The structure traditionally used in Adaptive Line Enhancer (ALE) applications is the transversal filter form of Widrow's Least Mean Square (LMS) algorithm. It has been reasoned that an ALE implemented with the Recursive Least Squares Lattice (RLS Lattice) Algorithm may offer advantages over LMS implementations. The expected advantages include faster convergence, improved tracking of dynamic signals, and reduced sensitivity to eigenvalue spread of the input data's correlation matrix. The work reported in this paper is a comparison of the detection and tracking performance of ALEs implemented with the traditional LMS Transversal and the RLS Lattice algorithms. The comparison is based on experimental results obtained from a real-time custom hardware system using 32-bit IEEE floating point format operating on stationary and non-stationary sinusoids with added broadband noise.

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EXPERIMENTAL RESULTS: DETECTION AND TRACKING OF LOW SNR SINUSOIDS USING REAL-TIME LMS AND RLS LATTICE ADAPTIVE LINE ENHANCERS

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ABSTRACT

The structure traditionally used in Adaptive Line Enhancer (ALE) applications is the transversal filter form of Widrow's Least Mean Square (LMS) algorithm. It has been reasoned that an ALE implemented with the Recursive Least Squares Lattice (RLS Lattice) Algorithm may offer advantages over LMS implementations. The expected advantages include faster convergence, improved tracking of dynamic signals, and reduced sensitivity to eigenvalue spread of the input data's correlation matrix. The work reported in this paper is a comparison of the detection and tracking performance of ALEs implemented with the traditional LMS Transversal and the RLS Lattice algorithms. This comparison is based on experimental results obtained from a real-time custom hardware system using 32-bit IEEE floating point format operating on stationary and non-stationary sinusoids with added broadband noise.

I. INTRODUCTION

A real-time adaptive filter test platform, called the Lattice Development System (LDS), was designed and built at the Naval Ocean System Center (NOSC) to support performance and behavior testing of adaptive lattice and adaptive transversal algorithms. Of particular interest are arithmetic and quantizing related stability questions and the interactions between filter length and time constant of exponential memory when tracking non-stationary signals. The LDS consists of pipelined microprogrammable Engine Boards performing 32-bit IEEE floating point arithmetic along with a Control Board which supports analog and digital input/output (I/O) during processing. The system analog converters have 16-bit resolution. The architecture and design of the LDS are described in [1]. All results reported here were obtained from ALEs implemented on this system.

This paper focuses on the detection and tracking capabilities of low Signal to Noise Ratio (SNR) sinusoids by ALEs implemented with the RLS lattice and the LMS transversal algorithms. While the performance of the LMS transversal algorithm is well documented [6-9] and to a lesser extent so is the performance of the RLS transversal algorithm [10], there is very little published on the performance of the RLS Lattice. This is due in part to the algorithm complexity which makes the implementation and analysis arduous.

Test conditions under which data were collected and processed is first described and then experimental results for stationary sinusoids in noise as well as for slowly varying sinusoids in noise are shown. Comparisons between LMS transversal and RLS Lattice algorithms are made along with their theoretical gains. Of particular interest for the non-stationary signal are the relationships between reliable detection for low SNR sinusoids and filter parameters such as filter length, and adaption speed as controlled by step size $\mu$ (LMS) or exponential decay factor $(1-W)$ (RLS Lattice).

II. EXPERIMENTAL METHOD

Figure 1. is a block diagram of the ALE and the measurement arrangement identifying the primary input $d(n)$, filter output $y(n)$, and prediction error output $e(n)$. As indicated, the input signal to the ALE is obtained as the sum of a synthesized sinusoid and filtered white noise. Input signal and noise levels are measured separately prior to summing, and are verified after summing with an FFT spectrum analyzer. Output signal and noise levels are determined with the spectrum analyzer and are presented as the ratio of signal power to noise power in an equivalent one-Hz bandwidth in units of dB/Hz.

The noise is bandlimited to 50 Hz by cascaded second order Butterworth filters and the composite signal is sampled at 140 S/S. Each test is conducted for a fixed time interval to permit the algorithms to achieve steady state performance after which 32 transforms of length 2048 points are formed and averaged to obtain stable spectral estimates.
Minimum detectable signal level for non-stationary sinusoids was determined by eye integration across a waterfall display consisting of successive spectral power estimates obtained from the spectrum analyzer. Reliable estimates were identified as a non-ambiguous trace on the CRT with signal components exceeding background noise over at least 50% of the spectral estimates.

III. PERFORMANCE FOR STATIONARY SINUSOID

The theoretical narrowband (NB) signal amplitude gain \([6,7,8,9]\) for the LMS algorithm is presented in (1).

\[
\text{NB GAIN} = \frac{(L/2) \cdot \text{SNR}}{1 + (L/2) \cdot \text{SNR}}
\]  

Figure 3 presents a curve of this relationship along with the measured performance of three different ALEs. These are the LMS transversal [5], the RLS Lattice [2,3], and the Direct Coefficient Updating RLS lattice [4]. The filters were each of length 600 taps or stages and the appropriate convergence factors or fading memory terms are indicated on the figure. As can be seen, the experimental data fits the theoretical curve within reasonable tolerances.

The theoretical output noise power gain for the LMS ALE \([6,7,8,9]\) is given in (2).

\[
\text{NOISE GAIN} = \mu \cdot L \cdot \sigma^2
\]  

where \(\sigma^2\) is the input noise power. A similar closed form expression does not exist for the RLS Lattice filter. Figure 4 presents a comparison of broadband noise reduction of the ALE implemented with the LMS and the RLS Lattice algorithms as a function of filter length (L) for the indicated values of \(\mu\) and W. The curves indicate that a relationship exists between the memory term (1-W) and the filter length for the RLS lattice similar to...
that indicated in (2) between the convergence factor \( \mu \) and the filter length of the LMS transversal algorithm.

Note that the RLS Lattice filter with parameter \( W = 0.99995 \) (with bandwidth \( 2(1-W) = 10^{-4} \)) exhibits nearly the same performance as the LMS filter with parameter \( \mu = 2^{-10} \) (or \( 10^{-2} \)). Thus, there appears to be an order of magnitude difference in the influence of the parameters \( \mu \) and \( 2(1-W) \).

![Figure 4. ALE BROADBAND NOISE REDUCTION WITH FILTER LENGTH FOR LMS TRANSVERSAL (\( \mu = 2^8, 2^{-10} \)) AND RLS LATTICE (\( W = 0.9999, 0.99995 \)) ALGORITHMS](image)

**IV. PERFORMANCE FOR NON-STATIONARY SINUSOID**

The two versions of the ALE were tested with sinusoids exhibiting linear FM slope to determine and compare the minimum detectable SNR of the filters. Filter lengths of 600 and 1200 (taps or stages) were used in the ALE. The ALE output was then processed by a fixed length spectrum analyzer with bandwidth adjusted to match the spectral resolution of the ALE. The sweep rate of the linear FM sweeps, normalized to the resolution of the analyzer, is presented in units of bins per 10,000 samples.

Figures 5a and 5b show the relationship between minimum input SNR for reliable frequency tracking and input sweep rate for LMS filters of length 600 and 1200 respectively. Note the general increase in required SNR as the input sweep rate increases. It is apparent that the long filter exhibits nearly a 3-dB advantage over the short filter and that for the longer filter the SNR is essentially independent of the algorithm's convergence factor \( \mu \) for the range shown.

Figures 6a and 6b show the relationship between minimum input SNR for reliable frequency tracking and input sweep rate for RLS Lattice filters of length 600 and 1200 respectively. We note the same increase in required SNR as the input sweep rate increases with approximately the same rate of increase for the shorter filter, but at a reduced rate of increase for the long fil-

![Figure 5. MINIMUM INPUT SNR FOR RELIABLE FREQUENCY TRACKING: LMS TRANSVERSAL FILTER](image)

![Figure 6. MINIMUM INPUT SNR FOR RELIABLE FREQUENCY TRACKING: RLS LATTICE FILTER](image)
ter. Here the long filter not only exhibits the 3-dB advantage over the short filter described earlier but an additional 3-dB advantage for the higher sweep rates relative to the LMS algorithm of the same filter length. For stationary sinusoids, the LMS algorithm has a 2 to 3 dB advantage for length 1200.

V. CONCLUSIONS

Narrowband gain for stationary inputs was measured for the LMS transversal filter and for two forms of the RLS Lattice filter. They were found to be the same within reasonable measurement criteria and agreed with the theoretical gain of the LMS ALE. For stationary signals the performance of the RLS Lattice exhibits similar variation with filter length L and memory fade factor (1-W) as does the LMS algorithm. For non-stationary signals, the LMS and the RLS Lattice algorithms perform the same for short filters and small FM sweep rates. For longer filters and for higher sweep rates, the RLS Lattice exhibits approximately a 3-dB advantage over the LMS algorithm. This advantage is expected due to the rapid convergence capabilities of the RLS Lattice structure. It is, however, 2 to 3 dB poorer for detecting stationary sinusoids with long filter lengths.

We also note that the RLS Lattice did not exhibit the numerical instability of the form observed and reported by others [11] for RLS transversal structures.

VI. ACKNOWLEDGEMENTS

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VII. REFERENCES

PARAMETER BOUNDS FOR RECURSIVE LEAST SQUARES LATTICE
ADAPTIVE NOISE CANCELLERS

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ABSTRACT

The performance of adaptive noise cancellers implemented with lattice algorithms is degraded by finite arithmetic effects, especially as the value of the exponential windowing parameter approaches unity. This degradation can be avoided if the window parameter value is kept within a certain range, and if the lattice has a properly implemented order expansion/contraction control mechanism. The region in parameter space where one can expect good cancellation of sinusoidal and random (broad-band) interference using 32-bit floating point arithmetic is defined.