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Development of a Non-Linear Smoothing Filter for the Processing of Eye-Movement Signals

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Abstract

The analysis of eye-movement (EM) signals poses problems for the designer of smoothing filters since many of the interesting types of EMs are bimodal. For example, optokinetic and/or vestibular stimulation results in an EM pattern called nystagmus consisting of alternating fast- and slow-phase components. Also, saccadic (refixation) EMs do not occur continuously, but are interspersed with periods of fixation. Conventional linear, low-pass filters (both finite impulse response (FIR) and infinite impulse response (IIR) types) smear the boundaries between the fast- and slow-phases of nystagmus and the fixation and fast components of saccadic EMs.

We have adapted a nonlinear smoothing filter (originally designed to optimize edge preservation in image processing applications) for the smoothing of EM signals. This filter is called a Predictive FIR-Median Hybrid (PFMH) filter. The PFMH filter operates on a moving window of data samples centered at the current point of interest. Several predictive FIR filters are applied to the “upper” and “lower” halves of the window and each is designed to predict the sample value at the center of the window. The median of these FIR filter outputs and the actual center data sample are taken as the PFMH filter output for each window position. By properly choosing the length and structure of the FIR sub-filters, a PFMH filter can be designed to smooth a bimodal EM signal without blurring the boundaries between the two signal components.

INTRODUCTION

In the clinical environment, eye-movements (EMs) are generally recorded using electrooculographic (EOG) methods. The EOG signal results from the nominal 0.4-0.7 mv potential difference between the cornea and retina. This small potential (cornea positive with respect to the retina) provides each eyeball with an electric dipole moment. A signal proportional to the angular position of the eyes can be recorded by placing electrodes at the outer canthi (outermost edge) of each eye and amplifying the voltage difference between the electrodes [8]. The resulting EM signal has a scale factor of about 20μV/degree. The EM signal level is about the same magnitude as EMG and large EEG potentials. Therefore, it is not surprising that EM signals recorded in this way are often corrupted by EMG signals from the facial muscles and sometimes EEG potentials. It has been standard practice to use linear low-pass filters to remove the high-frequency EMG noise from the EM recordings. The low-level, low-frequency EEG noise is usually tolerated.

The design of suitable smoothing filters is complicated by the fact that some EM signals are bimodal. Nystagmus consists of a sequence of slow- and fast-phase movements. The slow- and fast-phases of nystagmus are generated by different neural circuits and have different dynamic characteristics. Ordinary low-pass filters smear the boundaries between the slow- and fast-phases, and therefore, tend to distort the signal. Reducing the filter cutoff frequency enough to remove the noise can sometimes result in severe distortion. The same applies to saccadic EMs, each saccade is separated from the next by fixation intervals. The signal characteristics during fixation are different than during a saccade. Linear, low-pass filters blur the transition between the fixation and saccadic signal components.

KEY WORDS: Non-Linear Filters, Order-Statistic Filters, Eye Movements, Nystagmus, Signal Processing.

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FILTER DESIGN

We have developed a method for smoothing nystagmus and saccadic EM signals using a nonlinear filter called a Predictive FIR-Median Hybrid (PFMH) filter [4]. This filter design is based on the concept that both nystagmus and saccadic EMs can be modeled accurately as a sequence of piecewise continuous polynomial segments. The PFMH filter is designed to attenuate signal components that are not consistent with this signal model. Sharp changes between well behaved regions (such as the transition between slow- and fast-phases of nystagmus) are consistent with the signal model and are preserved. Oscillations and impulses are not consistent with the signal model and are suppressed. The PFMH filter smoothes the individual signal components and preserves the boundaries between them. The PFMH filter acts like a low-pass filter that respects the natural transitions in the signal and preserves them.

Order Statistic Filters

The PFMH filter is a member of a class of nonlinear filters called Order-Statistic (OS) filters [1, 6]. OS filters operate on a moving window of data samples. Generally, the window is of odd length, i.e., \( L = 2N + 1 \). The data samples in the window are rank ordered and \( L \) rank-ordered samples (order-statistics) are then linearly weighted. The linearly weighted combination of order statistics constitutes the filter output. For example, the standard median filter is an OS filter in which the center sample of the ordered window is given a weight of 1 and the other samples a weight of 0. Various other OS filters are implemented by choosing different weighting schemes. The OS filter concept includes the PFMH filter, a design developed by Heinonen and Neuvo [3, 4]. In the PFMH filter, the data samples in the window are first processed by a set of linear FIR predictive filters, each designed to predict the center value in the window. The PFMH filter output is the median of the FIR filter outputs and the acutal center data value. The distinctive feature of all OS filters is the rank-ordering process, a data dependent, nonlinear operation. OS filters are nonlinear in the sense that superposition does not apply. If we define the operation of OS filtering as \( OS\{ \} \), and \( u(n) \) and \( v(n) \) are different time varying signals then, in general, \( OS\{u(n) + v(n)\} \neq OS\{u(n)\} + OS\{v(n)\} \). OS filters are translation and offset invariant since, \( OS\{A \cdot u(n) + B\} = A \cdot OS\{u(n)\} + B \), where \( A \) and \( B \) are constants.

Some OS filters, such as the median and PFMH filters, possess a set of signals called root signals that are invariant to the filter [2, 4, 5, 7]. That is, root signals will pass through the filter unmodified. Also, any arbitrary signal will be reduced to a root signal after a finite number of passes through the filter.

Smoothing the Signal

One would expect that if an underlying signal were a root signal for a given OS filter, then that filter could extract the signal from noise without distortion. Repeated filtering should reduce the noise contaminated signal to a root signal, and thereby recover the signal and reject the noise. Since the number of root signals for a given OS filter is infinite, it is possible that the filtering process might not converge to the desired root signal. We have found, in practice, that is not necessary to reduce the EM signal to a root signal to obtain adequate smoothing; one or two passes through the filter provides a significant noise reduction. In any case, repeated filtering generally converges to a root signal very close to the desired signal in about five passes, or so.

Designing the PFMH Filter

The PFMH filter operates on a moving window, \( W(n) \), of data samples, \( x(j) \), of length \( L = 2N + 1 \), such that

\[
W(n) = \{x(n - N), \ldots, x(n), \ldots, x(n + N)\}
\]

The data samples before and after the center sample, \( x(n) \), are used to estimate the value at the center using predictive FIR substructures of the form:

Forward predictor,

\[
\hat{z}_F(n) = \sum_{i=1}^{N} h_i(i) \cdot x(n - i), \quad i = 1, \ldots, N
\]

Backward predictor,

\[
\hat{z}_B(n) = \sum_{i=1}^{N} h_i(i) \cdot x(n + i), \quad i = 1, \ldots, N
\]

Where \( \hat{z}_F(n) \) and \( \hat{z}_B(n) \) are the forward and backward predicted values and \( h_i(i) \) is the coefficient array for the FIR predictor of order \( r \). The output of the PFMH filter is the median of the predicted values and the data value \( x(n) \). Thus, if the median operation is defined as \( MED(\{ \}) \) then the filter output \( y(n) \) is given by

\[
y(n) = MED\{\hat{z}_F(n), \ldots, x(n), \ldots, \hat{z}_B(n)\}
\]

The zero-, first-, and second-order FIR predictor coefficients as given by Heinonen and Neuvo [4] are

\[
h_0(i) = 1/N, \quad i = 1, \ldots, N
\]

\[
h_1(i) = \frac{4N - 6i + 2}{N(N - 1)}, \quad i = 1, \ldots, N
\]

\[
h_2(i) = \frac{9N^2 + (9 - 36i)N + 30i^2 - 18i + 6}{N(N^2 - 3N + 2)}, \quad i = 1, \ldots, N
\]
These predictive FIR filter substructures are optimal predictors for rth order polynomials corrupted with additive Gaussian white noise. In practice, polynomials up to about third-order are used. We have been using four predictive substructures in our filters. For smoothing nystagmus, we use three first-order predictors of different lengths, and one second-order predictor. Saccadic EMs are smoothed using one zero-order, two first-order, and one second-order predictor. Several predictors of different length and/or order are used to insure having at least one that works well for each part of the signal. The median operation selects one of the predicted values (or the center data value) as the filter output.

**METHODS AND RESULTS**

We recorded EMs using standard EOG methods. The signals were recorded using silver-silver chloride bitemporal electrodes. The amplified electrode potentials were then passed through a 30-Hz, low-pass, 4-pole Butterworth filter and digitized at a sampling rate of 128 Hz. A PFMH filter was designed for nystagmus smoothing using three first-order predictors of sizes, \( N = 6 \), \( N = 15 \) and \( N = 24 \), and a second-order predictor of size, \( N = 24 \).

Fig. 1 shows the results of successively filtering a noisy nystagmus signal five times using the PFMH filter. After the second pass, the noise is nearly gone. After five passes, the signal is very near a root signal and almost noise free. The transitions between the slow- and fast-phases are actually enhanced by successive passes of the PFMH filter. This sharpening of the “corners” on the waveform is a result of the root structure of the filter. Repeated filtering using a linear low-pass filter would increasingly blur the transitions.

In order to provide a comparison of the PFMH filter to a standard low-pass filter, we corrupted an idealized nystagmus signal by adding 30-Hz low-pass filtered noise. The “ideal” signal consisted of a sequence of slow-phase segments represented by second-order polynomial segments, separated by linear high-velocity, fast-phase segments. We then processed the corrupted signal with each filter separately. The signal was passed through each filter once. The filtered signals were then compared to the uncorrupted original. The results are illustrated in Fig. 2. The mean-squared difference (MSD) for each filter was computed. This was done by calculating the sum of the squared differences between each filtered signal and the uncorrupted signal. The MSD for the PFMH filter was 8755 and the MSD for the low-pass filter was 8574. Thus, the linear low-pass filter had a 21% larger MSD than the PFMH filter. Examination of Fig. 2 discloses that one pass of the PFMH filter effectively smooths the slow-phase components and yet preserves the slow- to fast-phase transitions. By contrast, the low-pass filter allows considerable low-frequency noise to pass through while blurring the signal transitions.

**DISCUSSION**

In most applications, the standard linear low-pass filter is an efficient, effective smoothing filter. This is particularly true for signals generated by stationary processes when the spectrum of the corrupting noise does not significantly overlap the signal spectrum and the noise has a Gaussian amplitude distribution. Many practical data smoothing applications meet these conditions—at least approximately. In other instances, the signals are clearly multi-modal and are not generated by stationary processes; this occurs for many biological signals. These nonstationary signals can often be smoothed more effectively using a nonlinear filter such as the PFMH filter.

For the PFMH filter to be effective, the basic underlying signal to be smoothed must be a root signal for the filter. This requirement imposes a limitation for the method since only a limited number of PFMH filter designs have been discovered. To date, PFMH filters have been designed using linear FIR substructures and having root signals that are piecewise continuous, low-order polynomials.

Since the PFMH filter is nonlinear it does not have a transfer function or any other frequency domain description. The principle of superposition does not apply, so knowing the sine-wave response does not provide any information as to how complex signals will be processed. Therefore, there is no simple way to describe the “bandwidth” or “frequency response” of the PFMH filter.

Selecting the best filter parameters (i.e., length, order, and number of FIR substructures) is difficult. There are no design formulas to provide optimum parameters for any given application. The best approach is to use a Monte Carlo procedure to search a range of parameters using a large, representative data set. The data set may consist of simulated data. The parameter values that perform best according to some criteria, such as MSD, will generally work well for actual data.

Biological signals are often corrupted by non-Gaussian and sometimes non-stationary noise sources. For example, the typical electrocardiogram (ECG) signal has a non-Gaussian amplitude distribution, mainly due to the large QRS complex. In cases where the ECG signal is a noise source (as in EM recording), the corrupting noise is clearly non-Gaussian. Since the currently available PFMH filters have linear substructures, they are optimum smoothing filters for signals corrupted by additive Gaussian noise. These filter designs are less effective for removal of non-Gaussian noise such as impulse noise or noise having skewed or heavy-tailed amplitude distributions. Other, perhaps nonlinear, predictive FIR substructures could possibly be used to generate new PFMH filters having new root signal structures and improved performance for non-Gaussian noise. We are planning to explore several filter designs of this type.
References


Figure 1. The PFMH filter is used to filter a particularly noisy segment of nystagmus. Top trace is the noisy input signal, the five following traces are successively filtered versions of the signal. Each pass through the filter brings the nystagmus signal closer to a root signal. One or two passes produce a significant noise reduction.
FIGURE 2. An artificial nystagmus-like signal was created using second-order slow-phase components and first-order fast-phase components. This signal is a root signal for the PFMH filter. The ideal signal (top trace) was corrupted by adding Gaussian noise and the corrupted signal (second trace) was then filtered separately by the PFMH filter and a linear, 10 Hz low-pass filter. An error signal was calculated by subtracting the filtered signals from the uncorrupted signal. The PFMH filtered signal and the corresponding error signal (multiplied by 5) are shown in the center two traces. The last two traces show filtered signal and the X5 error signal for the 10-Hz low-pass filter.