. If $N_1$ is not correlated to $N_o$, i.e., if $N_1$ does not "look" somewhat like $N_o$, then no amount of filtering can make $Y$ look like $N_o$.

The factor $1/\sqrt{2}$ appears in Figure 3 because the input power splitter is assumed to evenly split the power associated with the signal and interference amplitudes $S$ and $N_o$. Since power is proportional to amplitude squared, reducing power by a factor of two means that amplitude is reduced by $\sqrt{2}$ at each output of the input power splitter.

As the adaptive algorithm iterates, it will cause the adaptive filter to form a bandpass around the interfering frequency. If the PTF has been properly designed, then the resulting bandpass filter will "pass" $F_{N_o}$ the interfering frequency and "reject" the intended signal frequency. Then the output of the adaptive filter (the filtered reference signal) will "look" even more like $N_o$ than input signal. When the output is subtracted from $S + N_o$, at the output power combiner, a signal very similar to $S$ will remain. The interference has been canceled. Thus the circuit shown in Figure 3 does indeed behave as an adaptive noise canceler.
Adaptive algorithms have been extensively discussed in a previous LABCOM technical report, in books and review articles. The theory of adaptive algorithms is too lengthy and involved to be reviewed in this report. The interested reader is referred to the references just cited.

The Least Mean Square (LMS) or Widrow's algorithm was chosen as the adaptive algorithm for use in the simulation. It is well understood, computationally simple and fast.

In the LMS algorithm, it is assumed that the adaptive filter is an adaptive linear combiner (see Figure 4). If data are acquired and input in parallel to an adaptive linear combiner, the structure in Figure 4a is used. For serial data input, the structure in 4b is used. Note that Figure 4b is just a tapped delay line or transversal filter. This is the filter structure that will be assumed in this report. It is further assumed that a "desired" response signal is available. For the adaptive noise canceler with a single input of Figure 3, the primary input is the "desired" response.

The LMS algorithm is given by:

$$W_{k+1} = W_k + 2 \mu e_k X_k$$

where:

- $W_k$ = the weight vector at the $k$th iteration, i.e., the set of tap weights used on the $k$th iteration.
$W_{k+1}$ = the weight vector at the $k+1$th iteration.

$\mu$ = a constant that regulates the step size or increment size of the weight vector change.

$\epsilon_k$ = canceler output error amplitude at the $k$th iteration.

$X_k$ = tap signal vector at the $k$th iteration, i.e., the set of tap signal amplitudes at the $k$th iteration.
Figure 4. The Adaptive Linear Combiner: (a) In General Form; (b) As a Transversal Filter (from Ref. 1, page 101).
DESCRIPTION OF THE ADAPTIVE NOISE CANCELER WITH
SINGLE INPUT SIMULATION

The circuit in Figure 3 was simulated. In the simulation, two sinusoids of different frequency (user specified) were input to the power splitter. For any given iteration k, the weight vector $W_k$ for that iteration was used to calculate the frequency transfer function of the adaptive filter or PTF. Knowledge of the PTF frequency transfer function and the input frequencies allows the input to the negative or inverting part of the power combiner from the lower "channel" of the adaptive noise canceler to be calculated. The output of the power combiner, the error amplitude $e_k$ and the error power $P_{e_k}$ can then be calculated. The signals $X_i$ at the tap outputs of the PTF are then calculated. Since $e_k$ and $X_i$ are known, Widrow's LMS algorithms (Equation 1) can be used to calculate a new set of tap weights $W_{k+1}$. The simulation can then be repeated with the new tap weight vector $W_{k+1}$ and new values for the error amplitude $e_k$ and the error power $P_{e_k}$ can then be calculated. After each iteration, the adaptive noise canceler error or output power for that iteration is displayed. When the output power has been reduced by a user specified number of dB, the simulation is stopped.

Once the adaptive filter processor speed of operation or equivalently its time per iteration is specified, then the product of the number of iterations necessary to reduce the output power by the user specified number of dB and the time per
iteration gives the adaptive convergence time. This is the time needed for the required reduction in output power to take place. So, in effect, when the simulation stops, the number of iterations it has performed tells us the value of the adaptive convergence time.

The simulation also displays the PTF frequency transfer function. In order for the canceler to work properly the interfering frequency must be "passed" and the intended signal "rejected" by the PTF. This can be observed directly from the PTF amplitude gain vs. frequency curve.
SIMULATION ASSUMPTIONS

The assumptions made in the course of the simulation are as follows:

1. The programmable transversal filter was assumed to have equally spaced taps. The frequency transfer function for a transversal filter with uniformly spaced taps is given by the following equations:

\[ H_k(\omega) = \sum_{i=1}^{N} W_{ik} e^{j\omega\Delta(-i)} \]  

where:

- \( H_k(\omega) \) = frequency transfer function at the \( k \)th iteration.
- \( \omega \) = frequency
- \( W_{ik} \) = weight on the \( i \)th tap at the \( k \)th iteration.
- \( \Delta \) = intertap delay
- \( N \) = number of taps
- \( j = \sqrt{-1} \)

amplitude gain = \(|H_k(\omega)| = [(\text{Real } H_k(\omega))^2 + (\text{Imag } H_k(\omega))^2]^{1/2} \) (3)

phase shift = \( \tan^{-1}(\text{Imag } H_k(\omega) / \text{Real } H_k(\omega)) \) (4)

2. The error amplitude output from the adaptive noise canceler was calculated using a "snapshot in time" approach, i.e., the interfering and intended signals input to the power splitter of the adaptive noise canceler were calculated at a single point in time.
These signals are then mathematically sent through the adaptive noise canceler. The upper channel of the canceler does not change either sinusoid. The PTF in the lower channel changes the amplitude and phase of each signal. The two channels are then subtracted at the power combiner to produce the error amplitude $r_k$.

3. At the input to the power splitter the intended and interfering signals are assumed to be in phase. An input parameter (called SIMTIME) determines at what point in time the snapshot is taken (see Figure 5).

4. At each iteration it was assumed that the snapshot was taken at the same relative point on the waveform, i.e., the same value of SIMTIME was used at each iteration.

5. It was found that the only way that the canceler output could be reduced by the specified number of dB was to choose the snapshot point very close (within a few degrees) to the maximum value of the interference. The snapshot point for all the simulation runs was therefore chosen at the maximum value of the interference. This is not as restrictive an assumption as it appears to be and will be further discussed in the parameters section of this report.

6. The output power for the power combiner was calculated as the RMS output of the two inputs.

7. The first or initial set of tap weights is always assumed to be all zero's. This was done so that all simulation runs have the same "starting point," i.e., initial weight vector
Thus, meaningful comparisons between runs can be made.

8. The simulation treats the two frequency hopping radios as fixed frequency radios, i.e., it does not hop or change the frequencies input by the user. This assumption was made in order to observe how many iterations of the adaptive algorithm were needed to reduce the interfering signal by the user specified number of dB. If the radios' frequencies were allowed to hop, then for a "bad" choice of design parameters the hop would occur before the power had been reduced by the specified amount. It would not be possible to tell just how bad the design parameters were. Frequency hopping would only complicate the simulation, it would not help in understanding adaptive algorithm performance. Therefore, it was not simulated.
Figure 5. Simulation Snapshot Point.
The user supplied input parameters to the simulation can be divided into two categories: scenario parameters and design parameters.

Scenario parameters are fixed or given by the exact nature of the scenario simulated. The analyst has no control over these parameters. Specifically the scenario input parameters required are:

- Intended frequency and power.
- Interfering frequency and power.
- Desired reduction of interfering signal power (in dB).
- PTF voltage loss per tap.

Design parameters can be varied. They are chosen to achieve the required adaptive noise canceler frequency transfer function and adaptive convergence time.

The design input parameters required are:

- The intertap delay time between taps of the PTF.
- The number of taps in the PTF.
- MU (or μ) the convergence parameter.

\( \Delta \) The intertap delay \( \Delta \) should be chosen to be consistent with the Nyquist criteria, i.e., if \( f_{\text{max}} \) is the highest frequency in the baseband signal, then

\[
f_{\text{max}} \leq \frac{1}{2\Delta}
\] (5)
\[ N, \text{ the number of taps, should be chosen on the basis of the} \]
\[ \text{required PTF frequency resolution, where frequency resolution is} \]
\[ \text{defined as the difference in frequency between the center of the} \]
\[ \text{PTF frequency transfer function and its first zero. Frequency} \]
\[ \text{resolution is given by}^{11}: \]
\[ \text{Frequency Resolution} = \frac{1}{N\Delta} = \frac{1}{(\text{total delay})} \quad (6) \]

\[ \text{The convergence parameter } \mu, \text{ which regulates the size of the} \]
\[ \text{weight vector increment at each iteration (see Equation 1) is} \]
\[ \text{the most important design parameter. Adaptive convergence time} \]
\[ \text{is a very sensitive function of } \mu. \text{ If } \mu \text{ is too small, the} \]
\[ \text{convergence time will be too long. If } \mu \text{ is too large, the LMS} \]
\[ \text{algorithm may "blow up" and no convergence will be achieved,} \]
\[ \text{i.e., there will be no reduction in the interfering signal power.} \]

\[ \text{It can be shown}^{12} \text{ that for a transversal filter with no tap} \]
\[ \text{loss, if the following inequality is satisfied:} \]
\[ 0 < \mu < \frac{1}{((N+1) \text{ (input power)})} \quad (7) \]
\[ \text{then the expected adaptive noise canceler weight vector will} \]
\[ \text{converge to the optimum or Weiner weight vector. The simulation} \]
\[ \text{requirements are somewhat different. Total elimination of the} \]
\[ \text{interference (the Wiener solution) is not required. It is only} \]
\[ \text{necessary to reduce the interfering signal by a user specified} \]
\[ \text{number of dB, but it must be done in a minimal number of itera-} \]
\[ \text{tions. Despite these differences, Inequality 7 has been found} \]
\[ \text{(empirically) to be a good guide for the selection of } \mu. \]

\[ \text{If the particular technology used has a loss at the PTF taps} \]
\[ \text{(e.g., SAW technology) then } \mu \text{ as determined from Inequality 7} \]
should be multiplied by \((1/\text{tap loss})\) to compensate for the tap loss. This works because in the LMS algorithm (Equation 1)

\[
\mathbf{w}_{k+1} = \mathbf{w}_k + 2\mu \varepsilon_k \mathbf{x}_k
\]

\(\mu\) and the tap signal vector \(\mathbf{x}_k\) only appear in a product. If \(\mathbf{x}_k\) is reduced by the same loss at each tap then multiplying \(\mu\) by \((1/\text{tap loss})\) cancels the tap loss in the product. So with no loss in generality in the analysis, it can be assumed that tap loss = 0. Therefore the analysis can be performed before tap loss data is available.

The only remaining parameter is SIMTIME, the parameter that determines the point in time at which the simulation snapshot is taken (see the simulation assumption section and Figure 5). Using typical cosite interference input parameters it was found that the only way that the canceler output could be reduced 30 dB (a "typical" cosite interference requirement) was to choose the snapshot point very close (within a few degrees) to the maximum amplitude value of the interference. So, in effect, SIMTIME is no longer an adjustable input parameter. All simulation runs will use the same value (specifically, SIMTIME = 1).

Choosing the snapshot point at the maximum amplitude value of the interfering signal can be justified on both mathematical and physical grounds.

Mathematically, the dominant contribution to canceler output is from the interfering signal. Any signal that is input to an
adaptive noise canceler with a single input (see Figure 3) is first split by the power splitter. Half the input power goes directly to the non-inverting input of the power combiner. The other half of the power is first filtered by the PTF and then input to the inverting input of the power combiner. In the course of the simulation, the canceler output power due to the interfering signal was shown to be equal to and was modeled as:

\[
\text{Canceler output power due to interfering signal} = [C_J^2 + C_J^2 |H_k(\omega_J)|^2 - 2 C_J^2 |H_k(\omega_J)| \cos(PSJSF_k)]/4
\]

where:

\[
C_J = \left[2 \cdot \text{Interfering power in watts} \right]^{1/3}
\]

\[
H_k(\omega_J) = \text{frequency transfer function at the } k\text{th iteration evaluated at the interfering or jamming frequency } \omega_J.
\]

\[
PSJSF_k = \text{PTF phase shift at the jammer signal frequency at the } k\text{th iteration.}
\]

Not being able to achieve the user specified reduction in the interfering power (when the simulation snapshot is not taken at the interference maximum) means that the power calculated in Equation 8 is too large. This means that either the PTF phase shift at the interfering frequency is too large or that \(|H_k(\omega_J)|\) is not numerically close enough to one. \(|H_k(\omega_J)|\), the PTF amplitude gain, must be close to one and the phase shift must be close to zero in order that the filtered interfering frequency signal cancels the unfiltered interfering frequency at the power combiner.
The PTF phase shift is given by Equation 4

\[ \text{PSJSF}_k = \tan^{-1} \left( \frac{\text{Imag } H_k(\omega_J)}{\text{Real } H_k(\omega_J)} \right) \] (4)

where \( H_k(\omega_J) \), the frequency transfer function at the interfering frequency \( \omega_j \), is given by Equation 2.

\[ H_k(\omega_j) = \sum_{I=1}^{N} W_{I,k} e^{j\omega \Delta (-I)} \] (2)

where:

- \( W_{I,k} \) = Weight on the \( I \)th tap at the \( k \)th iteration
- \( \omega_j \) = Interfering frequency
- \( \Delta \) = Intertap delay
- \( N \) = Number of taps
- \( J = \sqrt{-1} \)

In Equations 4 and 2, both the PTF phase shift and amplitude gain are functions of the tap weights. The tap weights are given by Widrow's LMS algorithm.

\[ \rightarrow W_{k+1} = \rightarrow W_k + 2\mu \epsilon_k X_k \] (1)

The value of the error amplitude \( \epsilon_k \) will depend on the snapshot time, i.e., \( \epsilon_k \) will be a function of SIMTIME. \( \epsilon_k \) is the amplitude output of the power combiner. Since the interfering signal is so much larger than the intended signal, \( \epsilon_k \) can be approximated by the difference between the unfiltered and filtered interfering signals. There is a phase shift between the unfiltered and filtered interfering signals that is given by
Equation 4. This phase shift affects the value of $\epsilon_k$.

Choosing the snapshot point at the maximum value of the interfering signal (i.e., $\text{SIMTIME} = 1$) minimizes the impact of a nonzero phase shift on $\epsilon_k$. In other words, when $\text{SIMTIME} = 1$, we can achieve a "small" value of $\epsilon_k$ even for a non-zero phase shift. This is due to the fact that at the maximum value of the interference, i.e., at the top of the sinewave, the sinewave is flat. Its first derivative is zero. Therefore, its rate of change is also zero. This is the point or region where the sinewave changes most slowly. This region of slow change is where a "large" phase shift can be most easily "tolerated," i.e., a phase shift produces the least change in the sinewave when the snapshot point is at the interfering frequency maximum. The phase shift does not move the sinewave very far off of its maximum. This implies that $\epsilon_k$ will be small. So, in effect, $\text{SIMTIME} = 1$ produces the smallest error amplitude $\epsilon_k$ for any given phase shift.

If $\epsilon_k$ is "small" (when $\text{SIMTIME} = 1$), then in Widrow's algorithm

$$W_{k+1} = W_k + 2\mu \epsilon_k x_k$$ (1)

the incremented term $2\mu \epsilon_k x_k$ will also be relatively small. A smaller incremental term means that the optimal set of weights can be approached more precisely, i.e., Equation 1 can get closer to the optimal set of weights with a "small" incremental term than with a "large" term. As we approach the optimal weight
vector, $|H_k(\omega_J)|$ approaches closer to one (via Equation 2) and $PSJSF_k$ approaches closer to zero (via Equation 4). This will in turn reduce (via Equation 8) the adaptive noise canceler output power due to the interfering signal. This is why the simulation worked best when $SIMTIME = 1$. To summarize: choosing the snapshot point at the interfering signal maximum ($SIMTIME = 1$) gives a "small" error output $e_k$. This gives a "small" incremental term $2\mu r_k x_k$ in Widrow's algorithm. This small incremental term lets Equation 1 get "very close" to the optimal tap weight vector. As we get close to the optimal tap weight vector, $|H_k(\omega_J)| \rightarrow 1$ and $PSJSF_k \rightarrow 0$. These two conditions reduce the canceler output power to the extent necessary to achieve the user specified power reduction.

Choosing the snapshot point at the maximum value of the interfering signal can also be justified on physical grounds. Real signal measurements are not made at a single point in time (as assumed in the simulation). They are made over an interval of time. Amplitude or power measurements are integrated over this period of time by the measuring instrument. This integration insures that measurements of strong signals are much larger than measurements of weak signals. Any simulation of a real measurement must be able to reproduce this behavior. Choosing the snapshot time in the simulation at the maximum value of the interfering sinewave guarantees this behavior. This is so because in the scenario the interfering signal will be much larger than the intended signal. This implies that the maximum
value of the strong amplitude will be much larger than the amplitude of the weak intended signal at the snapshot time. This is the behavior the simulation must and does reproduce.

If the snapshot time did not correspond to the maximum value of the interferer, then the dynamic range between the interferer and the intended signal would be compressed. If, for example, a randomly chosen snapshot point (actually a snapshot time) on the interferer is used instead of using the maximum value, then the "difference" between interfering and intended signals at the random point would not be as large as the "difference" calculated by assuming that the snapshot was taken at the interference maximum. Choosing the snapshot time at the maximum value of the interference is a way of insuring that the simulation can distinguish between strong and weak signals without compressing the input signal dynamic range.

As further physical justification, consider that any real signal measurement, whether defined via an average or RMS procedure, will be proportional to the maximum value of the waveform; i.e., for a sinusoid $V_p \sin(\omega T)$ (where $V_p$ is the maximum value of the waveform) the average value is $(2/\pi)V_p$, the RMS value is $V_p/\sqrt{2}$. So by choosing in the simulation the maximum value ($V_p$) of the interfering signal as our snapshot point we are getting an insight into the real signal behavior.
DESIGN EXAMPLE

In order to illustrate the use of the adaptive noise canceler simulation it will be applied to the analysis of two cosited frequency hopping radios. The canceler will be used to protect the receiving radio from the transmitting radio's signal. The simulation will be used to arrive at a set of design parameters for the canceler that minimizes the number of iterations necessary to achieve the user specified power reduction in the transmitter or interfering frequency signal while at the same time passing the receiver intended frequency.

The simulation treats the two frequency hopping radios as fixed radios. It will be assumed that the transmitter frequency differs from the receiver frequency by 1 MHz (i.e., the user selected intended and interfering frequencies will be chosen 1 MHz apart and then input to the simulation, the simulation does not assume a 1 MHz separation.) A one MHz frequency difference was chosen because a significant reduction in the interfering signal power 1 to 5 MHz away from the receiver frequency will significantly reduce the receiver bit error rate (BER) and increase the communications range. Transmitter signals less than 1 MHz away from the receiver frequency will not be considered because if the frequency hopping transmitter is hopping randomly, these "close-in" frequencies will occur so infrequently (compared to transmitter signal frequencies 1-5 MHz away from the receiver frequency) that their effect on the receiver BER will be minimal.
In a typical cosite scenario the transmitter power will be 50 watts (+47 dBm). The required receiver sensitivity will be -98 dBm. Propagation loss between the two radio antennas will be frequency dependent. It can vary between 13 and 27 dB. The canceler is intended to help the frequency hopping receiver filter out the interference. It is not expected to do all the filtering by itself, i.e., it is not expected to reduce the interferer below the radio sensitivity. This would require the canceler to produce 118-134 dB or rejection. The combination of the canceler plus the tuned circuitry of the hopping radio is expected to reduce the interferer below the radio sensitivity. A realistic requirement for adaptive noise canceler filtering is 30 dB. SAW technology can easily achieve 30 dB of filtering. Together, canceler filtering of 30 dB, the propagation loss between antennas, and the radio tuned circuitry will reduce the transmitter signal below the receiver sensitivity.

The frequency hopping radios will be assumed to hop between 30 and 90 MHz. For reasonable insertion loss, linear phase SAW devices are limited to 33% fractional bandwidth. The center frequency of the SAW device in the canceler will therefore be:

\[ f_c = \frac{\text{Bandwidth}}{\text{Fractional Bandwidth}} = \frac{60 \text{ MHz}}{0.33} = 180 \text{ MHz} \quad (9) \]

Since 180 MHz is the center frequency of the SAW device in the P'TF, it was chosen as the intended signal frequency in the simulation examples to be presented in this report. It is of course assumed that the hopping radio signals will be up-
converted to this frequency range, processed and then down-converted.

The interference power input to the receiver (transmitter power minus propagation loss) will be 20-34 dBm depending on frequency. This is too much power for the PTF/SAW device in the canceler. The maximum power that can be safely input to it is +20 dBm. One possible way of protecting the SAW device is by use of a frequency selective limiter that filters out high power signals and passes low power signals. Such a limiter can be simulated by simply setting the input interfering power at +20 dBm and the input intended signal at the receiver sensitivity of -98 dBm.

There is now enough information to specify the scenario input parameters:

- The intended frequency will be assumed to be at 180 MHz, the center frequency of the SAW device. It could have been chosen anywhere between 150 and 210 MHz.
- The intended signal power will be assumed to be at -98 dBm, the lowest power the receiver is capable of detecting.
- The interfering frequency will be assumed to be at 181 MHz, one MHz away from the intended signal at 180 MHz. It could have been assumed the interferer was at 179 MHz. It makes no difference whether 181 or 179 MHz is chosen.
The interfering signal power will be assumed to be at +20 dBm, the output of a frequency selective limiter.

The desired reduction of the interfering signal power will be 30 dB.

The PTF voltage loss per tap will be assumed to be zero. (See "Simulation Input Parameters" section for rationale.)

With the scenario parameters now given, the simulation will be used to optimize two of the three design parameters (the number of taps in the PTF and \(\mu\) the convergence parameter.) The third design parameter, the intertap delay time between taps, will be assumed to be 6.9444 nanoseconds. This corresponds to a sampling frequency of 144 MHz. This sampling frequency was chosen in order to model a SAW/PTF currently being built for ETDL by Texas Instruments under Contract DAAL01-88-C-0831. In that effort a sampling frequency of 144 MHz is being used. 144 MHz is slightly larger than the Nyquist sampling rate (120 MHz) necessary to sample a 60 MHz bandwidth signal. It was chosen by TI to provide some protection against aliasing.

To be more specific: discrete sampling of an analog waveform, which is what the PTF taps do (they form discrete samples of the input analog waveform), not only duplicates the input spectrum, but also replicates it around harmonics of the sampling rate. If the sampling rate (equal to 1/(intertap delay time)) is too low, the replicated spectrums overlap in the frequency domain. Thus, any frequencies higher than half the Nyquist rate
that are present will be aliased, i.e., appear (due to undersampling) as lower frequencies. This distorts or equivalently introduces interference into the "lower" (below half the Nyquist rate) frequency spectrum. The minimum sampling rate necessary to assure no overlap in the output spectrum is the Nyquist rate (equal to twice the highest frequency present in the input). To insure that the replicated spectrums do not overlap, it is considered good engineering practice to sample at a somewhat higher rate than the Nyquist rate. This is why 144 MHz was chosen as the sampling frequency rather than 120 MHz.

The simulation will now be used to determine N the number of taps needed in the SAW/PTF. Although the optimum value of $\mu$ the convergence parameter has not as yet been determined, a value to input as a design parameter is still needed. The value of $\mu$ that is used need not be its optimum value (that will be determined after the number of taps is determined). The convergence parameter, $\mu$, need only be close enough to the optimum (for a given N) so that the canceler output does not "blow up", but eventually converges. The adaptive convergence time is affected by $\mu$, but $\mu$ does not affect the "optimum" frequency transfer function of the PTF. For the purpose of determining a suitable value for N it does not matter how long it takes to arrive at the optimum frequency transfer function. Inequality (7)

$$0 < \mu < \frac{1}{(N+1)(\text{Input Power})}$$

(7)
can be used as a guide to guessing one or more values of $\mu$ to use in determining a suitable value of $N$. For the given scenario, as $N$ varies between 16 and 256, $\mu$, as given by Inequality 7, will vary between 0.625 and 0.039. Either Inequality 7 can be used to calculate a close to optimum $\mu$ for each value of $N$ (equal to the upper bound of the inequality), or an educated guess at $\mu$ between 0.625 and 0.039 can be taken. Using Inequality 7 is the more systematic method. The educated guess method, however, illustrates the effect of $\mu$ on adaptive convergence time.

An educated guess of $\mu = 0.1$ was made. With $\mu = 0.1$, the canceler output for $N = 16, 32, 64$ and 128 converged. The output for $N = 256$ did not converge. When $\mu$ was set equal to 0.04 the canceler output for $N = 256$ converged quite rapidly.

The simulation outputs for $N = 16, 32, 64$ and 128 and $\mu = 0.1$ are shown in Figures 6 (a,b,c), 7 (a,b,c), 8 (a,b,c), and 9 (a,b,c), respectively. The outputs for $N = 256$ and $\mu = 0.04$ are shown in Figure 10 (a,b,c). The "a" figure of each set shows the input parameters and the canceler power output for each iteration. The iterations are continued until the user specified reduction in interfering signal strength (30 dB) has been achieved. Notice that as the given value of $\mu (=0.1)$ gets closer to the optimum value of $\mu$ (by increasing $N$ in Inequality 7) the number of iterations necessary to achieve the user-specified reduction in interfering power decreases. Once the time necessary to complete a single iteration is known, or assumed, the total time necessary to reduce the interfering signal the re-
quired number of dB, the "adaptive convergence time", can be calculated.

The "a" figures give the canceler output power versus iteration number, but they tell nothing about output power versus frequency. To work properly the canceler must attenuate the interferer, but not the intended signal. The PTF must, therefore, pass the interferer with a zero phase shift and attenuate the intended signal. PTF amplitude gain and phase shift are given in the "b" and "c" figures, respectively, of each set.

The "b" figures show that as \( N \) increases, the width of the central lobe of the PTF amplitude gain curve decreases. PTF frequency resolution has been previously defined as the difference in frequency between the center of the PTF amplitude gain curve and its first zero. Resolution is given by Equation 6.

\[
\text{Frequency Resolution} = \frac{1}{N\Delta} = \frac{1}{(\text{total delay})}
\]

where:

\( N \) = number of taps  
\( \Delta \) = intertap delay

As \( N \) increases, the frequency resolution of the PTF gets better, i.e., the minimum frequency separation needed for the PTF to pass a strong interferer and reject a weak intended signal, decreases. Figures 6b, 7b, 8b, 9b, and 10b clearly show this relationship between \( N \), the number of PTF taps, and frequency resolution. For each curve, the frequency separation between the curve's center frequency and first zero is given by Equation 6.
Enter the number of taps in the transversal filter 16
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.1
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

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<tr>
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<th>Output Power (dBm)</th>
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<td>6.72E+0</td>
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<td>9.96E+0</td>
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<tr>
<td>22</td>
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Figure 6a. Iterated Canceler Output Power for N = 16 taps
MU = 0.1.
PTF Amplitude Gain vs Frequency

weights calculated via Widrows algorithm

Figure 6b. PTF Amplitude Gain vs Frequency for N = 16 taps
MU = 0.1.
Figure 6c. PTF Phase Shift vs Frequency for $N = 16$ taps

$\mu = 0.1$. 
Enter the number of taps in the transversal filter 32
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .1
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in dB) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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<tbody>
<tr>
<td>1</td>
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<td>1.32E+1</td>
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<td>1.05E+1</td>
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<td>7.31E+0</td>
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<td>4.09E+0</td>
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<td>9.62E-1</td>
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<td>-2.36E+0</td>
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<td>9</td>
<td>-8.79E+0</td>
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<td>10</td>
<td>-1.20E+1</td>
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</tbody>
</table>

OPTIMUM MU = 0.3125

Figure 7a. Iterated Canceler Output Power for N = 32 Taps, 
MU = 0.1.
Figure 7b. PTF Amplitude Gain vs Frequency for N = 32 taps
MU = 0.1.
Figure 7c. PTF Phase Shift vs Frequency for N = 32 taps
MU = 0.1.
run
Enter the number of taps in the transversal filter 64
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .1
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 131
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

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<th>Iteration</th>
<th>Output Power (dBm)</th>
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<td>-9.51E+0</td>
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<td>5</td>
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</table>

OPTIMUM MU=0.15625

Figure 8a. Iterated Canceler Output Power for N = 64 Taps, MU = 0.1.
Figure 8b. PTF Amplitude Gain vs Frequency for N = 64 taps
MU = 0.1.
Figure 8c. PTF Phase Shift vs Frequency for N = 64 taps
MU = 0.1.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .1
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -93
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
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<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.70E+1</td>
</tr>
<tr>
<td>2</td>
<td>5.81E+0</td>
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<td>-5.36E+0</td>
</tr>
<tr>
<td>4</td>
<td>-1.64E+1</td>
</tr>
</tbody>
</table>

OPTIMUM MU=0.078125

Figure 9a. Iterated Canceler Output Power for N = 128 Taps, MU = 0.1.
Figure 9b. PTF Amplitude Gain vs Frequency for N = 128 taps
MU = 0.1.
Figure 9c. PTF Phase Shift vs Frequency for N = 128 taps
MU = 0.1.
Enter the number of taps in the transversal filter 256
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.04
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency(in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
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<th>Iteration</th>
<th>Output Power (dBm)</th>
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<tbody>
<tr>
<td>1</td>
<td>1.70E+1</td>
</tr>
<tr>
<td>2</td>
<td>-1.69E+1</td>
</tr>
</tbody>
</table>

OPTIMUM MU=0.0390625

Figure 10a. Iterated Canceler Output Power for N = 256 taps
MU = 0.04.
Fig. 10b. PTF Amplitude Gain vs Frequency for N = 256 taps
MU = 0.04.
Therefore, in order to choose an appropriate value of \( N \), either use the required frequency resolution to solve for \( N \) (via Equation 6) or use the PTF amplitude gain vs. frequency curve generated by the simulation to give a more detailed look at the frequency transfer function (as a function of \( N \)).

For \( N = 128, 144 \) and 256, the respective frequency resolutions (via Equation 7) are 1.125 MHz, 1 MHz and 0.506 MHz. Setting \( N = 144 \) or 256 both achieves or betters the required resolution of 1 MHz. But they require more complicated and expensive PTFs and associated circuitry than for \( N = 128 \), simply because they have more taps. \( N \) was chosen equal to 128 because: (1) it almost achieves the "required resolution" of 1 MHz, (2) a 128 tap PTF will be somewhat simpler than a 144 or 256 tap filter and, (3) we are interested in modeling a Texas Instruments PTF being developed under Contract DAAL01-88-C-0831 that has 128 taps.

Figures 6c, 7c, 8c, 9c, and 10c show that the PTF phase shift versus frequency curve fluctuates faster as \( N \) increases. The PTF phase shift must behave in this manner in order for the canceler to be able to separate signals progressively closer in frequency. An adaptive noise canceler works by having the PTF pass the strong interfering frequency with a zero phase shift (so that the PTF filtered interferer subtracts in phase at the summer of the canceler from the non-filtered interferer). In addition, it attenuates the weak intended signal with a non-zero phase shift (so that the PTF filtered intended signal subtracts out of
phase at the summer from the non-filtered intended signal, thus minimizing the effect of this subtraction on the unfiltered input signal). The closer in frequency that the interfering and intended signals are, the faster the PTF phase shift will have to change in order for there to be a significant difference in the phase shift at the two frequencies. N has to be large enough so that the phase shift varies fast enough to insure signal separation.

Since N is now fixed equal to 128, the only design parameter left to be determined is the convergence parameter $\mu$.

With an educated guess of $\mu = 0.1$, four iterations were necessary to reduce the interference by 30 dB. A value of $\mu$ that reduces the number of iterations below four is needed in order to minimize the adaptive convergence time. Two iterations is the minimum number of iterations necessary for significant interference reduction. Significant interference reduction cannot be achieved in one iteration, since in the first iteration the simulation sets all tap weights equal to zero. As a result, there is only a 3 dB reduction in power due to the action of the power splitter in the adaptive noise canceler. In other words, in Figure 3, assuming all the tap weights are equal to zero implies there is no output from the PTF. In effect, half the input power is being lost and there is a 3 dB reduction in canceler output power relative to the input power.

Table 1 summarizes the results of a number of simulation runs with different values of $\mu$. The number of iterations
necessary to reduce the interference by 30 dB is a very sensitive function of $\mu$. In the approximate region of $\mu = 0.075$ to $\mu = 0.03$, 30 dB interference reduction is achieved in 2 iterations. This region corresponds to the upper bound of Inequality 7.

$$0 < \mu < \frac{1}{(N + 1) \text{ (input power)}} = \frac{1}{(128 + 1) 20 \text{ dBm}} = \frac{1}{129 \text{ (0.1 watt)}}$$

So the upper bound of Inequality 7 can be used to derive an optimal value (or range of values) for $\mu$.

From Table 1, the optimal range for $\mu$ is from 0.075 to 0.08. The convergence parameter will "arbitrarily" be assumed to be 0.08. The design of the adaptive filter is now complete.

An adaptive noise canceler with design parameters assumed here (number of PTF taps = 128, intertap delay = 6.944 nanoseconds, and $\mu = 0.08$) will reduce interfering or strong signals by 30 dB in two iterations while having a minimal affect on weak intended signals 1.125 MHz away from the interferer.

Figures 20a and 20b give the PTF amplitude gain vs. frequency and phase shift vs. frequency respectively for $N = 128$ and $\mu = 0.08$. Notice that these figures are identical to figures 9b and 9c for $N = 128$ and $\mu = 0.1$. This is so because $\mu$ does not affect the optimal PTF frequency transfer function. It only affects the number of iterations necessary to reach it.

It should be re-emphasized that Inequality 7 implies that if $\mu$ is chosen within the range of the Inequality then the expected weight vector will converge to the optimal or Wiener weight vector. It should not necessarily converge in a minimum number
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<th>$\mu$</th>
<th>Number of Iterations Necessary for 30 dB Reduction in Interference Power</th>
<th>Figure</th>
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<td>No convergence - simulation output &quot;blows up&quot;</td>
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<tr>
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<td>No convergence - output oscillates</td>
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of iterations. Yet, this is exactly what the simulation runs indicate. If $\mu = 1/((N+1)\text{(interfering power)})$ then the adaptive process converges in two iterations. The reason for this is not absolutely clear. What is probably happening is that when $\mu = 1/((N+1)\text{(interfering power)})$, the adaptive process is critically damped. Critical damping gives convergence in one iteration (two iterations in our simulation because the initial weight vector is assumed equal to zero and we are counting it as an iteration). In addition, if $\mu$ is greater than twice the $\mu$ value that gives critical damping, then the adaptive process does not converge. This is exactly what Table 1 shows.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.2
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in dB) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
</tr>
</thead>
<tbody>
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<td>6.43E+1</td>
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<td>8.63E+1</td>
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<td>11</td>
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<tr>
<td>12</td>
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</table>

Figure 11. Iterated Canceler Output Power for N = 128 taps

MU = 0.2.
<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
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<tbody>
<tr>
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<td>1.30E+2</td>
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<td>1.63E+2</td>
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<td>1.74E+2</td>
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<td>19</td>
<td>1.85E+2</td>
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<td>2.84E+2</td>
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<td>2.95E+2</td>
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<td>3.06E+2</td>
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<td>3.17E+2</td>
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<td>37</td>
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*** ERROR # 80 LINE # 970
Overflow

Figure 11. Iterated Canceler Output Power for N = 128 taps
MU = 0.2. (continued)
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.944
Enter MU the convergence parameter 0.16
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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<tbody>
<tr>
<td>1</td>
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<td>1.66E+1</td>
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Figure 12. Iterated Canceler Output Power for N = 128 taps
MU = 0.16.
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<td>1.56E+1</td>
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<td>1.69E+1</td>
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<td>39</td>
<td>1.67E+1</td>
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</table>

**Figure 12. Iterated Canceler Output Power for N = 128 taps MU = 0.16. (continued)**
run
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .15
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency(in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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<tbody>
<tr>
<td>1</td>
<td>1.70E+1</td>
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<tr>
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<td>1.54E+1</td>
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<td>1.46E+1</td>
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<td>1.31E+1</td>
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Figure 13. Iterated Canceler Output Power for N = 128 Taps, MU = 0.15.
Figure 13. Iterated Canceler Output Power for N = 128 Taps, 
MU = 0.15. (Continued)

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<td>2.93E+0</td>
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<td>8.75E+0</td>
<td>9.53E+0</td>
<td>1.03E+1</td>
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</table>
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .09
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in dB) 0

<table>
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<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.70E+1</td>
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<tr>
<td>2</td>
<td>4.23E-1</td>
</tr>
<tr>
<td>3</td>
<td>-1.60E+1</td>
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Figure 14. Iterated Canceler Output Power for N = 128 Taps, MU = 0.09.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .08
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

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<th>Iteration</th>
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</table>

Figure 15. Iterated Canceler Output Power for N = 128 taps
MU = 0.08.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.0775
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

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<td>-2.16E+1</td>
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</table>

Figure 16. Iterated Canceler Output Power for N = 128 taps
MU = 0.775.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.075
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

Iteration | Output Power (dBm)
--- | ---
1 | 1.70E+1
2 | -1.03E+1

Figure 17. Iterated Canceler Output Power for N = 128 taps
MU = 0.075.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.07
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
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<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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<tbody>
<tr>
<td>1</td>
<td>1.70E+1</td>
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<td>2</td>
<td>-2.43E+0</td>
</tr>
<tr>
<td>3</td>
<td>-2.14E+1</td>
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</tbody>
</table>

Figure 18. Iterated Canceler Output Power for N = 128 taps
MU = 0.07.
run

Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.06
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 20
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.70E+1</td>
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<tr>
<td>2</td>
<td>4.39E+0</td>
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<td>3</td>
<td>-8.19E+0</td>
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<tr>
<td>4</td>
<td>-2.04E+1</td>
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</tbody>
</table>

Figure 19. Iterated Canceler Output Power for N = 128 taps
MU = 0.06.
Figure 20a. PTF Amplitude Gain vs Frequency for N = 128 Taps, MU = 0.08.
The design example has assumed that the input interfering power has been limited to +20 dBm by a frequency selective limiter. Since, in our example, the input interfering power is always greater than or equal to +20 dBm, the output of the frequency selective limiter is a constant +20 dBm. This is the constant power that the adaptive noise canceler "sees."

Adaptive convergence time, i.e., the number of iterations necessary to reduce the interfering power by a user given amount, is a function of the interfering power level. "Large" interferers can be reduced a fixed amount (e.g., 30 dB) faster (i.e., a smaller number of iterations) than "small" interferers.

This is illustrated in Figure 21. Figure 21 shows a simulation run in which the design parameters of Figure 15 ($\mu = 0.08$, $N = 128$, intertap delay = 6.9444 nanoseconds) are used on an interfering signal that is 10 dB less than the interfering signal power used in Figure 15 (10 dBm vs. 20 dBm).

The output in Figure 15 dropped over 30 dB in 2 iterations. The output in Figure 21 took 30 iterations to drop 30 dB. This behavior can be explained using Equation 1, Widrow's LMS algorithm:

$$W_{k+1} = W_k + 2\mu e_k x_k$$  \hspace{1cm} (1)

where:

- $W_{k+1}$ = tap weight vector at the $k+1$th iteration.
- $W_k$ = tap weight vector at the $k$th iteration.
- $\mu$ = convergence parameter.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter .08
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 10
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
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<tbody>
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<td>1</td>
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<td>6.05E+0</td>
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<tr>
<td>10</td>
<td>-1.43E+0</td>
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</tbody>
</table>

Figure 21. Iterated Canceler Output Power for N = 128 taps
MU = 0.08, and Interfering Signal Strength = 10 dBm.
Figure 21. Iterated Canceler Output Power for $N = 128$ taps, $MU = 0.08$, and Interfering Signal Strength = 10 dBm. (continued)
\( \varepsilon_k \) = Canceler error output at the \( k \)th iteration

\( X_k \) = PTF tap amplitude vector at the \( k \)th iteration.

When the input interfering power is reduced both \( \varepsilon_k \) and \( X_k \) will be reduced. If the input interfering power is reduced by a factor of 10, then, since both \( \varepsilon_k \) and \( X_k \) are amplitudes, they will be reduced by \( \sqrt{10} \) and their product will be reduced by a factor of 10. As a result, the amount by which the weights are incremented at each iteration, \( 2 \mu \varepsilon_k X_k \), is decreased by a factor of 10. It now takes the adaptive process much longer to achieve the required power reduction.

Equation 1 also suggests a way of remedying the problem. Simply increase \( \mu \) in the same proportion that \( \varepsilon_k X_k \) was decreased, i.e., by a factor of 10. This was done in a simulation run illustrated in Figure 22 in which \( \mu \) was increased from \( \mu = 0.08 \) (in Figure 21) to \( \mu = 0.8 \). The output drops over 30 dB in just two iterations. The "problem" has been fixed.

Since input interfering power affects the adaptive convergence time, then, in any simulation of a frequency hopping co-site problem, either the adaptive noise canceler must "see" a sufficiently high interference power to minimize the adaptive convergence time (for a given \( \mu \)) or \( \mu \) must be recalculated via Equation 7:

\[
\mu = \frac{1}{(N + 1) \text{ (interfering power)}}
\]  

(7)

every time the input power is measured.
Enter the number of taps in the transversal filter 128
Enter the delay time between taps (in nanoseconds) 6.9444
Enter MU the convergence parameter 0.8
Enter intended signal frequency (in MHz) 180
Enter intended signal strength (in dBm) -98
Enter interfering frequency (in MHz) 181
Enter interfering signal strength (in dBm) 10
Enter desired reduction (in dB) of interfering signal strength 30
Enter SIMTIME (a dimensionless parameter between 0 and 4) 1
Enter the lowest expected signal frequency (in MHz) 150
Enter the highest expected frequency (in MHz) 210
Enter the frequency increment (in KHz) to be used for plotting 100
Enter the PTF voltage loss per tap (in db) 0

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Output Power (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.99E+0</td>
</tr>
<tr>
<td>2</td>
<td>-2.65E+1</td>
</tr>
</tbody>
</table>

Figure 22. Iterated Canceler Output Power for N = 128 taps
MU = 0.8, and Interfering Signal Strength = 10 dBm.
Up to this point the PTF was assumed to have no tap loss, as discussed in the "Simulation Input Parameters" section. Once the tap loss is known, then the optimal $\mu$ determined for zero tap loss should be multiplied by $(1/\text{tap loss})$ to get the actual value of $\mu$ to be used when building a real adaptive noise canceler.
CONCLUSION

The adaptive noise canceler simulation is able to handle the typical parameters encountered when two frequency hopping radios are co-located. It can identify design parameters that cause the canceler to reduce a +20 dBm interfering signal by 30 dB in one or two iterations, depending on the definition of an iteration. The simulation is suitable for use as an adaptive noise canceler design tool to evaluate the effect of design parameter changes on canceler performance by determining the adaptive convergence time and PTF frequency transfer function.
REFERENCES


[12] Reference 1, p. 103.


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