THESIS

ANALYSIS OF THE U.S. NAVY TERMINATION
MODEL FOR PROCUREMENT CONTRACTS

by

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September, 1990

Thesis Co-Advisors: Alan W. McMasters
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91-10677

91 9 16 006
**Title**: Analysis of the U.S. Navy Termination Model for Procurement Contracts

**Personal Author(s)**: Terence G. Smith

**Type of Report**: Master's Thesis

**Date of Report**: September 1990

**Page Count**: 69

**Abstract**

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Analysis of the U.S. Navy Termination Model for Procurement Contracts

by

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1990

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This thesis analyzes the Navy's procurement contract termination model, the mathematical model on which the Navy's Inventory Control Points will rely to determine whether to terminate procurement actions on items in long supply. The analysis focuses on which costs are relevant to the model and which costs are irrelevant. Suggestions for improvement are offered which include both eliminating irrelevant costs considered by the model and adding relevant costs not considered. Finally, the thesis evaluates the model's recommendations for terminations over a range of values of key model parameters to determine which parameters have the most impact on the model's decisions. Then it recommends further research in determining more precise values for those parameters with the largest effect on the model's decisions.
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INTRODUCTION

A. BACKGROUND

The Navy's Inventory Control Points (ICPs), the Ships' Parts Control Center (SPCC) and the Aviation Supply Office (ASO), rely on mathematical forecasting models to forecast demand for spare parts. The forecasts enable them to let procurement contracts with lead times as long as three years for the number of spares required at the end of the lead time period. Unfortunately, for various reasons, these forecasts are frequently incorrect, either overstating or understating the spares requirement at the end of the lead time period. When the requirement is overstated, the item is said to be in "long supply." An ICP has two basic options for items in long supply: 1) continue the procurement contract in effect and accept the costs associated with being overstocked (such as storage and obsolescence costs) or 2) cancel (terminate) the contract and accept the costs associated with cancellation (such as contractor termination fees and the administrative costs of letting a new contract sometime in the future). To make a good decision on whether or not to terminate a contract, decision makers at the ICPs must be able to consider and compare all relevant costs and choose the least-cost alternative.

The problem of deciding whether or not to terminate a contract has been with the Department of Defense (DOD) for many years. A Naval Postgraduate School Master's Thesis done by Gary Chapman [Ref. 1: pp. 8-26] gives an excellent review of the history of DOD (including U.S. Navy) contract terminations through June, 1988. As Chapman's thesis shows, for years prior to 1988, Government auditors had identified the need for cost-effective contract termination criteria. But even more recently, both the Department of Defense Inspector General (DODIG) and the General Accounting Office (GAO) have been critical of the military services' (particularly the Army's and Navy's)
failure to cost-effectively terminate procurement contracts for secondary items in long supply. A recent DODIG audit report faulted the services in three areas:

1. Item Managers did not accurately quantify the value of excess on-order assets before making termination decisions. [Ref. 2: p. 7]

2. Managers ... made uneconomical termination decisions because they did not have policies and procedures that specified how to make those decisions. As a result of these decisions, the inventory control points could incur $121.9 million a year in unnecessary expenditures. In addition, we concluded that the item managers at Navy's Aviation Supply Office established a dollar threshold for termination reviews that eliminated the on-order assets from termination considerations. [Ref. 2: p. 13]

3. The ICPs did not take advantage of the opportunities to use material from terminated contracts as Government-furnished material due to the lack of policies and procedures requiring excess on-order assets to be used as Government-furnished material. [Ref. 2: p. 31]

The DODIG estimated as a result of this audit, in which the Aviation Supply Office (ASO) was the only Navy ICP audited, that ASO alone could realize almost $50 million in recurring benefits and almost $135 million in one-time benefits. [Ref. 2: p. v]. Furthermore, Chapman, in his thesis, reported that past audits of DOD components had identified potential savings of anywhere from $20 million in 1957 [Ref. 1: p. 9] to almost $30 billion in 1987 [Ref. 1: p. 24] if contracts had been terminated for material in excess of requirements. And while there are many legitimate reasons why an ICP might want to keep material in excess of requirements--Chapman names eight reasons, some legiti-
mate and some not—as indicated above, recent DODIG and GAO auditors think that at least some contracts should be terminated that are not now (and vice versa.)

In the past, the services have terminated contracts using rules and policies that were somewhat arbitrary at best. The primary termination criteria have been monetary thresholds ranging from $50,000 [Ref. 3: p. 12] to $170,700 [Ref. 2: p. 14]. Below these thresholds, contracts were not considered for termination, causing uneconomical decisions (see finding 2 in the above audit.) DODIG auditors recommended as a result of findings 1 and 2 in the above audit (finding 3 will be addressed in Chapter III) that the ICPs initiate the following corrective actions:

1. ... we recommend that the Commander (sic), Aviation Supply Office, establish training programs that instruct item managers and their supervisors on the criteria that they should use to evaluate, review, and approve validations of excess on-order asset positions. [Ref. 2: p. 10]

2. ... we recommend that the Commander, Naval Supply Systems Command: provide specific guidance to Navy contracting officers describing how to accomplish and document cost comparison analyses necessary for termination decisions. [Ref. 2: p. 22]

B. OBJECTIVES

To implement these findings (and many other, similar findings from previous audits), the Naval Supply Systems Command (NAVSUP) Operations Research Group (Code 04E) has developed the Procurement Contract Termination Model (PCTM) [Ref. 4: pp. 1-2], which is currently being implemented by SPCC and programmed by the Fleet Material Support Office. NAVSUP operations researchers intend the PCTM to consider all costs associated with both contract termination and continuation and quantify them as a decision aid for the ICP's item managers. With millions of dollars potentially riding
on its decisions, it is important that the NAVSUP PCTM give the most correct results possible. To accomplish this, the model must be able to do three things:

1. Include all costs relevant to a contract termination.
2. Exclude any costs not relevant to a contract termination.
3. Correctly quantify important independent decision parameters, such as obsolescence rate, interest rate, inflation rate, storage cost rate, and administrative costs.

The objectives of this thesis are as follows:

1. To analyze the NAVSUP PCTM and determine which costs belong in the model and which costs do not belong.
2. Based on this analysis, to suggest changes to the current model that will improve the accuracy of the model's termination decisions.
3. To analyze which costs have the most impact on making a correct decision and which are relatively minor.
4. Based on this analysis, to suggest which costs deserve priority in future research to further enhance the model's accuracy.

Accomplishment of these objectives will enable operations researchers at NAVSUP to improve their PCTM so that it will provide a more accurate analysis of the costs involved in terminating a contract. In turn, a more accurate model should enable the U.S. Navy to realize a substantial proportion of the potential savings identified by DOD and GAO auditors by improving the contract termination decisions of its inventory managers and contracting officers.
C. ORGANIZATION

To achieve these objectives, this thesis is organized into four additional chapters.

Chapter II analyzes the current NAVSUP PCTM using the principles of economic analysis and inventory management to determine whether the costs included in the model are relevant or not. Costs found not to be relevant are recommended for deletion from the model.

Chapter III analyzes certain costs not included in the current NAVSUP PCTM to determine their relevance. Such costs found to be relevant are recommended for inclusion in the model.

Chapter IV analyzes the sensitivity of the model to variations in those key PCTM parameters on the following list which are still recommended for inclusion in the improved model: interest rate, storage cost rate, obsolescence rate, inflation rate, contractor termination fees, and administrative costs. Further research on those parameters which have the largest impact on the model's decisions is then recommended.

Chapter V is a summary of the findings with conclusions and recommendations, including recommendations of areas for future research.
II. THE PROPOSED MODEL - AN ANALYSIS

A. THE PROPOSED MODEL

1. Background

As mentioned in Chapter I, the Naval Supply Systems Command (NAVSUP) Operations Research Group (Code 04E) has developed the Procurement Contract Termination Model (PCTM) to optimize contract termination decisions made by NAVSUP Inventory Control Points (ICPs.) The PCTM includes three costs to continue and three costs to terminate a contract which the NAVSUP operations researchers considered both relevant and important to the termination decision and quantifiable for inclusion in a mathematical model.

The PCTM considers the following costs to continue a contract, which are a subset of the category of cost usually known as "holding costs" [Ref. 5: p. 14]:

1. The "capital cost", which "...reflects lost earning power or opportunity cost. If the funds were invested elsewhere, a return on investment would be expected. Capital cost is a charge that accounts for this unreceived return". [Ref. 5: p. 14]

2. The "shrinkage cost", which is defined as "...the decrease in inventory quantities over time from loss or theft". [Ref. 5: p. 14]

3. The "obsolescence cost", which reflects "...the risk that an item will lose value because of shifts in style or consumer preference" (In the case of the U.S. Navy, "shifts in style or consumer preference" are better characterized as advances in technology or as changes in doctrine prompted by responses to the Soviet threat, either one of which can render items in the supply system obsolete for their intended use; however, the principles suggested by Tersine still apply). [Ref. 5: p. 14]
"The usual simplifying assumption made in inventory management is that holding costs are proportional to the size of the inventory investment" [Ref. 5: p. 14].

The PCTM considers the following costs to terminate a contract:

1. The contractor's termination fee (CTF), the fee negotiated between the contractor and the government to reimburse the contractor for any work already performed. The CTF has no real analogue in the standard inventory model.

2. The administrative cost (ADM), the cost to remake the contract once the material becomes required again. The ADM is analogous to the "order cost" in the standard inventory model that must be paid for the next order placed. [Ref. 5: p. 14]

3. The inflation cost, the cost to order the new material added by the general inflation rate. This cost has no analogue anywhere.

No other costs to continue or terminate were considered because the NAVSUP Code 04E analysts who derived the model considered the model complete enough to cover most reasonable costs and complex enough to challenge most users of the model [Ref. 6].

The mathematics of the model are presented in the section below as equations and definitions of their variables. The following sections then present the logic the modelers used to build those equations. This is immediately followed by the author's analysis and recommendations for improvement.

2. The Model

The PCTM proposed by NAVSUP is as follows [Ref. 4: pp. 1-2]:

\[
\text{Cost to Continue} = (I)(R)(\Pi P_T) + (S)(Q_T \times P)(\Pi P_S) + (O)(Q_T \times P)(\Pi P_O),
\]

which is compared to the
Cost to Terminate = CTF+ADM+(F)(HPF) min(QT×P, Q×P),

where the model parameters are

I = yearly interest rate;
S = storage cost rate per year;
O = obsolescence rate per year;
F = yearly inflation rate.

The model inputs are:

QT = termination quantity;
P = unit price;
QT×P = total dollar value of the termination quantity;
R = dollar value returned to the Navy after termination (= contract price - CTF);
CTF = contractor termination fees (= contract price - R);
ADM = administrative cost both to award and terminate a contract;
Q = The procurement cycle order quantity (economic order quantity);
Q×P = total dollar value of the procurement cycle order quantity.

The relevant holding periods are calculated as follows (HP = holding period for horizon identified by subscript):

\[
\begin{align*}
HP_I &= T_{RL} + \frac{ALT}{4} - 1 \\
HP_S &= T_{RL} + \frac{PDLT_e + ALT}{4} \\
HP_Q &= T_{RL} + \frac{PDLT_e}{4} \\
HP_P &= T_{RL} + \frac{PDLT_e + ALT}{4},
\end{align*}
\]

where

\[
T_{RL} = \text{time to reach reorder level after termination (years)}
\]
ALT = administrative lead time (quarters)

PDLE = elapsed production lead time (i.e., the difference between the contract award date and the current julian date) (in quarters).

3. The Expected Time to Reach Reorder Level After Termination

The expected number of years to reach reorder level after termination is calculated as follows [Ref. 4: p. 2]:

\[ T_{RL} = \frac{[OH_A + OH_F \times SR + DI] - (RL + PPR + BO + DO) - QT}{4 \times (D - G) + \frac{4 \times PPR}{PCLT}} \]

where

\( OH_A \) = on-hand ready for issue (RFI) assets as of "today" in Figure 3 on page 19 and in Figure 4 on page 19

\( OH_F \) = on-hand non-RFI assets as of "today" in Figure 3 on page 19 and in Figure 4 on page 19

\( SR \) = survival rate (for repairable items)

\( DI \) = due-in assets from procurement

\( RL \) = reorder level

\( PPR \) = planned program requirements over procurement lead time

\( D \) = quarterly demand forecast

\( G \) = quarterly regenerations forecast (for repairable items)

\( BO \) = backorders

\( DO \) = due-out items

\( QT \) = termination quantity (as above)

\( PCLT \) = procurement lead time.
The formula for $T_{RL}$ was derived by first considering all current on-hand and due-in assets (the "inventory position") and subtracting both all requirements over the remaining lead time and the termination quantity. This result is then divided by the sum of annual recurring and nonrecurring demand rates. [Ref. 7: p. 1]

This formula seems correct to the author, provided over that same period the total assets include only those due-in assets considered over the remaining lead time for accurate comparison with requirements. The total assets less requirements over this lead time segment will give net assets available for issue. Subtracting the termination quantity gives net assets available for issue should that quantity be terminated from the contract. Dividing by the sum of annual recurring and nonrecurring demand rates gives the expected period of time during which assets will be issued and will fall to the reorder level should the termination quantity be terminated from the contract.

B. THE COST TO CONTINUE THE CONTRACT

1. The Cost to Invest in the Additional Inventory

   a. Background

   The first of the three components of the cost to continue the contract is the investment cost in any material furnished by the government for this contract or in any payments made on the contract which will be returned to the government after termination. The NAVSUP Code 04E analysts who derived the PCTM considered this investment cost to be the rate of interest that could be earned on the dollar value of this material returned to the Navy after termination for the applicable holding period for these assets. They believed that a straightforward product of the interest rate, the value of money and material, and the holding period was sufficient to get an idea of the cost. Compounding the interest, which is commonly done in any cost-based decision model, was assumed to not make enough difference to be worth including given the complexity it would add to the model. They also believed that, given the uncertain nature of the
estimates of the parameters, such a refinement of the computation was unwarranted. The cost portion of this term is analyzed in the section below, and the holding period is analyzed in the section following.

\textit{b. The Cost}

The investment cost formula, \((I)(R)(HP)\) (where \(I\), \(R\), and \(HP\) are defined as above), used in the NAVSUP PCTM more or less correctly expresses, in the opinion of the author, the investment cost. The interest rate times the dollar value of money and material which would be returned to the government should the contract be terminated (but which must be invested in should the contract be continued) times the applicable holding period may be an adequate approximation of the investment cost in such money and material. However, since the interest is calculated without being compounded, it is only an approximation, as the following example will show. Take the interest rate as a parameter equal to 10\% (as it is in the actual model in accordance with DOD Instruction 41-0.39), and assume that the amount of government-furnished money and material is $10,000 and that the relevant holding period is three years (both fairly typical amounts). Then we have the following calculation which will be performed by the NAVSUP PCTM:

Investment Cost = \((I)(R)(HP_t)\) = \((0.1)(10000)(3)\) = $3000.00.

But suppose we assume, more realistically, that interest is compounded daily instead of not at all. Then the correct investment cost calculation would be as follows:

Investment Cost = \((1+\frac{I}{365})(HP_t)(365)\times R = (1+\frac{0.1}{365})(3)(365)\times 10000-10000 = $3498.03.

The correct cost to continue due to increased investment cost, in this case, is almost 16 percent more than the amount calculated by the NAVSUP PCTM.\footnote{The above formula uses a holding period expressed in years. If the holding period \(HP_t\) is expressed in quarters, as it is in the PCTM, the formula becomes Investment Cost =}
In general, given that the model specifies a 10% rate of interest, the percent the PCTM formula understates the true interest cost is a function only of the holding period (HP). The general formula, where \( R \) = dollar value of material returned, is as follows:

\[
PCTM \text{ Investment Cost} = (I)(R)(\frac{HP}{1}) = (0.1)(R)(\frac{HP}{1}) ;
\]

and

\[
\text{Compounded Investment Cost} = (1 + \frac{I}{365})(\frac{HP}{R})(365) \times R - R = (1 + 0.00274)(\frac{HP}{R})(365) \times R - R
\]

\[
= [1.000274(\frac{HP}{R})(365) - 1] \times R.
\]

Combining these two formulas into a percent difference formula gives the following formula for percent difference as a function of the length of the holding period (HP):

\[
\frac{[(1.000274(\frac{365}{HP})) - 1] \times R - (0.1)(R)(\frac{HP}{R}) \times 100}{(0.000274(\frac{365}{HP})) \times R - R}
\]

\[
= \frac{[(1.000274(\frac{365}{HP})) - 1] - (0.1)(\frac{HP}{R}) \times 100}{(0.1)(\frac{HP}{R})}.
\]

Table I shows the values from this formula for holding periods from one to ten years. It is clear that for short holding periods and small amounts of money the simple interest approach is sufficient to get an idea of the cost involved. However, for longer holding periods and larger amounts of money it may no longer be sufficient.
Table 1. PERCENT DIFFERENCE BETWEEN PCTM AND COMPOUNDED INVESTMENT FORMULAS.

<table>
<thead>
<tr>
<th>Years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Difference</td>
<td>5.2</td>
<td>10.7</td>
<td>16.6</td>
<td>22.9</td>
<td>29.7</td>
<td>37.0</td>
<td>44.8</td>
<td>53.2</td>
<td>62.1</td>
<td>71.8</td>
</tr>
</tbody>
</table>

**c. The Relevant Period**

Determination of the relevant holding period over which to calculate the investment cost is straightforward (see the above formula for $HP_r$). It is simply the time from when the money and material would have been returned and savings would have started until the money and material may be required on the next contract for the cancelled item. If the contract were terminated and the money and material were returned immediately, this period would be the time to reach the reorder level (given the contract is terminated) plus the administrative lead time required to award the contract. The relationship can be formulated as $HP = T_{RL} + ALT$. However, the NAVSUP PCTM makes the assumption that the money and material will not be returned for (on average) one year. The PCTM therefore formulates the investment holding period as $HP = T_{RL} + \frac{ALT}{4}$, where ALT is expressed in quarters and is divided by 4 to convert it to years. In the author’s opinion, this formulation accurately represents the investment opportunity lost for the purposes of the model, provided the average one year return time for government-furnished money and material is correct.

**2. The Cost to Store the Additional Inventory**

a. **Background**

If the contract is continued, the quantity ($Q_r$) under consideration for termination must be stored as part of the inventory. Therefore, the second of the three components of the cost to continue the contract is the storage cost of holding that
quantity of inventory. NAVSUP Code 04E analysts believed that the termination quantity \( Q_T \), if the contract were not terminated, would be stored for the entire storage holding period as the difference between the non-terminated quantity and the terminated quantity. They therefore calculated the storage portion of the cost to continue the contract as a straightforward product of the termination quantity, the storage cost rate, and the storage holding period. They also assumed that any material terminated from a contract would eventually be reordered.

b. The Cost

In the opinion of the author, the NAVSUP PCTM formula for the storage cost, \( (S)(Q_r \times P)(HP_s) \), is incorrect. In the classical inventory model, annual storage (or holding) costs are based on the average inventory on hand during the order cycle, \( \frac{Q}{2} \) [Ref. 5: pp. 91-93]. Figure 1 on page 17 shows the "sawtooth" inventory curve resulting from terminating the quantity \( Q_T \), where \( Q_1 = \) inventory on hand if we do not terminate, \( Q_2 = \) inventory on hand if we do terminate, and \( Q_T = Q_1 - Q_2 \). It can be seen from Figure 1 that while the amount saved during the "holding" period \( HP_s \) proposed by the NAVSUP analysts (see the next section) is indeed \( (S)(Q_r \times P)(HP_s) \), that analysis does not take the entire process into account. The differential holding costs do not end at the time the item is reordered following termination; they continue until the time the item would have been reordered had it not been terminated. The reason for this is that when the contract is terminated, the item must be reordered back up to the new economic order quantity (EOQ) amount at the end of \( HP_s \), but when the item is not terminated, it is not reordered until the time \( HP_s + \frac{Q_T}{D} \), where \( D = \) the average quantity demanded, because the item does not run out as soon.

As Figure 1 shows, we are projecting future inventory levels assuming the only variable is the remaining inventory quantity at the start of \( HP_s \), namely \( Q_1 \) or \( Q_2 \). The projected expected demand rate must be the same in either case; hence the slopes
of the "sawteeth" are equal. Third, the most likely reason for the item to be under consider-
ation for termination is that it is in long supply as a consequence of the demand (D) having been reduc-
ed. This means that where the EOQ was previously \( Q_1 \), accounting for the fact that we had \( Q_1 \) units on order, it has now been reduced to \( Q_2 \) in accordance with the reduction in demand. Therefore, the "sawtooth" at the end of \( HP_s \) jumps back up only to \( Q_2 \). Fourth, the time period between \( HP_s \) and the time the item will be ordered if it is not terminated is \( \frac{Q_T}{D} \) because to use \( Q_T \) units at a rate of \( D \) units per time period takes \( \frac{Q_T}{D} \) units of time. And finally, the "sawtooth" at the end of \( HP_s \) fall by \( Q_T \) units at the end of the time period because using the item for \( \frac{Q_T}{D} \) units of time at a rate of \( D \) units per time period will deplete exactly \( Q_T \) units from the inventory.

The storage costs for these time periods are based on the average inventories held and are shown in Figure 1 as \( A_1 \), \( A_2 \), and \( A_3 \), where \( A_1 = \frac{1}{2} Q_s HP_s \), which is the inventory held for the reduced quantity \( Q_2 \) for the period \( HP_s \), \( A_2 = \) the inventory held based on the full quantity \( Q_1 \) for the period \( HP_s + \frac{Q_T}{D} \):

\[
A_2 = \frac{Q_1}{2} \left[ HP_s + \frac{Q_T}{D} \right] = \frac{Q_1 + Q_T}{2} \left[ HP_s + \frac{Q_T}{D} \right]
\]

(since \( Q_1 = Q_2 + Q_T \)), and \( A_3 = \) the inventory held based on the reduced quantity \( Q_2 \) for the period \( \frac{Q_T}{D} \):

\[
A_3 = \frac{Q_T}{D} \times Q_2 - \frac{1}{2} \times \frac{Q_T}{D} \times Q_T = \frac{Q_T}{D} \left[ Q_2 - \frac{Q_T}{2} \right].
\]

\[2\] It may not be true that the EOQ has changed to exactly \( Q_2 \); there may be some other factors that caused the model to optimize using that quantity. However, it is reasonable to assume that the new EOQ will be close to \( Q_2 \).
The difference between the average inventories when the contract is not
terminated and when the contract is terminated is

\[ A_2 - (A_1 + A_3) \]

\[
= \frac{Q_2 + Q_T}{2} \times \left( H P_S + \frac{Q_T}{D} \right) - \left( \frac{Q_2}{2} \times H P_S + \frac{Q_T}{D} \times \left( \frac{Q_2}{2} - \frac{Q_T}{2} \right) \right)
\]

\[
= \frac{Q_2}{2} \times H P_S + \frac{Q_2}{2} \times H P_S + \frac{Q_T^2}{2D} - \frac{Q_2}{2} \times H P_S - \frac{Q_2 \times Q_T}{D} + \frac{Q_T^2}{2D}
\]

\[
= \frac{Q_T}{2} \times H P_S + \frac{Q_T^2}{D} - \frac{Q_2 \times Q_T}{2D}.
\]

But from Figure 1 it can be seen that \( H P_s = \frac{Q_1}{D} \) so that \( \frac{Q_T}{2} \times H P_s = \frac{Q_T \times Q_1}{2D} \). Substituting this expression into the above formula gives the final result:

\[ A_2 - (A_1 + A_3) = \frac{Q_T^2}{D}. \]

This should be the correct formula for describing the differential inventory held when considering the current order period.
Figure 1. "Sawtooth" diagram for the on-hand inventory for the current period with and without termination.

Figure 2 shows the effect of termination on future order periods, under the assumption that demand will remain constant at its new rate. It can be seen from Figure 2 that future periods will have the same storage costs with only their time frames offset by the time \( \frac{Q_T}{D} \) whether or not the contract is terminated now. Thus, if demand does not change, future inventory held differentials will be 0. If demand is reduced again at some time in the future, the system will be in long supply again, and the termination problem will be resolved using the new data. If demand increases at some time in the future, more of the item will be placed on order, again requiring resolving the problem using the new data. In any case, it is appropriate to assume that demand will remain constant to solve the current problem.

In conclusion, the correct formula for the differential inventory held should be \( \frac{Q_I}{D} \) and the differences in future order periods are zero. Since \( \frac{Q_I}{D} = Q_T \times \frac{Q_T}{D} \), this formula can be interpreted to mean the differential inventory held equals the differential (termination) quantity \( Q_T \) times the differential time \( \frac{Q_I}{D} \). Therefore, the correct formula...
for the storage cost to continue is \( \frac{Q_T}{D} \times S \times P \) rather than \( S \times Q_T \times P \times H P_s \) for the differential inventory held when the contract is continued. This change should be incorporated into the model.

![Diagram](image)

Figure 2. "Sawtooth" diagram for future periods with and without termination.

c. The Relevant Period

The relevant period for the Navy's model was \( H P_s \), and its formula was given at the start of this chapter. For the discussion in this section (and also that in Section II.B .c.), the author relied in large part on an analysis provided by Mr. Jerry Zamer of the NAVSUP Operations Research Group [Ref. 6]. In his analysis, the period for which savings due to reduced storage costs are relevant begins when the buy from which the quantity \( Q_T \) was terminated is or would have been received (depending on whether it was a partial or a complete cancellation), because that is when the government would have begun paying storage costs. It ends when the next buy is received (if ever) because that is when the government will begin paying storage costs, given that the contract actually was terminated. The figure below is a time line representing points of time in the procurement process which will aid the reader in following this argument.
The point on the time line labelled "Buy$_1$ initiated" represents the point in time when the Supply Demand Review (SDR) program in the ICPs computer initiated the buy for which we are now considering a termination. The point labelled "Buy$_2$ initiated" represents the point in time when the SDR will initiate the next buy if we terminate the quantity Q$_T$ today. The point labelled "Buy$_1$ received" represents the point in time when the buy for which we are now considering a termination will be received if it is not terminated (hence the time difference between "Buy$_1$ initiated" and "Buy$_1$ received" is **Procurement Lead Time (PCLT)**.)

Figure 4 is the same time line as figure 3 above with the positions of **PCLT** and **ALT+PDLT+TRL** interchanged.

In other words, adding **PCLT** to the beginning of **ALT+PDLT+TRL** changes its position in time but not its value, and, after changing its position, its endpoints coincide with "Buy$_1$ initiated" and "Buy$_2$ initiated". Therefore, the time between when the buy from which Q$_T$ was terminated is (or would have been) received and the time when the next...
buy for that item is received is just $ALT + PDLT + T_{RL}$, and this represents the relevant holding period $HP_s$ for which savings due to reduced storage costs are relevant.

In the author's opinion, this portion of the formula would have correctly represented the relevant holding period if the only period under consideration were the time from when the termination is being considered until the time when the item is reordered if it has been terminated. But, as pointed out in the preceding section, the NAVSUP analysis does not take into account the period of time between when the item would have been reordered if termination had occurred and when the item would have been reordered if termination had not occurred. In the previous section, the differential inventory held was determined to be $Q_s$, an amount which is independent of any holding period. Therefore, in the author's opinion, a holding period $HP_s$ is unnecessary for the PCTM and should be dropped from the model.

3. The Obsolescence Cost of the Additional Inventory

a. Background

The third of the three components of the cost to continue the contract is the obsolescence cost of holding the quantity of inventory under consideration for termination. NAVSUP Code 04E analysts believed that, as with the storage costs, the termination quantity $Q_T$ would be obsolescent for the entire obsolescence holding period as the difference between the non-terminated quantity and the terminated quantity. They therefore calculated the obsolescence portion of the cost to continue the contract as a straightforward product of the termination quantity, the obsolescence cost rate, and the obsolescence holding period, just as was done with the storage cost. They also assumed, as with storage costs, that any material terminated from a contract would eventually be reordered.
b. The Cost

In the opinion of the author, the NAVSUP PCTM formula for the obsolescence cost, \((O)(Q_T \times P)(HP_o)\), incorrectly states the true obsolescence cost to continue just as it does for differential storage costs (and for the same reason). This can be shown using the same analysis of differential inventory held used in Section II.B.2.b. The result of this analysis is that, in the opinion of the author, the correct formula for the obsolescence cost to continue the contract is \(\frac{Q_T}{D} \times O \times P\), the differential inventory held when the contract is continued times the obsolescence rate, and this is the term which should be incorporated into the model for the obsolescence cost to continue the contract.

c. The Relevant Period

The analysis of the relevant holding period for obsolescence nearly parallels the analysis in Section II.B.2.b. The analysts at NAVSUP Code 04E believed that the period for which savings due to reduced obsolescence costs are relevant begins when the buy from which the quantity \(Q_T\) was terminated is or would have been received (depending on whether it was a partial or a complete termination), and it ends when the next buy is received (assuming the contract actually was terminated), just as it did for storage costs. Using the same analysis as in Section II.B.2.c. and referring to Figure 3 on page 19 and to Figure 4 on page 19, they showed that the time between \(Buy_1\) and \(Buy_2\) is still \(ALT + PDLT + T_{RL}\). However, they argued that since obsolescence of spare parts is a process driven by advancing technology, obsolescence does not actually begin until the contract is awarded and the process of choosing a design has actually begun. Before the design choice has been made, any technological advances can be taken advantage of, and obsolescence is not yet a problem. After the design choice has been made and the contract has been awarded, however, technological advances cannot be taken advantage of without rendering the equipment obsolete. Therefore, during ad-
should not be counted as part of the holding period for obsolescence. This is reflected in the formula they used in the NAVSUP PCTM, namely $PDLT_i + T_{RL}$.

This portion of the formula would have correctly represented the relevant holding period if the only period under consideration were the time from when the termination is being considered until the time when the item is reordered if it has been terminated just as was the case with the storage cost to continue the contract. But, as pointed out in sections II.B.2.b. and II.B.2.c., the NAVSUP analysis does not take into account the period of time between when the item would have been reordered if termination had occurred and when the item would have been reordered if termination had not occurred. In the previous section, the differential inventory held was determined to be $\frac{O_2}{D}$, an amount which is independent of any time period. Therefore a holding period $HP_o$ is unnecessary for the PCTM and should be dropped from the model.

C. THE COST TO TERMINATE THE CONTRACT

1. Contractor Termination Fees

   a. Background

   While contractor termination fees have a significant impact on the decision the model makes, they are input to the model primarily as a given parameter. The determination of how much to pay the contractor in termination fees rests with the termination contract officer at whatever Defense Contract Administrative Service Region has cognizance of the contract. Any dispute over such fees is decided by attorneys for the government and the contractor. But for the purposes of the NAVSUP PCTM, they should be predetermined. However, as Chapman, in his thesis, states,
there exists no motivation in the termination process for the contractor to estimate his fees in a realistic manner. If his proposed termination fees are higher then the eventual settlement, the R value will have been set too low. This low dollar value will artificially lessen the cost to continue while artificially inflating the cost to terminate. [Ref. 1: p. 88]

NAVSUP analysts agreed with his assessment, preferring to estimate the expected contractor fees using an empirical formula which is used at the ICPs.

b. The Costs

The formula used at the ICPs is as follows:

\[ CTF = \sqrt{\frac{PDLT_e}{PDLT}} \times CP = CP - R. \]

where the contract price (CP) is the original cost of the contract. Therefore the contractor termination fees are estimated as the square root of the fraction of production leadtime elapsed times the original cost of the contract and R is estimated as the difference between the termination fee and the original contract cost. As mentioned in the above section, the ICPs use this formula and believe it to be empirically correct. However, contracting officers should attempt to elicit reasonable responses from contractors to the question of contract termination fees, if only for comparison with the formula. Any significant deviations should be further investigated to obtain a reasonable compromise. Also, as this cost occurs instantaneously at the time the contract is cancelled, there is no applicable holding period. Therefore, no holding period has been proposed for these costs by the PCTM. This appears to be quite logical.

The sensitivity of the model's decision making to variations in the CTF will be tested in Chapter IV. If changes in the CTF have a significant impact on the model's
decision, the empirical formula should be thoroughly checked for correctness. For example, the ICPs could, using historical data, plot \( \frac{PDLT}{PDLT} \) (fraction of production lead time elapsed) against \( \frac{CTF}{Q_T} \) (fraction of termination fees paid on terminated material) to determine a suitable relationship between contractor termination fees and production time elapsed.

2. Administrative Costs

   a. Background

   Administrative costs were included as a cost to terminate the contract because when all or part of the contract is terminated, it is expected that the material will be reordered sometime in the future, with concomitant administrative order costs [Ref. 5: p.14]. The PCTM uses them strictly as an input parameter which has also been predetermined. For the purpose of the PCTM, the costs may have to be determined by a separate analysis or they may have to be guessed at. Therefore, NAVSUP analysts have included this cost as a given parameter.

   However, there is another administrative cost apparently not envisioned by the NAVSUP Operations Research Group that could impact the model’s decision and should therefore be considered for inclusion in the PCTM, namely the administrative cost to terminate the contract. This cost is real and should be included in the model’s logic because the work done to terminate the contract costs the government time and therefore money just as the work done later to reorder the material.

   b. The Costs

   As the administrative costs envisioned by NAVSUP analysts occur in the future, they must take into account the time value (opportunity cost) of money. In the author’s opinion, the correct way to do this is to discount this cost using an appropriate rate of interest. Since the time from now (when the contract is being considered for termination) until the material will be reordered should the contract be terminated is
\( T_{RL} \), it is the appropriate time period over which to compute the discount. The formula for computing the discounted administrative cost would then be as follows:

\[
(\text{Future}) \text{ Administrative Costs} = \frac{\text{ADM}}{1 + \left( \frac{T}{365} \right)^{T_{RL}}(365)}.
\]

Standard rates have been established for the costs attributable to contract awards. By contrast, to the author's knowledge no standard rates have been similarly determined for the cost to administratively terminate a contract. However, as argued above, such costs are real. As this cost occurs instantaneously at the time the contract is cancelled, there is no applicable holding period. Therefore, no holding period is been proposed. However, the author must give one caveat. If these costs do occur at some time in the future, they should be subject to discounting if that time is far enough in the future to make a significant difference in the model's decision. But, since the costs are likely to occur at the same time in the future each time a contract is terminated, such discounting need not be included in the model per se. It can be performed using a standard time period to determine a standard present value of the administrative cost to terminate external to the model and then that figure input to the model.

The sensitivity of the model's decision making to variations in administrative costs will be tested in Chapter IV. If changes in administrative costs have a significant impact on the model's decision, the amount used to represent administrative costs should be thoroughly checked for correctness.

3. The Cost to Reprocure Due to Inflation
   
   a. Background

   As mentioned above in Section II.B.2.a, NAVSUP Code 04E analysts assumed that should a quantity \( Q_T \) be terminated from a contract, eventually the material would have to be reordered. They further assumed that, if \( Q_T \) were larger than the economic order quantity (EOQ \( Q \)), only \( Q \) units would be ordered. They therefore formulated the cost due to inflation as \((F)(H)(P)\min(Q_T \times P, Q \times P)\). This formula reflects the
assumption that regardless of how much material is terminated from the contract, the ICP will never order more than the EOQ.

b. The Cost

The assumption that there is a cost to reprocure due to inflation is based on an analysis such as the following one. Without loss of generality for this argument, let $Q_T = Q$; then the inflation cost should be future cost minus present cost: 

$$(1+F)^{HP_F}(Q \times P) - Q \times P \approx (F)(Q \times P)(HP_F)$$

This formula, however, is only an approximation. For example, for $HP_F = 2$ years,

$$Cost_{inflation} = (1+F)^2(Q \times P) - Q \times P = (1+2F+F^2)(Q \times P) - Q \times P = (2)(F)\left(Q \times P\right) + (F^2)(Q \times P)$$

The formula neglects the effects of higher-order terms like $(F)(Q \times P)$. If a formula like this is to be used, the higher-order terms should be included as with the interest rate for $HP_F$ periods in the suggested changes to the costs to continue the contract presented above.

However, in the opinion of the author, the model’s use of a cost due to inflation fails to take into account the true (present) value of $Q$, $HP_F$ periods of time in the future, which must be discounted for inflation. The true equation is as follows:

$$(1+F)^{HP_F} \times \frac{Q \times P}{(1+F)^{HP_F}} - Q \times P = 0$$

(always!).

Therefore, since the cost of inflation is always 0, the inflation cost term is not needed. Another way of viewing this is that while we will certainly have to pay more money for future purchases of the item under consideration for termination if we do terminate the contract, we will be paying the increased price in inflated dollars which are worth less
than today's dollars by an amount which corresponds on the average to exactly the inflation rate. So, while the rate of inflation is certainly a concern for preparing the budget for the period in which the repurchase will be made, since budgets are prepared in nominal and not real dollars, it need not concern the operations analyst who is trying to identify the costs associated with terminating the contract. The cost term associated with general inflation should therefore be dropped from the model.

On the other hand, if there are *item-specific* cost increases that either the item manager, contracting officer, or someone else in the organization knows will occur, these would be legitimate costs to terminate the contract and should be included in the model as such. However, such costs would be likely to be incremental or even one-time cost increases rather than continuous increases compounded on a percentage basis, so this formula would not be likely to be of much use for these kind of costs. Such costs would be more likely to be determined external to the model and then added in as a given parameter, just as are the contractor termination fees and administrative costs.

c. The Relevant Period

Should the users of the model decide that item-specific cost increases are applicable to the contract termination and that these costs should be applied based on a continuous escalation rather than incrementally, they will need to determine the applicable holding period for which to apply them. In this case, the period for which increased termination costs due to inflation are relevant begins when the buy from which the quantity $Q_r$ was terminated is or would have been received (depending on whether it was a partial or a complete cancellation), because that is when $Q_r$ would have been paid for. It ends when the next buy is received because that is when it will be paid for, given that the contract was actually terminated. This period is the period during which the terminated quantity $Q_r$ will increase in cost due to inflation, and it is the same time period during which savings due to reduced storage and obsolescence costs are relevant.
The analysis in this section therefore parallels the analyses in Sections II.B.2.c. and II.B.3.c. Again referring to Figures 3 and 4, it can be shown that the time between $Buy_1$ and $Buy_2$ is $ALT + PDLT + T_{AI}$. This formula represents the correct holding period for any inflationary costs and is correctly represented in the NAVSUP PCTM.
III. ADDITIONS TO THE PROPOSED MODEL

A. THE CURRENT MODEL

The Naval Supply Systems Command (NAVSUP) Procurement Contract Termination Model (PCTM) currently weighs the cost to continue the contract, measured as the investment cost in the additional inventory plus the storage cost of the additional inventory plus the obsolescence cost of the additional inventory, and compares it to the cost to terminate the contract, measured as the cost of contractor termination fees plus the administrative cost to reorder the material later plus the inflation cost to reorder the material later. However, other costs could potentially have a significant impact on the model's decision. The two which, in the author's opinion, are the most important are addressed in the following sections.

B. COST TO REPAIR VERSUS COST TO PROCURE

1. Background

The PCTM may be considering terminating a contract for repairable items which have "F" condition 3 items in inventory awaiting repair. In such cases, the remaining cost to complete the contract may be less than the cost to repair the "F" condition assets. In the author's opinion, the PCTM should consider the cost of repairing such assets as a cost to terminate the contract, since it is clearly cost-effective to pay the remaining cost of the contract to obtain these "A" condition assets instead of paying more to repair a comparable number of "F" condition assets. However, remaining costs are the only costs which are appropriate to consider paying, since costs which have been or must already be paid are sunk and are not relevant to the decision at hand.

2. The Cost

For the purpose of the PCTM, the remaining costs have already been identified by the model as the cost to continue, since the cost to continue specifies all future costs which will be incurred should we continue the contract. The cost to continue is thus directly comparable to the cost to repair. The appropriate quantity to use to compute the cost to repair for the comparison with the cost to continue is $Q_r \times C_R$, where $C_R$ is the unit cost to repair the item and $Q_r$ is, as before, the termination quantity, unless $Q_r$

3 "F" condition refers to items that are not ready for issue (NRFI) because they require repair. "A" condition refers to items which are ready for issue (RFI).
is greater than the number of repair regenerations we expect to need during the lifetime of the item. If $Q_r$ is the expected number of regenerations during the lifetime of the item, then the appropriate formula for the cost to repair is as follows:

$$C_R \times \min(Q_T, Q_R).$$

If this cost is greater than the cost to continue the contract, it should be continued regardless of the comparison with the cost to terminate.

C. GOVERNMENT-FURNISHED MATERIAL AS AN ALTERNATIVE TO TERMINATION

1. Background

Department of Defense Inspector General (DODIG) auditors noted in their November 21, 1989 audit of contract terminations that

... the inventory control points could have arranged for excess on-order assets that were not terminated and that applied to higher assemblies in production to be used as Government-furnished material on [other] production contracts for the higher assemblies. The inventory control points did not take advantage of the opportunities due to the lack of policies and procedures requiring excess on-order assets to be used as Government-furnished material. As a result, we projected that the three inventory control points unnecessarily brought $156.9 million of excess on-order assets into inventory. [Ref. 2: p. 31]

There are two ways to take advantage of on-order assets in long supply as Government-furnished material (GFM) [Ref. 2: p. 32]. First, all excess on-order assets already completed which could be used as GFM under other contracts could be transferred as such to those contracts, thus reducing the termination cost of the contract under consideration for termination. Second, all work in-process on terminated contracts could be transferred as such to other contracts, thus reducing the cost of those contracts by reducing a portion of their work in-process.

2. The Cost

Both transferring excess on-order assets and transferring work in-process would result in lower costs on the contract under consideration for termination in two ways. First, termination costs would be reduced by the total (contract) price of these assets.
Second, if the contract is continued, inventory holding, storage, and obsolescence costs would be avoided on these assets just as it would if the assets were terminated. The first cost saving can be incorporated into the NAVSUP PCTM formula simply by either subtracting it from the cost to terminate the contract (or equivalently by adding it to the cost to continue.) This assumes that the material is available for transfer to the new contract immediately; otherwise, its value would have to be discounted appropriately. Subtracting the value of these assets from the cost to terminate the contract reflects the fact that using the assets to reduce the cost of another contract in effect reduces the cost of terminating the contract under consideration.

However, while the first cost saving applies as a reduction in the cost to terminate the contract, the second cost saving applies as a reduction in the cost to continue the contract. This is so because if it is known that at least some of the termination quantity \( Q_T \), say \( Q_o \), where \( Q_o \) = the dollar amount of material to be transferred, can be used as GFM on some other contract, then it is also known that no inventory-related costs, such as inventory holding, storage and obsolescence costs, will have to be paid on the quantity \( Q_o \) (since it will never become part of the inventory.) Therefore, the second cost is best incorporated into the PCTM formula as part of the cost to continue by subtraction from both the dollar value of material returned to the Navy after termination \( R \) and to the dollar value of the termination quantity: 

\[
\]

which can be compared to

\[
\text{Cost}_{\text{terminate}} = CTF+ADM-Q_G \times P.
\]

While the above discussion covers the transfer of finished product from a contract under consideration for termination to another ongoing government contract, a similar analysis can be done for the case where work in-process is transferred. The value of finished product shown as \( Q_o \) in the above formula can be replaced with the value of work in-process and the same comparison made.

Determining whether material can be transferred from one contract to another to reduce costs can be difficult. For this reason, implementing this DODIG finding will be difficult. However, if NAVSUP decision makers decide to implement it, the above
formulas used in the PCTM decision logic should provide the most cost-effective decision.
IV. SENSITIVITY OF THE MODEL TO PARAMETER ADJUSTMENTS

Once the the Naval Supply Systems Command (NAVSUP) Procurement Contract Termination Model (PCTM) “correctly” models the problem of whether or not to terminate a contract, obtaining a good decision from it still depends on the accuracy of the input parameters. The best-formulated model will not render an economical decision if the interest, storage, obsolescence, and inflation (should its users decide it is correct to include this cost) rates and the contractor termination fees and administrative costs on which the model depends are not accurately represented.

The ultimate goal of the users of the model should be to accurately specify all those parameters for which it is economical. For example, if a parameter can be found to a tolerance of 0.1% at a cost of just a few thousand dollars and such accuracy can enable the model to correctly make decisions saving millions of dollars, then clearly it is worth the effort to specify such a parameter to a tolerance of 0.1%. If, on the other hand, the situation is reversed and finding a parameter to within a tolerance of 0.1% costs millions and offers a savings of only thousands, then just as clearly it is not worth the effort. For such a parameter a tolerance of 5% might save almost as much money and cost thousands instead of millions. In this case, the model’s users should be satisfied with specifying this parameter to a tolerance of 5%. Four factors affect how much a parameter will impact the model’s decision: parameter determination, the relative magnitude of the parameter, the relative magnitude of the quantity impacted, and the relative lengths of the holding periods.

A. PARAMETER DETERMINATION

The more the cost parameter’s potential variability, the more the potential impact on the model’s decision. For example, obsolescence is probably the most potentially variable parameter because of the uncertainty of the rate of technological advance. Whether an item becomes obsolete within five years or 30 years depends on how fast its technology advances and also on the general classification of the item (for example, electronic components tend to become obsolete much faster than electric components which in turn tend to become obsolete faster than mechanical components.) In addition to obsolescence rates being difficult to specify exactly, they can be expected to be highly variable across different classes of items. By contrast, the interest rate, while it can vary over periods of years, will usually be relatively stable over shorter periods of time and
across item classifications. The interest rate can further usually be estimated with reasonable certainty using a benchmark like the treasury bill rate.

B. RELATIVE MAGNITUDE OF THE COST RATE PARAMETER

The greater the cost parameter as a fraction of the item's value, the greater will be its impact on the model's decision. For example, if obsolescence really costs $0.1Q_rP$ and storage really costs $0.01Q_rP$, then errors in the obsolescence rate will have ten times the impact on the model's decisions that errors in the storage rate will have.

C. RELATIVE MAGNITUDE OF THE QUANTITY IMPACTED

The greater the dollar value of the quantity associated with the parameter (i.e., the quantity impacted), the greater will be the impact of variations in its associated cost parameter on the model's decisions. For example, if the dollar value of the termination quantity $Q_rP$ is greater than the dollar value of the material returned, $R$, assuming the true values of $I$ and $O$ are both equal to their default values of 0.1, then the obsolescence of the item will have a greater impact on the model's decision than will the opportunity (interest) cost.

D. THE RELATIVE LENGTH OF THE HOLDING PERIOD

The greater the length of the holding period associated with a parameter relative to the lengths of the holding periods associated with other parameters, the greater will be its impact on the model's decision. For example, if opportunity cost holding periods are typically one year and obsolescence cost holding periods are typically two years, then all other factors being equal, the obsolescence rate will have twice as much impact on the model's decision as will the obsolescence rate.

E. THE RATE OF CHANGE OVER THE RANGE OF PARAMETER VALUES

The model was tested for sensitivity over a range of parameter values judged to be reasonable based on the experience of the author and on the "latest information personnel of the Ships Parts Control Center (SPCC) Operations Research Department (Code 0412) have on the model" [Ref: 8]. Table 2 presents the model's default values [Ref: 8] (the "base case"), except that, for the purpose of this analysis, the inflation rate has been set to 0. Based on the discussion in Section II.C.3.. the issue of whether to include an "inflation factor" has been decided in favor of no inflation factor; therefore no sensitivity analysis will be performed for this parameter.

In each case, the PCTM decision equation has been reduced to an equation in two variables using certain simplifying assumptions. First, all holding periods have been as-
sumed to be equal. While it is clear that the holding periods are not, in general, equal, since all holding periods have as their primary component $T_{RL}$, they should all be similar in length, making this a tenable assumption for analyzing the sensitivity of the model to parameter changes. Second, the termination amount has been assumed to be equal to one-half the contract price ($Q_T = \frac{1}{2} CP$). In other words, half the material under contract is to be cancelled. And third, the contract is assumed to be half-finished, so that $CTF = \sqrt{\frac{1}{2}} \times CP \approx 0.7071 CP$ implies that $R = \left(1 - \sqrt{\frac{1}{2}}\right) \times CP \approx 0.2929 CP$. Finally, administrative costs, a constant, will be assumed to be very small in comparison to the contract price, an assumption that is normally a very good one. These assumptions enable the model's decisions to be expressed as a function of both the parameter of interest and the approximate holding period. The analysis in the following three sections will then focus on the three parameters relevant to determining the cost to continue for the ranges of $Q_T P$ and $CTF$ specified above. In the final section, sensitivity analysis will focus on variations in the two important parameters, $CTF$ and $ADM$, of the cost to terminate.

### Table 2. PCTM “BASE CASE” PARAMETER VALUES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate</td>
<td>0.10</td>
</tr>
<tr>
<td>Storage Cost Rate</td>
<td>0.01</td>
</tr>
<tr>
<td>Obsolescence Rate</td>
<td>0.10</td>
</tr>
<tr>
<td>Contractor Termination Fee</td>
<td>No default value</td>
</tr>
<tr>
<td>Administrative Cost</td>
<td>$2,000</td>
</tr>
<tr>
<td>Inflation Rate</td>
<td>0.00</td>
</tr>
</tbody>
</table>

1. **Interest Rate and Holding Period**

In accordance with the assumptions stated above, the PCTM decision equation was reduced to a single equation with $I$ and $HP$ as the only variables as follows:

$$
Cost_{continue} = (I)(.29CP)(HP) + (.01)(.5CP)(HP) + (.10)(.5CP)(HP)
$$

$$
= (.29I + .055)CP(HP),
$$
which is compared to the cost to terminate:

\[
\text{Cost}_{\text{terminate}} = (.71)(CP) + \text{ADM}.
\]

For the model to decide to continue the contract, for example, we must have

\[
\text{Cost}_{\text{continue}} \leq \text{Cost}_{\text{terminate}} \quad \text{or}
\]

\[
(.29I + .055)(HP) \leq (.71)(CP) + \text{ADM}.
\]

This is equivalent to

\[
(.29I + .055)(HP) - (.71) \leq \frac{\text{ADM}}{CP}.
\]

Now suppose that the final assumption above is true; that is, administrative costs are very small compared to the contract price so that \( \frac{\text{ADM}}{CP} \approx 0 \). Then this last inequality reduces further to the following:

\[
(.29I + .055)(HP) - (.71) \leq 0.
\]

At the point where the model is indifferent between continuation and termination, the following equality must therefore hold:

\[
(.29I + .055)(HP) - (.71) = 0.
\]

Using this equation, \( I \) can be expressed as a function of the holding period as follows:

\[
I = \frac{.45}{HP} - .19.
\]

This indifference equality will be used below to compare the relative sensitivity of the model's decision with respect to changes in the continuation parameters.

2. Storage Cost Rate and Holding Period

As was done with interest rate in the above section, the PCTM decision equation was reduced to a single equation with \( S \) and \( HP \) as the only variables as follows:
\[ \text{Cost}_{\text{continue}} = (.1)(.29 \text{CP})(\text{HP}) + (.5 \text{CP})(\text{HP}) + (.1)(.5 \text{CP})(\text{HP}) \]

\[ = (.5S + .079)(\text{CP})(\text{HP}), \]

which is compared to the cost to terminate:

\[ \text{Cost}_{\text{terminate}} = (.71)(\text{CP}) + \text{ADM}. \]

For the model to decide to continue the contract, for example, we must have

\[ \text{Cost}_{\text{continue}} \leq \text{Cost}_{\text{terminate}}, \]

or

\[ (.5S + .079)(\text{CP})(\text{HP}) \leq (.71)(\text{CP}) + \text{ADM}. \]

This is equivalent to

\[ (.5S + .079)(\text{HP}) - (.71) \leq \frac{\text{ADM}}{\text{CP}}. \]

Now suppose, as in the section on interest rate above, that administrative costs are very small compared to the contract price so that \( \frac{\text{ADM}}{\text{CP}} \approx 0 \). Then this last inequality reduces further to the following:

\[ (.5S + .079)(\text{HP}) - (.71) \leq 0. \]

At the point where the model is indifferent between continuation and termination, the following equality must therefore hold:

\[ (.5S + .079)(\text{HP}) - (.71) = 0. \]

Using this equation, \( S \) can be expressed as a function of the holding period as follows:

\[ S = \frac{1.42}{\text{HP}} - 16. \]
This indifference equality will be used below to compare the relative sensitivity of the model's decision with respect to changes in the continuation parameters.

3. Obsolescence Rate and Holding Period

As in the two sections above, the PCTM decision equation was reduced to a single equation with $O$ and $HP$ as the only variables as follows:

\[
Cost_{\text{continue}} = (0.1)(0.29CP)(HP) + (0.01)(0.5CP)(HP) + (0)(0.5CP)(HP)
\]

\[
= (0.50 + 0.034)(CP)(HP),
\]

which is compared to the cost to terminate:

\[
Cost_{\text{terminate}} = (0.71)(CP) + ADM.
\]

For the model to decide to continue the contract, for example, we must have

\[
Cost_{\text{continue}} \leq Cost_{\text{terminate}},
\]

or

\[
(0.50 + 0.034)(CP)(HP) \leq (0.71)(CP) + ADM.
\]

This is equivalent to

\[
(0.50 + 0.034)(HP) - (0.71) \leq \frac{ADM}{CP}.
\]

Now suppose, as in the sections above, that administrative costs are very small compared to the contract price so that $\frac{ADM}{CP} \approx 0$. Then this last inequality reduces further to the following:

\[
(0.50 + 0.034)(HP) - (0.71) \leq 0.
\]

At the point where the model is indifferent between continuation and termination, the following equality must therefore hold:

\[
(0.50 + 0.034)(HP) - (0.71) = 0.
\]
Using this equation, O can be expressed as a function of the holding period as follows:

$$ O = \frac{1.42}{HP} - .07. $$

This indifference equality will be used below to compare the relative sensitivity of the model's decision with respect to changes in the continuation parameters.

4. Comparison of the Sensitivity of the Model Over a Range of HP Values

Figure 5, which shows the indifference curves for I, S, and O plotted together, illustrates the importance of the rate of change of the PCTM's decision over a range of the HP values. In this figure, the areas beneath and to the left of each curve represent decisions to continue the contract, and areas above and to the right of the curve represent decisions to terminate the contract. In Figure 5, the curve for I assumes S and O are held constant at .01 and .10, respectively, the curve for S assumes I and O are both held constant at .10, and the curve for O assumes I and S are held constant at .10 and .01, respectively.

Figure 5 clearly shows that for small values of HP and I, changes in I have little impact relative to S and O on the model's decision. In fact, for values of HP of less than 5.6 years, I would have to exceed .25 (since, from above, \( I = \frac{2.45}{HP} -.19 \) means that \( HP = \frac{2.45}{I+.19} \) and therefore \( HP_{.25} = \frac{2.45}{.25+.19} \approx 5.6 \) to change the model's decision from continue to terminate. This, in effect, means the model's decision is fixed at "continue" for holding periods of less than 5.6 years. By comparison, similar analysis for S and O shows that for values of HP of less than 3.5 and 4.4 years, respectively, S and O would have to exceed .25 to change the model's decision from continue to terminate.

At \( HP = 8.6 \) (since at that point, \( I = O \) means \( \frac{2.45}{HP} -.19 = \frac{1.42}{HP} -.07 \) and therefore \( HP = \frac{2.45-1.42}{.19-.07} \approx 8.6 \), the curve drops below the O curve. Below this intersection, for a given HP, whatever change in O is required to change the model's decision from continue to terminate, a smaller change in I is required to have the same effect on the decision. Ultimately, for holding periods of greater than 12.9 years, the interest rate would have to be negative to cause the model to make a decision to continue, which in effect means the model's decision is fixed at terminate for holding periods of greater than 12.9 years. For S and O, for holding periods of 8.9 and 20.3 years, respectively, the rates would have to be negative to cause the model to decide to continue the contract. In between the two extremes where the model will always decide to continue and where it will always decide to terminate, the faster the rate of change of the parameter with re-
spect to HP, the less likely changes in that parameter are to affect the model’s decision, because the indifference equation itself changes more quickly.

In view of the above discussion, it seems that because I changes faster with respect to HP, the model’s decision will be more likely to be fixed for extreme values of I than it will for these extreme values of S and O. It also seems that for values of HP (5.6 to 12.9 years) that fall within the reasonable range of I (0 to .25), a given change in I is more likely to change the model’s decision from continue to terminate (or vice versa) as the holding period increases. The smaller the difference in years between one fixed decision area to the other (for example, 8.9–3.5 = 5.4 years for S, the smallest difference), the less sensitive the model’s decision will be to changes in that parameter. Viewed graphically, this is the same as saying that the steeper the curve, the less changes in I, S, or O, being vertical changes, will be likely to change the decision from the continue area to the terminate area, and once the curve is in a fixed decision area, changes in the parameter will no longer have any impact at all on the model’s decision. Therefore, the faster the rate of change parameters have with respect to HP, the less impact they will have on the model’s decision.

For this particular example, an analysis of the rate of change of the parameters with respect to holding periods shows \( \frac{dI}{dHP} \approx -2.45 \) and \( \frac{dS}{dHP} = \frac{dO}{dHP} \approx -1.42 \). In general, the rate of change of the model’s decision with respect to the parameters can be derived, by considering the point at which we are indifferent between continuing and terminating the contract, as follows:

\[
(I)(R)(HP)+(S+O)(QTP)(HP)-CTF = 0
\]

which implies (after dividing both sides of the equation by CP and replacing R by the difference \( CP-CTF \))

\[
I = \frac{CTF \quad CP}{HP} \quad -(S+O) \left( \frac{QTP \quad CP}{CP} \right) ;
\]

which, in turn, means
Figure 5. Indifference curves for I, S, and O. $Q_r = 0.5CP$, $CTF = 0.71CP$. 

INDIFFERENCE CURVES

$Q=0.5CP$, $CTF=0.71CP$
Using a similar analysis for $S$ and $O$ gives the following formulas for the rates of change with respect to HP:

$$\frac{\partial S}{\partial H_P} = \frac{\partial O}{\partial H_P} = \frac{CTF}{CP} = -\frac{CTF}{Q_T P H_P^2}.$$  

So in general, I will have a faster rate of change with respect to the holding periods whenever

$$\frac{CTF}{(CP-CTF)H_P^2} > \frac{CTF}{Q_T P H_P^2}.$$  

or

$$Q_T P + CTF > CP.$$  

Therefore, if the total of the value of the termination quantity plus the contractor termination fees is greater than the original contract price, changes in I will be less likely to affect the decision of the model than changes in S and O, simply because, as discussed above, the model's decision with respect to changes in I is more likely to be fixed at the extremes and its indifference curve steeper in between the extremes.

5. Sensitivity of the Model's Decisions to Changes in Contractor Termination Fees Over a Range of HP Values

To assess the sensitivity of the model's decision to changes in CTF, we will retain the assumption that $\frac{ADM}{CP} \approx 0$. At the point of indifference between termination and continuation, we have the following relation:
\[ CTF = (I)(CP-CTF)(HP)+(S+O)(Q_T \times P)(HP) \]

which yields the following expression for contractor termination fee as a fraction of contract price (the actual model parameter):

\[
\frac{CTF}{CP} = \frac{\left[ I+(S+O)\left( \frac{Q_T}{CP} \right) \right](HP)}{1+(I)(HP)}.
\]

Therefore, the rate of change of \( \frac{CTF}{CP} \) with respect to holding periods can be expressed as follows:

\[
\frac{\partial \frac{CTF}{CP}}{\partial HP} = -\frac{I+(S+O)\left( \frac{Q_T}{CP} \right)}{[1+(HP)(I)]^2}.
\]

Comparing this expression with the expressions derived in the previous section for the rates of change of \( I \), \( S \), and \( O \) with respect to \( HP \), it is easy to see that the rate of change of \( CTF \) is higher for holding periods that are longer than a certain "breakpoint" holding period and also that the rate of change of \( CTF \) is smaller for holding periods that are shorter than that breakpoint holding period. To see this, note that the numerator of all four expressions is some fraction, while the denominator of the expressions for the rates of change of \( I \), \( S \), and \( O \) is the holding period squared. By contrast, the denominator of the expression for the rate of change of the fraction \( \frac{CTF}{CP} \) is the constant 1, plus the fraction 1 of the holding period, squared. Since the denominator for the rate of change of \( \frac{CTF}{CP} \) is therefore clearly much smaller than the denominators of the other expressions (for holding periods greater than the breakpoint), and since the numerators of all four expressions are approximately the same, the rate of change of the fraction \( \frac{CTF}{CP} \) must be greater for holding periods greater than the breakpoint. To estimate this "breakpoint" holding period, we can compare the expression for the rate of change of \( CTF \) with respect to \( HP \) with the expression for the rate of change of \( I \), the parameter from the cost to continue side of the equation with the highest rate of change, with respect to \( HP \). Setting these two equations equal gives the following result:
which gives, using the assumptions above that $CTF = 0.71CP$, $Q_TP = 0.5CP$, and I, S, and O equal their default values, the following equation:

$$- \frac{2.45}{HP^2} = - \frac{0.155}{(1+1HP)^2},$$

which can be rewritten as the following quadratic equation:

$$0.145hP^2 - 4.9hP - 2.45 = 0.$$

Solving using the quadratic formula and discarding the negative root, we are left with the solution for the "breakpoint" holding period $HP \approx 6.1$ years. Therefore, at least for the example case used in previous sections where $CTF = 0.71CP$, $Q_TP = 0.5CP$, and I, S, and O equal 0.10, 0.01, and 0.10, respectively, we can say that for holding periods of less than 6.1 years, the rate of change of CTF with respect to HP will be less (and probably much less for relatively small values of HP) than the comparable rate of change of I, S and O. We can also say, based on the discussion in section IV.E.4., that for holding periods of less than 6.1 years, that changes in the CTF will have more impact on the model's decisions than any of the parameters on the cost to continue side of the equation.

6. Sensitivity of the Model to Changes in Administrative Costs Over a Range of HP Values

We now drop the assumption that $\frac{ADM}{CP} \approx 0$ in order to assess the sensitivity of the model's decisions to changes in administrative costs. The decision indifference equation with administrative costs included is as follows (with $R = CP - CTF$):

$$(HP)(I)(CP-CTF)+(S+O)(Q_TP)(HP)-CTF = ADM$$

which can be reduced to the following expression for $\frac{ADM}{CP}$ as a linear function of $HP$:
\[
\frac{\Delta M}{CP} = (R)(I)+(S+O) \left( \frac{QTP}{CP} \right) (R) - (1+(R)(I))(\frac{CTF}{CP}).
\]

From this expression we can derive the rate of change of ADM with respect to HP as follows:

\[
\frac{\partial ADM}{\partial HP} = I+(S+O) \left( \frac{QTP}{CP} \right) - (I) \left( \frac{CTF}{CP} \right) = (S+O) \left( \frac{QTP}{CP} \right) + (I) \left( 1- \frac{CTF}{CP} \right) > 0,
\]

since \( \frac{CTF}{CP} \) is presumably a fraction less than 1 and all other terms are known to be positive. This seems logical, since as the length of the holding periods goes up, administrative costs will be allowed to be progressively larger without changing the decision from, say, terminate to continue. To determine how fast ADM changes with respect to HP as compared with the other parameters, using I as an example, we must compare the (absolute) rates of change of the two parameters with respect to HP as follows:

\[
\frac{\partial I}{\partial HP} = \frac{CTF}{HP \cdot (CP-CTF)} = \frac{\partial ADM}{\partial HP} = (S+O) \left( \frac{QTP}{CP} \right) + (I) \left( \frac{CP-CTF}{CP} \right).
\]

This equation is inconclusive without assuming values for some of the parameters. Letting, as in examples above, \( CTF = 0.71CP \), \( QTP = 0.5CP \), and I, S, and O equal 0.10, 0.01, and 0.10, respectively, reduces the above equation to the following:

\[
\frac{2.45}{HP^2} = .345,
\]

which implies that at the value of \( HP = 2.66 \) the slopes are equal. Therefore, for this example, for \( HP < 2.66 \), I has a faster rate of change and therefore ADM is the more influential parameter.

7. Comparison With the Sensitivity of the Author's Recommended Model

An analysis of the sensitivity of the author's recommended PCTM to changes in the parameters is complicated by the nonlinearity of the compound interest term and the fact that the author recommends no holding periods (which were assumed to be
equivalent in the above discussion to simplify the analysis.) For example, the expression for I in terms of the holding period is derived as follows, using for the sake of simplicity the continuous interest compounding formula to approximate the daily compounding formula recommended in Chapter II. At the point of indifference, cost to continue = cost to terminate:

\[
\left[ e^{(HP)(I)} - 1 \right] R + (S+O) \left( \frac{Q^2}{D} \right)(P) = CTF
\]

which reduces to (since \( R = CP - CTF \))

\[
e^{(HP)(I)} = 1 + \frac{CTF - (S+O) \left( \frac{Q^2}{D} \right)(P)}{CP - CTF}
\]

Expressing I as a function of HP gives

\[
I = \frac{\ln \left[ 1 + \frac{CTF - (S+O) \left( \frac{Q^2}{D} \right)(P)}{CP - CTF} \right]}{HP}
\]

which in turn gives

\[
\frac{\partial I}{\partial HP} = -\frac{\ln \left[ 1 + \frac{CTF - (S+O) \left( \frac{Q^2}{D} \right)(P)}{CP - CTF} \right]}{HP^2}
\]

Expressing S+O as a function of HP gives

\[
S+O = \frac{CP - R e^{(HP)(I)}}{\frac{Q^2}{D} \times P}
\]
which in turn gives

\[
\frac{\delta S}{\delta H_P} = \frac{\delta O}{\delta H_P} = -\frac{RIe^{(HP)(I)}}{Q_T^2 \times D \times P}.
\]

Clearly with expressions for I, S, and O as complex as these, setting them equal and solving for individual parameters will not yield any simple expressions that will be useful in comparing the relative sensitivities of I, S, and O except to say that the indifference curves of S and O will be the same. However, it is possible to compare the decisions of the two models empirically using, for example, the above assumptions about the values of \(Q_T\) and \(CTF\).

In general, the author’s model constrains continuation decisions more tightly and will recommend more termination decisions than the current NAVSUP PCTM. This can be seen specifically for the above example, where \(CTF=0.71CP\), \(Q_TP=0.5CP\), and I, S, and O equal 0.10, 0.01, and 0.10, respectively, using certain simplifying assumptions to place a lower bound on the cost to continue and an upper bound on the cost to terminate.

a. The Increase in the Cost to Continue the Contract

The NAVSUP PCTM formula for the cost to continue is (using the above assumptions)

\[
(1)(I)(HP) + (.01)(Q_TP)(HP) + (.1)(Q_TP)(HP) = (0.029CP)(HP) + (.055CP)(HP),
\]

while the author’s recommended formula for the cost to continue is

\[
\left[\left(1+\frac{1}{365}\right)^{HP}(365)-1\right] \cdot 29CP + (0.01)(.5CP)\left(\frac{Q_T}{D}\right) + (0.1)(.5CP)\left(\frac{Q_T}{D}\right).
\]

In comparing the two formulas, it is clear that the first term in the author’s recommended formula, as a compounded term, will be larger than the first term in the
NAVSUP formula. When we are comparing last term of the NAVSUP formula to the last term of the author's formula, we are comparing \((S+O)(Q_TP)(HP)\) to \((S+O)(Q_TP)(\frac{Q_T}{D})\), which reduces to comparing \(HP = T_{RL}\) to \(\frac{Q_T}{D}\).

Figure 6 below illustrates the limiting case for the holding period. As was noted near the beginning of this chapter, the main component of the holding period is the time to reach reorder level, \(T_{RL}\). However, \(T_{RL}\) falls somewhere in the period between now and the expected time the post-termination quantity \(Q - Q_T\) reaches the reorder level, which is the period of time of length \(\frac{Q - Q_T}{D}\). This implies that \(0 \leq T_{RL} \leq \frac{Q - Q_T}{D}\).

Now, the contract price \(CP\) can be plausibly argued to be equal to the dollar value of the economic order quantity, \(Q \times P\), since the economic order quantity is the most likely quantity to have been ordered. This means that, since \(Q_TP = \frac{1}{2} CP\), \(Q_T = \frac{1}{2} Q\) and therefore

\[
0 \leq T_{RL} \leq \frac{Q - Q_T}{D} = \frac{Q - \frac{1}{2} Q}{D} = \frac{\frac{1}{2} Q}{D} = \frac{Q_T}{D}.
\]

Therefore, \(\frac{Q_T}{D}\) is an upper limit on \(T_{RL}\), which is the major component of both \(HP_s\) and \(HP_o\). So we can conclude that the second term of the author's formula is at least as big as in the NAVSUP formula (and probably bigger), so that, in general, the cost to continue calculated by the author's formula will be bigger than that calculated by the NAVSUP formula.
Figure 6. Limiting case for the holding period.

This result can be extended to all cases where \( Q_r > \frac{1}{2} Q \) by noting that the upper bound on \( T_{rl} \) was \( \frac{Q - Q_r}{D} \). Therefore its values, for all \( Q_r > \frac{1}{2} Q \), will be something less than \( \frac{Q_r}{D} \).

Unfortunately, the analysis is inconclusive when \( Q_r < \frac{1}{2} Q \) since then the upper bound on \( T_{rl} \) will be something greater than \( \frac{Q_r}{D} \).

b. The Decrease in the Cost to Terminate the Contract

On the cost to terminate side of the equation, both models use \( C_T \), however determined, so that the first terms are identical. However, the author recommends that instead of using the absolute interest cost, the model use a discounted interest cost, particularly for those administrative costs related to reorder, which occur at some time in the future. These costs, as they are discounted, will therefore make the cost to terminate in the author's model smaller.

c. The Result of the Empirical Comparison

As the author's model has a greater cost to continue, at least for cases where \( Q_r > \frac{1}{2} Q \), and a smaller cost to terminate, the author's model will result in termination for more contracts than will the NAVSUP PCTM.

The above analysis did not take into account any "inflationary" costs. Should such costs be taken into account in the NAVSUP PCTM, they will probably cause its cost to terminate to be greater than the cost to terminate of the author's model.
However, as discussed in Section II.C.3., such costs are not real costs and should not affect the comparison between the models' decisions in any way.

d. The Graphical Comparison Between the Models

Figure 7 below illustrates graphically a comparison between the I versus HP indifference curves for the NAVSUP PCTM and the author's model. The curve labelled "NAVSUP PCTM" is the curve labelled "I" in Figure 5 repeated, and the curve labelled "Author's PCTM" used the formula derived as follows. Repeating the expression for I from the beginning of this section in terms of HP and dividing through by CP to convert the key parameters to fractions of CP, we have the following expression for I:

\[
I = \ln \left[ 1 + \frac{\frac{CTF}{CP} - (S + O) \left( \frac{Q_r P}{CP} \right) \left( \frac{Q_T}{D} \right)}{1 - \frac{CTF}{CP}} \right] \frac{1}{HP}
\]

After substituting the parameter values \( CTF = .71CP \), \( Q_r P = .5CP \), I, S, and O equal .10, .01, and .10 respectively, and, in accordance with the discussion above, \( \frac{Q_r}{D} = 12 \), so that \( 0 \leq HP \leq \frac{Q_r}{D} \), we get the following expression for I as a function of HP:

\[
I = \ln \left[ 1 + \frac{.71 - (.11)(.5)(12)}{.29} \right] \frac{1}{HP}
\]

which, in turn, reduces to the following:

\[
I = \frac{16}{HP}
\]

As in Figure 5, the areas above and to the left of the curve represent decisions to continue, and the areas above and to the right represent decisions to terminate. From Figure 7 it can be seen that, as discussed above, while \( 0 \leq HP \leq \frac{Q_r}{D} \), the author's model will terminate more often, but as HP approaches \( \frac{Q_r}{D} \), the decisions of the two models converge.

A similar analysis can be performed to show that for a holding period less than \( \frac{Q_r}{D} \), the indifference curves for S and O in the author's model will always lie below
Figure 7. Indifference curves for $I_1 Q_T = 0.5CP, CTF = 0.71CP, \frac{Q_T}{D} = 12$. 
the corresponding indifference curves from the NAVSUP PCTM. The author's model, using the values $CTF = 0.71CP$, $Q_TP = 0.5CP$, and $I$ and $O$ equal to 0.10, yields the following expression for the point at which we are indifferent between continuation and termination as a function of $S$ and $\frac{Q_T}{D}$:

$$0.029 I + 0.5S \left( \frac{Q_T}{D} \right) + \frac{1}{2} \left( \frac{Q_T}{D} \right) - 0.71 = 0.$$ 

Solving for $S$ gives

$$S = \frac{0.71 - 0.029HP - 0.05Q_T}{0.5 \frac{Q_T}{D}}.$$ 

This can be compared to the expression for $S$ as a function of HP derived above in section 2 (which is repeated in slightly altered form for easy comparison with the above expression):

$$S = \frac{0.71 - 0.029HP - 0.05HP}{0.05HP}.$$ 

Thus, when $HP < \frac{Q_T}{D}$ and $\frac{Q_T}{D}$ is constant, the indifference expression for $S$ in the author's model is linear in HP with a negative slope and always lies below the curve derived from the expression for $S$ using the NAVSUP PCTM. A similar analysis for $O$ shows the same result.

Many comparisons are possible varying the key parameters over a wide range of values. However, the basic results will not change; that is, the models will continue and terminate contracts at different rates depending on the length of the holding periods.

F. CONCLUSION

Based on considerations presented in the preceding five sections, the author believes parameter research should be prioritized as follows, regardless of which model is used:
1. Contractor Termination Fee formula $\frac{\sqrt{PDLT_e}}{PDLT}$;
2. Administrative Costs ADM;
3. Obsolescence rate O;
4. Storage rate S;
5. Interest rate I.

Parameters are recommended for research in the order they induce change in the model's decision, which, in accordance with the above discussion, is inversely proportional to the rate of change of their indifference curves with respect to holding periods.
V. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A. SUMMARY

This thesis has analyzed the current NAVSUP Procurement Contract Termination Model (PCTM) using the principles of economic analysis and inventory management. Five of the model's six terms were recommended for modification based on the analysis, while the sixth term, the cost to reprocure due to inflation, was recommended for deletion from the model based on the principles of economics. The cost to repair was formulated for comparison with the cost to continue, and continuation was recommended whenever the cost to repair was greater than the cost to continue. Government-furnished material was recommended for consideration as an alternative to termination, and a formula was derived to aid in that analysis. The model was tested for sensitivity to variations in key parameters, concluding with a prioritized list recommending future research. And finally, decision sensitivity of the current NAVSUP PCTM was compared to the sensitivity of the author's recommended model, where it was determined that the author's model decides to terminate a greater number of contracts under a wide range of common parameter values.

B. CONCLUSIONS

The NAVSUP PCTM is a good start in quantifying the costs associated with terminating procurement contracts for items in long supply. It takes a first step toward eliminating a problem that has been with the Navy for a long time. The model considers most of the relevant costs and excludes most that are not relevant. It also calculates the costs over the exact periods of time that are applicable for calculating such costs. The model in its present form would give ICP inventory managers and contracting officers
good guidance, much better than they currently have, as to which contracts are cost-effective to terminate.

However, the model in its current form needs a number of improvements. First, the time value of money should be taken into account correctly both using a compound interest formula for the investment cost to continue the contract and using a discount formula for the administrative cost to reorder in the future. These were discussed in sections II.B.1. and II.C.2., respectively. Second, the NAVSUP PCTM does not correctly take into consideration the full inventory cycle when computing both the storage cost and the obsolescence cost components of the cost to continue the contract, as discussed in sections II.B.2. and II.B.3. Third, it includes as a relevant cost the rate of inflation. As was demonstrated in Chapter II.B.2, the inflation rate is irrelevant for the already present-valued costs of termination. Fourth, as shown in Chapter III.B., it does not consider the future cost to repair an item as a savings from the cost of continuing an outstanding contract. Finally, as shown in Chapter III.C., the PCTM does not consider the possibility of using long-supply material as GFM on other contracts such as production contracts as an alternative to termination. Each of these improvements is suggested in an attempt to include any and all relevant costs and to exclude any and all irrelevant costs. Effecting these improvements should make the NAVSUP PCTM a much more accurate tool for use by ICP inventory managers and contracting officers and will make it capable of realizing the potential savings recognized by government auditors for many years.

Results of the sensitivity analysis of Chapter IV reveal that certain of the model's cost parameters are indeed more important and influential in its decision recommendations. Concentrating future research on the key parameters in the order of most influential to least influential to determine more precise values should also go a long way toward making the NAVSUP PCTM a more accurate tool.
C. RECOMMENDATIONS

In view of the findings of this thesis, as summarized above, it is clear that any and all relevant costs must be included in the model and any and all irrelevant costs must be excluded. Therefore, the following model is recommended for use in deciding contract terminations:

\[
\text{Cost}_{\text{continue}} = \left[ \left( 1 + \frac{I}{365} \right)^{(\text{HP}(365))} - 1 \right] \text{R} + \text{SP} \cdot \frac{Q_{\text{r}}^2}{D} + \text{OP} \cdot \frac{Q_{\text{r}}^2}{D},
\]

which can be compared to

\[
\text{Cost}_{\text{terminate}} = \text{CTF} + \frac{\text{ADM}}{1 + \left( \frac{I}{365} \right)^{(\text{TR}(365))}}.
\]

In addition, any savings from using material in long supply as GFM on other, production contracts must be included as savings from the cost to continue the contract. Therefore, when material is identified as being usable as government-furnished material on another contract, \( R, Q_{\text{r}}, \) and the cost to terminate the contract should be modified as shown in Section III.C.2. above to incorporate the additional costs into the formula.

Finally, for repairable items, the cost to repair, \( C_r \times \min[Q_{\text{r}}, Q_{\text{s}}] \) should be compared to the cost to continue the contract. If the cost to repair is greater than the cost to continue, the contract should be continued whatever the cost to terminate.

But making the PCTM an accurate tool also depends on determining more precise values for key cost parameters. It is therefore also recommended that research be conducted as soon as possible as detailed in the paragraph concluding Chapter IV.
LIST OF REFERENCES


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| 9.  | 1      | Commanding Officer  
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