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COMBINATION OF EVIDENCE IN C^3 SYSTEMS

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ABSTRACT

This paper has a threefold thrust:(1) a brief survey is presented of the development of approaches to modeling C^2/C^3 systems as given primarily in this forum-The MIT/ONR Workshop on C^3 Systems;(2) an outline of a theory of C^3 systems is developed which is compatible with previous efforts and which is rich enough for rigid, yet tractable, analysis;(3) as part of this theory, a procedure is exhibited for integrating subjective and objective/probabilistic/numerical information for C^3 system decisionmakers.

1. INTRODUCTION

The C^3 problem is a real-world problem and thus, analogous to theories in Chemistry, Physics, or Biology, a proposed theory for a C^3 system must be based on empirical, as well as sound, logical considerations. In addition, such a theory-following the usual pattern of change for scientific inquiries- will incorporate, overlap to some degree, or otherwise relate with, previously established models. Finally, the author's own biases and predilections will generally be reflected in the degree of detail granted to the various components of the overall model.

Compatible with the above philosophy, the goal of this paper is the development of a general C^3 theory which accounts for a systematic/comprehensive treatment of the combination of subjective information- such as linguistic-based descriptions- with the usual probabilistic or numerical type information. In conjunction with this effort, a literature search was conducted for previous work in this area. In addition to the premier collection of unclassified C^2/C^3 work- these Proceedings over the past eight years- other unclassified sources were also considered, including IEEE publications, Operations Research journals, Psychology publications, and separately published papers and books, among others. A brief survey of that portion of the literature relevant to the task here is presented in the next section. In section 3, general models of warfare and C^3 systems are proposed in the form of networks whose nodes represent decision makers/followers. These networks are also assumed to be time-varying. Section 4 is an abridged analysis of intranodal behavior, utilizing both probabilistic and possibilistic processes, analogous to the previous established PACT (Possibilistic Approach to Correlation and Tracking) program in Ocean Surveillance [55]..

2. BRIEF SURVEY OF RELEVANT C^3 WORK

A now extensive C^3 and related discipline literature exists solely within the first seven annual Proceedings of this journal (283 articles). Perhaps because of the great complexity of the overall C^3 problem, relatively few papers have been written establishing quantitative models of generic C^3 systems. Of course, this does not detract from the progress made for various aspects of the problem proper and for related issues. Foremost among the latter is Surveillance, and in particular, multi-target tracking and data association. To a lesser degree, Data Base Management and Communications within C^3 systems have also been extensively treated quantitatively. Similarly, limited portions of the C^3 problem proper have been thoroughly analyzed- under appropriate simplifying conditions- including command decision theory, viewed as a possible multiple player statistical decision game involving, typically,threat situations and system effectiveness reflected in the loss or objective functions, as e.g. in [1], or considering players' mental images of one another together with limited knowledge of rules of play , as in [2]. In a similar vein, distributed or decentralized decision theory appears to be a valuable tool for analyzing C^3 systems which may be spread out geographically or otherwise have loose communications structures. (See, e.g., Tenney [3]-[4'] and Sandell [5] for basic results in this direction.) Complexity of distributed decision problems relative to C^3 was presented in [6] in the form of NP-completeness. Other general results, including asymptotic forms, may be found in Tsitsiklis' general work [7].

Hierarchical games and systems were used as models for parts of C^3 systems by Castanon [8] and others [9]. Later, Castanon [10] applied rational aggregate theory to linear dynamic state processes to obtain sequential (relative to hierarchy level) solutions of systems with hierarchies defined by behavior tempo having also possible uncertain models. (See also Luh et al. [11] for other aspects of hierarchical systems useful in C^3.) Often,C^3 systems have been defined as essentially involving the management of military resources. In conjunction with this, a number of papers have considered resource allocation techniques([12],[13], e.g.) as the prime characterization of C^3 systems. In addition, as mentioned numerous times, C^3 analysis requires multidisciplinary usage. For example, Control Theory could be thought of as central to the problem([13'],e.g.). Many papers have concentrated on the human decision maker-in-the-loop aspect, as a perusal of the last two Proceedings of this journal will show. Such papers can vary in thrust of analysis from input-output node models [14] to various detailed (some, qualitative, others, quantitative in scope) internally analyzed systems as in [15] or Wohl's and others' extended SHOR(Sense, Hypothesize,Option,Response) paradigms [16]-[18], related to Lawson's proposals [19],[20].

Although- as mentioned above- few papers have attempted to analyze the overall C^3 problem quantitatively or qualitatively, those that have, have engendered much controversy. Consider first those qualitatively oriented papers attempting to define or analyze general C^3 systems. Lawson [19],[20] was among the first to propose a general theory of C^3, based to a degree on analogues with thermodynamic principles, motivated by the classic Lanchester equations of force attrition or increase. Later, he emphasized time as a critical factor in all aspects of a C^3 system [21], considered briefly C^3 systems from a knowledge-based systems viewpoint, among other items [22], and proposed generic experiments for analyzing C^3 systems [23]. Athans also has been active in attempting to define the general C^3 problem, beginning with the First Workshop [24]-[26] and culminating with his view of "expert team of experts" for commanders [27]. Other good qualitative overviews of the problem may be found in [28]-[31] as well as the short paper [32]. See also the more recent comments of Rona [33] and Metersky [34]. The latter emphasizes expanding Lawson's and others' concepts of C^3 and the integration in some systematic way of subjective and objective information. (This is compatible with section 4 here.) Strack [35] has compiled possibly the most far-reaching of qualitative analyses of C^2 problems in his recent report. In a related direction, development of measures-of-effectiveness (MOE's) for C^3 systems in general began in earnest with Lawson's concern for time/tempo of C^3 operations (such as in [21]) and Harmon and Brandenberg working on internodal and intranodal measures, among other topics [36]. Further work in this area has been carried out by Bouthonnier and Levis [37] (in conjunction with Levis' organizational approach - see below), Linsenmayer's countermeasure-oriented MOE paper [38], and recently, by Karam and Levis [39].

Recently, two additional approaches have been proposed for modeling general C^3 systems, which like Lawson's earlier proposals are most appropriate for large scale system behavior of C^3 components typically representing men in the field and supplies. Anthony [40] proposes four candidate, empirically-derived laws arising from other disciplines as governing C^3 systems. Mayk [41], somewhat similar to Lawson [1], presents a thermodynamics/uncertainty principle approach which regulates the more "irreducible primitive" components of C^3 systems. In addition, Rubin [42], following guidelines in [41], under semi-Markov and Markov assumptions, derived explicit forms for various stochastic processes acting as links among the components of a C^3 system. In particular, Lanchester's equations were shown to be a special case of this model.

The approach taken in this paper (section 3) follows to a degree the general view of Levis et al. [43]-[49]. There, a C^3 system is considered to be a collection of interacting decisionmakers, which as a whole, may follow (under appropriate limiting conditions) macroscopic principles (such as Lawson proposes, e.g.). However, critical to the analysis is the microscopic analysis of each decision maker or node representing a unit of decisionmakers acting through cooperation as a single individual. The structure of each decisionmaker follows the general pattern as the SHOR paradigm or variations. Then a quantitative (normative-descriptive) measure is obtained for each such decisionmaker in the form of the total workload-i.e., entropy-of all internal random variables connected with decision/action and choice of related algorithms, involving also possible interaction with other decisionmakers during this process, as well as accounting for memory. By simple summation over all decisionmakers, an overall C^3 system measure of workload G can be obtained. Alternatively, the overall joint workload can also be used. Another overall performance measure J is assumed obtainable, such as effectiveness of overall system in dealing with the enemy, so that both G and J are assumed to be dependent functionally - in a computable manner - on W, the internal variable strategies of the decisionmakers. Thus, possible tradeoffs or optimizations of G and J can be consider relative to W, subject to natural constraints on W resulting e.g. from bounded rationality involving G(W) and/or satisficing conditions connected with J(W).

The problem of processing and integrating subjective or linguistic-based information occurring within or between decision nodes and stochastic-based data in C^3 systems is an extension of that for the surveillance problem. In both situations, it may not be appropriate to model both types of information stochastically. In place of this, a possibilistic or multi-valued logic-based analysis may be the proper choice. (See [50] for motivations, background, and further details.) Zadeh [51],[52] originally proposed in these Proceedings use of possibilities in place of probabilities only, for decisions that could typically occur in a C^3 system. Similarly, Goodman employed such an approach- tying it in also with the coverage and incidence functions of stochastic set processes (i.e., random sets)- in addressing the data association problem in tracking [53]-[55]. Other approaches to the modeling of subjective information that occurs for C^3 systems have used forms of expert knowledge-based systems [56],[57]. Still others have considered use of neural network theory and the related area of self-organizing systems for C^3 analysis such as H. Szu has done at the most recent (8^{th}) MIT/ONR Workshop. (See also [58],[59] for background.)

3. OUTLINE OF A C^3 THEORY

This section outlines a C^3 theory which to some extent follows the spirit of Levis et al. ([46],e.g.) in considering a C^3 system dependent upon its local behaviors and analyzing the latter. (See also the discussion in section 2.)

First consider a warfare process. A warfare process V is a time-indexed process given for convenience as

$$V \stackrel{d}{=} (V_t)_{t\geq 0} \quad , \qquad (3.1)$$

where each V_t represents the overall warfare situation for some prescribed region at time t. (Note, that the term "process" and likewise all variables to be introduced below are to be interpreted in possibilistic terms in general, not necessarily probabilistic. Again, see [50] or [52] for background.) In turn, each warfare situation consists of a collection of C^3 systems

$$V_t \stackrel{d}{=} (C_{t,j} \mid j \text{ in } K_{t,1}) \;, \qquad (3.2)$$

where $K_{t,1}$ is some index set and each $C_{t,j}$ is some C^3 system of interest. These C^3 systems may in a sense (to be explained) overlap, be subsets of each other, or be disjoint, reflecting both the design of the individual systems and the choice of levels of analysis. V_t can be partitioned into

$$V_t = \bigcup_{j \text{ in } K_{t,2}} (V_{t,j}) \text{ (disjointly)} \qquad (3.3)$$

where $K_{t,2}$ is the index set of adversaries in conflict,

$$V_{t,j} \stackrel{d}{=} (C_{t,j'} \mid j' \text{ in } K_{t,1,j}) \;, \qquad (3.4)$$

and

$$K_{t,1} = \bigcup_{j \text{ in } K_{t,2}} (K_{t,1,j}) \qquad (3.5)$$

is a corresponding decomposition of index sets.

Often,

$$K_{t,2} = (1,2) \;, \qquad (3.6)$$

where 1 represents friendly forces and 2 that of hostile ones.

In general, each C^3 system is represented as a type of network through the following ordered quadruple:

$$C_{t,j} \stackrel{d}{=} (N_{t,j}, I_{t,j}, \overline{O}_{t,j}, M_{t,j}), \qquad (3.7)$$

where

$$N_{t,j} \stackrel{d}{=} (N_{t,j,k} \mid k \text{ in } K_{t,3,j}) \qquad (3.8)$$

is the set of all nodes of the network;

$$I_{t,j} \overset{d}{=} \{I_{t,j,k} | k \text{ in } K_{t,3,j}\} \quad (3.9)$$

is the set of all inputs (at t) of the network;

$$\bar{O}_{t,j} \overset{d}{=} \{\bar{O}_{t,j,k} | k \text{ in } K_{t,3,j}\} \quad (3.10)$$

is the set of all outputs (at t) of the network; and

$$M_{t,j} \overset{d}{=} (M_{t,j,j',k'} | k \text{ in } K_{t,3,j}, (j',k') \text{ in } K_{t,3,j,k}\} \quad (3.11)$$

is the set of all media/environment /noise involving any node in the network with any other node (of any other network), where $K_{t,3,j}$ is the index set of all nodes for $C_{t,j}$ and $K_{t,3,j,k}$ is an index set representing those possible nodes outside of $N_{t,j,k}$ to which an initial output can be directed (whether on purpose or due to general radiation patterns, distances, etc.). Thus

$$K_{t,3,j,k} \subseteq \{(j',k') | j' \text{ in } K_{t,2}, k' \text{ in } K_{t,3,j'}\} \quad (3.12)$$

and

$$\bar{O}_{t,j,k} \overset{d}{=} \{\bar{O}_{t,j,k,j',k'} | (j',k') \text{ in } K_{t,3,j,k}\} \quad (3.13)$$

is the decomposition of the output at node $N_{t,j,k}$ into possible outputs directed towards other nodes (for all adversaries.)

Hence, $(I_{t,j,k}, \bar{O}_{t,j,k})$ is the input-output pair for node $N_{t,j,k}$ at t. But the causal or semi-causal relation between inputs and outputs is given as: $I_{t_1,j,k}$ resulting in $\bar{O}_{t_2,j,k}$, for some $t_2 \geq t_1$ through

$$N_{t_1,t_2;j,k} \overset{d}{=} (N_{t,j,k})_{t_1 \leq t \leq t_2}, \quad (3.14)$$

due to processing delays within the node, as some version of the SHOR paradigm is carried out interacting possibly with other decisionmakers, etc. In (3.13), each $\bar{O}_{t,j,k,j',k'}$ is that output from $N_{t,j,k}$ directed towards $N_{t,j',k'}$ through medium $M_{t,j,k,j',k'}$. Thus, typically, the additive-like regression relation holds (where again, note that the values involved may be nonnumerical in nature - hence the use of $\oplus$)

$$I_{t_2,j',k'} = f_{t_1,t_2;j,k,j',k'}(\bar{O}_{t_1,j,k,j',k'}) \oplus R_{t_1,t_2;j,k,j',k'}, \quad (3.15)$$

where f represents some function and R some noise, where possibly the constraint

$$t_1 \leq t_2 \leq t_1 + \Delta t_1 \quad (3.16)$$

holds.

Next, each node is internally represented as an ordered quadruple

$$N_{t,j,k} \overset{d}{=} (S_{t,j,k}, \hat{S}_{t,j,k}, \hat{CS}_{t,j,k}, D_{t,j,k}), \quad (3.17)$$

where $S_{t,j,k}$ is the true state vector of $N_{t,j,k}$, possibly unknown to the decisionmaker complex $D_{t,j,k}$ of $N_{t,j,k}$ and evolving in time according to possibilistic, or, in particular, probabilistic transition values. Typically, $S_{t,j,k}$ can contain entries (possibly decoupled) for location and pattern of deployment of individuals within $N_{t,j,k}$, number of personnel there, equation of motion parameter values for that portion of the node involved in movement or going to battle, and weapon descriptions, if any weapons are present at the node. Similarly, $\hat{S}_{t,j,k}$ is the node's estimate of its own state, while $\hat{CS}_{t,j,k}$ is the node's estimate of all remaining relevant state vectors outside of the node. Finally, $D_{t,j,k}$ need not be a decisionmaker(s) in the narrow sense, but may also indicate a follower complex (such as a unit of soldiers ready for combat and following command orders). Use of $D_{t,j,k}$, possibly with $\hat{S}_{t,j,k}$ and $\hat{CS}_{t,j,k}$, if not vacuous, leads to the basic input-output mentioned around eq.(3.14). (One aspect of this will be given in section 4.)

Overall (real-or vector-valued) performance measures $J_{t,j,1}, J_{t,j,2}, \ldots$, can be constructed for each C^3 system $C_{t,j}$, generally through some function, such as addition, numerical averaging, or retaining the joint form of local performance measures at each node. Thus, e.g., one could have

$$J_{t,j,5} \overset{d}{=} \sum_{k \text{ in } K_{t,3,j}} (J_{t,j,5,k}), \quad (3.18)$$

where each $J_{t,j,5,k}$ is considered a function of the internal decision variable possibility functions of $D_{t,j,k}$ through the relation

$$J_{t,j,5,k} = \text{'E'} \left(\mathcal{J}_{t-\Delta,t;j,5,k}(W_{t-\Delta,t;j,k} | I_{t-\Delta,t;j,k})\right), \quad (3.19)$$

where $W_{t-\Delta,t;j,k}$ is a collection of internal variables of $D_{t,j,k}$ operating over time interval $[t-\Delta,t]$; similarly for the inputs $I_{t-\Delta,t;j,k}$; and where $\mathcal{J}$ is an appropriately chosen function. Quote marks surround the expectation since possibility functions may be involved, in which case a possibilistic measure of central tendency replaces ordinary probabilistic expectation [50].

Thus, as mentioned earlier, one can then determine tradeoffs between various performance measures of a given $C_{t,j}$ or even of V_t through admissible possibility functions, here, corresponding to W.

It is of some interest to determine if under reasonable conditions, as the number of nodes increase indefinitely, behaving in some "random" manner, that the proposed thermodynamic-type C^3 models can be obtained as limiting cases of the model presented here. At present, work is being carried out in this direction. Further details of the general theory presented here will be presented in a later publication. For the present, analysis will concentrate on intranodal use of subjective and objective information, in order to obtain the basic input-output equations.

4. COMBINATION OF EVIDENCE AT NODES

In this section, some quantitative results are derived for intranodal behavior of a C^3 system.

Consider any node $N_{t-\Delta,t;j,k}$ with internal variable set $W_{t-\Delta,t;j,k}$ and possible additional input set of variables during processing time $[t-\Delta,t]$, $\bar{I}_{t-\Delta,t;j,k}$, as well as original input set $I_{t-\Delta,j,k}$. Without loss of generality, suppose subjective components of the relevant quantities below are indicated by primes as superscripts, while objective/probabilistic ones are denoted by superscripted double primes:

$$\bar{I}_{t-\Delta,t;j,k} \overset{d}{=} (\bar{I}'_{t-\Delta,t;j,k}, \bar{I}''_{t-\Delta,t;j,k}) , \quad (3.20)$$

$$W_{t-\Delta,t;j,k} \overset{d}{=} (W'_{t-\Delta,t;j,k}, W''_{t-\Delta,t;j,k}) , \quad (3.21)$$

$$I_{t-\Delta,j,k} \overset{d}{=} (I'_{t-\Delta,j,k} , I''_{t-\Delta,j,k}) \quad . (3.22)$$

Furthermore, since no time integration will be carried out here (under simplistic assumptions for the current analysis), drop all subscripts in the above equations.

Following the development in [53],[54], all probabilistic information is modeled through some discretization/refinement level of probability density functions i.e., finite probability functions, while all subjective information is treated by possibility functions, which in general are not probability functions (not adding up to unity, since overlapping and vague concepts are being represented [50]). Thus both types of information are now modeled by possibility functions and may be manipulated through finite argument multi-valued logical operators. In particular, conjunction, replacing product for ordinary probability functions, is represented by a large class of operators, the t-norms, which include, as a special case, product. Similarly, disjunction extends the ordinary sum operator relative to probabilities and is represented through the class of t-conorms. Finally, negation or set complements is generalized by use of negation operators which include the more familiar classical operator 1-(·). (Again, see [50] for details.) More specifically, a t-norm $\phi_{\&}:[0,1]^n \to [0,1]$ is nondecreasing in all arguments, continuous, symmetric, associative (so that it may be extended recursively, unambiguously from n=2 arguments to an arbitrary number of), and a t-conorm $\phi_{or}:[0,1]^n \to [0,1]$ has formally the same properties, where both satisfy the boundary conditions for all $0 \le x,y \le 1$, for n=2 (the general case being similar)

$$\phi_{\&}(x,y) \le \min(x,y); \ \max(x,y) \le \phi_{or}(x,y) \ ;(3.23)$$

$$\phi_{\&}(0,x)=0 \ ;\phi_{\&}(1,x)=x=\phi_{or}(0,x) \ ; \ \phi_{or}(1,x)=1 \ . \quad (3.24)$$

Also, following the notation in (3.20)-(3.22) and the ensuing remarks, denote for probabilistic and subjective variables involved internally as

$$Z' \overset{d}{=} (W',\bar{I}') \ ; \ Z'' \overset{d}{=} (W'',\bar{I}'') \ . \quad (3.25)$$

It follows that analogous to ordinary probability function relations, denoting possibilities by ϕ [50], and finally noting that $\bar{O}$ as used here is an abbreviation for output $\bar{O}_{t,j,k}$,

$$\phi(\bar{O},W,\bar{I}|I)=\phi_{\&}(F(Z',Z''|I),\phi(Z'|Z'',I)) \ , \quad (3.26)$$

where

$$F(Z',Z''|I) \overset{d}{=} \phi_{\&}(\phi(\bar{O}|W,\bar{I},I),\phi(Z''|I))$$
$$=\phi_{\&}(\phi(\bar{O}|Z',Z'',I),\phi(Z''|I)). \quad (3.27)$$

Denote the discretization/refinement (including truncation, if needed) level by index p, so that from the above discussion, replace $\phi(Z'|Z'',I)$ by

$$\phi_p(Z'|Z'',I)=f(Z'|Z'',I)\cdot\Delta_p(Z'), \quad (3.28)$$

where $f(\cdot|\cdot\cdot)$ is a fixed p.d.f., not depending on p, and where the domain of f , assumed to be, say, $\mathbb{R}^m$, ϕ_p has a finitely discrete domain D_p' , so that in any natural sense

$$\lim_{p\to\infty} D_p' = \mathbb{R}^m \quad ; \ \lim_{p\to\infty} \Delta_p(Z') = 0 \ (\text{uniform}), (3.29)$$

In turn, it follows that

$$\phi_p(\bar{O}|I)= \underset{(\text{all } W,\bar{I})}{\phi_{or}} (\phi_p(\bar{O},W,\bar{I}|I))$$
$$= \underset{(\text{all } Z'')}{\phi_{or''}}(\underset{(\text{all } Z')}{\phi_{or'}} (\phi_p(\bar{O},Z',Z''|I))) \ , \quad (3.30)$$

where as in (3.26),(3.27)

$$\phi_p(\bar{O},Z',Z''|I)=\phi_{\&}(F(Z',Z''|I),\phi_p(Z'|Z'',I)) \ (3.31)$$

and it is assumed that ϕ_{or} represents a compound combination of $\phi_{or'}$ applied to probabilistic information followed by $\phi_{or''}$ applied to subjective information. In general, the two t-conorms may be different([50],Chp. 10).

With all of this established, the basic question arises as to the behavior of $\phi_p(\bar{O}|I)$ as more and more of the probabilistic information is used in terms of the discretization procedure, i.e., what is $\lim_{p\to\infty} \phi_p(\bar{O}|I)$?

The following theorem has an analogue for the PACT application([50],Chp.9); but differs somewhat in structure from the forms presented there.

<u>Theorem</u>

Suppose that all constructions hold as presented in (3.28) for any index p, where for convenience f is assumed to be also bounded. Suppose also the following:

1. $\phi_{\&}$ as a function of two arguments possesses continuous second order derivatives in some neighborhood of (0,0).

2. $\phi_{or'}$ is an Archimedean t-conorm, i.e., for the two argument case, for example,

$$\phi_{or'}(x,x) > x \ , \text{all } 0<x<1 \ . \quad (3.32)$$

(Many t-conorms are Archimedean and indeed it can be shown that arbitrary t-conorms can be written as affine types of mixtures (called ordinal sums) of Archimedeans and the non-Archimedean t-conorm max. Again, see[50] - Chp. 2.3.)

3. The corresponding generating function h to $\phi_{or'}$ (see Proof below for discussion) has a continuous second order derivative in some neighborhood $[1-\varepsilon,1]$ of 1 , $0<\varepsilon<1$.

Then ;

$$\lim_{p\to\infty} \phi_p(\bar{O}|I) = \phi_\infty(\bar{O}|I)$$
$$\overset{d}{=} \underset{(\text{all } Z'')}{\phi_{or''}}(\omega(Z''|I)), \quad (3.33)$$

where nondecreasing function ω is given in (3.46) in terms of the ordinary expectation of also nondecreasing function κ of $F(Z',Z''|I)$, with respect to $(Z'|Z'',I)$ now formally a random vector corresponding to p.d.f. f ; κ is given in (3.40).

<u>Proof:</u>

A. A relatively deep theorem from the theory of probabilistic metric spaces [60] shows first that any given Archimedean t-norm, say $\phi_{\&'}$, has an essentially unique generating function $h:[0,1] \to \mathbb{R}^+$, where $\mathbb{R}^+$ denotes the positive real line with $+\infty$ annexed. That is, h is continuous nonincreasing with

$$h(1)=0 \ ; \ h(0)\le+\infty \quad (3.34)$$

such that for all positive integers n and all $0\le x_1,\dots,x_n \le 1$,

$$\phi_{\&'}(x_1,\dots,x_n) = h^{-1}(\min(h(0), \sum_{j=1}^{n} h(x_j))). \quad (3.35)$$

The definition of an Archimedean t-norm is dual to that in (3.32) :

$$\phi_{\&'}(x,x) < x \quad , \text{all } 0<x<1 \ . \quad (3.36)$$

Although any pair of t-norm and t-conorm need not be DeMorgan, any t-conorm can be expressed as the DeMorgan transform of some corresponding t-norm. Furthermore, if one is Archimedean, then so is its DeMorgan transform. Thus, one can let $\phi_{or'}$ in (3.32) be written as , for all positive integers n ,etc.

$$\phi_{or'}(x_1,\ldots,x_n)=1-\phi_{\&'}(1-x_1,\ldots,1-x_n)$$
$$= \psi_h\Big(\sum_{j=1}^{n} h(1-x_j)\Big)\ , \quad (3.37)$$

where

$$\psi_h(x) \overset{d}{=} 1-h^{-1}(\min(h(0),x)),\text{all } 0\le x, \quad (3.38)$$

using assumption 2.

B. From assumptions 1 and 3 ,

$$\phi_{\&}(x,y) = \kappa(x)\cdot y + O(y^2;x) \quad (3.39)$$

where

$$\kappa(x) \overset{d}{=} (\partial\, \phi_{\&}(x,y)\ /\ \partial y)_{y=0}\ ; \quad (3.40)$$

and

$$h(1-z) = c_h\cdot z + O(z^2)\ , \quad (3.41)$$

where

$$c_h \overset{d}{=} -(dh(z)/dz)_{z=1} > 0\ . \quad (3.42)$$

$O(\cdot)$ denotes the usual "order of" relation, and x,y,z are arbitrary such that for ϵ_1,ϵ_2 fixed

$$0\le x,y\le \epsilon_2 \le 1\ ;\ 1-\epsilon_1\le z\le 1\ . \quad (3.43)$$

C. For any $0\le x_1,\ldots,x_n\le\epsilon_1$, using (3.37) and (3.41),

$$\phi_{or'}(x_1,\ldots,x_n)=\psi_h\Big(c_h\cdot \sum_{j=1}^{n}(x_j+O(x_j^2))\Big). \quad (3.44)$$

Apply (3.39) to (3.31) and (3.28), and then replace each x_j in (3.44) by $\phi_p(\bar{0},Z',Z''|I)$ with index Z' in D_p' replacing j, j=1,..,n. This yields

$$\phi_{or'}(\phi_p(\bar{0},Z',Z''|I))=\psi_h\Big(c_h\cdot \sum_{(Z' \text{ in } D_p')}\big(\kappa(F(Z',Z''|I))\Delta_p(Z'|Z'',I)\big)$$
(all Z' in D_p')
$$+ O\Big(\sum_{Z' \text{ in } D_p'} \Delta_p(Z')^2\Big)\Big), \quad (3.45)$$

The main result then follows using (3.45) in (3.30), where

$$\lim_{p\to\infty}\Big(\phi_{or'}(\phi_p(\bar{0},Z',Z''|I))\Big)\overset{d}{=}\omega(Z''|I)$$
$$(Z' \text{ in } D_p')$$
$$= \psi_h\big(c_h\cdot E(\kappa(F(Z',Z''|I))|Z'',I)\big) \quad (3.46)$$
$$(Z'|Z'',I)$$

Thus, up to essentially increasing transforms, the approach taken here to combining subjective information with objective information involves taking an expectation of the latter and using a multi-valued logical procedure on the former in a unified way, up to the level of discretization/refinement used for the probabilistic information. As a final remark, it should be noted that most common t-norms and t-conorms satisfy the rather mild analytic conditions required in the hypotheses of the theorem and max can also be used in place of an Archimedean form for $\phi_{or'}$, with appropriate modifications of the proof. (See Appendix A for an important example of this.)

5. CONCLUSIONS

An attempt at modeling the overall C^3 problem as a network of nodes has been outlined. Key to this is the local modeling, i.e., the modeling of input-output behavior at each node. A procedure was presented, analogous to the PACT algorithm for multi-target data association which treats in a unified manner subjective and objective information. Future efforts will elaborate further on both global and local aspects of combining such information.

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REFERENCES

In the following references, abbreviations are made for the Proceedings of the MIT/ONR Workshop on C^3 Systems by use of the notation $C^3(k)$ or $C^3(k,j)$, where k corresponds to the number of the Proceedings and j denotes the volume (in Roman numerals), if more than one was issued. Years are omitted for such references, noting the correspondences:

1978 ↔ k=1	1981 ↔ k=4	1984 ↔ k=7
1979 ↔ k=2	1982 ↔ k=5	1985 ↔ k=8
1980 ↔ k=3	1983 ↔ k=6	

1. Castanon,D.A.,Delaney,J.R.,Kramer,L.C.,Athans,M."A mathematical framework for the study of battle group position decisions", $C^3(5)$, 105-110.

2. Teneketzis,D.,Castanon,D.A.,"Information aspects of a class of subjective games of incomplete information", $C^3(6)$, 226-231.

3. Tenney,R.R., Distributed Decision Making Using a Distributed Model, Ph.D.Diss.(LIDS-TH-933),Sept.,1979.

4. Tenney,R.R.,"Distributed decision making with limited communication", $C^3(2,IV)$, 830-868.

4'. Tenney,R.R.,"A case study of distributed decision making", $C^3(4,IV)$, 235-244.

5. Sandell,N.R., "Distributed decision making processes in C^3", $C^3(3,II)$, 1-38.

6. Tsitsiklis,J.,Athans,M.,"On the complexity of distributed decision problems", $C^3(6)$, 232-237.

7. Tsitsiklis,J.N., Problems in Decentralized Decision Making and Computation,Ph.D.Diss.(LIDS-TH-1424),Dec,1984.

8. Castanon,D.A.,"Goal coordination for hierarchical structures in game theory",$C^3(3,II)$, 155-184.

9. Papavassilopoulos,G.P., "Directions and recent results in hierarchical decision making via game theoretic methods", $C^3(3,II)$, 185-198.

10. Castanon,D.A., "Games with uncertain models", $C^3(4,IV)$, 61-80.

11. Luh,P.B.,Ning,T.,"Dynamic, hierarchical decision problems", $C^3(6)$, 249-254.

12. Lauer,G.S.,Bertsekas,D.,Sandell,N.R.,"Solution of large dynamic resource allocation problems",$C^3(3,II)$,171-224.

13. Loparo,K.,Walker,B.,Griffiths,B.,"A framework for the design of survivable systems-Part II:Control and information structure", $C^3(4,IV)$, 295-308.

13'. Tenney,R.R.,"Optimal decentralized control of finite deterministic systems", $C^3(6)$, 238-242.

14. Sen,P.,Drenick,R.F.,"Information processing in man-machine systems", $C^3(4,IV)$, 81-102.

15. Kleinman,D.L.,Pattipati,K.B.,"Results toward developing a model of human decision making in C^3 systems", $C^3(3,III)$, 29-64.

16. Wohl, J.G., "Battle management decisions in Air Force Tactical Command", C^3(3,II), 87-126.

17. Wohl, J.G., Entin, E.E., "Modeling human decision processes in Command and Control", C^3(5), 124-126.

18. Wohl, J.G., "Rate of change of uncertainty as an indicator of Command and Control Effectiveness", C^3(4,IV), 1-18.

19. Lawson, J.S., "A unified theory of Command and Control", C^3(1), 10-125.

20. Lawson, J.S., "The state variables of a Command Control system", C^3(2,II), 462-510.

21. Lawson, J.S., "The role of time in a Command Control system, C^3(4,IV), 19-60.

22. Lawson, J.S., "Data bases and decisions", C^3(5), 1-7.

23. Lawson, J.S., "Doing C^2 experiments using war games", C^3(6), 3-9.

24. Athans, M., "Systems aspects of C^3 problems", C^3(1), ~~126-141~~.

25. Athans, M., "Overview of MIT/LIDS research in Naval C^3 systems", C^3(2,I), 1-34.

26 Athans, M., "System theoretic challanges and research opportunities in military C^3 systems, C^3(3,I), 1-8.

27. Athans, M., "The expert team of experts approach to C^2 organizations", C^3(4,IV), 207-234.

28. Brick, D.B., Jarvis, T., "C^3: systems, functions, problems,..", C^3(1), 142-214.

29. Blinkenberg, R., "Command, Control, and Communication (C^3) design implications for distributed data analysis and fusion activities", C^3(2,I), 134-157.

30. Schutzer, D.M., "Command, Control and Communications-some design aspects", C^3(2,III), 741-782.

31. Shanahan, E.J., Teates, H.B., Wise, B.B., "Defining and measuring C^2", C^3(3,III), 229-282.

31'. Tenney, R.R., "Modelling the C^3 decision process", C^3(3,II), 39-80.

32. Conley, R.E., "A command and control theory" C^3(3II), 81-86

33. Rona, T.P., "C^3 over the past 3-5 years- a personal learning experience", C^3(5), 8-13.

34. Metersky, M.E., "A C^2 process and an approach to design and evaluation", C^3(7), 11-22.

35. Strack, C.W., Elements of C^2 Theory, Report for DSI (McLean, Virginia), Jan. 30, 1985.

36. Harmon, S., Brandenburg, R., "Command, Control and Communications (C^3) systems model and measures of effectiveness (MOE's)", C^3(4,IV), 181-206.

37. Bouthonnier, V., Levis, A.H., "Effectiveness analysis of C^3 systems", C^3(5), 80-87.

38. Linsenmayer, G.R., "Measures-of-effectiveness for Command, Control, and Communications Countermeasures", C^3(5), 88-93.

39. Karam, J., Levis, A.H., "Effectiveness assessment of the METANET demonstration", C^3(7), 61-70.

40. Anthony, R.W., "Holistic patterns in command, control, communication and information systems". C^3(7), 71-78.

41. Mayk, I., Rosenstark, S., Frank, J., "Analysis of C^3 systems based on a proposed canonical reference model", C^3(7), 45-54.

42. Rubin, I., Queueing and Decision Theoretic Modeling and Mathematical Analysis of C^3 Systems and Networks, IRI Report No. IRI-CEC-REP-8501 (Tarzana, Cal.) Mar., 1985.

43. Levis, A.H., Boettcher, K.L., "Organization theory and C^3 systems", C^3(3,II), 127-154.

44. Boettcher, K.L., Levis, A.H., "Modeling the interacting decisionmaker with bound rationality", C^3(4,IV), 103-136.

45. Levis, A.H., Boettcher, K.L., "Decisionmaking organizations with acyclical information structures", C^3(5), 94-104.

46. Levis, A.H., "Information processing and decisionmaking organizations: a mathematical description", C^3(6), 30-38.

47. Hall, S.A., Levis, A.H., "Information theoretic models of memory in human decisionmaking models", C^3(6), 67-75.

48. Tomovic, M.M., Levis, A.H., "On the design of organizational structures for command and control", C^3(7), 131-138.

49. Tabak, D., Levis, A.H., "Petri net representation of decision models", C^3(7), 121-130.

50. Goodman, I.R., Nguyen, H.T., Uncertainty Models for Knowledge-Based Systems, North Holland Co., 1985.

51. Zadeh, L.A., "Possibility theory and its applications to decision analysis", C^3(1), 215-256.

52. Zadeh, L.A., "Fuzzy probabilities and their role in decision analysis", C^3(4,IV), 159-180.

53. Goodman, I.R., "An approach to the data association problem through possibility theory", C^3(5), 209-215.

54. Goodman, I.R., "A unified approach to modeling of evidence through random set theory", C^3(6), 42-47.

55. Goodman, I.R., "Modeling natural language information for use in the combination of evidence problem", C^3(7), 173-178.

56. Aldrich, J.R., "Applications of knowledge-based approaches to C^2I processes", C^3(3,III), 143-170.

57. Dillard, R.A., "Integration of Artificial Intelligence into tactical C^3 systems", C^3(7), 79-84.

58. Grossberg, S., Studies of Mind and Brain, Reidel Co., Boston, 1982.

59. Kohonen, T., Self Organization and Associative Memory, Springer-Verlag Co., Berlin, 1984.

60. Schweizer, B., Sklar, A., Probabilistic Metric Spaces North-Holland Co., 1983.

APPENDIX A.

A large and conveniently parameterized family of DeMorgan transform pair of Archimedean t-norms and t-conorms is due to Frank originally (see [50], Chp.2.3 for additional discussion) and satisfies uniquely the modular relation

$$\phi_{or}(x,y)=x+y-\phi_{\&}(x,y), \text{all } 0\le x,y\le 1. \quad (A-1)$$

The solution is given as, using parameter-index s,

$$\phi_{\&,s}(x_1,\dots,x_n)=\log_s\Big(1+\Big(\prod_{1\le j\le n}(s^{x_j}-1)/(s-1)^{n-1}\Big)\Big), \quad (A-2)$$

$$\phi_{or,s}(x_1,\dots,x_n)=1-\phi_{\&,s}(1-x_1,\dots,1-x_n), 0\le x_1,\dots,x_n\le 1, \quad (A-3)$$

where $0<s\le+\infty$; s otherwise any real number. In a limiting sense, it is natural to define for the non-Archimedean pair min, max ,

$$\phi_{\&,0}(x_1,\dots,x_n)=\min(x_1,\dots,x_n); \phi_{or,0}(x_1,\dots,x_n)=\max(x_1,\dots,x_n) \quad (A-4)$$

and to note the special cases $s=1, s=+\infty$ in (A-2),(A-3):

$$\phi_{\&,1}(x_1,\dots,x_n)=\prod_{1\le j\le n}(x_j); \phi_{or,1}(x_1,\dots,x_n)=1-\prod_{1\le j\le n}(1-x_j), \quad (A-5)$$

$$\phi_{\&,+\infty}(x_1,\dots,x_n)=\max\Big(\sum_{1\le j\le n}(x_j)-(n-1),0\Big); \phi_{or,+\infty}(x_1,\dots,x_n)=\min\Big(\sum_{1\le j\le n}(x_j),1\Big), \quad (A-6)$$

It then follows that Frank's family satisfies the hypotheses of the theorem in section 3, for all $s>0$, with $s=0$ also treatable as a special case. For all $s>0$, generator function h_s and (3.38),(3.40),(3.42) become.

$$h_s(x)=-\log((s^x-1)/(s-1)); c_{h_s}=(s\log(s))/(s-1) ; \quad (A-7)$$

$$\psi_{h_s}(x)=1-\log_s\big(1+((s-1)\cdot e^{-x})\big); \kappa_s^s(x)=(s^x-1)/(s-1). \quad (A-8)$$