DISTRIBUTED SENSOR SYSTEM DECISION ANALYSIS USING TEAM STRATEGIES

University of Virginia

Howard C. Choe and Dimitri Kazakos

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A distributed (or decentralized) multiple sensor system is considered under binary hypothesis environments. The system is deployed with a host sensor and multiple slave sensors. All sensors have their own independent decision makers (DM) which are capable of declaring local decisions based only on their own observation of the environment. The communication between the host sensor (HS) and the slave sensors (SS) is conditional upon the host sensor's command. Each communication that takes place involves a communication cost which plays an important role in approaches taken in this study. The conditional communication with cost initiates the team strategy in making the final decisions at the host sensor. The objectives are not only to apply the team strategy method in the decision making process, but also to minimize the expected system cost (or the probability or error in making decisions) by optimizing thresholds in the host sensor. The analytical expression of the expected system cost is numerically evaluated for Gaussian statistics over threshold locations in the host sensor to find an optimal threshold location for a given communication cost. The computer simulations of various sensor systems for Gaussian observations are also performed to understand the behavior of each system with respect to correct detections, false alarms, and target misses.
Acknowledgement

I wish to express my appreciation to my thesis advisor, Dr. D. Kazakos, for his guidance throughout this research. I am also indebted greatly to my committee chairperson, Dr. S. G. Wilson, and Dr. P. Papantoni-Kazakos for their careful review of this thesis and invaluable comments. I would like to thank fellow CSL students for their friendships and support throughout my years at the University of Virginia. Finally, I thank my parents, sister, and brother for their prayers, and for always being there.
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<th>Description</th>
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<tr>
<td>HS</td>
<td>Host sensor</td>
</tr>
<tr>
<td>HDM</td>
<td>Host sensor's decision maker</td>
</tr>
<tr>
<td>SS</td>
<td>Slave sensor</td>
</tr>
<tr>
<td>SDM</td>
<td>Slave sensor's decision maker</td>
</tr>
<tr>
<td>TL or TL2</td>
<td>Lower threshold at HS</td>
</tr>
<tr>
<td>TU or TU2</td>
<td>Upper threshold at HS</td>
</tr>
<tr>
<td>TSS</td>
<td>Decision threshold at SS</td>
</tr>
<tr>
<td>FT</td>
<td>Final threshold of system</td>
</tr>
<tr>
<td>UHS</td>
<td>Local decision of HS</td>
</tr>
<tr>
<td>USS</td>
<td>Local decision of SS</td>
</tr>
<tr>
<td>UF</td>
<td>Final decision of the system</td>
</tr>
<tr>
<td>C( )</td>
<td>The cost function of the system</td>
</tr>
<tr>
<td>YHS</td>
<td>Observation received at HS</td>
</tr>
<tr>
<td>YSS</td>
<td>Observation received at SS</td>
</tr>
<tr>
<td>Pe~s</td>
<td>Probability of error incurred by the HS only</td>
</tr>
<tr>
<td>PeTeam</td>
<td>Probability of error incurred by team decision</td>
</tr>
<tr>
<td>Z</td>
<td>( z = \begin{cases} 1, &amp; \text{if } TL \leq y_{HS} \leq TU \ 0, &amp; \text{Otherwise} \end{cases} )</td>
</tr>
<tr>
<td>cHS,SS</td>
<td>Communication cost constant</td>
</tr>
<tr>
<td>C</td>
<td>Expected system cost</td>
</tr>
<tr>
<td>H_0</td>
<td>Environment without a target</td>
</tr>
</tbody>
</table>
$H_1$ Environment with a target
$\Lambda(y_{HS})$ Likelihood ratio at HS
$\Lambda(y_{SS})$ Likelihood ratio at SS
$\lambda_t$ Pre-calculated threshold for HS
$\lambda_{tSS}$ Pre-calculated threshold for SS
$C_{\alpha\beta}$ Cost of deciding $\alpha$ given $\beta$
$T_0$ Ratio of a priori probabilities of environment
$f(U_{SS})$ Final threshold in function of $U_{SS}$
$Q(y)$ Q-function integrated from $y$ to $\infty$
$f_{HS0}(y_{HS})$ Gaussian PDF of $y_{HS}$ given $H_0$
$f_{HS1}(y_{HS})$ Gaussian PDF of $y_{HS}$ given $H_1$
$f_{SS0}(y_{SS})$ Gaussian PDF of $y_{SS}$ given $H_0$
$f_{SS1}(y_{SS})$ Gaussian PDF of $y_{SS}$ given $H_1$
$\mu_{HS0}$ Mean received at HS given $H_0$
$\mu_{HS1}$ Mean received at HS given $H_1$
$\sigma_{HS0}$ Standard deviation received at HS given $H_0$
$\sigma_{HS1}$ Standard deviation received at HS given $H_1$
$\mu_{SS0}$ Mean received at SS given $H_0$
$\mu_{SS1}$ Mean received at SS given $H_1$
$\sigma_{SS0}$ Standard deviation received at SS given $H_0$
$\sigma_{SS1}$ Standard deviation received at SS given $H_1$
List of Symbols Used in Chapter 3

Those symbols do not appear in this section are the symbols which are commonly used in Chapter 2 and Chapter 3. Please refer to “List of Symbols used in Chapter 2” for the symbols not listed here.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>SS1</td>
<td>Slave sensor 1</td>
</tr>
<tr>
<td>SS2</td>
<td>Slave sensor 2</td>
</tr>
<tr>
<td>SDM1</td>
<td>Slave sensor 2's decision maker</td>
</tr>
<tr>
<td>SDM2</td>
<td>Slave sensor 2's decision maker</td>
</tr>
<tr>
<td>TL or TL3</td>
<td>Lower threshold at HS</td>
</tr>
<tr>
<td>TU or TU3</td>
<td>Upper threshold at HS</td>
</tr>
<tr>
<td>TSS1</td>
<td>Decision threshold at SS1</td>
</tr>
<tr>
<td>TSS2</td>
<td>Decision threshold at SS2</td>
</tr>
<tr>
<td>USS1</td>
<td>Local decision of SS1</td>
</tr>
<tr>
<td>USS2</td>
<td>Local decision of SS2</td>
</tr>
<tr>
<td>YSS1</td>
<td>Observation received at SS1</td>
</tr>
<tr>
<td>YSS2</td>
<td>Observation received at SS2</td>
</tr>
<tr>
<td>f(USS1, USS2)</td>
<td>Final threshold in function of USS1 and USS2</td>
</tr>
<tr>
<td>fSS10(YSS1)</td>
<td>Gaussian PDF of YSS1 given H0</td>
</tr>
<tr>
<td>fSS11(YSS1)</td>
<td>Gaussian PDF of YSS1 given H1</td>
</tr>
<tr>
<td>fSS20(YSS2)</td>
<td>Gaussian PDF of YSS2 given H0</td>
</tr>
<tr>
<td>fSS21(YSS2)</td>
<td>Gaussian PDF of YSS2 given H1</td>
</tr>
<tr>
<td>( \mu_{SS1_0} )</td>
<td>Mean received at SS1 given ( H_0 )</td>
</tr>
<tr>
<td>( \mu_{SS1_1} )</td>
<td>Mean received at SS1 given ( H_1 )</td>
</tr>
<tr>
<td>( \sigma_{SS1_0} )</td>
<td>Standard deviation received at SS1 given ( H_0 )</td>
</tr>
<tr>
<td>( \sigma_{SS1_1} )</td>
<td>Standard deviation received at SS1 given ( H_1 )</td>
</tr>
<tr>
<td>( \mu_{SS2_0} )</td>
<td>Mean received at SS2 given ( H_0 )</td>
</tr>
<tr>
<td>( \mu_{SS2_1} )</td>
<td>Mean received at SS2 given ( H_1 )</td>
</tr>
<tr>
<td>( \sigma_{SS2_0} )</td>
<td>Standard deviation received at SS2 given ( H_0 )</td>
</tr>
<tr>
<td>( \sigma_{SS2_1} )</td>
<td>Standard deviation received at SS2 given ( H_1 )</td>
</tr>
</tbody>
</table>
List of Symbols Used in Chapter 4

For those symbols not appearing in this section, please refer to either "List of Symbols Used in Chapter 2" or "List of Symbols Used in Chapter 3" since those symbols are commonly used in Chapter 2, Chapter 3, and Chapter 4.

\[ P_{e_{T1}} \] Probability of error incurred by team decision with SS1 only

\[ P_{e_{T2}} \] Probability of error incurred by team decision with SS1 and SS2

TL1 or TL31 Lower threshold of HS for communicating with SS1 only

TL2 or TL32 Lower threshold of HS for communicating with SS1 and SS2

TU2 or TU32 Upper threshold of HS for communicating with SS1 and SS2

TU1 or TU31 Upper threshold of HS for communicating with SS1 only:

\[ z_{T1} = \begin{cases} 1, & \text{if } TL1 \leq y_{HS} \leq TL2, \text{ or } TU2 \leq y_{HS} \leq TU1 \\ 0, & \text{Otherwise} \end{cases} \]

\[ z_{T2} = \begin{cases} 1, & \text{if } TL2 \leq y_{HS} \leq TU2 \\ 0, & \text{Otherwise} \end{cases} \]

\[ c_{T1} \] Communication cost constant for communicating with SS1 only

\[ c_{T2} \] Communication cost constant for communicating with SS1 and SS2

\[ f(U_{SS1},U_{SS2}) \] Final threshold in function of \( U_{SS1} \) and \( U_{SS2} \)
CHAPTER 1

Introduction

1.1. Literature Review and Goals

The extension of classical detection theory to the case of distributed sensors is discussed in [1]; in particular, the problem of constructing decentralized Bayesian hypothesis testing rules is considered. In [2], the optimal data fusion structure is developed, when the global decision is obtained by weighting local decisions according to the reliability of detectors and comparing to a threshold. In that paper optimal fusion rules are derived when the decision rules per individual detector are known. Those rules are expressed in terms of the probability of false alarm and the probability of miss. The systems considered in [1] and [2] have a fusion center which always requires all the sensors to transmit their local decisions. But in certain applications, such continuous communication may not be desirable; such is the case in environments with adversaries. In [3], one of data fusion methods in distributed networks is to apply the Neyman-Pearson approach to find all of the optimal decision rules at each site (or detector). The optimal threshold for the system using those optimal decision rules found at each site is not stated. In [4], the problem of optimal data fusion in the sense of the Neyman-Pearson test is considered; uncertainty regions at the detectors are considered, but this information is used to enhance the decision at the data fusion center. A region where a definite decision cannot be made is called an uncertainty region. There are no communications between sensors when the
observation falls in the confident region. Papastavrou and Athans [5] evaluated a two-sensor network, consisting of a primary sensor and a consulting sensor using team strategy method, with performance criterion of the probability of error. They also provide numerical results for varying quality of observations at different sensors and a priori probabilities. The relationship between the position of threshold in the primary sensor and the system probability of error is not clearly stated.

Through this study, we applied team decision strategies to three different sensor systems and an analysis of each system was performed. The three different systems are a two-sensor-system (2SS), a three-sensor-system (3SS), and a two/three-sensor-system (2/3SS). The main goals of this study are to identify the level of risk which prohibits communication between sensors, to obtain the analytical expression of optimal global decision rules for each system considered, to investigate the behavior of decision thresholds and system performance for a given communication cost (or risk), and to compare the performance of each system through numerical evaluations and system simulations.

1.2. Overview of Chapters

In Chapter 1, general concepts are discussed. The team strategy method in decision processes and the communication cost involved in the team strategy are described in this chapter. The binary hypothesis environment and assumptions made in deriving the expected system cost are also stated. A couple of examples, concerning the interpretation of the communication cost constant in real system, are also presented.
In Chapter 2, a two-sensor-system (2SS) similar to the system studied by Papastavrou and Athans [5] is considered. The model consists of a host sensor (HS) and a slave sensor (SS). The model used the team strategy method in making final decisions, depending upon the local decisions made by the host sensor. The expected system cost is expressed in general probabilistic terms. This expression is numerically evaluated based on the assumption of Gaussian observation.

Adding an additional sensor to the system, a three-sensor-system (3SS) is treated in Chapter 3. In this system both slave sensors return their binary information to the host sensor when a request of information is made by the host sensor. The analytical expression as well as the numerical evaluation of the expected system cost are also performed.

Chapter 4 contains an analysis of a two/three-sensor-system (2/3SS). This system can be considered as a combination of 2SS and 3SS since 2/3SS switches from 2SS to 3SS, or vice versa, depending upon the local decision of the host sensor. Most of the 2/3SS model criteria are the same as in the previous chapters. Numerical evaluation is also done for this system.

The results from the numerical evaluation of the expected cost of each system are presented in Chapter 5. The data are available in both tables and plots. The plots are attached at the end of Chapter 2, 3, and 4. Each system is compared to other systems based on the results. In the numerical evaluation of the expected system cost, \( \bar{C} \), FORTRAN programs are written and these are attached in Appendix A, B, and C in order of 2SS, 3SS, and 2/3SS, respectively.
In Chapter 6, simulation results of the systems analyzed in Chapter 2, 3, and 4 are presented. Tables and plots are used to show the data from the simulation. A FORTRAN program is also written to carry out the simulation. The program is attached in Appendix E.

Finally, an overall summary of this study and the conclusion are written in Chapter 7.

1.3. Environment

A binary hypothesis environment, $H_0$ and $H_1$, is considered. $H_0$ indicates that there is no target present. $H_1$ indicates that a target is present.

1.4. Team Strategies

The sensor communications occur only when the host sensor declares lack of confidence in its local observation. When a slave sensor transmits only a binary decision to the host sensor, some information received at the slave sensor may be lost, but the risk of interception by adversaries is then reduced. Examples of the costs or the risks in real systems are given in section 1.6. The final threshold in the host sensor is evaluated using the binary decisions transmitted from the slave sensors and a prior probabilities of the hypothesis. The final threshold (FT), then, is compared against the observation at the host sensor to make the final decision. In other words, the final decision at the host sensor is declared by using its local analog data and the binary decisions transmitted from the slave sensors.
The team strategy allows collaboration of sensors in the distributed (multiple) sensor system. The differences between distributed sensor systems which use the team strategies and those that do not are;

(1) The host sensor in team strategies has an overall control of communication between the host sensor (HS) and the rest of the sensors, namely the slave sensors (SSs).

(2) All the sensors including HS and SSs are capable of making their own local decisions utilizing observations from their local environment.

(3) The host sensor carries multiple thresholds which divide the decision space into either three regions (2 thresholds) or five regions (4 thresholds). It uses them to recruit inputs from other sensors accordingly, which means that the communication schemes between the host sensor and the slave sensor are determined, depending upon the decisions of the HS.

(4) The systems do not have a central data fusion center. The host sensor is capable of making the final decision either based on its local observations only, or through communication with the slave sensors.

1.5. Assumptions

(1) The observations received at different sensors are mutually independent conditioned on each hypothesis.
(2) All the sensors used in the model are considered identical in performance.

(3) The influence of the number of observations available at each sensor is ignored.

1.6. Communication Cost Constant (CCC)

There may be many ways to interpret an application of the communication cost constant (CCC) in real systems. An example would be the communication between sensors in a hostile environment, where interception is possible. Then, the communication cost constant can be interpreted as the probability of interception, for example.

The other way to interpret the communication cost constant is that a limitation of bandwidth, a duration of time delay in communication, quality of information obtained by communication, etc.

This study can be applied with a modification when the environment is non-hostile and the cost is known. In this case, the communication cost constant can represent a physical value, such as a dollar cost, etc. For example, if there is an allocated asset or capital for the communication between each party, the asset (or budget) should be wisely used to obtain the information from the other party. If the asset is $100.00/month and the communication cost is $10.00/communication, there are only 10 communications per month allowed. Thus the system should use the communication capability when it is really required. On the other hand, if the communication cost constant is $1.00/communication, the system have 100 communications per month. This case the system can use the communication capability more frequently.
In this paper, the communication cost constant is interpreted as the probability of interception. The larger communication cost constant indicates greater risk in communication with other parties. Thus when the communication cost constant is null, the communication between parties, i.e., the host sensor and the slave sensors, are encouraged and desirable; however, as the communication cost constant increases, the exchange of information is restricted to the cases of "must communicate" only.
CHAPTER 2

Analysis of a Two-Sensor-System (2SS)

2.1. The Model and Configuration

2.1.1. Host Sensor (HS) and its Decision Maker (HDM)

The host sensor has two decision boundaries providing three decision regions. One of the boundaries is called a lower threshold (TL) and the other is called an upper threshold (TU). When an observation falls below TL, HDM will declare "No Target Detected". When an observation is between TL and TU, HDM declares "Not sure and communication necessary". Finally, when an observation falls above TU, HDM declares "Target Detected". Throughout this paper, the three decisions mentioned in the above will be denoted by a set \( U_{HS} = \{0, ?, 1\} \), respectively. These decision regions are shown in Figure 2.1. These thresholds can be varied from one mission to another, depending on the specific requirements and constraints. The thresholds control decision accuracies and the frequency of communication between the host sensor and the slave sensors. The narrower the gap between TL and TU is, the less communication between HS and SS would occur. This is because the gap between the thresholds is directly related to the uncertainty decision region in the host sensor. The decision region between TL and TU may be called a dubious region or uncertainty region.
2.1.2. Slave Sensor (SS) and its Decision Maker (SDM)

The slave sensors have a single decision threshold ($T_{SS}$) providing two decision regions. The slave sensor does not have an uncertainty region, meaning the SDMs are forced to make a decision either "0" or "1".

When an observation falls below $T_{SS}$, SDM declares "No Target Detected". When an observation falls above $T_{SS}$, SDM declares "Target Detected". In this paper these decisions are represented by a set $U_{SS} = \{0, 1\}$.

2.1.3. Host Sensor's Final Decision Threshold

When the communication between the host sensor and the slave sensors occurs, the analog data of the host sensor and the slave sensor's binary data are used to determine the final decision. The threshold for the final decision is evaluated utilizing the
binary information from the slave sensors and *a priori* probabilities. Then this final threshold is compared against the observation received by the HS to determine whether there is a target or not. The final decision is denoted by $U_F$. $FT$ can vary from one evaluation to another since $U_{SS}$ provided from the SDMs may differ from one communication to the other.

2.1.4. The Overall Process of The Model

An observation, $y_{HS}$, which is received at the host sensor, is mutually independent from the observations received by other sensors. When the observation is greater than or equal to $TU$ ($TU_2$ in Figure 2.1) or is less than or equal to $TL$ ($TL_2$ in Figure 2.1), the host sensor’s local decision, $U_{HS}$, 1 or 0, respectively, becomes the final decision, $U_F$. When $y_{HS}$ is between $TL_2$ and $TU_2$, a request of assistance from the host sensor to the slave sensor is transmitted. The slave sensor returns the local decision, $U_{SS}$, to the host sensor’s team processing unit upon the request. This communication process involves a communication cost constant (CCC). $U_{SS}$ is also determined based upon an independent observation at the slave sensor. The slave sensor makes a binary decision since it only has one decision threshold. Refer to Figure 2.2.

2.2. Definition of the System Cost Function

The system cost is defined by the total system probability of error. The following is the cost function of the system, $C(...)$, which is represented by error probabilities of the individual sensor as well as the error caused by the team process.
$C(z, y_{HS}, TL, TU, c_{HS:SS}) = (1 - z) \cdot P_{e_{HS}} + z \cdot (P_{e_{Team}} + c_{HS:SS})$ \hspace{1cm} (2.2.1)

In (2.2.1), $z$ is a value that determines whether the communication should be made or not. $z$ takes a binary number either 0 or 1. When the host sensor makes "?"
decision, \( z \) becomes 1. In case the host sensor makes a confident decision, \( z \) becomes 0. It is obvious from the expected system cost function that the team decision operation takes a role only when a communication channel is open, i.e. \( z = 1 \), between the host sensor and the slave sensor. When \( z = 0 \), meaning that there will be no communication between the host sensor and the slave sensor, the cost function becomes that of a centralized system of single sensor with a possibility of smaller error.

2.3. Evaluation of an System's Expected Cost, \( \bar{C} \)

Since the system cost function is defined, it is possible to evaluate an system's expected cost.

\[
\bar{C} = E\{C(z,TL,TU,c_{HS:ss})\}
\]

\[
= C(0,TL,TU,c_{HS:ss}) \cdot P_r(z=0) + C(1,TL,TU,c_{HS:ss}) \cdot P_r(z=1)
\]

\[
= C(0,TL,TU,c_{HS:ss}) \cdot (1-P_r(z=1)) + C(1,TL,TU,cc) \cdot P_r(z=1)
\]

\[
= C(0,TL,TU,c_{HS:ss}) + \{C(1,TL,TU,c_{HS:ss}) - C(0,TL,TU,cc)\} \cdot P_r(z=1) \tag{2.3.1}
\]

By evaluating \( C(0,...) \) and \( C(1,...) \), and using (2.2.1); the following is obtained;

\[
C(0,TL,TU,c_{HS:ss}) = P_{e_{HS}} \tag{2.3.2}
\]

\[
C(1,TL,TU,c_{HS:ss}) = P_{e_{Team}} + c_{HS:ss} \tag{2.3.3}
\]

Therefore the expression of equation becomes as follows:

\[
\bar{C} = P_{e_{HS}} + (P_{e_{Team}} + c_{HS:ss} - P_{e_{HS}}) \cdot P_r(z=1) \tag{2.3.4}
\]

This equation is further developed in detail such as;

\[
P_{e_{HS}} = P_r(\text{false local decision at HS})
\]
\[
\begin{align*}
\Pr(D_{\text{hi}} | H_0) & = \Pr(D_{\text{li}} | H_0) + \Pr(D_{\text{li}} | H_1) \\
\Pr(D_{\text{hi}} | H_1) & = \Pr(D_{\text{li}} | H_0) + \Pr(D_{\text{li}} | H_1) \\
\Pr(D_{\text{hi}} | H_0) & = \Pr(D_{\text{li}} | H_0) + \Pr(D_{\text{li}} | H_1) \\
\Pr(D_{\text{hi}} | H_1) & = \Pr(D_{\text{li}} | H_0) + \Pr(D_{\text{li}} | H_1)
\end{align*}
\]

(2.3.5)

\[
\Pr(\text{z}=1) = \Pr(\text{uncertainty in decision; communication channel open})
\]

\[
\begin{align*}
\Pr(\text{z}=1) & = \Pr(\text{TL} < \text{y}_{\text{HS}} < \text{TU}) \\
& = \Pr(\text{TL} < \text{y}_{\text{HS}} | H_0) \cdot \Pr(H_0) + \Pr(\text{TL} < \text{y}_{\text{HS}} | H_1) \cdot \Pr(H_1)
\end{align*}
\]

(2.3.6)

\[
\Pr(\text{error}) = \Pr(\text{error resulted by communication using team strategy}) = \Pr(\text{error})
\]

\[
\begin{align*}
\Pr(\text{error}) & = \Pr(\text{error} | y_{\text{HS}} \in [\text{TL}, \text{TU}]) \cdot \Pr(y_{\text{HS}} \in [\text{TL}, \text{TU}]) \\
& = \Pr(\text{error} | y_{\text{HS}} \in [\text{TL}, \text{TU}] | H_0) \cdot \Pr(H_0) + \Pr(\text{error} | y_{\text{HS}} \in [\text{TL}, \text{TU}] | H_1) \cdot \Pr(H_1)
\end{align*}
\]

(2.3.7)

where \( \Pr(H_0) \) and \( \Pr(H_1) \) are \textit{a priori} probabilities of the environment \( H_0 \) and \( H_1 \) respectively. \( U_{SS} \) is the local decision determined by the slave sensor.

In the (2.3.7) it is quite reasonable that a decision of the slave sensor, the binary data, takes a part since the communication between the host sensor and the slave sensor is established as a team effort. This is shown in the equation by giving the probability terms conditioned on the decision of the slave sensor. To evaluate

\[
\Pr(\text{error} | y_{\text{HS}} \in [\text{TL}, \text{TU}] | U_{SS}, H_i), \quad i=0,1
\]

(2.3.8)

it is necessary to compute the likelihood ratio, \( \Lambda(y_{HS}, U_{SS}) \). This probability is the probability of error induced in the host sensor due to the communication with the slave sensor (refer to (2.3.7)). Thus, the probability of error in the slave sensor will contribute to the probability of error evaluated in the host sensor.
2.4. The Likelihood Ratio Test

In evaluating the Likelihood Ratio (LR), it is assumed that the individual observations received at each sensor are independent from each other. Thus, their local decisions are also independent. In other words, the local decision of the slave sensor is statistically independent from (or not coupled with) either the local decision or the observation of the host sensor. Then the LR of this system can be written as

\[
\Lambda(y_{HS}, U_{SS}) = \frac{P_r(y_{HS}, U_{SS} | H_1)}{P_r(y_{HS}, U_{SS} | H_0)}
\]

and the above equation is re-written as follows:

\[
\Lambda(y_{HS}, U_{SS}) = \frac{P_r(y_{HS} | H_1)}{P_r(y_{HS} | H_0)} \cdot \frac{P_r(U_{SS} | H_1)}{P_r(U_{SS} | H_0)} \cdot \frac{P_r(H_1)}{P_r(H_0)}
\]

where \( \lambda_t = \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \) : a pre-calculated threshold, and

\[
C_{\alpha\beta} : \text{a cost of deciding } \alpha \text{ given } \beta
\]

For the most of cases it is assumed that a cost of making a false decision and cost of missing target are the same, i.e.

\[
C_{01} = C_{10},
\]

and this also applies to a cost of making a correct decision, for example,

\[
C_{00} = C_{11}.
\]

These conditions will give the pre-calculated threshold \( \lambda_t = 1 \). Re-writing (2.4.2) into (2.4.4), which plays an important role in evaluating (2.3.8) is obtained.
and now defining the following,

\[
g(Y_{HS}) = \frac{P_r(Y_{HS} \mid H_1)}{P_r(Y_{HS} \mid H_0)}.
\]  \hspace{1cm} \text{(2.4.5)}

\[
T_0 = \frac{P_r(H_0)}{P_r(H_1)}, \quad \text{and}
\]  \hspace{1cm} \text{(2.4.6)}

\[
f(U_{SS}) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{SS} \mid H_0)}{P_r(U_{SS} \mid H_1)}.
\]  \hspace{1cm} \text{(2.4.7)}

then (2.4.4) becomes as

\[
\frac{P_r(Y_{HS} \mid H_1)}{P_r(Y_{HS} \mid H_0)} > \lambda_t \cdot P_r(H_0) \cdot \frac{P_r(U_{SS} \mid H_0)}{P_r(U_{SS} \mid H_1)} \hspace{1cm} \text{if } U_F = 1
\]

\[
< \lambda_t \cdot P_r(H_0) \cdot \frac{P_r(U_{SS} \mid H_0)}{P_r(U_{SS} \mid H_1)} \hspace{1cm} \text{if } U_F = 0
\]  \hspace{1cm} \text{(2.4.8)}

\[g(Y_{HS})\] is the decision-statistic and \(f(U_{SS})\) is depends on the slave sensor's decision. The function \(f(U_{SS})\) represents the final threshold (FT) in the host sensor after a communication is exchanged. As is seen in (2.4.8), the function \(f(U_{SS})\) takes two different values (thresholds) depending on the decision of the slave sensor, \(U_{SS}\). Since the slave sensor is forced to make a decision based only upon its observation, \(U_{SS}\) is going to be either "0" or "1". More explicit expression of the function \(f(U_{SS})\) is

\[
f(U_{SS} = 0) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{SS} = 0 \mid H_0)}{P_r(U_{SS} = 0 \mid H_1)},
\]  \hspace{1cm} \text{(2.4.9)}

\[
f(U_{SS} = 1) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{SS} = 1 \mid H_0)}{P_r(U_{SS} = 1 \mid H_1)}.
\]  \hspace{1cm} \text{(2.4.10)}

Then, the final decision, \(U_F\), rules can be written as
All equations needed to evaluate (2.3.8) which is from (2.3.7) are obtained. By evaluating (2.3.8) further, the following is derived:

\[
P_r(E, y_{HS} \in [TL, TU]) = P_r(H_0) \cdot \sum_{U_{SS}=0}^{U_{SS}=1} P_r(E, y_{HS} \in [TL, TU] \mid U_{SS}, H_0) \cdot P_r(U_{SS} \mid H_0) \\
+ P_r(H_1) \cdot \sum_{U_{SS}=0}^{U_{SS}=1} P_r(E, y_{HS} \in [TL, TU] \mid U_{SS}, H_1) \cdot P_r(U_{SS} \mid H_1)
\]

\[
= P_r(H_0) \cdot \sum_{U_{SS}=0}^{U_{SS}=1} P_r(g(y_{HS}) > f(U_{SS}) \text{ and } y_{HS} \in [TL, TU] \mid U_{SS}, H_0) \cdot P_r(U_{SS} \mid H_0)
\]

\[
+ P_r(H_1) \cdot \sum_{U_{SS}=0}^{U_{SS}=1} P_r(g(y_{HS}) < f(U_{SS}) \text{ and } y_{HS} \in [TL, TU] \mid U_{SS}, H_1) \cdot P_r(U_{SS} \mid H_1)
\]

Thus, when (2.3.5), (2.3.6), (2.3.7), and (2.4.11) are substitute into (2.3.4), the general expression of the expected system cost is obtained.

2.5. $\overline{C}$ under Gaussian Assumption

The Gaussian distribution are applied to the probabilistic expression of $\overline{C}$ so that numerical method can be used to evaluate the $\overline{C}$ for various thresholds in the host sensor that effects system performances.

2.5.1. The $Q(y)$-function

In evaluating the probabilities which are involved in the analytical expression of the expected system cost, an integration of Gaussian probability density function is
required. We define the $Q(y)$ function to be

$$Q(y) = \int_{y}^{\infty} e^{-\frac{z^2}{2}} \, dz$$  \hspace{1cm} (2.5.1.1)$$

2.5.2. Probability Density Functions

The probability density functions under $H_0$ and $H_1$ at each sensors are written below. For the symbols used in the expression, please refer to the beginning of the thesis under "Symbols used in Chapter 2".

2.5.2.1. Gaussian PDFs at the Host Sensor

$$f_{HS_0}(y_{HS}) = \frac{1}{\sqrt{2\pi} \sigma_{HS_0}} e^{-\frac{(y_{HS} - \mu_{HS_0})^2}{2\sigma_{HS_0}^2}}$$ \hspace{1cm} (2.5.2.1.1)$$

$$f_{HS_1}(y_{HS}) = \frac{1}{\sqrt{2\pi} \sigma_{HS_1}} e^{-\frac{(y_{HS} - \mu_{HS_1})^2}{2\sigma_{HS_1}^2}}$$ \hspace{1cm} (2.5.2.1.2)$$

2.5.2.2. Gaussian PDFs at the Slave Sensor

$$f_{SS_0}(y_{SS}) = \frac{1}{\sqrt{2\pi} \sigma_{SS_0}} e^{-\frac{(y_{SS} - \mu_{SS_0})^2}{2\sigma_{SS_0}^2}}$$ \hspace{1cm} (2.5.2.2.1)$$

$$f_{SS_1}(y_{SS}) = \frac{1}{\sqrt{2\pi} \sigma_{SS_1}} e^{-\frac{(y_{SS} - \mu_{SS_1})^2}{2\sigma_{SS_1}^2}}$$ \hspace{1cm} (2.5.2.2.2)$$

2.5.3. Decision Boundary of Slave Sensor (SS)

A LRT is used to find SDM's optimal decision threshold, $T_{SS}$. 

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\[\Lambda_{ss}(y_{ss}) = \frac{P_r(H_0)}{P_r(H_1)} \cdot \frac{C_{10} - C_{00}}{C_{01} - C_{11}} = \lambda_{ss} \cdot \frac{P_r(H_0)}{P_r(H_1)} \] 

(2.5.3.1)

and the LR can also be represented as

\[\Lambda_{ss}(y_{ss}) = \frac{P_r(y_{ss} \mid H_1)}{P_r(y_{ss} \mid H_0)} = \frac{1}{\sqrt{2\pi \sigma_{ss1}}} \cdot \exp \left\{ \frac{(y_{ss} - \mu_{ss1})^2}{2\sigma_{ss1}} \right\} \]

(2.5.3.2)

Equating (2.5.3.1) and (2.5.3.2) and taking the natural logarithm on both sides of the equation, we obtain

\[
\log_e \left[ \lambda_{ss} \cdot \frac{P_r(H_0)}{P_r(H_1)} \right] = \log_e \frac{\sigma_{ss0}}{\sigma_{ss1}} + \frac{\left( \sigma_{ss1}^2 - \sigma_{ss0}^2 \right) y_{ss}^2 + 2(\mu_{ss1} \cdot \sigma_{ss0} - \mu_{ss0} \cdot \sigma_{ss1}) y_{ss} + \mu_{ss0} \cdot \sigma_{ss0} - \mu_{ss1} \cdot \sigma_{ss1}}{2\sigma_{ss0} \cdot \sigma_{ss1}}
\]

By re-arranging the terms, the above equation can be written as (2.5.3.3).

\[
(\sigma_{ss1}^2 - \sigma_{ss0}^2) y_{ss}^2 + 2(\mu_{ss1} \cdot \sigma_{ss0} - \mu_{ss0} \cdot \sigma_{ss1}) y_{ss} + \left[ \mu_{ss0} \cdot \sigma_{ss1}^2 - \mu_{ss1} \cdot \sigma_{ss0}^2 - 2\sigma_{ss0} \cdot \sigma_{ss1} \log_e \left\{ \lambda_{ss} \cdot \frac{P_r(H_0)}{P_r(H_1)} \right\} \right] = 0
\]

(2.5.3.3)

Solving (2.5.3.3) for \(y_{ss}\), the optimal threshold for the slave sensor, \(T_{ss}\) is found.

\[
T_{ss} = \frac{-(\mu_{ss1} \cdot \sigma_{ss0}^2 - \mu_{ss0} \cdot \sigma_{ss1}^2)}{(\sigma_{ss1}^2 - \sigma_{ss0}^2)} + \frac{1}{(\sigma_{ss1}^2 - \sigma_{ss0}^2)} \cdot \sqrt{(\mu_{ss1} \cdot \sigma_{ss0}^2 - \mu_{ss0} \cdot \sigma_{ss1}^2)^2 - (\sigma_{ss1}^2 - \sigma_{ss0}^2) \left[ \mu_{ss0} \cdot \sigma_{ss1}^2 - \mu_{ss1} \cdot \sigma_{ss0}^2 - 2\sigma_{ss0} \cdot \sigma_{ss1} \log_e \left\{ \lambda_{ss} \cdot \frac{P_r(H_0)}{P_r(H_1)} \right\} \right]}
\]

(2.5.3.4)

In (2.5.3.4) we require \(\sigma_{ss0}^2\) not equal \(\sigma_{ss1}^2\). For the case \(\sigma_{ss0} = \sigma_{ss1} = \sigma\), (2.5.3.3) becomes
2\sigma^4 \cdot \log e \left[ \lambda_{TSS} \cdot \frac{P_r(H_0)}{P_r(H_1)} \right] = 2\sigma^2 (\mu_{SSS} - \mu_{SSS0}) y_{SS} - (\mu_{SSS} - \mu_{SSS0})^2

Again, solving for \( y_{SS} \),

\[
T_{SS} = \frac{\mu_{SSS0} + \mu_{SSS1}}{2} + \frac{\sigma^2}{\mu_{SSS1} - \mu_{SSS0}} \log e \left[ \lambda_{TSS} \cdot \frac{P_r(H_0)}{P_r(H_1)} \right] \tag{2.5.3.5}
\]

For the numerical evaluation performed later in this chapter, the threshold for the slave sensor is evaluated by using (2.5.3.5). The decision at the slave sensor is carried out as below:

\[
U_{SS} = \begin{cases} 
0, & \text{if } y_{SS} < T_{SS} \\
1, & \text{if } y_{SS} \geq T_{SS} 
\end{cases} \tag{2.5.3.6}
\]

2.5.4. Calculation of \( \bar{C} \) for Gaussian Model

Let's represent the equations derived in the previous sections using Gaussian-distributed data.

\[
P_{eHS} = P_r(y_{HS} \geq TU \mid H_0) \cdot P_r(H_0) + P_r(y_{HS} \leq TL \mid H_1) \cdot P_r(H_1)
\]

\[
= P_r(H_0) \cdot \int_{TU}^{TL} f_{HS0}(y_{HS}) \, dy_{HS} + P_r(H_1) \cdot \int f_{HS1}(y_{HS}) \, dy_{HS}
\]

\[
= P_r(H_0) \cdot Q \left( \frac{TU - \mu_{HS0}}{\sigma_{HS0}} \right) + P_r(H_1) \cdot \left[ 1 - Q \left( \frac{TL - \mu_{HS1}}{\sigma_{HS1}} \right) \right]. \tag{2.5.4.1}
\]

\[
P_r(z=1) = P_r(TL < y_{HS} < TU \mid H_0) \cdot P_r(H_0) + P_r(TL < y_{HS} < TU \mid H_1) \cdot P_r(H_1)
\]

\[
= P_r(H_0) \cdot \int_{TL}^{TU} f_{HS0}(y_{HS}) \, dy_{HS} + P_r(H_1) \cdot \int_{TL}^{TU} f_{HS1}(y_{HS}) \, dy_{HS}
\]
\[
P_t(H_0) \cdot \left\{ Q \left[ \frac{TL - \mu_{HS}}{\sigma_{HS}} \right] - Q \left[ \frac{TU - \mu_{HS}}{\sigma_{HS}} \right] \right\} \\
+ P_t(H_1) \cdot \left\{ Q \left[ \frac{TL - \mu_{HS_1}}{\sigma_{HS_1}} \right] - Q \left[ \frac{TU - \mu_{HS_1}}{\sigma_{HS_1}} \right] \right\},
\]

and

\[
P_{true} = P_t(H_0) \cdot \sum_{y_{SS}=0}^{U_{SS}=1} P_t(g(y_{HS}) > T(U_{SS}) \text{ and } y_{HS} \in [TL, TU] \mid U_{SS}, H_0) \cdot P_t(U_{SS} \mid H_0) \\
+ P_t(H_1) \cdot \sum_{y_{SS}=0}^{U_{SS}=1} P_t(g(y_{HS}) < T(U_{SS}) \text{ and } y_{HS} \in [TL, TU] \mid U_{SS}, H_1) \cdot P_t(U_{SS} \mid H_1)
\]

\[
= P_t(H_0) \cdot \int_{T(U_{SS} = 0)}^{T(U_{SS} = 1)} f_{HS_0}(y_{HS}) \text{ dy}_{HS} \cdot \int f_{SS_0}(y_{SS}) \text{ dy}_{SS} \\
+ P_t(H_0) \cdot \int_{f(U_{SS} = 0)}^{T(U_{SS} = 1)} f_{HS_0}(y_{HS}) \text{ dy}_{HS} \cdot \int f_{SS_0}(y_{SS}) \text{ dy}_{SS}
\]

\[
+ P_t(H_1) \cdot \int_{f(U_{SS} = 0)}^{T(U_{SS} = 1)} f_{HS_1}(y_{HS}) \text{ dy}_{HS} \cdot \int f_{SS_1}(y_{SS}) \text{ dy}_{SS} \\
+ P_t(H_1) \cdot \int_{f(U_{SS} = 1)}^{T(U_{SS} = 1)} f_{HS_1}(y_{HS}) \text{ dy}_{HS} \cdot \int f_{SS_1}(y_{SS}) \text{ dy}_{SS}
\]

\[
= P_t(H_0) \cdot \left\{ Q \left[ \frac{f(U_{SS} = 0) - \mu_{HS_0}}{\sigma_{HS_0}} \right] - \frac{T_{SS} - \mu_{SS_0}}{\sigma_{SS_0}} \right\} \cdot \left\{ 1 - \frac{T_{SS} - \mu_{SS_0}}{\sigma_{SS_0}} \right\} \\
+ P_t(H_0) \cdot \left\{ Q \left[ \frac{f(U_{SS} = 1) - \mu_{HS_0}}{\sigma_{HS_0}} \right] \right\} \cdot \left\{ T_{SS} - \mu_{SS_0} \right\} \\
+ P_t(H_0) \cdot \left\{ \frac{T_{SS} - \mu_{SS_0}}{\sigma_{SS_0}} \right\} \\
+ P_t(H_0) \cdot \left\{ \frac{T_{SS} - \mu_{SS_0}}{\sigma_{SS_0}} \right\}
\]
Substituting (2.5.4.1), (2.5.4.2), and (2.4.5.3) into (2.3.4), the expected system cost is obtained.

2.6. Numerical Evaluation of $\bar{C}$

In the previous section $\bar{C}$ is represented in terms of $Q(y)$-function, which makes possible to evaluate $\bar{C}$ numerically. The purpose of the numerical evaluation is to determine the expected system cost at the various thresholds position in the host sensor, meaning the position of $TL$ and $TU$, so that the optimal thresholds for the system can be realized with different communication cost constants. At the optimal thresholds the expected system cost is minimum.

The a priori probabilities of the environment are considered equiprobable, $P_r(H_0) = P_r(H_1) = 0.5$. For the statistics of the observations we take $\sigma_{HS_0} = \sigma_{HS_1} = \sigma_{SS_0} = \sigma_{SS_1} = \sigma = 1$. The mean values of the observations at each sensor are $\mu_{HS_0} = \mu_{SS_0} = -1$ and $\mu_{HS_1} = \mu_{SS_1} = 1$. The communication cost constant is held a constant value until all the expected system costs are evaluated at the desired threshold positions. The thresholds, $TL$ and $TU$, are varied with a relationship of $TU = -TL$. This threshold relationship is selected because of the symmetric nature of Gaussian PDF and its observations. The results from the numerical
evaluation of $\bar{C}$ are plotted in Figure 2.3, Figure 2.4, Figure 2.5, and Figure 2.6. For the tabulated data of Figure 2.5, please refer to Table 5.1 in Chapter 5. The computer program is written for (2.3.4), substituted with $P_{e_{HS}}$, $P_{e_{T}(z=1)}$, and $P_{e_{Team}}$, which are expressed in (2.5.4.1), (2.5.4.2), and (2.5.4.3), respectively. The program is listed in Appendix A.

2.6.1. Comments on Numerical Evaluation

The system expected costs are evaluated over the threshold positions on the host sensor's observation space for a given communication cost constant. The following figures are plotted from the results obtained through the numerical evaluation of $\bar{C}$.

Figure 2.3 shows the expected system cost as the threshold is departing from the origin (position 0.0) for each communication cost constant, $CCC_1$. TU moves in the positive direction and TL moves in the negative direction. When $CCC_1 \geq 0.5$, in the curve of the expected system cost vs. HS threshold position, the expected system cost never gets smaller than the cost at threshold position 0.0. The dotted curve indicates the maximum $CCC_1$ which has a minimum other than at the threshold position of 0.0. Figure 2.4 is the enlarged version of Figure 2.3, where the minima of the curves are shown clearly.

Figure 2.5 is a plot of extracted information from Figure 2.3. It shows the behavior of the minimum expected system cost due to the change of the communication cost constant.

Figure 2.6 involves all the vital information obtained in this evaluation. It represents the 2SS's minimum expected system cost vs. the optimum threshold.
position and the communication cost constant.

More detailed observation of these data is performed in Chapter 5 where the systems are compared.
Two-Sensor-System

Figure 2.3 Expected System Cost vs. HS Threshold Position
Figure 2.4 Expected System Cost vs. HS Threshold Position
(Enlarged Version of Figure 2.3)

Two-Sensor-System

$c_{c1} = $0.25
$c_{c1} = $0.20
$c_{c1} = $0.15
$c_{c1} = $0.10
$c_{c1} = $0.05
$c_{c1} = $0.00

$c_{c1}$: One-Sensor Communication Cost Constant

HS Threshold Position

Expected System Cost
Two-Sensor-System

Figure 2.5 Min. Expected System Cost vs. Communication Cost Constant

Figure 2.6 Summary of Data
3.1. The Model and Configuration

The environment and other elements in modeling this system are closely related to those in Chapter 2, the two-sensor-system. The only difference in this chapter is that the host sensor communicates with two slave sensors, instead of one, when the host sensor makes an uncertain decision. The system cost is also defined the same way as in (2.2.1) of Chapter 2. Thus, the expected system cost expression is the same as (2.3.4). All the expressions of terms in (2.3.4) are directly applied for this system.

3.1.1. Host Sensor’s Decision Boundaries

The design of thresholds in the host sensor in the three-sensor-system is very similarly done as in the two-sensor-system (refer to Figure 3.1). The thresholds divide the observation space into three decision regions, No Target (0), No Decision (?), and Target Detected (1).

3.1.2. The Overall Process of The Model

The host sensor’s confident local decision, either 0 or 1, will become the final decision of the system. In case the decision of the host sensor is dubious ($y_{HS}$ falls between TL and TU, or TL3 and TU3 in Figure 3.1), the host sensor will request
binary informations, $U_{SS1}$ and $U_{SS2}$, from both of the slave sensors, which are also independently generated according to their observation, $y_{SS1}$ and $y_{SS2}$, at the slave sensor 1 (SS1) and the slave sensor 2 (SS2). These communication process also involves a communication cost constant as in 2SS. The illustration of the process is in Figure 3.2.

3.2. Evaluation of Error Caused by Team Strategies

$P_{e_{Team}}$ of 3SS has the same probabilistic expression as that of 2SS except now the expression is conditioned on two slave sensors, not one. The expression is shown below.

$$P_{e_{Team}} = P_t(error\ resulted\ by\ communication\ using\ team\ strategy) = P_t(E)$$
$y_{SS1}$

Environment
$H_0 \& H_1$

$y_{HS}$

Host Sensor (HS)

HS Decision Maker (HDM)

Local Decision, $U_{HS}$

$0$ or $1$

Final Decision, $U_F$

$0$ or $1$

$y_{SS2}$

Slave Sensor 2 (SS2)

SS2 Decision Maker

Local Decision, $U_{SS2}$

$0$ or $1$

$y_{SS1}$

Slave Sensor 1 (SS1)

SS1 Decision Maker

Local Decision, $U_{SS1}$

$0$ or $1$

Local Decision of SS1 & SS2 Returned

Information Requested

Team Decision Process

Figure 3.2 Model Configuration of 3SS
+ P(E, y_{HS} \in \{TL, TU\} | H_1, U_{SS1}, U_{SS2}) \cdot P_r(U_{SS1} | H_1) \cdot P_r(U_{SS2} | H_1) \cdot P_r(H_1) \tag{3.2.1}

3.3. The Likelihood Ratio Test

In evaluating the Likelihood Ratio Test (LRT), it is assumed that the individual observations received at each sensor are independent from each other. Thus, their local decisions are also independent from other sensors. In other words, the local decision of the slave sensors is statistically independent from (or not coupled with) either the local decision or the observation of the host sensor.

\[
A(y_{HS}, U_{SS1}, U_{SS2}) = \frac{P_r(U_{HS} \cdot U_{SS1}, U_{SS2} | H_1)}{P_r(U_{HS} | H_0) \cdot P_r(U_{SS1} | H_0) \cdot P_r(U_{SS2} | H_0) \cdot P_r(H_0)} \begin{cases} \text{UF} = 1 & \text{UF} > \lambda_t \\ \text{UF} = 0 & \text{UF} < \lambda_t \end{cases} \tag{3.3.1}
\]

Using the above assumption, (3.3.1) can be written as following.

\[
\frac{P_r(U_{HS} | H_1) \cdot P_r(U_{SS1} | H_1) \cdot P_r(U_{SS2} | H_1) \cdot P_r(H_1)}{P_r(U_{HS} | H_0) \cdot P_r(U_{SS1} | H_0) \cdot P_r(U_{SS2} | H_0) \cdot P_r(H_0)} \begin{cases} \text{UF} = 1 & \text{UF} > \lambda_t \\ \text{UF} = 0 & \text{UF} < \lambda_t \end{cases} \tag{3.3.2}
\]

\[
\frac{P_r(U_{HS} | H_1)}{P_r(U_{HS} | H_0)} \begin{cases} \text{UF} = 1 & \text{UF} > \lambda_t \cdot \frac{P_r(U_{SS1} | H_0)}{P_r(U_{SS1} | H_1)} \cdot \frac{P_r(U_{SS2} | H_0)}{P_r(U_{SS2} | H_1)} \cdot \frac{P_r(H_0)}{P_r(H_1)} \\ \text{UF} = 0 & \text{UF} < \lambda_t \end{cases} \tag{3.3.3}
\]

\[
f(U_{SS1}, U_{SS2}) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{SS1} | H_0)}{P_r(U_{SS1} | H_1)} \cdot \frac{P_r(U_{SS2} | H_0)}{P_r(U_{SS2} | H_1)} \tag{3.3.4}
\]

\[
T_0 = \frac{U_{F}}{g(y_{HS})} \begin{cases} \text{UF} = 1 & \text{UF} > f(U_{SS1}, U_{SS2}) \\ \text{UF} = 0 & \text{UF} < f(U_{SS1}, U_{SS2}) \end{cases}
\]

\[
T_0 \text{ is a ratio of a priori probabilities. The function } f(U_{SS1}, U_{SS2}) \text{ represents the final threshold in the host sensor after communication between the host sensor and the}
\]
slave sensors. As it is seen in (3.3.3), the function $f(U_{ss1}, U_{ss2})$ can have four different values (thresholds) depending on the decision of the slave sensors, $U_{ss1}$ and $U_{ss2}$, since the decisions of SS, $U_{ss1}$, $U_{ss2}$ are always either "0" or "1". This gives more explicit expression of the function $f(U_{ss1}, U_{ss2})$ which is listed below.

$$f(U_{ss1}=0, U_{ss2}=0) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{ss1}=0|H_0)}{P_r(U_{ss1}=0|H_1)} \cdot \frac{P_r(U_{ss2}=0|H_0)}{P_r(U_{ss2}=0|H_1)} \quad (3.3.5)$$

$$f(U_{ss1}=0, U_{ss2}=1) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{ss1}=1|H_0)}{P_r(U_{ss1}=0|H_1)} \cdot \frac{P_r(U_{ss2}=1|H_0)}{P_r(U_{ss2}=1|H_1)} \quad (3.3.6)$$

$$f(U_{ss1}=1, U_{ss2}=0) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{ss1}=1|H_0)}{P_r(U_{ss1}=1|H_1)} \cdot \frac{P_r(U_{ss2}=0|H_0)}{P_r(U_{ss2}=0|H_1)} \quad (3.3.7)$$

$$f(U_{ss1}=1, U_{ss2}=1) = \lambda_t \cdot T_0 \cdot \frac{P_r(U_{ss1}=1|H_0)}{P_r(U_{ss1}=1|H_1)} \cdot \frac{P_r(U_{ss2}=1|H_0)}{P_r(U_{ss2}=1|H_1)} \quad (3.3.8)$$

Using (3.3.4), we can write the final decision, $U_F$, rules of the system as

$$U_F = \begin{cases} 
1 & \text{if } y_{HS} \geq TU \\
1 & \text{if } g(y_{HS}) \geq f(U_{ss1}, U_{ss2}) \\
0 & \text{if } g(y_{HS}) < f(U_{ss1}, U_{ss2}) \\
0 & \text{if } y_{HS} < TL 
\end{cases}$$

Then, it is possible to express $P_{e_{team}}$ as shown below.

$$P_r(E, \ y_{HS} \in [TL, \ TU])$$

$$= P_r(H_0) \cdot \sum_{U_{ss1}=1}^{U_{ss1}=1} \sum_{U_{ss2}=0}^{U_{ss2}=0} P_r(E, \ y_{HS} \in [TL, \ TU] | U_{ss1}, U_{ss2}, H_0) \cdot P_r(U_{ss1} \ | H_0) \cdot P_r(U_{ss2} \ | H_0)$$

$$- P_r(H_0) \cdot \sum_{U_{ss1}=0}^{U_{ss1}=0} \sum_{U_{ss2}=0}^{U_{ss2}=0} P_r(E, \ y_{HS} \in [TL, \ TU] | U_{ss1}, U_{ss2}, H_1) \cdot P_r(U_{ss1} \ | H_1) \cdot P_r(U_{ss2} \ | H_1)$$
\[ P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} \sum_{U_{SS2}=0}^{U_{SS2}=1} P_r(g(y_{HS})) U_r=1 > + P_r(H_1) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} \sum_{U_{SS2}=0}^{U_{SS2}=1} P_r(g(y_{HS})) U_r=0 > \]

\[ f(U_{SS1}, U_{SS2}) \text{ and } y_{HS} \in [TL, TU] \mid U_{SS1}, U_{SS2}, H_0 \} \cdot P_r(U_{SS1} \mid H_0) \cdot P_r(U_{SS2} \mid H_0) \]

\[ f(U_{SS1}, U_{SS2}) \text{ and } y_{HS} \in [TL, TU] \mid U_{SS1}, U_{SS2}, H_1 \} \cdot P_r(U_{SS1} \mid H_1) \cdot P_r(U_{SS2} \mid H_1) \]  

(3.3.9)

3.4. Calculation of $\overline{C}$ for Gaussian Models

As in Chapter 2, the Gaussian distribution function is used to give examples in expressing $\overline{C}$ so that it can be evaluated numerically.

3.4.1. Gaussian Probability Density Function

The probability density functions are shown below. They show PDF of "0" and "1" at HS, SS1, and SS2.

\[ f_{HS0}(y_{HS}) = \frac{1}{\sqrt{2\pi} \sigma_{HS0}} e^{-\frac{(y_{HS} - y_{HS0})^2}{2\sigma_{HS0}^2}} \]  

(3.4.1.1)

\[ f_{HS1}(y_{HS}) = \frac{1}{\sqrt{2\pi} \sigma_{HS1}} e^{-\frac{(y_{HS} - y_{HS1})^2}{2\sigma_{HS1}^2}} \]  

(3.4.1.2)

\[ f_{SS10}(y_{SS1}) = \frac{1}{\sqrt{2\pi} \sigma_{SS10}} e^{-\frac{(y_{SS1} - y_{SS10})^2}{2\sigma_{SS10}^2}} \]  

(3.4.1.3)

\[ f_{SS11}(y_{SS1}) = \frac{1}{\sqrt{2\pi} \sigma_{SS11}} e^{-\frac{(y_{SS1} - y_{SS11})^2}{2\sigma_{SS11}^2}} \]  

(3.4.1.4)

\[ f_{SS20}(y_{SS2}) = \frac{1}{\sqrt{2\pi} \sigma_{SS20}} e^{-\frac{(y_{SS2} - y_{SS20})^2}{2\sigma_{SS20}^2}} \]  

(3.4.1.5)
3.4.2. Decision Boundary of SS1 & SS2

An equation of the decision boundary for the SSs was derived, (2.5.3.5), in Chapter 2. The analytical expression of threshold in SS1 and SS2 follows the same as in Chapter 2.

\[
T_{SS1} = \frac{\mu_{SS10} + \mu_{SS11}}{2} + \frac{\sigma^2}{2\mu_{SS11} - \mu_{SS10}} \cdot \log_e \left\{ \frac{\lambda_{SSS1}}{P_r(H_0)} \cdot \frac{P_r(H_1)}{1 - P_r(H_1)} \right\} \quad (3.4.2.1)
\]

\[
T_{SS2} = \frac{\mu_{SS20} + \mu_{SS21}}{2} + \frac{\sigma^2}{2\mu_{SS21} - \mu_{SS20}} \cdot \log_e \left\{ \frac{\lambda_{SSS2}}{P_r(H_0)} \cdot \frac{P_r(H_1)}{1 - P_r(H_1)} \right\} \quad (3.4.2.2)
\]

Then, the decisions at the slave sensors are stated as below:

For SS1,

\[
U_{SS1} = \begin{cases} 
0, & \text{if } y_{SS1} < T_{SS1} \\
1, & \text{if } y_{SS1} \geq T_{SS1}
\end{cases} \quad (3.4.2.3)
\]

and for SS2,

\[
U_{SS2} = \begin{cases} 
0, & \text{if } y_{SS2} < T_{SS2} \\
1, & \text{if } y_{SS2} \geq T_{SS2}
\end{cases} \quad (3.4.2.4)
\]

Since the probability of error incurred by communication and the final decision boundaries are different from Chapter 2, they are written in this section. The Gaussian expression of the FTs are written:
The corresponding probability of error in the team process is, then,

\[
P_{e_{\text{team}}} = P_t(\text{error resulted by communicating with all SSs using team strategy})
\]

\[
= P_t(H_0) \cdot \left\{ \int_{T_{t_1}} f_{SS10}(y_{SS1}) dy_{SS1} \cdot \int_{T_{t_2}} f_{SS20}(y_{SS2}) dy_{SS2} \cdot \int f_{HS10}(y_{HS}) dy_{HS} \cdot f(T_{t_1} = 0, \ U_{t_1} = 0) \right\} + \int_{T_{t_1}} f_{SS10}(y_{SS1}) dy_{SS1} \cdot \int_{T_{t_2}} f_{SS20}(y_{SS2}) dy_{SS2} \cdot \int f_{HS10}(y_{HS}) dy_{HS} \cdot f(T_{t_1} = 0, \ U_{t_1} = 1) \right\}
\]
\[
\begin{align*}
&= \mathbb{P}(H_0) \cdot \\
&\quad \cdot \left( 1 - Q \left( \frac{T_{SS11} - \mu_{SS11}}{\sigma_{SS11}} \right) \right) \cdot \left( 1 - Q \left( \frac{T_{SSS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \right) \cdot \mathbb{Q} \left( \frac{f(U_{SS11}=0, U_{SS2}=1) - \mu_{HIS0}}{\sigma_{HIS0}} \right) \\
&\quad + \left( 1 - Q \left( \frac{T_{SS11} - \mu_{SS11}}{\sigma_{SS11}} \right) \right) \cdot Q \left( \frac{T_{SSS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \cdot \mathbb{Q} \left( \frac{f(U_{SS11}=0, U_{SS2}=1) - \mu_{HIS0}}{\sigma_{HIS0}} \right) \\
&\quad + Q \left( \frac{T_{SS11} - \mu_{SS11}}{\sigma_{SS11}} \right) \cdot \left( 1 - Q \left( \frac{T_{SSS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \right) \cdot \mathbb{Q} \left( \frac{f(U_{SS11}=1, U_{SS2}=0) - \mu_{HIS0}}{\sigma_{HIS0}} \right)
\end{align*}
\]
Thus, $\bar{C}$ of 3SS in the Gaussian case is obtained by substituting (3.4.2.9), (2.5.4.1), and (2.5.4.2) into (2.3.4). A program listing is attached in Appendix B.

3.4.3. Numerical Evaluation of $\bar{C}$

The method of performing the numerical evaluation for $\bar{C}$ is quite similar to those done in Chapter 2. The parameter values used in this section are as follows: All the standard deviations, $\sigma$, are set to 1.0, the mean of "0" observation is -1.0, and the mean of "1" observation is 1.0. The communication cost constant is varied from 0.0 to 1.0 in steps of 0.05. The thresholds in the host sensor are moved away from the origin. TL moves to the negative direction and TU moves to the positive direction with the relationship of $TU = -TL$. 

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3.4.4. Comments on Numerical Evaluation

The results of the numerical evaluation are plotted in Figure 3.3, Figure 3.4 and Figure 3.5. Figure 3.3 shows that the curves of the system expected cost over the threshold positions with different communication cost constant (CCC1). When CCC1 is greater or equal to 0.55, the curves are monotonically increasing, giving minima at the threshold position of 0.0. Figure 3.4 is an enlarged version of Figure 3.3 which shows the minima of the curves with CCC1 less than 0.55 clearly. Figure 3.5 can be interpreted that the minimum expected system cost increases as CCC1 increases; however, near CCC1 = 0.5, the minimum expected system cost tends to be flattening since the thresholds (TL and TU) are collapsed into the threshold position of 0.0. This is shown in Figure 3.6 where we plot the minimum expected system cost vs. communication cost constant and vs. the optimal threshold position. More discussion of the results are carried in Chapter 5.
Figure 3.3 Expected System Cost vs. HS Threshold Position
Three-Sensor-System

![Graph showing the expected system cost vs. HS threshold position.](image)

**Figure 3.4** Expected System Cost vs. HS Threshold Position
(Enlarged Version of Figure 3.3)
Three-Sensor-System

Figure 3.5 Min. Expected System Cost vs. Communication Cost Constant

Minimum Expected System Cost

Optimum HS Threshold Position

Figure 3.6 Summary of Data
4.1. The Model and Configuration

The difference between this chapter and the previous chapters is that here the host sensor chooses a communication scheme, based on quality of its own decision. For example, when the host sensor's observation, $y_{HS}$, falls in a certain region of uncertainty, it communicates with only one slave sensor. It communicates with two slave sensors when the observation falls in the other region of uncertainty. Contrary to the previous chapters, two different communication cost constants are considered; one for communicating with one slave sensor, and another for communicating with two slave sensors.

4.1.1. The Host Sensor's Thresholds and Decision Regions

In the host sensor's observation space, there are four thresholds that divide the space into four decision regions. (Actually, there are five decision regions but two out of five regions yield the same decision.) When $y_{HS}$ falls below TL1 (TL31 in Figure 4.1) or above TU1 (TU31 in Figure 4.1), the host sensor decides 0 or 1, respectively. When the observation falls between TL1 and TL2 (TL31 and TL32 in Figure 4.1) or between TU2 and TU1 (TU32 and TU31 in Figure 4.1), the host sensor's decision, $U_{HS}$, becomes uncertain (?1). In case of the observation lies between TL2 and TU2 (TL32 and TU32 in Figure 4.1), the host sensor makes a dubi-
ous decision (?2). Thus there are level of confidence in making uncertain decision. In ?1 decision region, the probability of making a correct decision is much greater than the probability of making a false decision; then, a minimum help from the slave sensors is needed. In ?2 decision region, the probability of making a correct decision and the probability of making a false detection are compatible; thus, this situation requires more information to make a correct decision.

When the host sensor makes a binary decision (either 0 or 1), it becomes the final decision of the system. In case the decision of the host sensor is ?1, the host sensor requests information only from one of the slave sensors, say slave sensor 1 (SS1). On the other hand, when the host sensor determines ?2, it asks an assistance from both of the slave sensors, slave sensor 1 (SS1) and slave sensor 2 (SS2).

\[
\begin{array}{|c|c|c|c|c|}
\hline
U_{HS} = 0 & U_{HS} = ?1 & U_{HS} = ?2 & U_{HS} = ?1 & U_{HS} = 1 \\
\hline
\text{TL31} & \text{TL32} & \text{TU32} & \text{TU31} & y_{HS} \\
\hline
\end{array}
\]

**Figure 4.1 Decision Boundaries of HS for 2/3SS**
This process is illustrated in Figure 4.2.

Figure 4.2 Model Configuration of 2/3SS
4.2. Definition of the System Cost Function

The following is the cost function of the system, \( C(\ldots) \), which is represented by error probabilities of the individual sensors. For descriptions of symbols used in this chapter, please refer to the beginning of this paper under "Symbols used in Chapter 4".

\[
C(f) = C(ZT_1, ZT_2, TL_1, TL_2, TU_2, TU_1, c_{T_1}, c_{T_2})
= (1 - ZT_1) \cdot (1 - ZT_2) \cdot P_{e_{HS}}
+ ZT_1 \cdot (1 - ZT_2) \cdot (P_{e_{T_1}} + c_{T_1})
+ ZT_2 \cdot (1 - ZT_1) \cdot (P_{e_{T_2}} + c_{T_2})
\]

(4.2.1)

As shown in the above equation, the communication schemes are dependent upon the values of \( ZT_1 \) and \( ZT_2 \). \( ZT_1 \) and \( ZT_2 \) take binary numbers depending on the type of host sensor's uncertain decision. When \( U_{HS} = 1 \), \( ZT_1 \) becomes 1. When \( U_{HS} = 2 \), \( ZT_2 \) becomes 1. This is shown in the table below.

<table>
<thead>
<tr>
<th>Communication Scheme</th>
<th>( ZT_1 )</th>
<th>( ZT_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Communication</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Communication with SS1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Communication with SS1 &amp; SS2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.1 Communication Scheme of 2/3SS

4.3. Evaluation of an Expected System's Total Cost, \( \bar{C} \)

Let's evaluate the expected cost of the system.

\[
\bar{C} = E(C(f))
\]
\[ C(f)_{z_{T1}=0, z_{T2}=0} \cdot P_r(z_{T1}=0) \cdot P_r(z_{T2}=0) + C(f)_{z_{T1}=1, z_{T2}=0} \cdot P_r(z_{T1}=1) \cdot P_r(z_{T2}=0) + C(f)_{z_{T1}=1, z_{T2}=1} \cdot P_r(z_{T1}=1) \cdot P_r(z_{T2}=1) \]

\[ = P_{e_{HS}} \cdot P_r(z_{T1}=0) \cdot P_r(z_{T2}=0) + (P_{e_{T1}} + c_{T1}) \cdot P_r(z_{T1}=1) \cdot P_r(z_{T2}=0) + (P_{e_{T2}} + c_{T2}) \cdot P_r(z_{T1}=1) \cdot P_r(z_{T2}=1) \]

\[ = P_{e_{HS}} + (P_{e_{T1}} + c_{T1} - P_{e_{HS}}) \cdot P_r(z_{T1}=1) + (P_{e_{T2}} + c_{T2} - P_{e_{HS}}) \cdot P_r(z_{T2}=1) + (P_{e_{HS}} - P_{e_{T1}} - P_{e_{T2}} - c_{T1} - c_{T2}) \cdot P_r(z_{T1}=1) \cdot P_r(z_{T2}=1) \]

The terms, \( P_{e_{HS}}, P_r(z_{T1}=1), \) and \( P_r(z_{T2}=1), \) are written in generalized probabilistic expressions:

\[ P_{e_{HS}} = P_r(\text{false local decision at HS}) \]

\[ = P_r(\text{Decide } H_1 | H_0) \cdot P_r(H_0) + P_r(\text{Decide } H_0 | H_1) \cdot P_r(H_1) \]

\[ = P_r(y_{HS} \geq TU1 | H_0) \cdot P_r(H_0) + P_r(y_{HS} \leq TL1 | H_1) \cdot P_r(H_1) \]

(4.3.2)

\[ P_r(z_{T1}=1) = P_r(\text{uncertain decision } = ?1; \text{ communication channel open only with SS1}) \]

\[ = P_r(TL1 < y_{HS} < TL2) + P_r(TL2 < y_{HS} < TU1) \]

\[ = P_r(H_0) \cdot (P_r(TL1 < y_{HS} < TL2 | H_0) \cdot P_r(H_0) + P_r(TL2 < y_{HS} < TU1 | H_0)) \]

\[ + P_r(H_1) \cdot (P_r(TL1 < y_{HS} < TL2 | H_1) \cdot P_r(H_0) + P_r(TL2 < y_{HS} < TU1 | H_1)) \]

(4.3.3)

\[ P_r(z_{T2}=1) = P_r(\text{uncertain decision } = ?2; \text{ communication channel open with SS1 & SS2}) \]

\[ = P_r(TL2 < y_{HS} < TU2) \]

\[ = P_r(TL2 < y_{HS} < TU2 | H_0) \cdot P_r(H_0) + P_r(TL2 < y_{HS} < TU2 | H_1) \cdot P_r(H_1) \]

(4.3.4)
There are two different costs (probability of error) incurred in communication, $P_{eT1}$ and $P_{eT2}$, since the system has two different modes of communication, communicating with one slave sensor (SS1), or with two slave sensors (SS1 and SS2), respectively.

$$P_{eT1} = P_r(\text{error resulting after communication with SS1 only}) = P_r(E1)$$

$$= P_r(E1 | y_{HS} \in [TL1,TL2] \text{ or } [TU2,TU1]) \cdot P_r(y_{HS} \in [TL1,TL2] \text{ or } [TU2,TU1])$$

$$= P_r(E1, y_{HS} \in [TL1,TL2] \text{ or } [TU2,TU1] | H_0) \cdot P_r(H_0)$$

$$+ P_r(E1, y_{HS} \in [TL1,TL2] \text{ or } [TU2,TU1] | H_1) \cdot P_r(H_1)$$

$$= P_r(E1, y_{HS} \in [TL1,TL2] \text{ or } [TU2,TU1] | H_0, U_{SS1}) \cdot P_r(U_{SS1} | H_0) \cdot P_r(H_0)$$

$$+ P_r(E1, y_{HS} \in [TL1,TL2] \text{ or } [TU2,TU1] | H_1, U_{SS1}) \cdot P_r(U_{SS1} | H_1) \cdot P_r(H_1)$$

(4.3.5)

$$P_{eT2} = P_r(\text{error resulting after communication with SS1 & SS2}) = P_r(E2)$$

$$= P_r(E2 | y_{HS} \in [TL2,TU2]) \cdot P_r(y_{HS} \in [TL2,TU2])$$

$$= P_r(E2, y_{HS} \in [TL2,TU2] | H_0) \cdot P_r(H_0) + P_r(E2, y_{HS} \in [TL2,TU2] | H_1) \cdot P_r(H_1)$$

$$= P_r(E2, y_{HS} \in [TL2,TU2] | H_0, U_{SS1}, U_{SS2}) \cdot P_r(U_{SS1} | H_0) \cdot P_r(U_{SS2} | H_0) \cdot P_r(H_0)$$

$$+ P_r(E2, y_{HS} \in [TL2,TU2] | H_1, U_{SS1}, U_{SS2}) \cdot P_r(U_{SS1} | H_1) \cdot P_r(U_{SS2} | H_1) \cdot P_r(H_1)$$

(4.3.6)

4.4. The Likelihood Ratio Test

In this chapter, it is necessary to evaluate two kinds of LRT, since the LRT for the different communication schemes differs. These evaluations closely follow those derived in Chapters 2 and 3.
4.4.1. LRT for Communicating with SS1 Only

Let LR of this case be

$$\Lambda_{T1}(y_{HS}, U_{SS1}) = \frac{P_r(y_{HS}, U_{SS1} | H_1)}{P_r(y_{HS}, U_{SS1} | H_0)}$$  (4.4.1.1)

Since the observations received at different sensors are mutually independent, (4.4.1.1) can be written as

$$\Lambda_{T1}(y_{HS}, U_{SS1}) = \frac{P_r(y_{HS} | H_1) \cdot P_r(U_{SS1} | H_1) \cdot P_r(H_1)}{P_r(y_{HS} | H_0) \cdot P_r(U_{SS1} | H_0) \cdot P_r(H_0)}$$

Thus,

$$\frac{P_r(y_{HS} | H_1)}{P_r(y_{HS} | H_0)} > \lambda_t \cdot \frac{P_r(H_0)}{P_r(H_1)} \cdot \frac{P_r(U_{SS1} | H_0)}{P_r(U_{SS1} | H_1)}$$  (4.4.1.2)

Recalling the definitions made in Chapter 2, (2.4.5), (2.4.6), and (2.4.7), then (4.4.1.3) can be written as below, provided that we substitute $g(y_{HS}) = g_{T1}(y_{HS})$ and $f(U_{SS}) = f(U_{SS1})$.

$$U_F = 1$$

$$g_{T1}(y_{HS}) < f(U_{SS1})$$  (4.4.1.4)

The function $f(U_{SS1})$ represents the final threshold at the host sensor after communication with one slave sensor, SS1. $f(U_{SS1})$ can be two different values (thresholds) depending on the decision of the slave sensor, $U_{SS1}$. More explicit expression of the function $f(U_{SS1})$ is listed below.
Then, the probability of error caused by the team process with SS1 only can be expressed in probabilistic terms as below.

\[
P_r(E_1, y_{HS} \in [TL1, TL2] \text{ or } [TU2, TU1])
\]

\[
= P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} P_r(E_1, y_{HS} \in [TL1, TL2] \text{ or } [TU2, TU1] \mid U_{SS1}, H_0) \cdot P_r(U_{SS1}=1 \mid H_0)
\]

\[
+ P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} P_r(E_1, y_{HS} \in [TL1, TL2] \text{ or } [TU2, TU1] \mid U_{SS1}, H_1) \cdot P_r(U_{SS1}=1 \mid H_1)
\]

\[
= P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} P_r(g(Y_{HS}) > f(U_{SS1}) \text{ and } y_{HS} \in [TL1, TL2] \text{ or } [TU2, TU1] \mid U_{SS1}, H_0) \cdot P_r(U_{SS1}=1 \mid H_0)
\]

\[
+ P_r(H_1) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} P_r(g(Y_{HS}) > f(U_{SS1}) \text{ and } y_{HS} \in [TL1, TL2] \text{ or } [TU2, TU1] \mid U_{SS1}, H_1) \cdot P_r(U_{SS1}=1 \mid H_1)
\] (4.4.1.7)

4.4.2. LRT for Communicating with SS1 and SS2

This section is very similar to the section 3.3 of Chapter 3. The LRT when communicating with two slave sensors had been derived in Chapter 3. Adapting (3.3.1) and (3.3.2), we obtain \(\Lambda_{T2}(y_{HS}, U_{SS1}, U_{SS2}) = \Lambda(y_{HS}, U_{SS1}, U_{SS2})\). The (3.3.3) can be directly applicable in this section as well. By replacing \(g(y_{HS})\) in (3.3.4) with \(g_{T2}(y_{HS})\), we have the description of the two-helper LRT as
From (4.4.1.4) and (4.4.2.1), the final decision, $U_F$, rule of the system can be written as

$$U_F = \begin{cases} 
1 & \text{if } y_{HS} \geq TU1 \\
1 & \text{if } g_{T2}(y_{HS}) \geq f(U_{SS1}, U_{SS2}) \\
1 & \text{if } g_{T1}(y_{HS}) \geq f(U_{SS1}) \\
0 & \text{if } g_{T1}(y_{HS}) < f(U_{SS1}) \\
0 & \text{if } g_{T2}(y_{HS}) < f(U_{SS1}, U_{SS2}) \\
0 & \text{if } y_{HS} < TL1
\end{cases}$$

An explicit expression of the final threshold, $f(U_{SS1}, U_{SS2})$, is dependent upon the decision on the slave sensors as mentioned in Chapter 3. The explicit expressions are given in section 3.3, (3.3.5), (3.3.6), (3.3.7), and (3.3.8). Then, it is possible to express the probability of error caused by using data from two slave sensors. This is shown in (4.4.2.2).

$$P_r(E2, y_{HS} \in \{TL2, TU2\})$$

$$= P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} \sum_{U_{SS2}=0}^{U_{SS2}=1} P_r(E2, y_{HS} \in \{TL2, TU2\} | U_{SS1}, U_{SS2}, H_0) \cdot P_s(U_{SS1} | H_0) \cdot P_s(U_{SS2} | H_0)$$

$$+ P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} \sum_{U_{SS2}=0}^{U_{SS2}=1} P_r(E2, y_{HS} \in \{TL2, TU2\} | U_{SS1}, U_{SS2}, H_1) \cdot P_s(U_{SS2} | H_1) \cdot P_s(U_{SS2} | H_1)$$

$$= P_r(H_0) \cdot \sum_{U_{SS1}=0}^{U_{SS1}=1} \sum_{U_{SS2}=0}^{U_{SS2}=1} P_r(g(y_{HS}) >$$

$$f(U_{SS1}, U_{SS2}) \text{ and } y_{HS} \in \{TL2, TU2\} | U_{SS1}, U_{SS2}, H_0) \cdot P_s(U_{SS1} | H_0) \cdot P_s(U_{SS2} | H_0)$$
\[ f(USS_1, USS_2) \text{ and } y_{HS} \in [TL_2, TU_2] | USS_1, USS_2, H_1) \cdot P_Y(USS_1 | H_1) \cdot P_Y(USS_2 | H_1) \] (4.4.2.2)

4.5. Calculation of $\bar{C}$ under Gaussian Models

We again assume the probability density function on the observation of the host sensor and the slave sensors given in Section 3.4.1 of Chapter 3. The decision boundary for the local decision on the slave sensors is also given in Section 3.4.2 of Chapter 3, namely set the binary decision threshold at 0.

\[
P_e_{HS} = P_r(H_0) \cdot \int_{TU_1} f_{HS_0}(y_{HS})dy_{HS} + P_r(H_1) \cdot \int_{TL_1} f_{HS_1}(y_{HS})dy_{HS}
\]

\[
= P_r(H_0) \cdot \left\{ TL_1 \frac{TL_1 - \mu_{HS_0}}{\sigma_{HS_0}} \right\} + P_r(H_1) \cdot \left\{ 1 - Q \left\{ TL_1 \frac{TL_1 - \mu_{HS_1}}{\sigma_{HS_1}} \right\} \right\} \tag{4.5.1}
\]

\[
P_i(z_{TI} = i) = P_r(H_0) \cdot \left\{ TL_2 \int_{TL_1} f_{HS_0}(y_{HS})dy_{HS} + \int_{TU_2} f_{HS_0}(y_{HS})dy_{HS} \right\}
\]

\[
+ P_r(H_1) \cdot \left\{ TL_2 \int_{TL_1} f_{HS_1}(y_{HS})dy_{HS} + \int_{TU_2} f_{HS_1}(y_{HS})dy_{HS} \right\}
\]

\[
= P_r(H_0) \left\{ \left\{ TL_1 \frac{TL_1 - \mu_{HS_0}}{\sigma_{HS_0}} \right\} - Q \left\{ TL_2 - \mu_{HS_0} \right\} + Q \left\{ TU_2 - \mu_{HS_0} \right\} - Q \left\{ TU_1 - \mu_{HS_0} \right\} \right\}
\]

\[
+ P_r(H_1) \left\{ Q \left\{ TL_1 \frac{TL_1 - \mu_{HS_1}}{\sigma_{HS_1}} \right\} - Q \left\{ TL_2 - \mu_{HS_1} \right\} + Q \left\{ TU_2 - \mu_{HS_1} \right\} - Q \left\{ TU_1 - \mu_{HS_1} \right\} \right\} \tag{4.5.2}
\]
\[ P_r(z_{T2}=1) = P_r(H_0) \cdot \int_{T_{L2}}^{T_{U2}} f_{HS_0}(y_{HS})dy_{HS} + P_r(H_1) \cdot \int_{T_{L2}}^{T_{U2}} f_{HS_0}(y_{HS})dy_{HS} \]

\[ = P_r(H_0) \left\{ Q \left( \frac{T_{L2} - \mu_{HS_0}}{\sigma_{HS_0}} \right) - Q \left( \frac{T_{U2} - \mu_{HS_0}}{\sigma_{HS_0}} \right) \right\} \]

\[ + P_r(H_1) \left\{ Q \left( \frac{T_{L2} - \mu_{HS_1}}{\sigma_{HS_1}} \right) - Q \left( \frac{T_{U2} - \mu_{HS_1}}{\sigma_{HS_1}} \right) \right\} \]

(4.5.3)

\[ P_{e_{T1}} = P_r(H_0) \left\{ \int_{(U_{SS1}=0)} f_{HS_0}(y_{HS})dy_{HS} \cdot \int_{T_{SS1}}^{T_{USS1}} f_{SS1_0}(y_{SS1})dy_{SS1} \right\} \]

\[ + \int_{(U_{SS1}=1)} f_{HS_0}(y_{HS})dy_{HS} \cdot \int_{T_{SS1}}^{T_{USS1}} f_{SS1_0}(y_{SS1})dy_{SS1} \]

\[ + P_r(H_1) \left\{ \int_{(U_{SS1}=0)} f_{HS_1}(y_{HS})dy_{HS} \cdot \int_{T_{SS1}}^{T_{USS1}} f_{SS1_1}(y_{SS1})dy_{SS1} \right\} \]

\[ + \int_{(U_{SS1}=1)} f_{HS_1}(y_{HS})dy_{HS} \cdot \int_{T_{SS1}}^{T_{USS1}} f_{SS1_1}(y_{SS1})dy_{SS1} \]

\[ = P_r(H_0) \cdot \left\{ Q \left( \frac{f(U_{SS1}=0)-\mu_{HS_0}}{\sigma_{HS_0}} \right) \cdot \left[ 1 - Q \left( \frac{T_{SS1} - \mu_{SS1_0}}{\sigma_{SS1_0}} \right) \right] \right\} \]

\[ + \left\{ Q \left( \frac{f(U_{SS1}=1)-\mu_{HS_0}}{\sigma_{HS_0}} \right) \cdot Q \left( \frac{T_{SS1} - \mu_{SS1_0}}{\sigma_{SS1_0}} \right) \right\} \]

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\[ P_{r_2} = P_r(H_0) \cdot \left[ 1 - Q \left( \frac{T_{SS1} - \mu_{SS10}}{\sigma_{SS10}} \right) \right] \cdot \left( 1 - Q \left( \frac{T_{SS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \right) \cdot \left( 1 - Q \left( \frac{f(U_{SS1=0}, U_{SS2=1}) - \mu_{HS0}}{\sigma_{HS0}} \right) \right) \]

\[ + \left[ 1 - Q \left( \frac{T_{SS1} - \mu_{SS10}}{\sigma_{SS10}} \right) \right] \cdot \left( 1 - Q \left( \frac{T_{SS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \right) \cdot \left( Q \left( \frac{f(U_{SS1=0}, U_{SS2=0}) - \mu_{HS0}}{\sigma_{HS0}} \right) \right) \]

\[ + Q \left( \frac{T_{SS1} - \mu_{SS10}}{\sigma_{SS10}} \right) \cdot \left( 1 - Q \left( \frac{T_{SS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \right) \cdot \left( Q \left( \frac{f(U_{SS1=1}, U_{SS2=0}) - \mu_{HS0}}{\sigma_{HS0}} \right) \right) \]

\[ + Q \left( \frac{T_{SS1} - \mu_{SS10}}{\sigma_{SS10}} \right) \cdot \left( Q \left( \frac{T_{SS2} - \mu_{SS20}}{\sigma_{SS20}} \right) \right) \cdot \left( Q \left( \frac{f(U_{SS1=1}, U_{SS2=1}) - \mu_{HS0}}{\sigma_{HS0}} \right) \right) \]

\[ + P_r(H_1) \cdot \left[ 1 - Q \left( \frac{T_{SS1} - \mu_{SS11}}{\sigma_{SS11}} \right) \right] \cdot \left( 1 - Q \left( \frac{T_{SS2} - \mu_{SS21}}{\sigma_{SS21}} \right) \right) \cdot \left( 1 - Q \left( \frac{f(U_{SS1=0}, U_{SS2=1}) - \mu_{HS1}}{\sigma_{HS1}} \right) \right) \]

\[ + \left[ 1 - Q \left( \frac{T_{SS1} - \mu_{SS11}}{\sigma_{SS11}} \right) \right] \cdot \left( Q \left( \frac{T_{SS2} - \mu_{SS21}}{\sigma_{SS21}} \right) \right) \cdot \left( 1 - Q \left( \frac{f(U_{SS1=0}, U_{SS2=0}) - \mu_{HS1}}{\sigma_{HS1}} \right) \right) \]

\[ + Q \left( \frac{T_{SS1} - \mu_{SS11}}{\sigma_{SS11}} \right) \cdot \left( 1 - Q \left( \frac{T_{SS2} - \mu_{SS21}}{\sigma_{SS21}} \right) \right) \cdot \left( 1 - Q \left( \frac{f(U_{SS1=1}, U_{SS2=0}) - \mu_{HS1}}{\sigma_{HS1}} \right) \right) \]

\[ + Q \left( \frac{T_{SS1} - \mu_{SS11}}{\sigma_{SS11}} \right) \cdot \left( 1 - Q \left( \frac{T_{SS2} - \mu_{SS21}}{\sigma_{SS21}} \right) \right) \cdot \left( 1 - Q \left( \frac{f(U_{SS1=1}, U_{SS2=1}) - \mu_{HS1}}{\sigma_{HS1}} \right) \right) \]
\[ + Q \left( \frac{T_{SS1} - \mu_{SS1}}{\sigma_{SS1}} \right) \cdot Q \left( \frac{T_{SS2} - \mu_{SS2}}{\sigma_{SS2}} \right) \cdot \left[ 1 - Q \left( \frac{f(U_{SS1} = 1, U_{SS2} = 1) - \mu_{HS1}}{\sigma_{HS1}} \right) \right] \]  

(4.5.5)

4.5.1. Numerical Evaluation of \( \bar{C} \)

The same method is used as in the previous chapters in evaluating \( \bar{C} \) numerically. The difference is that the host sensor in this system has 4 thresholds, unlike 2SS and 3SS. The thresholds are varied with a relationship of \( TU_{31} = -TL_{31}, TU_{32} = -TL_{32}, \) and \( TU_{32} = \frac{1}{2} TU_{31} \). This threshold relationship is selected arbitrarily.

For the threshold configuration, refer back to Figure 4.1. The program written for this evaluation is attached under Appendix C. As shown in Figure 4.3, depending on the communication cost constants, CCC1 and CCC2, individual curves are obtained. The dotted curve which is generated using \( CCC1 = 0.325 \) and \( CCC2 = 0.65 \) is the last curve with a minimum other than at the threshold position of 0.0. Figure 4.4 is an enlarged version of Figure 4.3. In Figure 4.3 and 4.4, the curves have ripples, unlike the set of curves shown in the previous chapters. This phenomenon is induced from the arbitrary choice of thresholds, giving a suboptimal threshold locations, and from the changes of the system’s communication scheme from one to another. Figure 4.5 shows the minimum expected system cost holds at a constant beyond the communication cost constant of 0.65. This is because that the optimum thresholds at the host sensor, \( TU_{31}, TU_{32}, TL_{31}, \) and \( TL_{32} \), eventually become zero. This is more clearly represented in Figure 4.6.
Two/Three-Sensor-System

Figure 4.3 Expected System Cost vs. HS Threshold Position
Two/Three-Sensor-System

Figure 4.4 Expected System Cost vs. HS Threshold Position
Enlarged Version of Figure 4.3
**Two/Three-Sensor-System**

![Graph](image)

**Figure 4.5** Min. Expected System Cost vs. Communication Cost Constant

![Graph](image)

**Figure 4.6** Summary of Data
CHAPTER 5

Comparison of C of 2SS, 3SS, and 2/3SS

5.1. Comparison of C

In this section numerically-evaluated expected system costs in Chapter 2, Chapter 3, and Chapter 4 are compared against each other. The comparison are made based upon the data obtained using Gaussian models for the different sensor systems.

The system expected costs are evaluated over the various threshold locations on the host sensor’s observation space and different communication cost constant incurred in communication between the host sensor and the slave sensors. Data are collected from the results obtained through the C expressed in terms of Q(y)-functions. These informations are plotted and attached at the end of Chapter 2, Chapter 3, and Chapter 4. The summarized data are tabulated in Table 5.1, Table 5.2, and Table 5.3 in following sections.

5.1.1. C of 2SS

Figure 2.3 shows that the total expected system costs are evaluated as the thresholds, TL and TU, are departing from the origin with various communication cost constant, CCC1. As in Figure 2.3 or 2.4, some of the curves have minima other than at the threshold position of 0.0, some don’t. It is roughly seen that a minimum of curve occurs at the threshold position of 0.0 when CCC1 ≥ 0.5. If CCC1 ≥ 0.5, the communications between sensors are prohibited and the final decision is made by the
host sensor alone. The exact value of the communication cost constant that may not give minimum (other than the threshold position of 0.0) is included between 0.45 and 0.50 leaning more toward to 0.45. The dotted curve indicates that the communication cost constant is 0.45. As the thresholds move away from 0.0, the cost is increasing beyond the optimal threshold position. It begins to stop increasing near the threshold position of 4.0.

Figure 2.5 is a plot of extracted information from Figure 2.3. It shows the behavior of the minimum expected system cost due to the change of the communication cost constant. As the communication cost constant becomes greater, the minimum expected system cost increases; however, the cost starts saturating at CCC of 0.45. The percentage change in the expected system cost when the communication cost constants are varied from 1.0 to 0.0 is 43.7 %. This shows that the communication cost constant takes a very important role in the system.

The relationship among the minimum expected cost, the optimal threshold position, and the communication cost constant is shown in Figure 2.6. The numerical tabulated data are given in Table 5.1. In Table 5.1, when the minimum expected system cost is 0.1587, this means there are no communications between sensors. This number, thus, represents the cost of making a final decision by the host sensor only.

5.1.2. $\bar{C}$ of 3SS

As in the previous section, Figure 3.3 shows that the expected system cost vs. the threshold position in the host sensor's observation space. The dotted curve indi-
The information in Figure 3.3 are summarized in Figure 3.5 and Figure 3.6. The numerical tabulated data of these figures are listed in Table 5.2. From the table there is 51.5% difference in the expected system cost when communication cost is varied from 1.0 to 0.0.
Table 5.2 Tabulated Data of 3SS

5.1.3 \( \bar{C} \) of 2/3SS

The numerical tabulated data of Figure 4.3, Figure 4.4, and Figure 4.5 is in Table 5.3. In Figure 4.3, it is noted that the dotted curve occurs when CCC1 = 0.325 and CCC2 = 0.65. The percentage change in the expected system cost when CCC1 changes from 0.5 to 0.0, meaning CCC2 changes from 1.0 to 0.0, is 55.3%. It is clearly shown in Figure 4.3 that the curves are leveling off near the threshold position of 8.0.
Communication Cost Constant with Two Sensor CCC2 Communication Cost Constant with One Sensors CCC1 Optimum Inner Threshold Position TU32 Optimum Outer Threshold Position TU31 Minimum Expected System Cost

<table>
<thead>
<tr>
<th>Communication Cost Constant with Two Sensor CCC2</th>
<th>Communication Cost Constant with One Sensors CCC1</th>
<th>Optimum Inner Threshold Position TU32</th>
<th>Optimum Outer Threshold Position TU31</th>
<th>Minimum Expected System Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
<td>0.5150</td>
<td>1.030</td>
<td>0.0712</td>
</tr>
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<td>0.05</td>
<td>0.025</td>
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</tr>
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<td>0.510</td>
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</tr>
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</tr>
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</table>

Table 5.3 Tabulated Data of 2/3SS

5.2. Comparison of Systems

Since each system's numerical evaluation results are collected, and 5.1.3, it is possible to carry out the performance comparison of these systems. Mainly the systems' expected cost and the optimal threshold position at different communication cost constant are considered for the comparison. The method used to compare the systems in this section is that, first, the 2SS is compared with the rest of systems, 3SS and 2/3SS. Secondly, the 2SS is compared to 2/3SS. For the convenience, the com-
munication cost constants of 0.0 and 0.45 are chosen to be the bases of comparison. The communication cost constant of 0.0 is selected since it means that there is no risk in communication between sensors, in other words, the communication between the host sensor and the slave sensors is encouraged. The communication cost constant of 0.45 are chosen because it is the largest communication cost constant of 2SS which gives an optimal threshold position other than 0.0.

Using the table presented in the previous sections, at the communication cost constant of 0.0, the expected system cost of 3SS is 13.24 % less than that of 2SS. Comparing 2SS to 2/3SS, 2/3SS outperforms 2SS by 20.10 % in the expected system cost. In comparing with 2/3SS, the outer threshold location is selected for the comparison. 2/3SS has 47.14 % larger width (or size) of the dubious decision region in systems observation space. This paragraph is summarized in Table 5.4.

<table>
<thead>
<tr>
<th>Type of Sensor System</th>
<th>CCC1 = 0.0</th>
<th>CCC2 = 0.0</th>
<th>2SS</th>
<th>3SS</th>
<th>2/3SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement in Expected System Cost</td>
<td>0.0 %</td>
<td>13.24 %</td>
<td>20.10 %</td>
<td></td>
<td></td>
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<tr>
<td>Improvement in Optimal Threshold Position</td>
<td>0.0 %</td>
<td>12.14 %</td>
<td>47.14 %</td>
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</tbody>
</table>

CCC1 = Communication Cost Constant of communicating with one sensor
CCC2 = Communication Cost Constant of communicating with two sensors

Table 5.4 Comparison of 2SS to the Others with CCC=0.0
In aspects of the optimal threshold position, 2/3SS has a wider uncertain decision region than 3SS by 31.21%. With CCC1 = CCC2 = 0.0, 2/3SS performs about 7.90% better than 3SS in the expected system cost. This is because 2/3SS requests information from the slave sensors more frequent than 3SS since 2/3SS has a wider uncertain decision region. This information are contained in Table 5.5.

Now we consider system improvements in the expected system cost and in optimal threshold location with the communication cost constant of 0.45 is considered. In 2/3SS this communication cost constant is used when the host sensor communicates with two slave sensors; when the host sensor communicates with only one sensor, the communication cost constant in this case is a half of the prior case, 0.225. In aspects of the expected system cost, the difference of system cost between 2SS and 3SS is 0.5% in favour of 3SS. For the optimal threshold location, 3SS has a wider uncertainty region by 100%. In comparison of the 2SS to 2/3SS, 2/3SS performs better in the expected system cost by 5.43%. These are listed in Table 5.6.

<table>
<thead>
<tr>
<th>Type of Sensor System</th>
<th>3SS</th>
<th>2/3SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCC1 = 0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCC2 = 0.0</td>
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<td></td>
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<tr>
<td>Improvement in</td>
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<td>7.90%</td>
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<tr>
<td>Expected System Cost</td>
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<tr>
<td>Improvement in</td>
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<td>31.21%</td>
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<tr>
<td>Optimal Threshold</td>
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<tr>
<td>Position</td>
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</tbody>
</table>

CCC1 = Communication Cost Constant of communicating with one sensor  
CCC2 = Communication Cost Constant of communicating with two sensors

Table 5.5 Comparison of 2SS to 2/3SS with CCC=0.0
<table>
<thead>
<tr>
<th>Type of Sensor System</th>
<th>CCC1 = 2.25</th>
<th>CCC2 = 4.5</th>
<th>2SS</th>
<th>3SS</th>
<th>2/3SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement in Expected System Cost</td>
<td>0.0 %</td>
<td>0.5 %</td>
<td>5.43 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Improvement in Optimal Threshold Position</td>
<td>0.0 %</td>
<td>100 %</td>
<td>584.71 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CCC1 = Communication Cost Constant of communicating with one sensor
CCC2 = Communication Cost Constant of communicating with two sensors

Table 5.6 Comparison of 2SS to the Others with CCC ≠ 0.0

2/3SS performs about 4.95 % better than 3SS in the expected system cost. In aspects of the optimal threshold position, 2/3SS has a wider uncertain decision region than 3SS by 242.86 %. This information is contained in Table 5.7.

It is noted that the width of the threshold location is shrinking relatively faster for 2SS and 3SS than 2/3SS as the communication cost constant increases. As far as

<table>
<thead>
<tr>
<th>Type of Sensor System</th>
<th>CCC1 = 2.25</th>
<th>CCC2 = 4.5</th>
<th>3SS</th>
<th>2/3SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement in Expected System Cost</td>
<td>0.0 %</td>
<td>4.95 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Improvement in Optimal Threshold Position</td>
<td>0.0 %</td>
<td>242.86 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CCC1 = Communication Cost Constant of communicating with one sensor
CCC2 = Communication Cost Constant of communicating with two sensors

Table 5.7 Comparison of 2SS to 2/3SS with CCC ≠ 0.0
the expected system cost is concerned, there is not a great difference as in the position of optimal threshold. Moreover, at a higher communication cost constant say 0.45 (refer to Table 5.6), there are insignificant differences in the expected system cost among the systems.

It is interesting to observe the relationship between optimal thresholds and \( P_r(U_{HS} = ?) \) since the probability of an observation landing in the dubious region is closely related to the optimal thresholds location in the host sensor. \( P_r(U_{HS} = ?) \) represents that the probability of the host sensor’s observation falls in the uncertainty region, \( TL \leq y_{HS} \leq TU \), inducing the host sensor’s local decision to be "?". This relationship is tabulated in Table 5.8, Table 5.9, and Table 5.10. It is obvious, without looking at the tables, that \( P_r(U_{HS} = ?) \) decreases as the optimum threshold approaches to zero. When the tables are plotted (See Figure 5.1, 5.2, and 5.3), a linear relationship is found between the optimal threshold positions and the probabil-

<table>
<thead>
<tr>
<th>Optimal Threshold Position</th>
<th>( P_r(U_{HS} = ?) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.700</td>
<td>0.3375</td>
</tr>
<tr>
<td>0.585</td>
<td>0.2826</td>
</tr>
<tr>
<td>0.490</td>
<td>0.2369</td>
</tr>
<tr>
<td>0.405</td>
<td>0.1959</td>
</tr>
<tr>
<td>0.330</td>
<td>0.1597</td>
</tr>
<tr>
<td>0.265</td>
<td>0.1282</td>
</tr>
<tr>
<td>0.200</td>
<td>0.0968</td>
</tr>
<tr>
<td>0.140</td>
<td>0.0678</td>
</tr>
<tr>
<td>0.085</td>
<td>0.0411</td>
</tr>
<tr>
<td>0.035</td>
<td>0.0169</td>
</tr>
<tr>
<td>0.000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 5.8 \( P_r(U_{HS} = ?) \) given the Optimal Thresholds of 2SS
ity of $U_{HS} = ?$. The program which evaluates the probability of communication with given the optimal thresholds is attached in Appendix D.

<table>
<thead>
<tr>
<th>Optimal Threshold Position</th>
<th>$P_r(U_{HS} = ?)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.785</td>
<td>0.3778</td>
</tr>
<tr>
<td>0.655</td>
<td>0.3161</td>
</tr>
<tr>
<td>0.550</td>
<td>0.2658</td>
</tr>
<tr>
<td>0.460</td>
<td>0.2225</td>
</tr>
<tr>
<td>0.380</td>
<td>0.1838</td>
</tr>
<tr>
<td>0.310</td>
<td>0.1500</td>
</tr>
<tr>
<td>0.240</td>
<td>0.1161</td>
</tr>
<tr>
<td>0.180</td>
<td>0.0871</td>
</tr>
<tr>
<td>0.125</td>
<td>0.0605</td>
</tr>
<tr>
<td>0.070</td>
<td>0.0339</td>
</tr>
<tr>
<td>0.015</td>
<td>0.0073</td>
</tr>
<tr>
<td>0.000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 5.9 $P_r(U_{HS} = ?)$ given the Optimal Thresholds of 3SS

<table>
<thead>
<tr>
<th>Optimal Threshold Position</th>
<th>$P_r(U_{HS} = ?)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.030</td>
<td>0.4908</td>
</tr>
<tr>
<td>0.885</td>
<td>0.4245</td>
</tr>
<tr>
<td>0.770</td>
<td>0.3707</td>
</tr>
<tr>
<td>0.675</td>
<td>0.3256</td>
</tr>
<tr>
<td>0.585</td>
<td>0.2826</td>
</tr>
<tr>
<td>0.510</td>
<td>0.2465</td>
</tr>
<tr>
<td>0.435</td>
<td>0.2104</td>
</tr>
<tr>
<td>0.365</td>
<td>0.1766</td>
</tr>
<tr>
<td>0.305</td>
<td>0.1476</td>
</tr>
<tr>
<td>0.240</td>
<td>0.1161</td>
</tr>
<tr>
<td>0.180</td>
<td>0.0871</td>
</tr>
<tr>
<td>0.125</td>
<td>0.0605</td>
</tr>
<tr>
<td>0.070</td>
<td>0.0339</td>
</tr>
<tr>
<td>0.015</td>
<td>0.0073</td>
</tr>
<tr>
<td>0.000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 5.10 $P_r(U_{HS} = ?)$ given the Optimal Thresholds of 2/3SS
Two-Sensor-System

Figure 5.1 Dubious Decision Probability at HS vs. HS Optimum Threshold

Three-Sensor-System

Figure 5.2 Dubious Decision Probability at HS vs. HS Optimum Threshold

Two/Three-Sensor-System

Figure 5.3 Dubious Decision Probability at HS vs. HS Optimum Threshold
CHAPTER 6

System Simulations

6.1. Simulation Method

Simulation of the systems evaluated in Chapter 2, 3, and 4 are performed to understand how these systems behave in a realistic environment. The same assumptions as those made in the beginning of this work (Refer Chapter 1) are also used in the simulation. One additional system is simulated in addition to three systems with which we have dealt. This system consists of one sensor that has a single threshold and no slave sensors.

To the signal, either -1 or 1, Gaussian noise is added at the host sensor and the slave sensors. In each system, different slave sensors receive independent observation. However, in all systems, each host sensor receives the same observation so that the performance of each system can be compared easily. In the simulations, different communication constants were used and the number of iterations performed was 10,000. The iterations can be interpreted as the number of observations taken by the host sensor and the slave sensors. In our Gaussian random number generation routine, 10,000 iterations provide with well distributed Gaussian random numbers. The outputs of the different system are compared in terms of the percentage of correct detections (CD), false alarms (FA), and target misses (TM). The total detection error is, then, FA + TM. These are listed in Table 6.1 and Table 6.2.
<table>
<thead>
<tr>
<th>CCC</th>
<th>CD (%)</th>
<th>FA (%)</th>
<th>TM (%)</th>
<th>CD (%)</th>
<th>FA (%)</th>
<th>TM (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>84.17</td>
<td>7.64</td>
<td>8.18</td>
<td>90.38</td>
<td>4.60</td>
<td>5.01</td>
</tr>
<tr>
<td>0.05</td>
<td>83.71</td>
<td>8.09</td>
<td>8.82</td>
<td>89.02</td>
<td>5.45</td>
<td>5.52</td>
</tr>
<tr>
<td>0.10</td>
<td>83.90</td>
<td>7.96</td>
<td>8.13</td>
<td>88.76</td>
<td>5.51</td>
<td>5.72</td>
</tr>
<tr>
<td>0.15</td>
<td>83.94</td>
<td>8.00</td>
<td>8.05</td>
<td>88.85</td>
<td>5.60</td>
<td>5.52</td>
</tr>
<tr>
<td>0.20</td>
<td>83.89</td>
<td>8.03</td>
<td>8.07</td>
<td>88.26</td>
<td>5.83</td>
<td>5.90</td>
</tr>
<tr>
<td>0.25</td>
<td>83.32</td>
<td>8.51</td>
<td>8.16</td>
<td>87.24</td>
<td>6.54</td>
<td>6.21</td>
</tr>
<tr>
<td>0.30</td>
<td>83.52</td>
<td>8.31</td>
<td>8.16</td>
<td>86.31</td>
<td>6.88</td>
<td>6.80</td>
</tr>
<tr>
<td>0.35</td>
<td>84.73</td>
<td>7.66</td>
<td>7.60</td>
<td>86.73</td>
<td>6.72</td>
<td>6.45</td>
</tr>
<tr>
<td>0.40</td>
<td>84.02</td>
<td>8.11</td>
<td>7.86</td>
<td>84.95</td>
<td>7.69</td>
<td>7.35</td>
</tr>
<tr>
<td>0.45</td>
<td>84.03</td>
<td>8.01</td>
<td>7.95</td>
<td>84.70</td>
<td>7.63</td>
<td>7.66</td>
</tr>
<tr>
<td>0.50</td>
<td>84.08</td>
<td>7.98</td>
<td>7.93</td>
<td>84.08</td>
<td>7.98</td>
<td>7.93</td>
</tr>
<tr>
<td>0.55</td>
<td>84.45</td>
<td>7.94</td>
<td>7.60</td>
<td>84.84</td>
<td>7.94</td>
<td>7.60</td>
</tr>
<tr>
<td>0.60</td>
<td>83.88</td>
<td>8.17</td>
<td>7.94</td>
<td>83.88</td>
<td>8.17</td>
<td>7.94</td>
</tr>
<tr>
<td>0.65</td>
<td>84.16</td>
<td>7.97</td>
<td>7.86</td>
<td>84.16</td>
<td>7.97</td>
<td>7.86</td>
</tr>
<tr>
<td>0.70</td>
<td>84.81</td>
<td>7.75</td>
<td>7.43</td>
<td>84.81</td>
<td>7.75</td>
<td>7.43</td>
</tr>
</tbody>
</table>

CCC = communication cost constant

Table 6.1 Simulation Results for 1SS & 2SS
### Simulation of Systems

<table>
<thead>
<tr>
<th>Type of Sensor Systems</th>
<th>3SS</th>
<th>2/3SS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CCC</strong></td>
<td><strong>CD (%)</strong></td>
<td><strong>FA (%)</strong></td>
</tr>
<tr>
<td>0.00</td>
<td>90.36</td>
<td>2.07</td>
</tr>
<tr>
<td>0.05</td>
<td>89.44</td>
<td>3.02</td>
</tr>
<tr>
<td>0.10</td>
<td>88.95</td>
<td>3.34</td>
</tr>
<tr>
<td>0.15</td>
<td>89.38</td>
<td>3.80</td>
</tr>
<tr>
<td>0.20</td>
<td>88.76</td>
<td>4.28</td>
</tr>
<tr>
<td>0.25</td>
<td>87.80</td>
<td>5.23</td>
</tr>
<tr>
<td>0.30</td>
<td>86.88</td>
<td>5.73</td>
</tr>
<tr>
<td>0.35</td>
<td>87.19</td>
<td>5.87</td>
</tr>
<tr>
<td>0.40</td>
<td>85.76</td>
<td>6.86</td>
</tr>
<tr>
<td>0.45</td>
<td>84.96</td>
<td>7.28</td>
</tr>
<tr>
<td>0.50</td>
<td>84.43</td>
<td>7.74</td>
</tr>
<tr>
<td>0.55</td>
<td>84.45</td>
<td>7.94</td>
</tr>
<tr>
<td>0.60</td>
<td>83.88</td>
<td>8.16</td>
</tr>
<tr>
<td>0.65</td>
<td>84.16</td>
<td>7.97</td>
</tr>
<tr>
<td>0.70</td>
<td>84.81</td>
<td>7.75</td>
</tr>
</tbody>
</table>

CCC = communication cost constant

**Table 6.2** Simulation Results for 3SS & 2/3SS

6.2. Simulation Results and Discussion

The simulation results are quite reasonable. In general, the results show that 2/3SS performs the best and followed by 3SS, 2SS, and 1SS in declining performance. As shown in the previous chapter the optimal threshold of 2SS collapses to 0.0 when the communication cost constant is 0.5. This is also shown in Table 6.1 as the CD of 2SS equals to that of 1SS when CCC becomes 0.50. Also, CD of 3SS and 2/3SS become that of 1SS when CCC is equal to 0.55 and 0.70, respectively. This is because the host sensors in the different systems receive the same observations.

There are about 6% difference in CD between 1SS and other systems; however, the difference among the 2SS, 3SS, and 2/3SS is rather insignificant when CCC is 0.0. This is because there are no influence of communication cost constant to each system. As the CCC increases, the differences in CD among systems become
noticeable even though the largest difference are about 3.5%. For 1SS CD, FA, and TM are virtually remain constant over all the CCCs were used since the performance of 1SS is independent from CCC. CD in 2SS decreases as CCC increases. FA and MT are increasing as CCC increases. In 3SS and 2/3SS FA is about 3.5 and 2.3 times less than TM, respectively, when CCC = 0.0. As CCC increases the ratio of FA and MT approaches to 1.0. These informations are tabulated in Table 6.1 and Table 6.2. The plotted version of these data are in Figure 6.1, Figure 6.2, and Figure 6.3. The program for this simulation is attached in Appendix E.
Correct Detections (%)

![Figure 6.1 CD vs. CCC](image1)

False Alarms (%)

![Figure 6.2 FA vs. CCC](image2)

Target Misses (%)

![Figure 6.3 TM vs. CCC](image3)

Legend:

- $\bigcirc$ --- 1SS;  $\times$ --- 2SS;  $\triangle$ --- 3SS;  $\square$ --- 2/3SS
CHAPTER 7

Conclusion

One objective of this study was to characterize team strategy decision methods in terms of analytical derivation, numerical evaluations, and system simulations. The optimization of the system in terms of minimization of either the expected system cost or the probability of error in decision is another objective.

The team strategy is applied to three different systems, and the performance of each system is characterized. Cost functions for each system are defined. From the cost function, the expected system cost, $\overline{C}$, is derived. The $\overline{C}$ is represented in general probabilistic terms as well as for Gaussian statistics using $Q(y)$ functions. The numerical evaluations are performed for Gaussian models. The numerical evaluation shows, subject to communication cost, that $2/3SS$ is the most efficient. The next most desirable system is $3SS$ and the least is $2SS$. Simulation of the three systems was also carried out. The simulation results confirm the above order of desirability.

The communication cost constant plays an important role in the global decisions of team strategies. Changes in the communication cost constant influence the frequency of communication between sensors. Selection of optimum thresholds in the host sensor is heavily dependent upon the communication cost constant since the frequency of the communication allowance determines the optimal threshold positions.

The simulation results show that there is a communication cost constant which makes all the systems perform the same. The communication cost constant in this
situation is 0.7. Thus when the communication between sensors becomes very risky or expensive, meaning a high probability of interception, the sensors avoid communication. This fact is shown by comparing the system to 1SS because 1SS has no communication capability, i.e., there are no slave sensors involved. Refer to Figure 6.1, 6.2, and 6.3.

As the communication cost increases, the system with three sensors apparently make better chances of detection than the system with two sensors. When the communication cost is large so that communications between sensors are prohibited, then the performance of 2SS, 3SS, and 2/3SS is compatible to that of 1SS.
APPENDIX A

Program for Cost Evaluation of 2-Sensor-System

This appendix contains a FORTRAN program listing which evaluates (2.3.4) substituted with (2.5.4.1), (2.5.4.2), and (2.5.4.3) numerically. It consists of a main routine called "TWOSENSYS" and two subroutines, "QfunE" and "FindMin". Program TWOSENSYS (TWO SEhors SYStem) is responsible for iterations (generation of Gaussian observations), main calculations, calling subroutines, writing outputs to files, etc. Subroutine QfunE evaluates Q(y)-function (refer to (2.5.1.1)) when limits of integration are provided. Subroutine FindMin sorts a minimum in output data. This routine is used to find an optimal threshold in HS where the expected system cost is minimum.

The program computes expected system costs over threshold positions in HS for a given communication cost constant. Descriptions of variables used and comments are embedded in the program.

Figure 2.3, Figure 2.4, Figure 2.5, and Figure 2.6 are plotted version of outputs from this program. The tabulated data are contained in Table 5.1.
PROGRAM TWOSENSYS

c*******************************************************************************
c
Author: Howard C. Choe
Organization: Department of Electrical Engineering
The University of Virginia, Charlottesville

c This program numerically evaluates expected costs of a
two-sensor-system which uses team strategies for Gaussian statistics.

c Variable Description
p0 : a priori probability of "No target exists", H0
pl : a priori probability of "Target exists", H1

For Host Sensor
mh0 : mean value of H0 received by HS
mh1 : mean value of H1 received by HS
sh0 : standard deviation of H0 received by HS
sh1 : standard deviation of H1 received by HS

For Slave Sensor
ms0 : mean value of H0 received by SS
ms1 : mean value of H1 received by SS
ss0 : standard deviation of H0 received by SS
ss1 : standard deviation of H1 received by SS

Thresholds
TL : lower threshold
TU : upper threshold

For Slave Sensor
Tss : LRT optimal threshold

Pre-cost constant
c00 : deciding H0 given H0
c10 : deciding H1 given H0
c11 : deciding H1 given H1
c01 : deciding H0 given H1

Team effort cost
cccc : communication cost constant

*******************************************************************************

Declaration
REAL p0, pl
REAL mh0, mh1, ms0, ms1
REAL sh0, sh1, ss0, ss1
REAL c00, c10, c11, c01
REAL Tss

REAL ccc(21), cfmin(21), opthr(21)
REAL TU(1001), TL(1001)
REAL pehs(1001), pzcom(1001), cf(1001)
REAL cbar(21,1001)

DATA ccc/0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 1.00/

* The input data
* a priori probability of the binary hypothesis environment
  p0 = 0.5
  p1 = 0.5

* The statistics of the received information
  mh0 = -1.0
  mh1 = 1.0
  ms0 = -1.0
  ms1 = 1.0
  sigma = 1.0

* Assign all standard deviation to the same value
  sh0 = sigma
  sh1 = sigma
  ss0 = sigma
  ss1 = sigma

* pre-cost values
  c00 = 0.0
  c10 = 1.0
  c11 = 0.0
  c01 = 1.0

* Evaluation of the pre-calculated threshold for the team strategy and
  for the slave sensor
  planms = (c10 - c00)/(c01 - c11)
  planm = planms

* Evaluation of the ratio between a priori probabilities and LRT threshold
c for the host sensor and the slave sensor, considering each sensor is
centralized individually.

t0 = p0/p1
Ths = (mh0 + mh1)/2.0 + (sigma**2/(mh1 - mh0))*LOG(plamt*t0)
Tss = (ms0 + ms1)/2.0 + (sigma**2/(ms1 - ms0))*LOG(plamss*t0)

c Evaluation of the final threshold after communication

a0s = (Tss - ms0)/ss0
als = (Tss - ms1)/ss1

CALL QfunE(a0s, Qa0s)
CALL QfunE(als, Qals)

fus0 = plamss * t0 * (1.0 - Qa0s)/(1.0 - Qals)
fus1 = plamss * t0 * Qa0s/Qals

c Evaluation of Q(y)-function values with fus0 and fus1

fus0h0 = (fus0 - mh0)/sh0
fus1h0 = (fus1 - mh0)/sh0
fus0h1 = (fus0 - mh1)/sh1
fus1h1 = (fus1 - mh1)/sh1

CALL QfunE(fus0h0, Qfus0h0)
CALL QfunE(fus1h0, Qfus1h0)
CALL QfunE(fus0h1, Qfus0h1)
CALL QfunE(fus1h1, Qfus1h1)

c Calculation of an error probability by the team effort

pteam = (Qfus0h0*(1.0-Qa0s) + Qfus1h0*Qa0s) * p0
* + ((1.0-Qfus0h1)*(1.0-Qals) + (1.0-Qfus1h1)*Qals) * p1

c Calculation of the host sensor error probability and that of
communication probability, pchs and pzcom, respectively.
These probabilities are TL and TU dependent which means that
whenever TL and TU changes, values of pchs and pzcom also change.

unc = 0.02
DO 10 ia = 1, 21
ib = 0
DO 30 thr = Ths, Ths + 4.0, unc
ib = ib + 1

78
TU(ib) = thr
TL(ib) = Ths - (FLOAT(ib) - 1.0)*tunc

c Evaluation for pehs for various TL and TU

tuh0 = (TU(ib) - mh0)/sh0
tlh1 = (TL(ib) - mh1)/sh1

CALL QfunE(tuh0, Qtuh0)
CALL QfunE(tlh1, Qtlh1)

pehs(ib) = Qtuh0*p0 + (1.0 - Qtlh1)*p1

c Evaluation for pzcom for various TL and TU

tlh0 = (TL(ib) - mh0)/sh0
tuh1 = (TU(ib) - mh1)/sh1

CALL QfunE(tlh0, Qtlh0)
CALL QfunE(tuh1, Qtuh1)

pzcom(ib) = (Qtuh0 - Qtuh0)*p0 + (Qtlh1 - Qtuh1)*p1

c Evaluation for the expected probability of error of the system, COST

cbar(ia~ib) = pehs(ib) + (peteam-pehs(ib)+ccc(ia))*pzcom(ib)
cf(ib) = cbar(ia,ib)

30 CONTINUE

c Extract the minimum system cost for each case of ccc(ia)

CALL FINDMIN(cf,ib,mini)
cfmin(ia) = cf(mini)
opthr(ia) = TU(mini)

10 CONTINUE

c Write OUTPUT DATA .....
DO 50 ic = 1, ib
    WRITE (11,1000) TL(ic), TU(ic), (cbar(id,ic),id=1,10)
50 CONTINUE

DO 70 ie = 1, ib
    WRITE (12,1000) TL(ie), TU(ie), (cbar(ig,ie),ig=11,20)
70 CONTINUE

DO 90 ih = 1, 21
    WRITE (13,*), ccc(ih), cfmin(ih)
90 CONTINUE

DO 110 ii = 1, 21
    WRITE (14,*), opthr(ii), cfmin(ii)
110 CONTINUE

DO 130 ij = 1, 21
    WRITE (15,*), ccc(ij), opthr(ij)
130 CONTINUE

c Format statements

1000 FORMAT ("",F6.3,1X,11(F6.4,IX))

STOP
END

SUBROUTINE QfunE(xx, erfcx)
c This function calculates the error function and the complimentary
c error function for the value "xx"
c Accuracy is to within 1.5E-07.

REAL x, xx, erfcx
REAL a1, a2, a3, a4, a5, p, pi, t

pi = 3.141592654
x = ABS(xx)

a1 = 0.319381530
a2 = -0.356563782
a3 = 1.781477937
a4 = -1.821255978
a5 = 1.330274429

p = 0.2316419
t = 1.0/(1.0 + p*x)

s1 = a1*t + a2*t**2 + a3*t**3 + a4*t**4 + a5*t**5
s2 = s1*EXP(-(x**2)/2.0)

IF (xx .GE. 0.0) THEN
  erfcx = s2/SQRT(2.0*pi)
ELSE IF (xx .LT. 0.0) THEN
  erfcx = 1.0 - s2/SQRT(2.0*pi)
END IF

RETURN
END

SUBROUTINE FINDMIN(array,isize,minindex)

REAL array(isize)
INTEGER minindex

int = 1
11 CONTINUE
   DO 10 i = int+1, isize
      IF (array(int) .GT. array(i)) THEN
         minindex = i
         int = minindex
      END IF
   10 CONTINUE
RETURN
END
APPENDIX B

Program for Cost Evaluation of 3-Sensor-System

In Appendix B, a FORTRAN program used for numerical evaluations of (2.3.4) substituted with (2.5.4.1), (2.5.4.2), and (3.4.2.9) is contained. The main program is called "THREESENSYS" (THREE SENsor SYStem). As is in Chapter 2, the same subroutines, QfunE and FindMin, are also used.

Outputs of this program are represented by Figure 3.3, Figure 3.4, Figure 3.5, and Figure 3.6. Some of the data are also available from Table 5.2.
PROGRAM THRESENSYS

Author: Howard C. Choe
Organization: Department of Electrical Engineering
The University of Virginia, Charlottesville
Purpose: Master of Science Research

This program numerically evaluates expected system costs of a three-sensor-system which uses team strategies for Gaussian statistics.

Variable Description
- \( p_0 \): a priori probability of "No target exists", \( H_0 \)
- \( p_1 \): a priori probability of "Target exists", \( H_1 \)

For Host Sensor
- \( m_{h0} \): mean value of \( H_0 \) received by HS
- \( m_{h1} \): mean value of \( H_1 \) received by HS
- \( s_{h0} \): standard deviation of \( H_0 \) received by HS
- \( s_{h1} \): standard deviation of \( H_1 \) received by HS

For Slave Sensor 1
- \( m_{s10} \): mean value of \( H_0 \) received by SS1
- \( m_{s11} \): mean value of \( H_1 \) received by SS1
- \( s_{s10} \): standard deviation of \( H_0 \) received by SS1
- \( s_{s11} \): standard deviation of \( H_1 \) received by SS1

For Slave Sensor 2
- \( m_{s20} \): mean value of \( H_0 \) received by SS2
- \( m_{s21} \): mean value of \( H_1 \) received by SS2
- \( s_{s20} \): standard deviation of \( H_0 \) received by SS2
- \( s_{s21} \): standard deviation of \( H_1 \) received by SS2

Thresholds
- \( T_{L0} \): lower threshold
- \( T_{U0} \): upper threshold

For Slave Sensor 1
- \( T_{s10} \): LRT optimal threshold

For Slave Sensor 2
- \( T_{s20} \): LRT optimal threshold

Pre-cost constant
- \( c_{00} \): deciding \( H_0 \) given \( H_0 \)
- \( c_{10} \): deciding \( H_1 \) given \( H_0 \)
- \( c_{11} \): deciding \( H_1 \) given \( H_1 \)
- \( c_{01} \): deciding \( H_0 \) given \( H_1 \)

Team effort cost
- \( c \): communication cost constant for communicating with two sensors.
Other variables
- \( cf\text{min}() \): minimum cost in a single case of run, i.e., for a \( ccc() \)
- \( opth\text{r}() \): threshold where \( cf\text{min}() \) is occurred
- \( pehs() \): expected error of the system when there is no communication
- \( pz\text{com}() \): probability of communication would occur
- \( cf() \): expected cost of the system at various of thresholds
- \( cb\text{ar}() \): same as \( cf() \) but saved in 2-D array

Other variables are commented as program is progressed.

Declaration

\[
\begin{align*}
\text{REAL } & p0, p1 \\
\text{REAL } & mh0, mh1, ms10, ms11, ms20, ms21 \\
\text{REAL } & sh0, sh1, ss10, ss11, ss20, ss21 \\
\text{REAL } & c00, c10, c11, c01 \\
\text{REAL } & Tss1, Tss2 \\
\text{REAL } & ccc(21) \\
\text{REAL } & cf\text{min}(21), opth\text{r}(21) \\
\text{REAL } & TU(1001), TL(1001) \\
\text{REAL } & pehs(1001), pz\text{com}(1001), cf(1001) \\
\text{REAL } & cb\text{ar}(21,1001) \\
\text{DATA } & ccc/0.00, 0.05, 0.10, 0.15, 0.20, \\
& \quad \times 0.25, 0.30, 0.35, 0.40, 0.45, \\
& \quad \times 0.50, 0.55, 0.60, 0.65, 0.70, \\
& \quad \times 0.75, 0.80, 0.85, 0.90, 0.95, 1.00/ \\
\end{align*}
\]

Determination of communication cost constant for communicating with 2 slave sensors

The input data

\[
\begin{align*}
p0 &= 0.5 \\
p1 &= 0.5 \\
mh0 &= -1.0 \\
mh1 &= 1.0 \\
ms10 &= -1.0 \\
ms11 &= 1.0 \\
ms20 &= -1.0 \\
ms21 &= 1.0
\end{align*}
\]
\[ \sigma = 1.0 \]
\[ c_{00} = 0.0 \]
\[ c_{10} = 1.0 \]
\[ c_{11} = 0.0 \]
\[ c_{01} = 1.0 \]

```
c Assign all standard deviation to the same value

    sh0 = sigma
    sh1 = sigma
    ss01 = sigma
    ss11 = sigma
    ss20 = sigma
    ss21 = sigma
```

```
c Evaluation of the pre-calculated threshold for the team strategy and
for the slave sensor

    plamss1 = (c_{10} - c_{00})/(c_{01} - c_{11})
    plamss2 = (c_{10} - c_{00})/(c_{01} - c_{11})
    plamt = plamss1
```

```
c Evaluation of the ratio between a priori probabilities and LRT threshold
for the host sensor and the slave sensor, considering each sensor is
centralized individually.

    t0 = p0/p1
    Ths = (m_{h0} + m_{h1})/2.0 + (\sigma^2/(m_{h1} - m_{h0})) \cdot \text{LOG}(plamt \cdot t0)
    Tss1 = (m_{s10} + m_{s11})/2.0
    \quad + (\sigma^2/(m_{s11} - m_{s10})) \cdot \text{LOG}(plamss1 \cdot t0)
    Tss2 = (m_{s20} + m_{s21})/2.0
    \quad + (\sigma^2/(m_{s21} - m_{s20})) \cdot \text{LOG}(plamss2 \cdot t0)
```

```
c Slave sensor 1

c bs10 : integration limit for Q-function under H0

c bs11 : integration limit for Q-function under H1

c Qbs10 : probability of making uss1=1 under H0

c Qbs11 : probability of making uss1=1 under H1

    bs10 = (Tss1 - m_{s10})/ss10
    bs11 = (Tss1 - m_{s11})/ss11

    CALL QfunE(bs10, Qbs10)
    CALL QfunE(bs11, Qbs11)
```
Slave sensor 2
bs20 : integration limit for Q-function under H0
bs21 : integration limit for Q-function under H1
Qbs20 : probability of making uss2=1 under H0
Qbs21 : probability of making uss2=1 under H1

bs20 = (Tss2 - ms20)/ss20
bs21 = (Tss2 - ms21)/ss21

CALL QfunE(bs20, Qbs20)
CALL QfunE(bs21, Qbs21)

FOR COMMUNICATING WITH 2 SLAVE SENSORS
Evaluation of the final threshold after communication
f00 : final threshold when uss1=0 and uss2=0
f01 : final threshold when uss1=0 and uss2=1
f10 : final threshold when uss1=1 and uss2=0
f11 : final threshold when uss1=1 and uss2=1

f00 = plainss1*t0*(1.0-Qbs10)*(1.0-Qbs20)/((1.0-Qbs11)*(1.0-Qbs21))
f01 = plainss1*t0*(1.0-Qbs10)*Qbs20/((1.0-Qbs11)*Qbs21)
f10 = plainss2*t0*Qbs10*(1.0-Qbs20)/(Qbs11*(1.0-Qbs21))
f11 = plainss2*t0*Qbs10*Qbs20/(Qbs11*Qbs21)

Evaluation of Q(y)-function values using the above values
f00h0 : integration limit with f00 under H0
f01h0 : integration limit with f01 under H0
f10h0 : integration limit with f10 under H0
f11h0 : integration limit with f11 under H0

f00h0 = (f00 - mh0)/sh0
f01h0 = (f01 - mh0)/sh0
f10h0 = (f10 - mh0)/sh0
f11h0 = (f11 - mh0)/sh0

Qf00h0 : probability of making uf=1 with f00 under H0
Qf01h0 : probability of making uf=1 with f01 under H0
Qf10h0 : probability of making uf=1 with f10 under H0
Qf11h0 : probability of making uf=1 with f11 under H0

CALL QfunE(f00h0, Qf00h0)
CALL QfunE(f01h0, Qf01h0)
CALL QfunE(f10h0, Qf10h0)
CALL QfunE(f11h0, Qf11h0)
c f00h0 : integration limit with f00 under H1
c f01h0 : integration limit with f01 under H1
c f10h0 : integration limit with f10 under H1
c f11h0 : integration limit with f11 under H1

\[
\begin{align*}
  f00h1 &= (f00 - mh1)/sh1 \\
  f01h1 &= (f01 - mh1)/sh1 \\
  f10h1 &= (f10 - mh1)/sh1 \\
  f11h1 &= (f11 - mh1)/sh1
\end{align*}
\]

c Qf00h1 : probability of making uf=1 with f00 under H1
c Qf01h1 : probability of making uf=1 with f01 under H1
c Qf10h1 : probability of making uf=1 with f10 under H1
c Qf11h1 : probability of making uf=1 with f11 under H1

CALL QfunE(f00h1, Qf00h1)
CALL QfunE(f01h1, Qf01h1)
CALL QfunE(f10h1, Qf10h1)
CALL QfunE(f11h1, Qf11h1)

c Calculation of an error probability by the team strategy
c with communicating with 2 slave sensors

\[
\begin{align*}
  \text{petcam} &= (\ Qf00h0*(1.0-Qbs10)*(1.0-Qbs20) \\
  &+ Qf01h0*(1.0-Qbs10)*Qbs20 \\
  &+ Qf10h0*Qbs10*(1.0-Qbs20) \\
  &+ Qf11h0*Qbs10*Qbs20 )*p0 \\
  &+ ( (1.0-Qf00h1)*(1.0-Qbs11)*(1.0-Qbs21) \\
  &+ (1.0-Qf01h1)*(1.0-Qbs11)*Qbs21 \\
  &+ (1.0-Qf10h1)*Qbs11*(1.0-Qbs21) \\
  &+ (1.0-Qf11h1)*Qbs11*Qbs21 )*p1
\end{align*}
\]

c Calculation of the host sensor error probability, and
c that of communication frequency probability with 2 sensors-pehs,
c pzcom-respectively.

unc = 0.02
DO 10 ia = 1, 21
  ib = 0
  DO 30 thr = Ths, Ths + 4.0, unc
    ib = ib + 1
    TU(ib) = thr
    TL(ib) = Ths - (FLOAT(ib) * 1.0)*unc

  DO 30

 10 CONTINUE
 30 CONTINUE

c Evaluation for pehs for various TL and TU
tuh0 = (TU(ib) - mh0)/sh0
tlh1 = (TL(ib) - mh1)/sh1

CALL QfunE(tuh0, Quh0)
CALL QfunE(tlh1, Qlh1)

pehs(ib) = Qtuh0*p0 + (1.0 - Qtlh1)*p1

c Evaluation for pzcom for various TL and TU

th0 = (TL(ib) - mh0)/sh0
tuh1 = (TU(ib) - mh1)/sh1

CALL QfunE(th0, Qth0)
CALL QfunE(tuh1, Quh1)

pzcom(ib) = (Qtlh0 - Qtuh0)*p0 + (Qth1 - Qtlh1)*p1

c Evaluation for the expected probability of error of the system, COST

ctbar(ia,ib) = pehs(ib) + (peteam-pehs(ib)+ccc(ia))*pzcom(ib)
cf(ib) = ctbar(ia,ib)

30 CONTINUE

c Extract the minimum system cost for each case of ccc(ia)

CALL FINDMIN(cf,ib,mini)
cfmin(ia) = cf(mini)
opthr(ia) = TU(mini)

10 CONTINUE

c Write OUTPUT DATA ....

WRITE (10,*) 'Ratio of a priori probabilities: ', i0
WRITE (10,*) 'LRT Threshold for SS1 --------: ', tss1
WRITE (10,*) 'LRT Threshold for SS2 --------: ', tss2
WRITE (10,*) 'FT at HS when Us1=0 and Us2=0 : ', f00
WRITE (10,*) 'FT at HS when Us1=0 and Us2=1 : ', f01
WRITE (10,*) 'FT at HS when Us1=1 and Us2=0 : ', f10
WRITE (10,*) 'FT at HS when Us1=1 and Us2=1 : ', f11
WRITE (10,*) 'Error caused by Team Strategy : ', peteam

DO 50 ic = 1, ib
WRITE (11,1000) TL(ic), TU(ic), (cbar(id,ic),id=1,10)
50 CONTINUE

DO 70 ie = 1, ib
  WRITE (12,1000) TL(ie), TU(ie), (cbar(ig,ie),ig=11,20)
70 CONTINUE

DO 90 ih = 1, 21
  WRITE (13,*) ccc(ih), cfmin(ih)
90 CONTINUE

DO 110 ii = 1, 21
  WRITE (14,*) opthr(ii), cfmin(ii)
110 CONTINUE

DO 130 ij = 1, 21
  WRITE (15,*) ccc(ij), opthr(ij)
130 CONTINUE

c Format statements

1000 FORMAT (' ',F6.3,IX,11(F6.4,1X))

STOP
END

SUBROUTINE QfunE(xx, erfcx)

This function calculates the error function and the complimentary error function for the value "xx"

Accuracy is to within 1.5E-07.

REAL x, xx, erfcx
REAL a1, a2, a3, a4, a5, p, pi, t

pi = 3.141592654
x = ABS(xx);

a1 = 0.319381530
a2 = -0.356563782
a3 = 1.781477937

89
\[
a_4 = -1.8 \times 1255978 \\
a_5 = 1.330274429 \\
\]
\[
p = 0.2316419 \\
t = 1.0 / (1.0 + p \times x) \\
\]
\[
s_1 = a_1 \times t + a_2 \times t^2 + a_3 \times t^3 + a_4 \times t^4 + a_5 \times t^5 \\
s_2 = s_1 \times \exp(-(x^2)/2.0) \\
\]
\[
\text{IF (} xx \ \text{.GE. 0.0) THEN} \\
\quad \text{erfcx} = s_2 / \sqrt{2.0 \times \pi} \\
\text{ELSE IF (} xx \ \text{.LT. 0.0) THEN} \\
\quad \text{erfcx} = 1.0 - s_2 / \sqrt{2.0 \times \pi} \\
\text{ENDIF} \\
\]
\[
\text{RETURN} \\
\text{END} \\
\]

```fortran
SUBROUTINE FINDMIN(array, isize, minindex)

REAL array(isize)
INTEGER isize, minindex

int = 1
11 CONTINUE
  DO 10 i = int + 1, isize
    IF (array(int) .GT. array(i)) THEN
      minindex = int
      int = minindex
    END IF
  GOTO 11
10 CONTINUE

RETURN
END
```

90
APPENDIX C

Program for Cost Evaluation of 2/3-Sensor-System

The program listed in Appendix C is used for numerical evaluation of 2/3-Sensor-System’s expected costs which is described by (4.2.1) substituted with (4.5.1), (4.5.2), (4.5.3), (4.5.4), and (4.5.5).

Figure 4.3, Figure 4.4, Figure 4.5, Figure 4.6, and Table 5.3 are constructed by using outputs from this program.
PROGRAM TWO3SENSYS

Author: Howard C. Choe
Organization: Department of Electrical Engineering
The University of Virginia, Charlottesville

This program numerically evaluates expected costs of a three-sensor-system which uses team strategies for Gaussian statistics.

Variable Description
p0 : a priori probability of "No target exists", H0
p1 : a priori probability of "Target exists", H1

For Host Sensor
mh0 : mean value of H0 received by HS
mh1 : mean value of H1 received by HS
sh0 : standard deviation of H0 received by HS
sh1 : standard deviation of H1 received by HS

For Slave Sensor 1
ms10 : mean value of H0 received by SS1
ms11 : mean value of H1 received by SS1
ss10 : standard deviation of H0 received by SS1
ss11 : standard deviation of H1 received by SS1

For Slave Sensor 2
ms20 : mean value of H0 received by SS2
ms21 : mean value of H1 received by SS2
ss20 : standard deviation of H0 received by SS2
ss21 : standard deviation of H1 received by SS2

Thresholds
For Host Sensor
TL1 : lower threshold 1
TL2 : lower threshold 2
TU1 : upper threshold 1
TU2 : upper threshold 2

For Slave Sensor 1
Tss1 : LRT optimal threshold

For Slave Sensor 2
Tss2 : LRT optimal threshold

Pre-cost constant
c c00 : deciding H0 given H0
c c10 : deciding H1 given H0
c c11 : deciding H1 given H1
c c01 : deciding H0 given H1
c Team effort cost

c ccc1 : communication cost constant for communicating with one sensor

c ccc2 : communication cost constant for communicating with two sensor

c

Declaration

REAL p0, pl
REAL mh0, mh1, ms10, ms11, ms20, ms21
REAL sh0, sh1, ss10, ss11, ss20, ss21
REAL c00, c10, c11, c01
REAL Tss1, Tss2

REAL ccc1(21), ccc2(21)
REAL cfmin(21), opthr(21)
REAL TU1(1001), TU2(1001), TL1(1001), TL2(1001)
REAL pehs(1001), pzcom1(1001), pzcom2(1001), ef(1001)
REAL cbar(21,1001)

DATA ccc2/0.00, 0.05, 0.10, 0.15, 0.20,
  * 0.25, 0.30, 0.35, 0.40, 0.45,
  * 0.50, 0.55, 0.60, 0.65, 0.70,
  * 0.75, 0.80, 0.85, 0.90, 0.95, 1.00/

c Determination of communication cost constant for communicating
with 2 slave sensors

DO 5 i = 1, 21
  ccc1(i) = 0.5*ccc2(i)
5 CONTINUE

c The input data

p0 = 0.5
pl = 0.5

mh0 = -1.0
mh1 = 1.0
ms10 = -1.0
ms11 = 1.0
ms20 = -1.0
ms21 = 1.0

sigma = 1.0
c00 = 0.0
c10 = 1.0
c11 = 0.0
c01 = 1.0

c Assign all standard deviation to the same value

sh0 = sigma
sh1 = sigma
ss10 = sigma
ss11 = sigma
ss20 = sigma
ss21 = sigma

c Evaluation of the pre-calculated threshold for the team strategy and
for the slave sensor

plamss1 = (c10 - c00)/(c01 - c11)
plamss2 = (c10 - c00)/(c01 - c11)
plamt = plamss1

c Evaluation of the ratio between a priori probabilities and LRT threshold
for the host sensor and the slave sensor, considering each sensor is
centralized individually.

t0 = p0/p1
Ths = (mh0 + mh1)/2.0 + (sigma**2/(mh1 - mh0))*LOG(plamt*t0)
Tss1 = (ms10 + ms11)/2.0
* + (sigma**2/(ms11 - ms10))*LOG(plamss1*t0)
Tss2 = (ms20 + ms21)/2.0
* + (sigma**2/(ms21 - ms20))*LOG(plamss2*t0)

c Slave sensor 1

bs10 = (Tss1 - ms10)/ss10
bs11 = (Tss1 - ms11)/ss11

CALL QfunE(bs10, Qbs10)
CALL QfunE(bs11, Qbs11)

c Slave sensor 2

bs20 = (Tss2 - ms20)/ss20
bs21 = (Tss2 - ms21)/ss21
CALL QfunE(bs20, Qbs20)
CALL QfunE(bs21, Qbs21)

c FOR COMMUNICATING WITH 1 SLAVE SENSOR
c Evaluation of the final threshold after communication

\[
fs0 = \text{plamss1} \times t0 \times (1.0 - \text{Qbs10})/(1.0 - \text{Qbs11})
\]
\[
fs1 = \text{plamss1} \times t0 \times \text{Qbs10}/\text{Qbs11}
\]

c Evaluation of Q(y)-function values using the above values

\[
fs0h0 = (fs0 - \text{mh0})/\text{sh0}
\]
\[
fs1h0 = (fs1 - \text{mh0})/\text{sh0}
\]

calculation via the function QfunE

CALL QfunE(fsoh0, Qfs~h0)
CALL QfunE(fslh0, Qfslh0)

\[
fs0h1 = (fs0 - \text{mh1})/\text{sh1}
\]
\[
fs1h1 = (fs1 - \text{mh1})/\text{sh1}
\]

calculation via the function QfunE

CALL QfunE(fsoh1, Qfs~h1)
CALL QfunE(fslh1, Qfslh1)


c Calculation of an error probability by the team strategy

c with communication with 1 slave sensor

\[
\text{peteam1} = (Qfs0h0 \times (1.0 - \text{Qbs10}) + Qfs1h0 \times \text{Qbs10}) \times p0
\]
\[
+ ((1.0 - Qfs0h1) \times (1.0 - \text{Qbs11}) + (1.0 - Qfs1h1) \times \text{Qbs11}) \times p1
\]


c FOR COMMUNICATING WITH 2 SLAVE SENSORS

c Evaluation of the final threshold after communication

\[
f00 = \text{plamss1} \times t0 \times (1.0 - \text{Qbs10}) \times (1.0 - \text{Qbs20})/((1.0 - \text{Qbs11}) \times (1.0 - \text{Qbs21}))
\]
\[
f01 = \text{plamss1} \times t0 \times (1.0 - \text{Qbs10}) \times \text{Qbs20}/((1.0 - \text{Qbs11}) \times \text{Qbs21})
\]
\[
f10 = \text{plamss2} \times t0 \times \text{Qbs10} \times (1.0 - \text{Qbs20})/(\text{Qbs11} \times (1.0 - \text{Qbs21}))
\]
\[
f11 = \text{plamss2} \times t0 \times \text{Qbs10} \times \text{Qbs20}/(\text{Qbs11} \times \text{Qbs21})
\]

c Evaluation of Q(y)-function values using the above values

\[
f00h0 = (f00 - \text{mh0})/\text{sh0}
\]
\[
f01h0 = (f01 - \text{mh0})/\text{sh0}
\]
\[
f10h0 = (f10 - \text{mh0})/\text{sh0}
\]
\[
f11h0 = (f11 - \text{mh0})/\text{sh0}
\]
CALL QfunE(f00h0, Qf00h0)
CALL QfunE(f01h0, Qf01h0)
CALL QfunE(f10h0, Qf10h0)
CALL QfunE(f11h0, Qf11h0)

f00h1 = (f00 - mh1)/sh1
f01h1 = (f01 - mh1)/sh1
f10h1 = (f10 - mh1)/sh1
f11h1 = (f11 - mh1)/sh1

CALL QfunE(f00h1, Qf00h1)
CALL QfunE(f01h1, Qf01h1)
CALL QfunE(f10h1, Qf10h1)
CALL QfunE(f11h1, Qf11h1)

c Calculation of an error probability by the team strategy
with communicating with 2 slave sensors

peam2 = ( Qf00h0*(1.0-Qbs10)*(1.0-Qbs20) *
* + Qf01h0*(1.0-Qbs10)*Qbs20
* + Qf10h0*Qbs10*(1.0-Qbs20) *
* + Qf11h0*Qbs10*Qbs20 ) * p0 *
* + ( (1.0-Qf00h1)*(1.0-Qbs11)*(1.0-Qbs21)
* + (1.0-Qf01h1)*(1.0-Qbs11)*Qbs21
* + (1.0-Qf10h1)*Qbs11*(1.0-Qbs21)
* + (1.0-Qf11h1)*Qbs11*Qbs21 ) * p1

Calculation of the host sensor error probability, that of
communication frequency probability with 1 sensor, and that
c of communication frequency probability with 2 sensors-pehs,
c pzcom1, and pzcom2-respectively.

c pehs is TL1 and TU1 dependent, pzcom1 depends on TL1 and TL2,
c and TU2 and TU1, and pzcom2 is dependent upon TL2 and TU2.

unc = 0.02
DO 10 ia = 1, 21
   ib = 0
   DO 30 thr = Ths, Ths + 4.0, unc
      ib = ib + 1
      TU1(ib) = thr
      TL1(ib) = Ths - (FLOAT(ib) - 1.0)*unc
      TU2(ib) = (TU1(ib)-Ths)/2.0
      TL2(ib) = TL1(ib)+(Ths-TL1(ib))/2.0
      96
c Evaluation for pehs for various TL and TU

tu1h0 = (TU1(ib) - mh0)/sh0
t11h1 = (TL1(ib) - mh1)/sh1

CALL QfunE(tu1h0, Qu1h0)
CALL QfunE(t11h1, Qd1h1)

pehs(ib) = Qu1h0*p0 + (1.0 - Qd1h1)*p1

c Evaluation for pzcom1 for various TL1, TL2, TU1, and TU2

t11h0 = (TL1(ib) - mh0)/sh0
t2h0 = (TL2(ib) - mh0)/sh0
tu1h0 = (TU1(ib) - mh0)/sh0
tu2h0 = (TU2(ib) - mh0)/sh0

CALL QfunE(t11h0, Qu1h0)
CALL QfunE(t2h0, Qu2h0)
CALL QfunE(tu1h0, Qu1h0)
CALL QfunE(tu2h0, Qu2h0)

t11h1 = (TL1(ib) - mh1)/sh1
t2h1 = (TL2(ib) - mh1)/sh1
tu1h1 = (TU1(ib) - mh1)/sh1
tu2h1 = (TU2(ib) - mh1)/sh1

CALL QfunE(t11h1, Qu1h1)
CALL QfunE(t2h1, Qu2h1)
CALL QfunE(tu1h1, Qu1h1)
CALL QfunE(tu2h1, Qu2h1)

pzcom1(ib) = (Qd1h0 - Qu2h0 + Qu2h0 - Qu1h0) * p0
+ (Qd1h1 - Qu2h1 + Qu2h1 - Qu1h1) * p1

c Evaluation for pzcom2 for various TL2 and TU2

pzcom2(ib) = (Qd2h0 - Qu2h0)*p0 + (Qd2h1 - Qu2h1)*p1

c Evaluation for the expected probability of error of the system, COST

cbar(ia,ib) = pchs(ib)
+ (peteam1-pchs(ib)+ccc1(ia))*pzcom1(ib)
+ (peteam2-pchs(ib)+ccc2(ia))*pzcom2(ib)
+ (pehs(ib)-peteam1-peteam2-ccc1(ia)-ccc2(ia))
pfcom1(ib)*pfcom2(ib)

cf(ib) = cbar(ia,ib)

30 CONTINUE

c Extract the minimum system cost for each case of ccc(ia)

CALL FINDMIN(cf,ib,mini)
cfmin(ia) = cf(mini)
opthr(ia) = TU1(mini)

10 CONTINUE

c Write OUTPUT DATA ....

WRITE (10,*), 'Ratio of a priori probabilities --: ', t0
WRITE (10,*), 'LRT Threshold of SS1 --------: ', Tss1
WRITE (10,*), 'LRT Threshold of SS2 --------: ', Tss2
WRITE (10,*), 'Error caused by TS using SS1 only : ', peteam1
WRITE (10,*), 'FT at HS when Usl=0 --------: ', f00
WRITE (10,*), 'FT at HS when Usl=1 --------: ', f01
WRITE (10,*), 'FT at HS when Usl=1 and Us2=1 ----: ', f11

DO 45 ja = 1, ib
   WRITE (9,*), TLI(ja), TL2(ja), TU2(ja), TU1(ja)
45 CONTINUE

DO 50 ic = 1, ib
   WRITE (11,1000), TUI(ic), (cbar(id,ic),id=1,10)
50 CONTINUE

DO 70 ie = 1, ib
   WRITE (12,1000), TU1(ic), (cbar(ig,ie),ig=1,20)
70 CONTINUE

DO 90 ih = 1, 21
   WRITE (13,*), ccc2(ih), cfmin(ih)
90 CONTINUE
DO 110 ii = 1, 21
   WRITE (14,*) opthr(ii), cfmin(ii)
110 CONTINUE

DO 130 ij = 1, 21
   WRITE (15,*) ccc2(ij) opthr(ij)
130 CONTINUE

c Format statements

1000 FORMAT (' ',1(F6.4,1X))

STOP
END

SUBROUTINE QfunE(xx, erfcx)

This function calculates the error function and the complimentary error function for the value "xx"

Accuracy is to within 1.5E-07.

REAL x, xx, erfcx
REAL a1, a2, a3, a4, a5, p, pi, t
pi = 3.141592654
x = ABS(xx)

a1 = 0.319381530
a2 = -0.356563782
a3 = 1.781477937
a4 = -1.821255978
a5 = 1.330274429

p = 0.2316419

IF (xx .GE. 0.0) THEN
   IF (x .GT. 3.0) THEN
      erfcx = 0.5
   ELSE
      t = 1.0/(1.0 + p*x)
      s1 = a1*t + a2*t**2 + a3*t**3 + a4*t**4 + a5*t**5
      s2 = s1*EXP(-(x**2)/2.0)
      erfcx = s2/SQRT(2.0*pi)
   END IF
END IF

erfcx = s2/SQRT(2.0*pi)
ELSE IF (xx .LT. 0.0) THEN
   erfcx = 1.0 - s2/SQR(2.0*pi)
END IF

RETURN
END

c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SUBROUTINE FINDMIN(array,isize,minindex)

REAL array(isize)
INTEGER minindex

int = 1
11 CONTINUE
   DO 10 i = int+1, isize
      IF (array(int) .GT. array(i)) THEN
         minindex = i
         int = minindex
      END IF
   GOTO 11
10 CONTINUE

RETURN
END

c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
APPENDIX D

Program for Calculation of Dubious Decision Probabilities

In this appendix, a program UNPRO (UNcertainty PRObability) is attached. This program evaluates the dubious decision probability at the host sensor in 2SS, 3SS, and 23SS when the optimal thresholds for given communication cost constants are known. These thresholds can be obtained from the programs attached in Appendix A, Appendix B, and Appendix C.

The information obtained by this program are plotted in Figure 5.1, Figure 5.2, and Figure 5.3. The tabulated data can also be found in Table 5.8, Table 5.9, and Table 5.10.
PROGRAM UNPRO

This program UNPRO (UNcertainty PRObability) is written to evaluate the probability of the observation that falls in the uncertainty region of the host sensor. This program reads in the optimal threshold locations which are evaluated using programs such as 2sensys.f, 3sensys.f, and 2/3sensys.f (These programs are listed in Appendix A, B, and C, respectively.).

Declaration

REAL otp(3,21), error(3,21)
REAL qlimR(3,21), qlimL(3,21)
REAL yintR(3,21), yintL(3,21)
REAL prob(3,21)

Open data file and rewind

OPEN (UNIT=11,FILE='fort.11')
OPEN (UNIT=12,FILE='fort.12')
OPEN (UNIT=13,FILE='fort.13')

REWIND (11)
REWIND (12)
REWIND (13)

Statistics for Gaussian observation

avg = 1.0
sigma = 1.0

Read in input data from files opened

DO 10 i = 1, 21
   READ (11,*), otp(1,i), error(1,i)
   READ (12,*), otp(2,i), error(2,i)
   READ (13,*), otp(3,i), error(3,i)
10 CONTINUE

Calculation of limits (qlimR and qlimL) for Q(y)-function
c and evaluate the corresponding probabilities.

DO 30 i = 1, 3
   DO 50 j = 1, 21
      qlimR(i,j) = (otp(i,j)-avg)/sigma
      qlimL(i,j) = (-otp(i,j)-avg)/sigma
   
   xxR = qlimR(i,j)
   CALL QfunE(xxR,erfcxR)

   xxL = qlimL(i,j)
   CALL QfunE(xxL,erfcxL)

   yintR(i,j) = erfcxR
   yintL(i,j) = erfcxL
50  CONTINUE
30  CONTINUE

c Evaluate the probability of dubious decision for one hypothesis

DO 70 i = 1, 3
   DO 90 j = 1, 21
      prob(i,j) = yintL(i,j) - yintR(i,j)
   90  CONTINUE
70  CONTINUE

c prob(,) is multiplied by 2.0 since prob(,) is the probability of
a an observation that lies in the uncertainty region of HS given H1.
c The dubious decision probability, when HO is considered, is the same.

DO 110 i = 1, 21
   WRITE (21,1000) otp(1,i), prob(1,i)*2.0
   WRITE (22,1000) otp(2,i), prob(2,i)*2.0
   WRITE (23,1000) otp(3,i), prob(3,i)*2.0
110  CONTINUE

STOP

1000 FORMAT (' ',F8.3,F8.4)

END
SUBROUTINE QfunE(xx, erfcx)

! This function calculates the error function and the complimentary
! error function for the value "xx"

! Accuracy is to within 1.5E-07.

REAL x, xx, erfcx
REAL a1, a2, a3, a4, a5, p, pi, t

pi = 3.14159265
x = ABS(xx)

a1 = 0.319381530
a2 = -0.356563782
a3 = 1.781477937
a4 = -1.821255978
a5 = 1.330274429

p = 0.2316419
q = 1.0/(1.0 + p*x)

s1 = a1*q + a2*q**2 + a3*q**3 + a4*q**4 + a5*q**5
s2 = s1*EXP(-x**2/2.0)

IF (xx .GE. 0.0) THEN
    erfcx = s2/SQRT(2.0*pi)
ELSE IF (xx .LT. 0.0) THEN
    erfcx = 1.0 - s2/SQRT(2.0*pi)
END IF

RETURN
END
The program, SENSIM (SENsor SIMulation), simulates 2SS, 3SS, and 2/3SS for Gaussian observations. This program incorporates the programs listed in Appendix A, B, and C. These programs - TWOSENSYM, THREESENSYM, and TWO3SENSYM - become subroutines named SETBAND2, SETBAND3, and SETBAND23, respectively. These subroutines return the optimal thresholds location and the final decision thresholds at the host sensor for a given communication cost constant. There are other subroutines which are used to generate Gaussian random observation (or Gaussian random number). Because of the mutually independent observations among sensors, each sensor is provided with its own Gaussian random observation generator.

Outputs from this simulation are presented in Figure 6.1, Figure 6.2, and Figure 6.3. Tabulated data of these figures are in Table 6.1 and Table 6.2.

In the subroutine SETBAND2, SETBAND3, and SETBAND23, all the comments are omitted since they are the same as the programs attached in Appendix A, B, and C.
PROGRAM SENSIM

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*ORGANIZATION: Department of Electrical Engineering
University of Virginia, Charlottesville

This program simulates the sensor systems (2, 3, and 2/3 sensor system) which uses team strategies. The host sensor and the slave sensors receive independent observations from the binary hypothesis environment under Gaussian model.

*Find the optimum thresholds of the host sensor (TL & TU or TL1, TL2, TU2, and TU1) for the different system.

WRITE (6,*) 'ENTER ccc for each system, 2, 3, & 23'
READ (5,*) ccc2, ccc3, ccc23

WRITE (6,*) 'ENTER # of iterations desired'
READ (5,*) nter

WRITE (6,*) 'ENTER # seed for a random # generation'
READ (5,*) iseed

TLRT = 0.0
CALL SETBAND2(ccc2, TL2, TU2, F20, F21, T2)
CALL SETBAND3(ccc3, TL3, TU3, F300, F301, F310, F311, T31, T32)
CALL SETBAND23(ccc23, TL31, TL32, TU32, TU31, *F0, F1, F00, F01, F10, F11, T231, T232)

WRITE (10,*) '********* ONE-SENSOR-SYSTEM ************'
WRITE (10,*) 
WRITE (10,*) 'LRT Threshold --------------- ', TLRT
WRITE (10,*) '********* TWO-SENSOR-SYSTEM ************'
WRITE (10,*) 'Lower Threshold --------------- ', TL2
WRITE (10,*) 'Upper Threshold --------------- ', TU2
WRITE (10,*) 'LRT threshold of Slave Sensor: ', T2
WRITE (10,*) 'Final Threshold when U = 0 : ', F20
WRITE (10,*) 'Final Threshold when U = 1 : ', F21
WRITE (10,*)
WRITE (10,*) '********* THREE-SENSOR-SYSTEM ************'

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WRITE (10,*) 'Lower Threshold ----------: ', TL3
WRITE (10,*) 'Upper Threshold -------------: ', TU3
WRITE (10,*) 'LRT threshold of SS1 ---------: ', T31
WRITE (10,*) 'LRT threshold of SS2 ---------: ', T32
WRITE (10,*) 'FT when Us1 = 0 & Us2 = 0 ---: ', F300
WRITE (10,*) 'FT when Us1 = 0 & Us2 = 1 ---: ', F301
WRITE (10,*) 'FT when Us1 = 1 & Us2 = 0 ---: ', F310
WRITE (10,*) 'FT when Us1 = 1 & Us2 = 1 ---: ', F311
WRITE (10,*) '******** TWO/THREE-SENSOR-SYSTEM ********
WRITE (10,*) 'Lower Threshold 1 --------: ', TL31
WRITE (10,*) 'Lower Threshold 2 --------: ', TL32
WRITE (10,*) 'Upper Threshold 2 --------: ', TU32
WRITE (10,*) 'Upper Threshold 1 --------: ', TU31
WRITE (10,*) 'LRT threshold of SS1 --------: ', T231
WRITE (10,*) 'LRT threshold of SS2 --------: ', T232
WRITE (10,*) 'FT when Us1 = 0 -----------: ', F0
WRITE (10,*) 'FT when Us1 = 0 & Us2 = 0 ---: ', F00
WRITE (10,*) 'FT when Us1 = 0 & Us2 = 1 ---: ', F01
WRITE (10,*) 'FT when Us1 = 1 & Us2 = 0 ---: ', F10
WRITE (10,*) 'FT when Us1 = 1 & Us2 = 1 ---: ', F11

c Standard deviation of observation

sigma = 1.0

c ITERATION STARTS

ic0 = 0
ie1 = 0

icd1 = 0
ifa1 = 0
imt1 = 0

icd2 = 0
ifa2 = 0
imt2 = 0

icd3 = 0
ifa3 = 0
imt3 = 0

icd23 = 0

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ifa23 = 0
int23 = 0

c Get system clock time for random seeds

c im = mclock()
c WRITE (6,*) 'im = ', im

c iseed = 74591 + 2*MOD(1000*im,500)

CALL SRAND(iseed)

DO 10 ia = 1, ntr

c Generate Environment

11 CALL GENENV(ienv)
    env = FLOAT(ienv)
    IF (env .EQ. -1.0) THEN
        ie0 = ie0 + 1
    ELSE IF (env .EQ. 1.0) THEN
        ie1 = ie1 + 1
    ELSE
        WRITE (6,*) '### Generated ENV is NOT either -1 or 1 ###'
        GO TO 11
    END IF

END IF

c Generate Observations at each sensors

c For 1-Sensor-System
    CALL HS1(env,sigma,yh1)

    CALL HS2(env,sigma,yh2)
    CALL SS2(envsigma,ys2)

c For 2-Sensor-System
    CALL HS2(env,sigma,yh3)

c For 3-Sensor-System
    CALL HS3(env,sigma,yh3)
    CALL SS31(env,sigma,ys31)
    CALL SS32(env,sigma,ys32)

c For 23-Sensor-System
    CALL HS23(env,sigma,yh23)
    CALL SS231(env,sigma,ys231)
    CALL SS232(env,sigma,ys232)

    yh2 = yh1
y3 = y1  
yh2 = yh1

ys1 = y2  
ys23 = y2

ys232 = y32

c WRITE (33,1100) env,yh1,yh2,ys2,ys31,ys32,yh23,ys231,ys232
c1100 FORMAT (' ',10(F7.3,1X))

c Single Sensor using LRT Threshold
   IF (yh1 .LE. TLRT) THEN
      uhl = -1.0
   ELSE IF (yh1 .GT. TLRT) THEN
      uhl = 1.0
   END IF
   iuhl = INT(uhl)

c Count False alarm, missing target, and correct detection
   IF (iuhl .EQ. ienv) THEN
      icd1 = icd1 + 1
   ELSE IF (iuhl.EQ.1 .AND. ienv.EQ.-1) THEN
      ifal = ifal + 1
   ELSE IF (iuhl.EQ.-1 .AND. ienv.EQ.1) THEN
      imtl = imtl + 1
   END IF

c Use Team Strategies to make Decision

c For 2-Sensor-System
   IF (yh2 .LE. TL2) THEN
      uh2 = -1.0  
   ELSE IF (yh2 .GE. TU2) THEN  
      uh2 = 1.0
   ELSE IF (yh2.GT.TL2 .AND. yh2.LT.TU2) THEN
      IF (ys2 .LE. T2) THEN
         us2 = -1.0
         IF (yh2 .LE. F20) THEN
            uh2 = -1.0
         ELSE IF (yh2 .GT. F20) THEN
            uh2 = 1.0
         END IF
      ELSE IF (ys2 .GT. T2) THEN
         us2 = 1.0
         IF (yh2 .LE. F21) THEN
            uh2 = -1.0
      ELSE
         uh2 = uh2
      END IF
   ELSE IF (ys2 .GT. T2) THEN
      us2 = 1.0
      IF (yh2 .LE. F21) THEN
         uh2 = -1.0
   ELSE
      uh2 = uh2
   END IF
ELSE IF (yh2 .GT. F21) THEN
  uh2 = 1.0
END IF
END IF
END IF

iuh2 = INT(uh2)

c Count False alarm, missing target, and correct detection
IF (iuh2 .EQ. ienv) THEN
  icd2 = icd2 + 1
ELSE IF (iuh2 .EQ. 1 .AND. ienv .EQ. -1) THEN
  ifa2 = ifa2 + 1
ELSE IF (iuh2 .EQ. -1 .AND. ienv .EQ. 1) THEN
  imt2 = imt2 + 1
END IF

c For 3-Sensor-System
IF (yh3 .LE. TL3) THEN
  uh3 = -1.0
ELSE IF (yh3 .GE. TU3) THEN
  uh3 = 1.0
ELSE IF (yh3 .GT. TL3 .AND. yh3 .LT. TU3) THEN
  IF (ys31 .LE. T31 .AND. ys32 .LE. T32) THEN
    us31 = -1.0
    us32 = -1.0
  IF (yh3 .LE. F300) THEN
    uh3 = -1.0
  ELSE IF (yh3 .GT. F300) THEN
    uh3 = 1.0
END IF
ELSE IF (ys31 .GT. T31 .AND. ys32 .LE. T32) THEN
  us31 = 1.0
  us32 = -1.0
  IF (yh3 .LE. F310) THEN
    uh3 = -1.0
  ELSE IF (yh3 .GT. F310) THEN
    uh3 = 1.0
END IF
ELSE IF (ys31 .LT. T31 .AND. ys32 .GT. T32) THEN
  us31 = -1.0
  us32 = 1.0
  IF (yh3 .LE. F310) THEN
    uh3 = -1.0
  ELSE IF (yh3 .GT. F310) THEN
    uh3 = 1.0
END IF

END IF
ELSE IF (ys31.GT.T31 .AND. ys32.GT.T32) THEN
    us31 = 1.0
    us32 = 1.0
    IF (yh3 .LE. F311) THEN
        uh3 = -1.0
    ELSE IF (yh3 .GT. F311) THEN
        uh3 = 1.0
    END IF
END IF
ELSE IF (ys32 .GT. T32) THEN
    us32 = 1.0
    IF (yh3 .LE. F321) THEN
        uh3 = -1.0
    ELSE IF (yh3 .GT. F321) THEN
        uh3 = 1.0
    END IF
END IF
END IF
END IF
END IF
iuh3 = INT(uh3)
c Count False alarm, missing target, and correct detection
IF (iuh3 .EQ. ienv) THEN
    icd3 = icd3 + 1
ELSE IF (iuh3.EQ.1 .AND. ienv.EQ.-1) THEN
    ifa3 = ifa3 + 1
ELSE IF (iuh3.EQ.-1 .AND. ienv.EQ.1) THEN
    imt3 = imt3 + 1
END IF
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ENDIF
us231 = -1.0
us232 = -1.0
IF (yh23 .LE. F00) THEN
  uh23 = -1.0
ELSE IF (yh23 .GT. F00) THEN
  uh23 = 1.0
END IF
ELSE IF (ys231 .LE. T231 .AND. ys232 .GT. T232) THEN
  us231 = -1.0
  us232 = 1.0
  IF (yh23 .LE. F01) THEN
    uh23 = -1.0
  ELSE IF (yh23 .GT. F01) THEN
    uh23 = 1.0
  END IF
ELSE IF (ys231 .GT. T231 .AND. ys232 .LE. T232) THEN
  us231 = 1.0
  us232 = -1.0
  IF (yh23 .LE. F10) THEN
    uh23 = -1.0
  ELSE IF (yh23 .GT. F10) THEN
    uh23 = 1.0
  END IF
ELSE IF (ys231 .GT. T231 .AND. ys232 .GT. T232) THEN
  us231 = 1.0
  us232 = 1.0
  IF (yh23 .LE. F11) THEN
    uh23 = -1.0
  ELSE IF (yh23 .GT. F11) THEN
    uh23 = 1.0
  END IF
END IF
END IF
END IF
iuh23 = INT(uh23)
c Count False alarm, missing target, and correct detection
IF (iuh23 .EQ. ienv) THEN
  icd23 = icd23 + 1
ELSE IF (iuh23 .EQ. 1 .AND. ienv .EQ. -1) THEN
  ifa23 = ifa23 + 1
ELSE IF (iuh23 .EQ. -1 .AND. ienv .EQ. 1) THEN
  imt23 = imt23 + 1
END IF
10 CONTINUE
c Find the percentage of 0's and 1's in total environment generated

\[
\begin{align*}
\text{pev0} &= 100.0 \times \text{FLOAT}(\text{ie0})/\text{FLOAT}(\text{ia}) \\
\text{pev1} &= 100.0 \times \text{FLOAT}(\text{ie1})/\text{FLOAT}(\text{ia}) \\
\end{align*}
\]

WRITE (10,*), ' WRITE (10,*), '@@@@@@ % of 0 or 1 of the Environment @@@@@@' WRITE (10,*), '% of 0s :', pev0 WRITE (10,*), '% of 1s :', pev1

c Find the percentage of correct detection, false alarm, and missing target for each system.

c 1-Sensor-System
\[
\begin{align*}
\text{pcd1} &= 100.0 \times \text{FLOAT}(\text{icd1})/\text{FLOAT}(\text{ia}) \\
\text{pfa1} &= 100.0 \times \text{FLOAT}(\text{ifa1})/\text{FLOAT}(\text{ia}) \\
\text{pmt1} &= 100.0 \times \text{FLOAT}(\text{imt1})/\text{FLOAT}(\text{ia}) \\
\end{align*}
\]

c 2-Sensor-System
\[
\begin{align*}
\text{pcd2} &= 100.0 \times \text{FLOAT}(\text{icd2})/\text{FLOAT}(\text{ia}) \\
\text{pfa2} &= 100.0 \times \text{FLOAT}(\text{ifa2})/\text{FLOAT}(\text{ia}) \\
\text{pmt2} &= 100.0 \times \text{FLOAT}(\text{imt2})/\text{FLOAT}(\text{ia}) \\
\end{align*}
\]

c 3-Sensor-System
\[
\begin{align*}
\text{pcd3} &= 100.0 \times \text{FLOAT}(\text{icd3})/\text{FLOAT}(\text{ia}) \\
\text{pfa3} &= 100.0 \times \text{FLOAT}(\text{ifa3})/\text{FLOAT}(\text{ia}) \\
\text{pmt3} &= 100.0 \times \text{FLOAT}(\text{imt3})/\text{FLOAT}(\text{ia}) \\
\end{align*}
\]

c 23-Sensor-System
\[
\begin{align*}
\text{pcd23} &= 100.0 \times \text{FLOAT}(\text{icd23})/\text{FLOAT}(\text{ia}) \\
\text{pfa23} &= 100.0 \times \text{FLOAT}(\text{ifa23})/\text{FLOAT}(\text{ia}) \\
\text{pmt23} &= 100.0 \times \text{FLOAT}(\text{imt23})/\text{FLOAT}(\text{ia}) \\
\end{align*}
\]

c WRITE the percentages
WRITE (10,*), ' WRITE (10,*), '// % of CD, FA, and MT for 1-Sensor-System \ WRITE (10,*), 'Correct Decision %: ', pcd1 WRITE (10,*), 'False Alarm % ---: ', pfa1 WRITE (10,*), 'Missing Target % : ', pmt1 WRITE (10,*), 'CD + FA + MT in % : ', pcd1+pfa1+pmt1 WRITE (10,*), ' WRITE (10,*), '// % of CD, FA, and MT for 2-Sensor-System \ WRITE (10,*), 'Correct Decision %: ', pcd2 WRITE (10,*), 'False Alarm % ---: ', pfa2
WRITE (10,*) 'Missing Target % : ', pmt2
WRITE (10,*) 'CD + FA + MT in % : ', pcd2+pfa2+pmt2
WRITE (10,*)
WRITE (10,*) '// % of CD, FA, and MT for 3-Sensor-System \'
WRITE (10,*) 'Correct Decision %: ', pcd3
WRITE (10,*) 'False Alarm % ----: ', pfa3
WRITE (10,*) 'Missing Target % : ', pmt3
WRITE (10,*) 'CD + FA + MT in % : ', pcd3+pfa3+pmt3
WRITE (10,*)
WRITE (10,*) '// % of CD, FA, and MT for 23-Sensor-System \'
WRITE (10,*) 'Correct Decision %: ', pcd23
WRITE (10,*) 'False Alarm % ----: ', pfa23
WRITE (10,*) 'Missing Target % : ', pmt23
WRITE (10,*) 'CD + FA + MT in % : ', pcd23+pfa23+pmt23

STOP

END

c%% Subroutine for the generation of environment %%%%

SUBROUTINE GENENV(ienv)

irn = irand0()
xrn = FLOAT(ir)/FLOAT(2**15 - 1)
renv = xrn - 0.5

IF (renv .LE. 0.0) THEN
   ienv = -1
ELSE IF (renv .GT. 0.0) THEN
   ienv = 1
END IF

RETURN
END

c%% Subroutine for Single-Sensor-System %%%%

SUBROUTINE HS1(env,sigma,yhl)

a = 0.0
DO 10 i = 1, 12
   irn = irand0()
xrn = FLOAT(ir)/FLOAT(2**15 - 1)
a = a + xrn

114
c%% Subroutines for 2-Sensor-System %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

SUBROUTINE HS2(env,sigma,yh2)

a = 0.0
DO 10 i = 1, 12
  irn = irand0
  xrn = FLOAT(irn)/FLOAT(2**15 - 1)
  a = a + xrn
10 CONTINUE

yh2 = (a - 6.0)*sigma + env

RETURN
END

SUBROUTINE SS2(env,sigma,ys2)

a = 0.0
DO 10 i = 1, 12
  irn = irand0
  xrn = FLOAT(irn)/FLOAT(2**15 - 1)
  a = a + xrn
10 CONTINUE

ys2 = (a - 6.0)*sigma + env

RETURN
END

SUBROUTINE HS3(env,sigma,yh3)

a = 0.0
DO 10 i = 1, 12
irn = irand0
xrn = FLOAT(irn)/FLOAT(2**15 - 1)
a = a + xrn
10 CONTINUE

yh3 = (a - 6.0)*sigma + env

RETURN
END

C-------------------------------------------------------------------
SUBROUTINE SS31(env,sigma,ys31)
a = 0.0
DO 10 i = 1, 12
   irn = irand0
   xrn = FLOAT(irn)/FLOAT(2**15 - 1)
a = a + xrn
10 CONTINUE

ys31 = (a - 6.0)*sigma + env

RETURN
END

C-------------------------------------------------------------------
SUBROUTINE SS32(env,sigma,ys32)
a = 0.0
DO 10 i = 1, 12
   irn = irand0
   xrn = FLOAT(irn)/FLOAT(2**15 - 1)
a = a + xrn
10 CONTINUE

ys32 = (a - 6.0)*sigma + env

RETURN
END

c%% Subroutines for 23-Sensor-System %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SUBROUTINE HS23(env,sigma,yh23)

a = 0.0
DO 10 i = 1, 12
  irn = irand()
  xrn = FLOAT(irn)/FLOAT(2**15 - 1)
  a = a + xrn
10  CONTINUE

yh23 = (a - 6.0)*sigma + env

RETURN
END

SUBROUTINE SS231(env,sigma,ys231)

a = 0.0
DO 10 i = 1, 12
  irn = irand()
  xrn = FLOAT(irn)/FLOAT(2**15 - 1)
  a = a + xrn
10  CONTINUE

ys231 = (a - 6.0)*sigma + env

RETURN
END

SUBROUTINE SS232(env,sigma,ys232)

a = 0.0
DO 10 i = 1, 12
  irn = irand()
  xrn = FLOAT(irn)/FLOAT(2**15 - 1)
  a = a + xrn
10  CONTINUE

ys232 = (a - 6.0)*sigma + env

RETURN
END
SUBROUTINE SETBAND2(ccc2,TL2,TU2,F20,F21,T2)

REAL p0, p1
REAL mh0, mh1, ms0, ms1
REAL sh0, sh1, ss0, ss1
REAL c00, c10, c11, c01
REAL Tss

REAL ccc2, cfmin, oplth, oputhr
REAL TU(1001), TL(1001)
REAL pehs(1001), pzcom(1001)
REAL cbar(1001)

p0 = 0.5
p1 = 0.5

mh0 = -1.0
mh1 = 1.0
ms0 = -1.0
ms1 = 1.0

sigma = 1.0

c00 = 0.0
c10 = 1.0
c11 = 0.0
c01 = 1.0

sh0 = sigma
sh1 = sigma
ss0 = sigma
ss1 = sigma

plamss = (c10 - c00)/(c01 - c11)
plamt = plamss

t0 = p0/p1
Ths = (mh0 + mh1)/2.0 + (sigma**2/(mh1 - mh0))*LOG(plamt*t0)
Tss = (ms0 + ms1)/2.0 + (sigma**2/(ms1 - ms0))*LOG(plamss*t0)

ao0 = (Tss - ms0)/ss0
al0 = (Tss - ms1)/ss1
CALL QfunE(a0s, Qa0s)
CALL QfunE(a1s, Qa1s)

\[ f_{u0} = \text{plamss} \times t_0 \times (1.0 - Qa0s)/(1.0 - Qa1s) \]
\[ f_{u1} = \text{plamss} \times t_0 \times Qa0s/Qa1s \]

\[ f_{u0h0} = (f_{u0} - mh0)/sh0 \]
\[ f_{u1h0} = (f_{u1} - mh0)/sh0 \]
\[ f_{u0h1} = (f_{u0} - mh1)/sh1 \]
\[ f_{u1h1} = (f_{u1} - mh1)/sh1 \]

CALL QfunE(fus0h0, Qfus0h0)
CALL QfunE(fus1h0, Qfus1h0)
CALL QfunE(fus0h1, Qfus0h1)
CALL QfunE(fus1h1, Qfus1h1)

peteams = \( (Qfus0h0 \times (1.0 - Qa0s) + Qfus1h0 \times Qa0s) \times p0 + ((1.0 - Qfus0h1)/(1.0 - Qa1s) + (1.0 - Qfus1h1)/(1.0 - Qa1s)) \times p1 \)

tinc = 0.02
ib = 0
DO 30 thr = Ths, Ths + 2.0, tinc
  ib = ib + 1
  TL(ib) = thr
  TL(ib) = Ths \times (\text{FLOAT}(ib) - 1.0) \times \text{tinc}

\[ t_{uh0} = (\text{TL}(ib) - mh0)/sh0 \]
\[ t_{lh1} = (\text{TL}(ib) - mh1)/sh1 \]

CALL QfunE(tu0h0, Qtuh0)
CALL QfunE(tu1h1, Qtuh1)

pehs(ib) = Qtuh0 \times p0 + (1.0 - Qtuh1) \times p1

\[ t_{lh0} = (\text{TL}(ib) - mh0)/sh0 \]
\[ t_{uh1} = (\text{TL}(ib) - mh1)/sh1 \]

CALL QfunE(tl0h0, Qtuh0)
CALL QfunE(tu1h1, Qtuh1)

pzcom(ib) = (Qtuh0 - C_tuh0) \times p0 + (Qtuh1 - Qtuh1) \times p1

cbar(ib) = pehs(ib) + (peteams-pehs(ib) + \text{ccc2}) \times pzcom(ib)

30 CONTINUE
CALL FINDMIN(cbar,ib,mini)
cfmin = cbar(mini)
opth = TL(mini)
opurh = TU(mini)

TL2 = opth
TU2 = opurh
F20 = fus0
F21 = -fus0
T2 = Tss

RETURN
END

SUBROUTINE SETBAND3(ccc3,TL3,TU3,F300,F301,F310,F311,T31,T32)

REAL p0, p1
REAL mh0, mh1, ms10, ms11, ms20, ms21
REAL sh0, sh1, ss10, ss11, ss20, ss21
REAL c00, c10, c11, c01
REAL Tss1, Tss2

REAL ccc3
REAL cfmin, opth, opurh
REAL TU(1001), TL(1001)
REAL pchs(1001), pzcom(1001)
REAL cbar(1001)

p0 = 0.5
p1 = 0.5

mh0 = -1.0
mh1 = 1.0
ms10 = -1.0
ms11 = 1.0
ms20 = -1.0
ms21 = 1.0

sigma = 1.0

c00 = 0.0
c10 = 1.0
c11 = 0.0
c01 = 1.0

sh0 = sigma
sh1 = sigma
ss10 = sigma
ss11 = sigma
ss20 = sigma
ss21 = sigma

plamss1 = (c10 - c00)/(c01 - c11)
plamss2 = (c10 - c00)/(c01 - c11)
plamt = plamss1

t0 = p0/p1

Tss1 = (mh0 + mh1)/2.0 + (sigma**2/(mh1 - mh0))*LOG(plamt*t0)

Tss1 = (ms10 + ms11)/2.0

Tss2 = (ms20 + ms21)/2.0

bsl1 = (Tss1 - ms11)/ss11

bsl1 = (Tss1 - ms11)/ss11

bs20 = (Tss2 - ms20)/ss20

bs21 = (Tss2 - ms21)/ss21

bs10 = (Tss1 - ms10)/ss10

CALL QfunE(bs10, Qbs10)

CALL QfunE(bs11, Qbs11)

bs20 = (Tss2 - ms20)/ss20

bs21 = (Tss2 - ms21)/ss21

CALL QfunE(bs20, Qbs20)

CALL QfunE(bs21, Qbs21)

f00 = plamss1*t0*(1.0-Qbs10)*(1.0-Qbs20)/(1.0-Qbs11)*(1.0-Qbs21)

f01 = plamss1*t0*(1.0-Qbs10)*Qbs20/(1.0-Qbs11)*Qbs21

f10 = plamss2*t0*(1.0-Qbs10)*Qbs10/(1.0-Qbs20)/(Qbs11*(1.0-Qbs21))

f11 = plamss2*t0*Qbs10*Qbs20/(Qbs11*Qbs21)

f00h0 = (f00 - mh0)/sh0

f01h0 = (f01 - mh0)/sh0

f10h0 = (f10 - mh0)/sh0

f11h0 = (f11 - mh0)/sh0

CALL QfunE(f00h0, Qf00h0)

CALL QfunE(f01h0, Qf01h0)
CALL QfunE(f10h0, Qf10h0)
CALL QfunE(f11h0, Qf11h0)

f00h1 = (f00 - mhl)/sh1
f01h1 = (f01 - mhl)/sh1
f10h1 = (f10 - mhl)/sh1
f11h1 = (f11 - mhl)/sh1

CALL QfunE(f00h1, Qf00h1)
CALL QfunE(f01h1, Qf01h1)
CALL QfunE(f10h1, Qf10h1)
CALL QfunE(f11h1, Qf11h1)

peteam = ( Qf00h0*(1.0-Qbs10)*(1.0-Qbs20)
    * + Qf01h0*(1.0-Qbs10)*Qbs20
    * + Qf10h0*Qbs10*(1.0-Qbs20)
    * + Qf11h0*Qbs10*Qbs20 ) * p0
    * + ( (1.0-Qf00h1)*(1.0-Qbs11)*(1.0-Qbs21)
    * + (1.0-Qf01h1)*(1.0-Qbs11)*Qbs21
    * + (1.0-Qf10h1)*Qbs11*(1.0-Qbs21)
    * + (1.0-Qf11h1)*Qbs11*Qbs21 )*p1

tinc = 0.02
ib = 0
DO 30 thr = Ths, Ths + 2.0, tinc
    ib = ib + 1
    TU(ib) = thr
    TL(ib) = Ths - (FLOAT(ib) - 1.0)*tinc

    tuh0 = (TU(ib) - mh0)/sh0
    tlh1 = (TL(ib) - mhl)/sh1

CALL QfunE(tuh0, Quh0)
CALL QfunE(tlh1, Qth1)

pehs(ib) = Quh0*p0 + (1.0 - Qth1)*p1

tlh0 = (TL(ib) - mh0)/sh0
    tuh1 = (TU(ib) - mhl)/sh1

CALL QfunE(tlh0, Qth0)
CALL QfunE(tuh1, Quh1)

pzcom(ib) = (Qthl0 - Quh0)*p0 + (Qtlh1 - Qtu1h1)*p1
cbar(ib) = pehs(ib) + (peteam-pehs(ib)+ccc3)*pzcom(ib)

30 CONTINUE

CALL FINDMIN(cbar,ib,mini)
cfmin = cbar(mini)
oplthr = TL(mini)
oputhr = TU(mini)

TL3 = oplthr
TU3 = oputhr
F300 = f00
F301 = f01
F310 = f10
F311 = -f00
T31 = Tss1
T32 = Tss2

RETURN
END

SUBROUTINE SETBAND23(ccc23,TL3,TL32,TU32,TU31,  
* F0,F1,F00,F01,F10,F11,T231,T232)

REAL p0, p1
REAL mh0, mh1, ms10, ms11, ms20, ms21
REAL sh0, sh1, ss10, ss11, ss20, ss21
REAL c00, c10, c11, c01
REAL Tss1, Tss2

REAL ccc1, ccc2
REAL cfmin
REAL opu1, opd2, optu2, optu1
REAL TU1(1001), TU2(1001), TL1(1001), TL2(1001)
REAL pehs(1001), pzcom1(1001), pzcom2(1001)
REAL cbar(1001)

ccc1 = ccc23/2.0
ccc2 = ccc23

p0 = 0.5
p1 = 0.5
mh0 = -1.0
mh1 = 1.0
ms10 = -1.0
ms11 = 1.0
ms20 = -1.0
ms21 = 1.0

sigma = 1.0

c00 = 0.0
c10 = 1.0
c11 = 0.0
c01 = 1.0

sh0 = sigma
sh1 = sigma
ss10 = sigma
ss11 = sigma
ss20 = sigma
ss21 = sigma

plamss1 = (c10 - c00)/(c01 - c11)
plamss2 = (c10 - c00)/(c01 - c11)
plamt = plamss1

t0 = p0/p1

Ths = (mh0 + mh1)/2.0 + (sigma**2/(mh1 - mh0))*LOG(plamt*t0)
Tssl1 = (ms10 + ms11)/2.0
* + (sigma**2/(ms11 - ms10))*LOG(plamss1*t0)
Tss2 = (ms20 + ms21)/2.0
* + (sigma**2/(ms21 - ms20))*LOG(plamss2*t0)

bs10 = (Tssl1 - ms10)/ss10
bs11 = (Tssl1 - ms11)/ss11

CALL Qfune(bs10, Qbs10)
CALL Qfune(bs11, Qbs11)

bs20 = (Tss2 - ms20)/ss20
bs21 = (Tss2 - ms21)/ss21

CALL Qfune(bs20, Qbs20)
CALL Qfune(bs21, Qbs21)

fs0 = plamss1*t0*(1.0-Qbs10)/(1.0-Qbs11)
fsl = plamss1*t0*Qbs10/Qbs11

fs0h0 = (fs0 - mh0)/sh0
fs1h0 = (fs1 - mh0)/sh0

CALL QfunE(fs0h0, Qfs0h0)
CALL QfunE(fs1h0, Qfs1h0)

fs0h1 = (fs0 - mh1)/sh1
fs1h1 = (fs1 - mh1)/sh1

CALL QfunE(fs0h1, Qfs0h1)
CALL QfunE(fs1h1, Qfs1h1)

ptideam1 = (Qfs0h0*(1.0-Qbs10) + Qfs1h0*Qbs10) * p0
* + ((1.0-Qfs0h1)*(1.0-Qbs11) + (1.0-Qfs1h1)*Qbs11) * p1

f00 = plamss1*t0*(1.0-Qbs10)*(1.0-Qbs20)/(1.0-Qbs11)*(1.0-Qbs21)
f01 = plamss1*t0*(1.0-Qbs10)*Qbs20/(1.0-Qbs11)*Qbs21
f10 = plamss2*t0*Qbs10*(1.0-Qbs20)/(Qbs11*(1.0-Qbs21))
f11 = plamss2*t0*Qbs10*Qbs20/(Qbs11*Qbs21)

f00h0 = (f00 - mh0)/sh0
f01h0 = (f01 - mh0)/sh0
f10h0 = (f10 - mh0)/sh0
f11h0 = (f11 - mh0)/sh0

CALL QfunE(f00h0, Qf00h0)
CALL QfunE(f01h0, Qf01h0)
CALL QfunE(f10h0, Qf10h0)
CALL QfunE(f11h0, Qf11h0)

f00h1 = (f00 - mh1)/sh1
f01h1 = (f01 - mh1)/sh1
f10h1 = (f10 - mh1)/sh1
f11h1 = (f11 - mh1)/sh1

CALL QfunE(f00h1, Qf00h1)
CALL QfunE(f01h1, Qf01h1)
CALL QfunE(f10h1, Qf10h1)
CALL QfunE(f11h1, Qf11h1)

ptideam2 = ( Qf00h0*(1.0-Qbs10)*(1.0-Qbs20)
* + Qf01h0*(1.0-Qbs10)*Qbs20
* + Qf10h0*Qbs10*(1.0-Qbs20)
\[ \begin{align*}
&+ Q_{f1h0} Q_{bs10} Q_{bs20} p0 \\
&+ (1.0-Q_{f00h1}) (1.0-Q_{bs11}) (1.0-Q_{bs21}) \\
&+ (1.0-Q_{f01h1}) (1.0-Q_{bs11}) Q_{bs21} \\
&+ (1.0-Q_{f10h1}) Q_{bs11} (1.0-Q_{bs21}) \\
&+ (1.0-Q_{f11h1}) Q_{bs11} Q_{bs21} p1
\end{align*} \]

\[ \text{tinc} = 0.02 \]

\[ \text{ib} = 0 \]

\[ \text{DO 30 thr = Ths, Ths + 2.0, tinc} \]

\[ \text{ib = ib + 1} \]

\[ \text{TU1(ib) = thr} \]

\[ \text{TL1(ib) = Ths - (FLOAT(ib) - 1.0) \times \text{tinc}} \]

\[ \text{TU2(ib) = (TU1(ib) + Ths) / 2.0} \]

\[ \text{TL2(ib) = TL1(ib) + (Ths - TL1(ib)) / 2.0} \]

\[ \text{tu1h0 = (TU1(ib) - mh0) / sh0} \]

\[ \text{tu1h1 = (TL1(ib) - mh1) / sh1} \]

\[ \text{CALL QfunE(tu1h0, Qu1h0)} \]

\[ \text{CALL QfunE(tu1h1, Qu1h1)} \]

\[ \text{pehs(ib) = Qu1h0 \times p0 + (1.0 - Qu1h1) \times p1} \]

\[ \text{tu1h0 = (TL1(ib) - mh0) / sh0} \]

\[ \text{tu2h0 = (TL2(ib) - mh0) / sh0} \]

\[ \text{tu1h0 = (TU1(ib) - mh0) / sh0} \]

\[ \text{tu2h0 = (TU2(ib) - mh0) / sh0} \]

\[ \text{CALL QfunE(tu1h0, Qu1h0)} \]

\[ \text{CALL QfunE(tu2h0, Qu2h0)} \]

\[ \text{CALL QfunE(tu1h1, Qu1h0)} \]

\[ \text{CALL QfunE(tu2h0, Qu2h0)} \]

\[ \text{tu1h1 = (TL1(ib) - mh1) / sh1} \]

\[ \text{tu2h1 = (TL2(ib) - mh1) / sh1} \]

\[ \text{tu1h1 = (TU1(ib) - mh1) / sh1} \]

\[ \text{tu2h1 = (TU2(ib) - mh1) / sh1} \]

\[ \text{CALL QfunE(tu1h1, Qu1h1)} \]

\[ \text{CALL QfunE(tu2h1, Qu2h1)} \]

\[ \text{CALL QfunE(tu1h1, Qu2h1)} \]

\[ \text{CALL QfunE(tu2h1, Qu2h1)} \]

\[ \text{pzcom1(ib) = (Qu1h0 - Qu2h0 + Qu2h0 - Qu1h0) \times p0} \]
* + (Qd1h1 - Qd2h1 + Qu2h1 - Qtu1h1) * p1

\[ \text{pzcom2}(ib) = (Qd2h0 - Qu2h0) * p0 + (Qd2h1 - Qtu2h1) * p1 \]

cbar(ib) = pehs(ib)
* + (peteam1-pehs(ib)+ccc1)*pzcom1(ib)
* + (peteam2-pehs(ib)+ccc2)*pzcom2(ib)
* + (pehs(ib)-peteam1-peteam2-ccc1-ccc2)
* *pzcom1(ib)*pzcom2(ib)

30 CONTINUE

CALL FINDMIN(cbar,ib,mini)
cfmin = cbar(mini)
optl1 = TL1(mini)
optl2 = TL2(mini)
optu2 = TU2(mini)
optu1 = TU1(mini)

TL31 = opt11
TL32 = optu2
TU32 = optu2
TU31 = optu1
F0 = fs0
F1 = -fs0
F00 = f00
F01 = f01
F10 = f10
F11 = -f00
T231 = Tss1
T232 = Tss2

RETURN
END

SUBROUTINE QfunE(xx, erfcx)

REAL x, xx, erfcx
REAL a1, a2, a3, a4, a5, p, pi, t

pi = 3.141592654
x = ABS(xx)
al = 0.319381530
a2 = -0.356563782
a3 = 1.781477937
a4 = -1.821255978
a5 = 1.330274429

p = 0.2316419

\[ t = \frac{1.0}{1.0 + p\cdot x} \]

\[ s_1 = a_1\cdot t + a_2\cdot t^2 + a_3\cdot t^3 + a_4\cdot t^4 + a_5\cdot t^5 \]
\[ s_2 = s_1\cdot \exp\left(-\frac{x^2}{2.0}\right) \]

\[ \text{IF (} xx \GE 0.0 \text{) THEN} \]
\[ \text{erfcx = } s_2/\sqrt{2.0\pi} \]
\[ \text{ELSE IF (} xx \LT 0.0 \text{) THEN} \]
\[ \text{erfcx = } 1.0 - s_2/\sqrt{2.0\pi} \]
\[ \text{END IF} \]

RETURN
END

SUBROUTINE FINDMIN(array,isize,minindex)
REAL array(isize)
INTEGER minindex

int = 1
CONTINUE
11  CONTINUE
   DO 10 i = int+1, isize
      IF (array(int) .GT. array(i)) THEN
         minindex = i
         int = minindex
      END IF
10  CONTINUE
RETURN
END
References


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