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PROBLEMS IN NONLINEAR ACOUSTICS:

PULSED FINITE AMPLITUDE SOUND BEAMS, NONLINEAR PROPAGATION OF SOUND IN LAYERED MEDIA, TIME DOMAIN SOLUTIONS FOR FOCUSED SOUND BEAMS, FOCUSING OF SOUND WITH AN ELLIPSOIDAL MIRROR, AND MODELING FINITE AMPLITUDE PROPAGATION IN WAVEGUIDES

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Five projects are discussed in this annual summary report, all of which involve basic research in physical acoustics, mainly nonlinear acoustics. (1) Pulsed Finite Amplitude Sound Beams are investigated numerically with a time domain computer algorithm, and experiments are underway to study nonlinear self-demodulation. (2) Nonlinear Propagation of Sound in Layered Media involves studies of shallow water above a penetrable bottom (both theory and experiment) and the SOFAR channel (only theory). (3) Time Domain Solutions for Axial Pressures in Focused Sound Beams is a theoretical comparison of three different models. (4) Focusing of Sound with an Ellipsoidal Mirror is a theoretical study of the reflected field, typically short pulses, along the axis of the mirror. (5) Modeling Finite Amplitude Propagation in Waveguides with a Nonlinear Parabolic Wave Equation is a new project that involves both theory and experiment.

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CONTENTS

INTRODUCTION

This annual summary report describes research performed from 1 October 1990 through 31 July 1991 (10 months) with support from ONR under grant N00014-89-J-1003. The following projects are discussed in this report:

I. Pulsed Finite Amplitude Sound Beams

II. Nonlinear Propagation of Sound in Layered Media

III. Time Domain Solutions for Focused Sound Beams

IV. Focusing of Sound with an Ellipsoidal Mirror

V. Finite Amplitude Propagation Models for Sound in Waveguides

Contributions to these projects were made by the following individuals:

Senior Personnel

- M. F. Hamilton, principal investigator (projects I–V)
- E. A. Zabolotskaya, visiting scientist (project II)

Graduate Students

- M. A. Averkiou, Ph.D. student in Mechanical Engineering (project I)
- C. E. Bruch, Ph.D. student in Physics (project II)
- Y.-S. Lee, Ph.D. student in Mechanical Engineering (project I)
- D. E. Reckamp, M.S. student in Mechanical Engineering (project II)
- T. W. VanDoren, Ph.D. student in Mechanical Engineering (project V)

Professor Zabolotskaya spent approximately five months (early May through the middle of October, 1990) in the Mechanical Engineering Department while on leave from the Institute of General Physics in Moscow, USSR. Her financial support was provided by the David and Lucile Packard Fellowship for Science and Engineering. All of the graduate students received partial financial support from other funding agencies, as described in the following sections.

The following manuscripts and abstracts, which describe work supported at least in part by ONR, have been published (or submitted for publication) since 1 October 1990.

Refereed Publications

- M. F. Hamilton and D. T. Blackstock, "On the linearity of the momentum equation for progressive plane waves of finite amplitude," *J. Acoust. Soc. Am.* **88**, pp. 2025–2026 (1990).
- S. J. Lind and M. F. Hamilton, "Noncollinear interaction of a tone with noise," *J. Acoust. Soc. Am.* **89**, pp. 583–591 (1991).
- C. M. Darvennes, M. F. Hamilton, J. Naze Tjøtta, and S. Tjøtta, "Effects of absorption on the nonlinear interaction of sound beams," *J. Acoust. Soc. Am.* **89**, pp. 1028–1036 (1991).
- M. F. Hamilton and E. A. Zabolotskaya, "Nonlinear propagation of sound in a liquid layer between a rigid and a free surface," *J. Acoust. Soc. Am.* **90**, pp. 1048–1055 (1991).
- M. F. Hamilton, "Comparison of three time domain solutions for the axial pressure in a focused sound beam," accepted for publication in *J. Acoust. Soc. Am.*
- C. M. Darvennes and M. F. Hamilton, "Parametric reception near a reflecting surface," submitted for publication in *J. Acoust. Soc. Am.*

Publications in Conference Proceedings

- M. A. Averkiou, I. R. S. Makin, and M. F. Hamilton, "Reflection of weakly nonlinear focused sound beams," Compilation of Abstracts, XIV Scandinavian Cooperation Meeting in Acoustics/Hydrodynamics, edited by H. Hobæk (Scientific/Technical Report No. 1991-09, Department of Physics, University of Bergen, Norway, 1991), pp. 99–103.
- Y.-S. Lee and M. F. Hamilton, "Nonlinear effects in pulsed sound beams," to appear in *Ultrasonics International 91 Conference Proceedings* (Butterworth Scientific Ltd, England, 1991).

Oral Presentation Abstracts

- T. W. VanDoren and M. F. Hamilton, "Scattering of sound by sound in a waveguide," *J. Acoust. Soc. Am.* **88**, S75 (1990).
- Y.-S. Lee and M. F. Hamilton, "A time domain computer algorithm for solving the KZK equation," *J. Acoust. Soc. Am.* **88**, S76 (1990).
- I. R. S. Makin and M. F. Hamilton, "Harmonic generation in focused sound reflected from a curved surface," *J. Acoust. Soc. Am.* **89**, 1928 (1991).

I. PULSED FINITE AMPLITUDE SOUND BEAMS

The numerical work on this project was performed by Y.-S. Lee, who is expected to receive his Ph.D. in Mechanical Engineering in May 1992. Salary support for Lee was provided entirely by ONR during the period covered by this report. Computing resources were provided by The University of Texas System Center for High Performance Computing. Experimental work on this project will be performed by M. A. Averkiou, a Ph.D. student in Mechanical Engineering who passed his qualifying exam in March 1991. Averkiou received salary support from ONR during the period 1 March 1991 through 31 July 1991. Previous salary support for Averkiou was provided by NSF.

A. Background

Lee has developed a computer algorithm that, with operations performed entirely in the time domain, solves the KZK (Khokhlov–Zabolotskaya–Kuznetsov) nonlinear parabolic wave equation for pulsed, axisymmetric, finite amplitude sound beams. The KZK equation accounts for the combined effects of nonlinearity, diffraction and thermoviscous absorption on the propagation of directive sound beams. The algorithm is particularly suitable for modelling the propagation of tone bursts, such as those typically used in a variety of underwater applications. Moreover, the sound beams may be weakly focused, as is usually the case in biomedical applications of ultrasound. Details of the algorithm were presented at the November 1990 Meeting of the Acoustical Society of America in San Diego,[1] and preliminary numerical results were first published in the Second Annual Summary Report[2] under the present ONR grant.

B. Results

The following numerical results have been excerpted from a paper[3] that was presented at the July 1991 Ultrasonics International Conference in Le Touquet, France. This paper received an award for outstanding work by a graduate student.

The acoustic source is assumed to be a circular piston in the plane $\sigma = 0$, where σ is dimensionless range along the axis of the beam (i.e., axial distance z divided by the Rayleigh distance z_0 at the characteristic source frequency ω_0). Results are presented as a function of distance along the axis.

For the first example we consider the following source conditions:

$$p = p_0 \cos(\omega_0 t/9) \sin(\omega_0 t) \qquad A = 0.1\,, \quad N = 2.0 \tag{1}$$

where $A = \alpha_0 z_0$ is dimensionless absorption (α_0 is the absorption coefficient at frequency ω_0) and $N = z_0/z_s$ is the nonlinearity parameter (z_s is the shock formation distance for a plane wave with peak source pressure p_0 and frequency ω_0). These conditions were chosen because they are the same as those used by Naze Tjøtta, Tjøtta, and Vefring to produce Fig. 7 of Ref. 4 with a frequency domain numerical solution of the KZK equation. However, since our algorithm performs all calculations in the time domain, the source waveform must have finite duration. We thus modify Eq. (2) by setting the source pressure equal to zero at times $|\omega_0 t| > 45\pi/2$. As a result, the source waveform shown in Fig. 1 (at $\sigma = 0$) is repeated exactly twice at both earlier and later times, in order that the propagation of the middle segment approximate the case of continuous radiation (i.e., infinite pulse length). Comparison of the time waveforms in Fig. 1 with those in Fig. 7 of Ref. 4 shows that our results are virtually identical to those obtained with frequency domain calculations. In the right column of Fig. 1 are the corresponding frequency spectra $S(\Omega)$, where $S(\Omega)$ is normalized to yield a maximum amplitude of unity at the source, and $\Omega = \omega/\omega_0$. To calculate $S(\Omega)$ we assumed the time waveforms shown in Fig. 1 to be repeated indefinitely at earlier and later times (to simulate continuous radiation) before performing the FFT. From the spectrum at $\sigma = 0$ it can be seen that the source radiates at two discrete primary frequencies, which corresponds to the operation of a parametric array that ultimately produces a dominant difference frequency component at $\sigma = 100$.

For the second example we consider the following frequency-modulated source waveform:

$$p = p_0 \exp\left[-(\omega_0 t/25\pi)^8\right] \sin\left[(1 + \omega_0 t/150\pi)\omega_0 t\right] \qquad A = 1.0, \quad N = 1.0 \quad (2)$$

The exponential is an envelope function that corresponds to a pulse duration of approximately 25 cycles at the center frequency ω_0 of the tone burst. The instantaneous angular frequency of the tone is $\omega = \omega_0(1+\omega_0 t/75\pi)$, which increases linearly with time and approximately doubles over the duration of the pulse. At $\sigma = 3$ in Fig. 2, absorption is seen to have a greater effect at the trailing, high frequency edge of the pulse, and at $\sigma = 10$, the nonlinear effect of self-demodulation is noticeable at the trailing edge. At $\sigma = 45$, the leading, low frequency edge of the pulse is finally demodulated. The frequency spectra in Fig. 2 show the corresponding shift of energy from the high frequencies that characterize the tone burst at the source to, in the farfield, the low frequencies that characterize the envelope.

Under an acceleration of the current grant, work has begun on experimental verification of the numerical results described above, in particular for self-demodulation. We have identified the appropriate source parameters (size and resonance frequency) and fluid (glycerin) which should permit observation of the full self-demodulation process within the confines of our newly constructed, computer controlled, water tank facility in the Department of Mechanical Engineering. Construction of a vessel to hold the glycerin is currently underway, and Averkiou will begin the experiments during fall 1991.

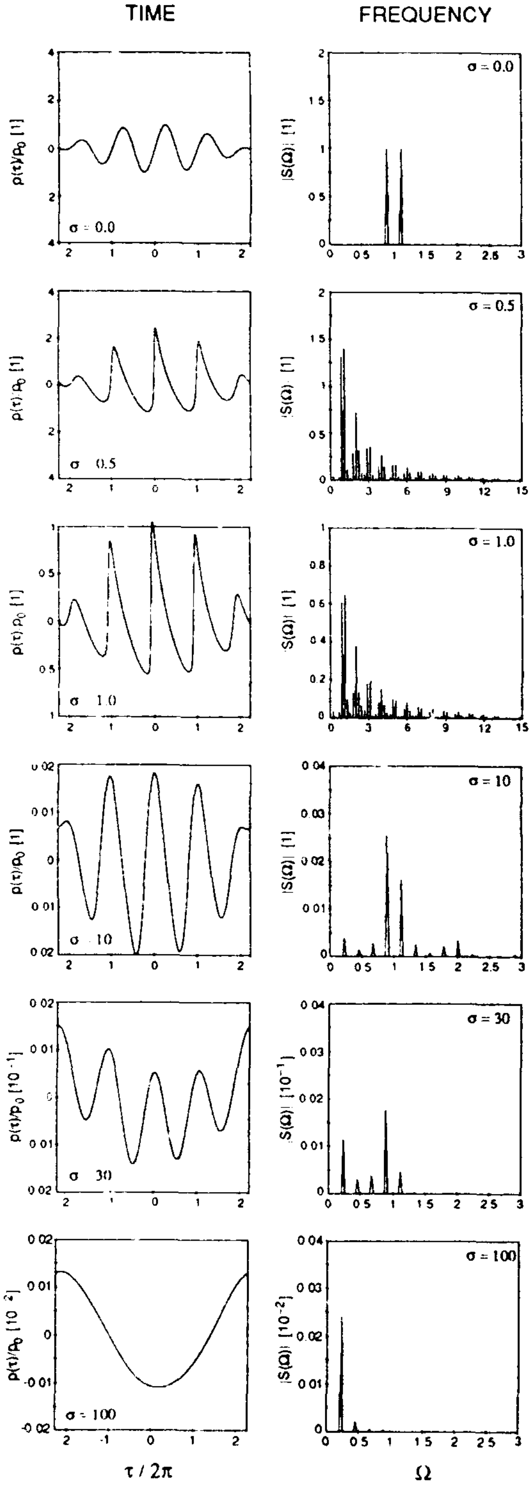

Figure 1. Computed on-axis time waveforms and frequency spectra for the source condition in Eq. (1).

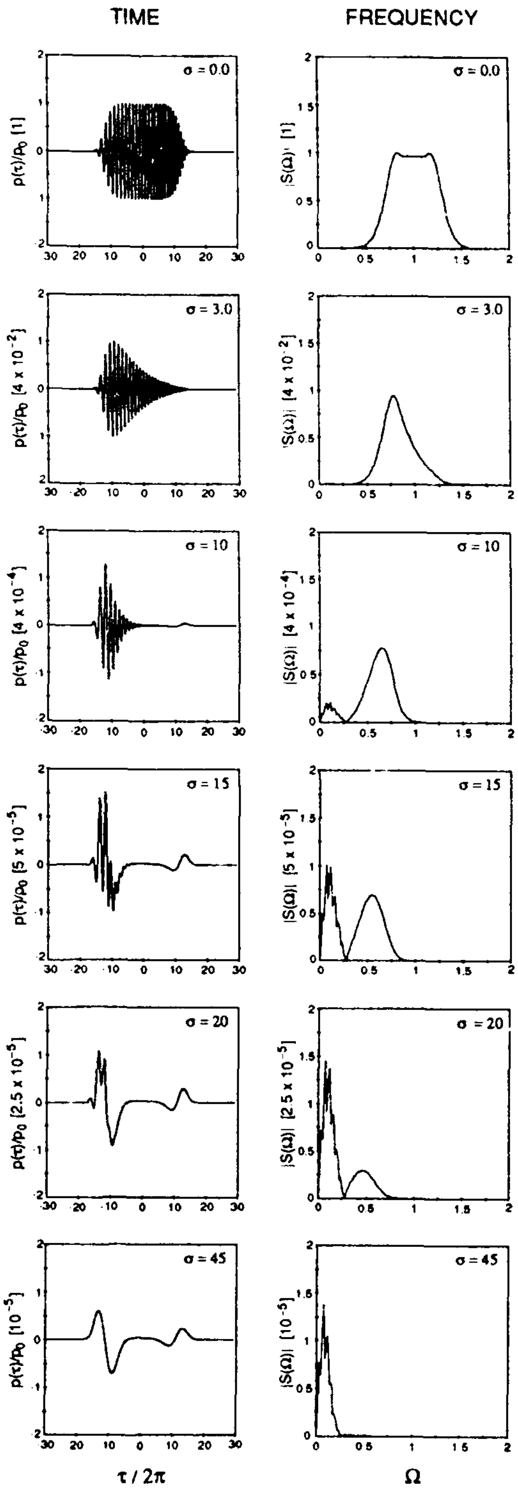

Figure 2. Computed on-axis time waveforms and frequency spectra for the source condition in Eq. (2).

II. NONLINEAR PROPAGATION OF SOUND IN LAYERED MEDIA

Under an acceleration of ONR Grant N00014-89-J-1003, financial support has been provided for C. E. Bruch, a Ph.D. student in Physics, to perform experimental work on this project. Bruch is scheduled to take his Ph.D. qualifying exam in August 1991. Bruch received salary support from ONR during the period 1 March 1991 through 31 July 1991. Previous salary support for Bruch was provided by the David and Lucile Packard Foundation. Not supported by ONR, but also working under the supervision of the principal investigator, is Ensign D. E. Reckamp, USN, an M.S. student in Mechanical Engineering who receives his financial support from Applied Research Laboratories. Reckamp began his studies at the University of Texas at Austin in September 1990, and he is expected to receive his M.S. degree in May 1992. Professor Zabolotskaya is cosupervisor for both Bruch and Reckamp.

A. Background

Previous work on guided nonlinear acoustic wave propagation is based primarily on the assumption that the sound speed does not vary throughout the medium, and that the sound does not penetrate the walls of the waveguide. Much of this work is reviewed in the Introduction of an article by Hamilton and TenCate.[5] In a real ocean environment, however, not only does sound penetrate the ocean bottom, but the sound speed may vary with depth in the water column.

A general theory for the nonlinear propagation of sound in a layered medium with variable sound speed profiles and penetrable boundaries has been developed recently by Zabolotskaya and Shvartsburg.[6,7] This theory makes it possible to study, with a unified approach, a wide variety of problems that involve finite amplitude wave propagation in liquid or gaseous layers which may be either discretely or continuously stratified. The method is particularly well suited to the investigation of narrow-band pulses.

In the first application of this theory, Zabolotskaya[8] investigated second harmonic generation in a fluid layer that is bounded by two identical fluid half-spaces with acoustical properties that are different from those of the layer in between. Her analytical results demonstrate the effect of varying the relative properties of the two fluids on the second harmonic mode shapes generated within the layer. In the second application, Hamilton and Zabolotskaya[9] investigated the propagation of narrow-band pulses in a layer of water that is bounded above by a free surface and below by a rigid surface. Analytical results revealed that only dark (kink) envelope solitons, and not bright envelope solitons, can exist in this case.

B. Results

Funding provided under the ONR grant acceleration was used to purchase equipment for an experimental investigation of nonlinear acoustic wave propagation in a layer of water above a penetrable bottom. Specifically, we have purchased a digitizing oscilloscope, power amplifier, hydrophone, and computer controlled actuator for measurement of the vertical mode shapes. The waveguide facility has already been constructed by Bruch. The walls of the waveguide are made of stainless steel which forms a channel that is 8 m long and 20 cm deep. The source configuration consists of a vertical stack of eight piezlelectric elements which can radiate sound in several individual modes at frequencies near 80 kHz. Penetrable bottoms will be formed with sheets of rubber.

The first experiments to be performed by Bruch are designed to check the theoretical results obtained by Hamilton and Zabolotskaya[9] for second harmonic generation and pulse propagation in a liquid layer above a rigid bottom. Later experiments will include a penetrable bottom in the waveguide, and theory will be developed for this case.

The investigation performed by Reckamp is entirely theoretical. He is looking at second harmonic generation and pulse propagation in a water column with a parabolic sound speed profile [i.e., $c^{-2} = c_0^{-2} - \alpha z^2$, where $c(z)$ is the sound speed as a function of depth z]. This profile is frequently used to model the sound speed dependence near the axis of the SOFAR channel. Reckamp has derived a simple expression for second harmonic generation in the sound channel, and his preliminary analytical results suggest a possibility for the existence of bright envelope solitons in the higher order modes.

The results for second harmonic generation in a waveguide with a parabolic sound speed dependence may be expressed as follows. The mode shapes for the linear solution are Gauss-Hermite functions (see Ref. 10 for a review) of the form

$$\phi_m(z) = \exp\left(-\alpha\omega z^2/2\right) H_m\left(\sqrt{\alpha\omega}\, z\right) \tag{3}$$

where H_m is the Hermite polynomial of order m. It is assumed that the fundamental wave of angular frequency ω propagates in a single mode m. The mode shape for the resulting second harmonic component that is generated in the fluid is found to be of the following form:

$$p_2(z) = \exp\left(-\alpha\omega z^2\right) \sum_{n=0}^{m} b_n H_{2n}\left(\sqrt{2\alpha\omega}\, z\right) \tag{4}$$

Note that the second harmonic component is generated only in even modes. Results based on Eqs. (3) and (4) are shown in Fig. 3. At the left is the profile $c(z)$ of the parabolic sound speed dependence on depth, and the remaining plots show mode shapes for the fundamental wave (in mode m and at ten times the cutoff frequency of that mode; dashed lines) and for the resulting second harmonic wave (solid lines). For $m = 0$, both the fundamental and second harmonic mode shapes are Gaussian. For $m = 1$, the second harmonic wave is generated in both modes $m = 0$ and $m = 2$, with most of its energy in the higher mode. In general, when the fundamental wave propagates in mode m, most of the second harmonic energy is generated in mode $2m$, with the remainder distributed among all of the lower even modes.

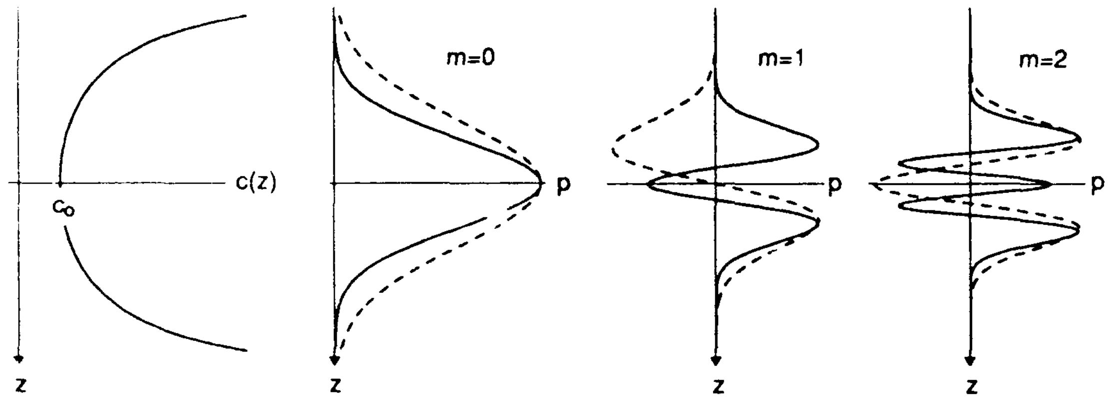

Figure 3: Parabolic sound speed profile (at left), mode shapes for the fundamental wave in mode m (dashed lines), and mode shapes for the corresponding nonlinearly generated second harmonic component (solid lines).

III. TIME DOMAIN SOLUTIONS FOR FOCUSED SOUND BEAMS

This work was performed by M. F. Hamilton, the principal investigator, with salary support from ONR during the period 16 January 1991 through 31 July 1991. Personal computer support was provided by the David and Lucile Packard Foundation and NSF.

A. Background

The purpose of this work is to compare three simple time domain solutions for the axial pressure in a focused sound beam. The acoustic source is assumed to be an axisymmetric spherical cap that vibrates with uniform motion, the velocity component of which is everywhere normal to the spherical surface. Using different approximations, several authors[11-14] have previously obtained solutions for the case in which the source vibrates at a single frequency.

B. Results

The same approximations[11–14] yield simple time domain solutions for the axial sound pressure. Time domain solutions are particularly useful for modeling the propagation of pulses. A manuscript[15] in which the time domain solutions are derived and compared has been accepted for publication in the *Journal of the Acoustical Society of America*. First, the Rayleigh integral (which is strictly valid only for planar sources) is applied to the exact velocity condition on the curved surface. The resulting time domain solution was obtained previously by Djelouah et al.[16] from the impulse response based on the Rayleigh integral.[17,18] Second, the velocity condition on the curved surface is replaced by a pressure distribution in a plane, and the Rayleigh integral is applied to the approximate planar source condition. The solution is a generalization of the result obtained by Levin et al.[14] for monofrequency sources. Third, the parabolic approximation is applied to both the Rayleigh integral and the source condition, and a solution derived previously by Frøysa et al.[19] is obtained.

All three solutions for the pressure p as a function of axial location z may be written in the form

$$\frac{p(z,\tau)}{\rho_0 c_0 u_0} = \frac{f[\tau] - A(z) f[\tau - \phi(z)]}{1 - z/d} \tag{5}$$

where u_0 is the characteristic velocity of the source, $f(t)$ is the time dependence of the source velocity, $\rho_0 c_0$ is the specific acoustic impedance of the fluid, d is the location of the geometric focus, and τ is a retarded time. Simple expressions are obtained for the amplitude factor $A(z)$ and the phase factor $\phi(z)$. The first term in the numerator is associated with the signal that arrives from the center of the source, the second with the signal from the edge of the source. In the first and third cases described in the previous paragraph one obtains $A(z) = 1$, and the amplitudes of the center wave and edge wave are identical (although reversed in sign). All three cases are compared in both the time and frequency domains. For small aperture angles, the three solutions yield similar results in the focal region and beyond—only near the source do significant differences arise, which are manifested through the descriptions of the edge wave. Each solution predicts a pressure waveform at the focus which is the time derivative of the velocity waveform at the source. The paper concludes with calculations for tone bursts and N waves.

IV. FOCUSING OF SOUND WITH AN ELLIPSOIDAL MIRROR

This work was also performed by M. F. Hamilton, the principal investigator, with salary support from ONR during the period 16 January 1991 through 31 July 1991. Personal computer support was provided by the David and Lucile Packard Foundation and NSF.

A. Background

In certain lithotripters, the acoustic source is a spark discharge at the near focus of an axisymmetric ellipsoidal mirror. The outgoing spherical wave produced by the spark is reflected from the mirror, and the intensity of the reflected field is maximized in the vicinity of the far focus, where the kidney stone is located. An analytical solution has been derived for the impulse response of the reflected pressure field along the axis of symmetry. Solutions for arbitrary incident waveforms are obtained by convolution, and a simple expression results for the pressure at the far focus. The solution is limited to high frequencies, i.e., to wavelengths that are short in comparison with the minimum radius of curvature of the mirror. Otherwise, there is no restriction on either the depth or eccentricity of the mirror. It is also assumed that the fluid is lossless and homogeneous, and the effects of nonlinearity are ignored.

The analysis is based on a solution of the Kirchhoff integral. Grünewald et al.[20] also used the Kirchhoff integral to investigate the reflection of a pulse from an ellipsoidal mirror. Although their mathematical statement of the boundary condition on the mirror is equivalent to ours, their solution for the reflected pressure field is expressed in the frequency domain as a sum of spherical harmonics, and the surface integral was evidently solved numerically. Our time domain analysis is a direct extension of theory developed by Cornet and Blackstock[21] for the axial pressure due to the reflection of an N wave from a spherical mirror. The Cornet-Blackstock solution involves two replicas of the incident waveform which are opposite in sign and different in amplitude, and which are associated with signals that arrive from the center of the mirror and from the edge of the mirror. The Cornet-Blackstock solution may therefore be written in the form of Eq. (5) with $A(z) = 1$. Our solution reveals, in addition to terms for the center wave and edge wave, a third term that results from the amplitude variation of the reflected field on the surface of the mirror. The third term, which requires convolution in the time domain, is also present in an axial solution derived by Naze Tjøtta and Tjøtta[22] for the pressure radiated by an axisymmetric planar source with arbitrary amplitude shading. The Tjøttas referred to the third term as the "wake." In the limiting case of a spherical mirror, the wake disappears, and we recover the solution of Cornet and Blackstock.

Relevant experimental work on the reflection of spherical waves from ellipsoidal mirrors has been reported by Müller[23–25] for spark sources in water, and by Wright and Blackstock[26] for spark sources in air. Cornet and Blackstock[21] also reported measurements in air for the special case of reflection of a spherical wave from a concentric spherical mirror.

B. Results

The solution for the reflected pressure p_r as a function of location z along the axis of symmetry may be written in the form

$$p_r = H_c(z)f(\tau_c) + H_e(z)f(\tau_e) + \int_{-\infty}^{\infty} H_w(z,t')f(t-t')\,dt' \qquad (6)$$

where $f(t)$ is the time dependence of the incident spherical wave, τ is a retarded time, and the H functions are algebraic expressions for the impulse responses associated with the center wave (H_c), the edge wave (H_e), and the wake (H_w). The solution at the far focus is proportional to the time derivative of $f(t)$, and the constant of proportionality is a simple function of the geometric parameters associated with the mirror. Numerical simulations based on Eq. (6) are currently being run and compared with published experimental results. An abstract has been submitted for presentation of these results at the November 1991 Meeting of the Acoustical Society of America in Houston.

V. FINITE AMPLITUDE PROPAGATION MODELS FOR SOUND IN WAVEGUIDES

This project involves T. W. VanDoren, a Ph.D. student in Mechanical Engineering who is scheduled to take his qualifying exam in September 1991. VanDoren received salary support from ONR during the period 1 March 1991 through 31 July 1991. Previous salary support for VanDoren was provided by the David and Lucile Packard Foundation. VanDoren received his M.S. degree in Mechanical Engineering in December 1990.

A. Background

The objective of this investigation, which involves both theory and experiment, is to assess the accuracy of the KZK nonlinear parabolic wave equation for applications to waveguide problems, particularly the propagation of finite amplitude sound in shallow water or in the deep sea (SOFAR) channel. The advantage of using the KZK equation is the availability of efficient computer algorithms which solve the equation. For example, the NPE code, which solves an equation similar to the KZK equation, has been applied recently to waveguide problems.[27] Analytical solutions for not only the linear fundamental component but also the nonlinearly generated second harmonic component can be obtained from both the parabolic and exact nonlinear wave equations on the basis of normal mode theory. In a linear analysis by McDaniel,[28] it is pointed out that errors introduced by the parabolic approximation occur in predictions of the phase speeds, but not the amplitudes, of the waves that propagate in the various modes. As a result, the propagation curves obtained from the two equations typically exhibit different fine structure but nearly identical large scale structure. We therefore anticipate that the nonlinear generation of higher harmonic components, which is a cumulative effect that tends to average out fine scale properties, should be reasonably well predicted in the parabolic approximation. Comparisons will be made not only with an exact quasilinear analytical solution, but also with experiment.

B. Results

VanDoren will perform the experiments in an air-filled waveguide that he constructed as part of his M.S. research. The waveguide is 7.5 m long and has interior dimensions of 66 mm by 38 mm. During the period he received ONR support, VanDoren designed a new source configuration for piezoelectric elements that will operate in air at either 33 kHz or 50 kHz and thus excite a variety of modes in the waveguide. The source amplitudes will be sufficient to produce substantial harmonic generation as the sound propagates down the waveguide.

Theoretical predictions for the linear field, without absorption, are shown in Fig. 4. The parameters were chosen to agree with the experiment that will be performed by VanDoren with a 50 kHz source that fills approximately one third of the cross sectional area of the waveguide. In Fig. 4(a) is shown the exact solution (along one wall of the waveguide, where there are microphone ports) based on the Helmholtz equation, and in Fig. 4(b) is shown the approximate solution based on the parabolic wave equation. Whereas the fine scale oscillations are not in agreement, the large scale oscillations are in very good agreement. This type of correlation is typical when the energy is concentrated in modes that are far from cutoff.[28]

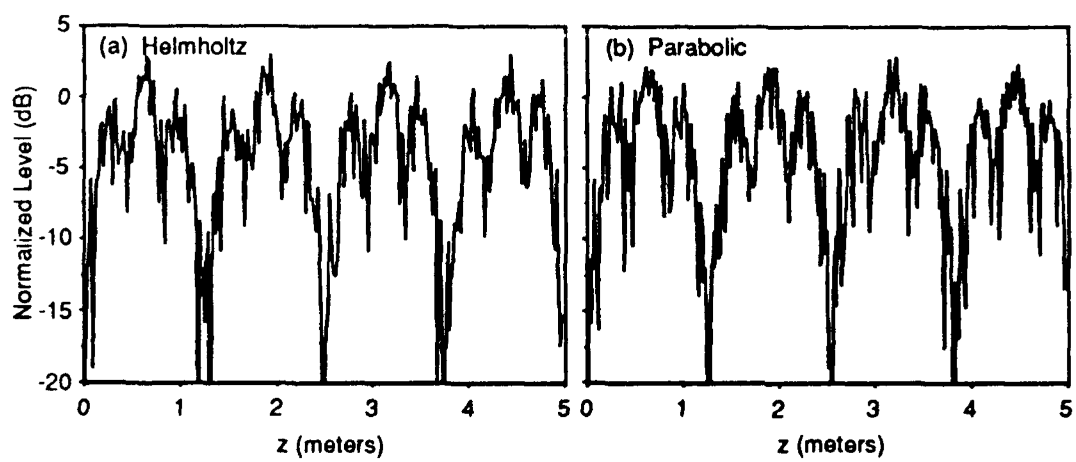

Figure 4: Propagation curves based on (a) the Helmholtz equation and (b) the parabolic approximation.

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