**4. TITLE AND SUBTITLE**
The Use and Misuse of Aircraft and Missile RCS Statistics

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**11. SUPPLEMENTARY NOTES**

Both static and dynamic RCS measurements are used for RCS predictions, but the static data are less complete than the dynamic. Integrated dynamic RCS data also have limitations for prediction radar detection performance. When raw static data are properly used, good first-order detection estimates are possible. The research to develop more-useful RCS statistics is reviewed, and windowing techniques for creating probability density functions from static RCS data are discussed.

**14. SUBJECT TERMS**
Low Observable, Radar Cross Section, Dynamic RCS, Static RCS, Probability of Detection, Integrated Dynamic RCS

**15. NUMBER OF PAGES**
9

**16. PRICE CODE**

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THE USE AND MISUSE OF AIRCRAFT AND MISSILE RCS STATISTICS

TECHNICAL MEMORANDUM

July 31, 1991

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DSN 349-3319
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"The number of pitfalls that may be encountered in the use of the radar equation is almost without limit." Marcum

INTRODUCTION

There are two basic RCS measurement types, static and dynamic. In distinguishing between the two, recall that objects in Cartesian space can have 6 degrees of freedom. There are three degrees of freedom in translation along the X, Y, and Z axes: There are three degrees of rotational freedom about each of these axes. When RCS data are taken from an aircraft that is rotating, but not translating, those data are referred to as 'static.' RCS data taken from an aircraft that is both rotating and translating are referred to as 'dynamic.' Static RCS measurements are normally made with the aircraft mounted on a column or pylon. Either the aircraft is rotated, or the radar is moved around the aircraft. Dynamic measurements are made with the aircraft in flight, and in addition to 6 (not necessarily independent) degrees of freedom, also exhibit Doppler, vibration, and flexing effects.

Both static and dynamic RCS measurements are used for radar-detection predictions, but detection theory assumes dynamic RCS conditions. The same statistical terms are used to describe both static and dynamic RCS measurements. This practice creates confusion, because the medians from windowed-static and dynamic RCS measurements do not describe identical processes. The situation is analogous to comparing median wage to median income. An uncritical application of statistics from static, or even integrated dynamic, RCS data can lead to unexpected vehicle detection results.

This paper focuses on RCS descriptive statistics. The approach taken is (a) recount the historical approach to radar performance prediction, and (b) relate the evolution of single-valued number presentations of RCS data to classical detection theory. The goal is (a) to show how confusion over RCS statistics can develop, and (b) make clear to the reader that medianized or percentile static-RCS data are a very good first-order approximation for predicting detection performance.

Integration, coherent and non-coherent, is an averaging process. By the central limit theorem, "the distribution of averages of equal samples drawn from any distribution, approaches normality as a limit." With dynamic RCS measurements using integration, the original RCS distribution is lost: The resultant is a distribution of integrated values approaching a normal distribution that is narrower than the original distribution. The mean-of-means for the integrated data approaches the median value of the distribution of averages. Nonetheless, a dynamic RCS average based on integrated data does reflect 6 degrees of freedom and the effects of flight on the vehicle. Integrated dynamic RCS data are most useful for mean-RCS specification verification and follow-on RCS integrity checks.
BACKGROUND

Radar detection theory is well understood. No shortage of texts and papers on the subject exists. Given the radar range equation, an appropriate probability density function (PDF), i.e., the distribution of the RCS, and the time-correlation properties of the RCS distribution (auto-correlation function, (ACF), radar detection can be calculated for any probability of detection and false alarm rate. Unfortunately, technical, sitting, and economic considerations assure a shortage of the all-band dynamic RCS data needed to produce the required PDF and ACF. Without dynamic RCS data, the next best approach is using windowed static RCS data, but even these data have limited availability. All too frequently radar analysts have but a single RCS number and must guess at the statistical distribution of the RCS about the aspect angle of interest and an ACF. A likely result is a performance prediction, particularly at high and low probabilities of detection, differing markedly from reality.

MEASURING THE RCS

Dynamic measurements are usually made with an instrumentation radar slaved to a tracking radar that follows the vehicle being measured. The test aircraft is instrumented for recording or telemetering roll, pitch, heading, and time information. It will maintain a flight profile that addresses the aspect angles of interest. The instrumentation radar, previously calibrated--from a dropped or towed sphere--records the amplitude and time of each returned pulse. Additionally, the range, azimuth, and elevation of the target relative to the instrumentation radar are recorded. During data reduction, aircraft and radar data are tied together with time. The finished product is zero-dimensional (0-D) RCS, i.e., RCS at a given aspect angle. Care must be taken to control the time period that is ascribed to one aspect angle.

Static RCS data are measured either in a chamber or on an outdoor range. The target is mounted on column(s) or a pylon and rotated to display the aspect angles of interest to the instrumentation radar. The radar is calibrated with an object of known cross section. The target pitch and roll are known, and the azimuth of the rotator is recorded during the test. The finished product--providing the object is not undersampled--is the actual lobe pattern (raw RCS) developed by the vehicle at the frequency of interest. Ideally, the raw static-RCS data are exceedingly fine 0-D RCS data.
One of the most vital services provided by RCS test ranges is the collection of enough RCS information to assess compliance with an RCS specification. That specification usually requires that the RCS not exceed a certain level over certain angular sectors in both the pitch and yaw planes... of the target. That level, in turn, is usually the result of a study of the vulnerability and survivability of the target in a tactical environment. The pitch and yaw sector boundaries define a solid angle over which the specification must be met, forcing us to consider the question, “What intervals must we use to adequately insure that we have sampled the spatial pattern?”

Static RCS measurements with full-scale vehicles on a large ground range permit a high assurance of vehicle survivability before the actual flying vehicle is ever built.

THE PROBLEM

Neither static RCS measurements, nor even those dynamic RCS measurements placing integrated data in bins covering small angular segments, provide the more-complete RCS-distribution data needed for solving the radar detection problem. This is because radar detection occurs while a radar is viewing a target over a limited range of aspect angles. As an aircraft flexes, pitches, yaws, and rolls along its flight path, RCS values scintillate in a time-variant manner that is a function of scattering centers, flight dynamics, and the nominal aspect angle presented during the measurement time interval. The mean, distribution, and time correlation properties of the RCS over a solid-angle centered on the aspect angle of interest, are required for the detection problem.

Both windowed static and dynamic RCS measurements yield ‘distributions,’ as any repeated measurement yields a distribution. For detection predictions, however, we want the distribution most appropriate to the problem: This is the RCS distribution at the nominal aspect angle of interest of a flying vehicle. An effort to adapt classical statistical distributions to describing RCS distribution data began early in the study of detection theory. Statistically described distributions make closed-form solutions of the detection problem possible. Work to adapt classical statistics to the description of RCS distributions continues to this day.

Today’s high-speed computing and low-cost mass storage technology offer an alternative to closed-form-solution detection predictions. If the RCS distribution around the angle or sector of interest can be empirically determined, and the radar detection process modeled, computer simulation can be applied to the detection problem. Hovanessian has published a hybrid version of this approach. His computer program modifies probability of detection curves for a non-fluctuating target having the same mean RCS as the measured distribution. This approach can accommodate any distribution, and frees the analyst from trying to decide just how well normal, log-normal, Chi-square, Weibull, or Gamma distributions fit the RCS data.
SOLUTIONS IN USE TODAY

Initially, radar performance analysts will consider four vehicle aspect-angle sectors to be of interest: the nose; the tail; the two side aspects. Instead of using mean and distribution data from the aspect of interest (these data are not commonly published), the performance analyst uses single-valued RCS statistics such as medians or percentiles. Whether the RCS statistics were generated by static or aspect-angle-binned dynamic measurements is at times overlooked.

In air defense vehicle penetration analysis, median or percentile, nose-aspect RCS data are used to determine the initial penetration range. A question that logically arises is "how can a single number replace the several dictated by classical detection theory?" In the following paragraphs, we will examine the evolution of the single-valued RCS substitute for the dynamic data described in classical radar detection theory and the influence the single-valued approach has had on RCS reduction statistics.

THE MEDIAN SOLUTION

The wide use of the median as an RCS descriptor stems from the fact (supported by physics) that RCS distributions about a particular vehicle aspect angle can display Rayleigh characteristics. Actually, the RCS distribution of the commonly referred to Rayleigh-target is negative exponential: The voltage output from a linear detector with a negative exponential power input is Rayleigh distributed. Hence, the name 'Rayleigh target.' While the Rayleigh nature of RCS is justifiable as a very first approximation, Nathanson is replete with examples of dynamically-measured RCS distributions that are anything but Rayleigh. This writer has dynamically measured the RCS distribution of a stores-free F-4 about the nose aspect: The RCS distribution was log-normal. Nonetheless, the theoretical study of Rayleigh detection statistics has produced practical results useful to the analyst, and provided a stimulus for further investigation of radar detection phenomena.

Nathanson and Wilson show that if a signal is Rayleigh distributed (Chi-square with 2 degrees of freedom), use of the median rather than the mean provides the best solution for the 50-percent probability of detection case. Even if the actual distribution were Chi-square with degrees of freedom between 0.6 and 4.0, we would still have less prediction error using the median rather than the mean. Hence, the long love affair with the median. For the low probability of detection ($P_d$) situation however -- less than 30 percent -- the mean becomes the better estimator of detection performance, if we assume that the Chi-square family of statistics is applicable.
THE PERCENTILE SOLUTION

Percentile statistics developed as a way to provide single-number values of dynamic-RCS for use in the radar range equation. They are used as a lower bound for the RCS value in the radar range equation. To create percentiles, measure RCS about an aspect angle and order the RCS values from lowest to highest.

Typically, RCS values for the 20, 50 (median), and 80 percentiles are reported for dynamic data: Static RCS data are typically reported at the 10, 50, and 90 percentile points. The 80th percentile is the RCS value exceeded 20 percent of the time (highest reported value) and the 20th percentile is the RCS value exceeded 80 percent of the time (lowest reported value).

The percentile approach to estimating probability of detection at the calculated range deliberately ignores receiver noise in the interests of simplicity. By ignoring noise and representing the aircraft or missile RCS probability density function as \( p(\sigma) \), probability of detection can be simply expressed as

\[
P_d = \int_{\sigma_{\text{min}}}^{\sigma} p(\sigma) d\sigma
\]

For example, if a 20th percentile RCS value is used as the lower limit of integration, one would expect the \( P_d \) to equal 80 percent. Even though (1) is not exact (since receiver noise is ignored), it gives excellent results at signal-to-noise ratios above 10 dB. The simplicity of the percentile method, of which the median solution is a subset, makes it attractive.

RCS DATA REDUCTION

Dynamic RCS data are recorded and binned as a function of aspect angle. The boundaries of a bin enclose the aspect angle of interest. Sorting the RCS values in the bin enables a PDF for that aspect angle to be developed. Representative mean, median, or percentile data can be developed provided there are sufficient samples (>100) in the bin. If the bin contains integrated RCS samples, the number of samples in the bin can be smaller (10 - 15), but the original PDF is lost. The mean of the integrated values in a bin is an excellent indicator of mean dynamic RCS about an aspect angle, and is best suited for final specification verification.

When a typical threat radar scans across a flying target, its circuitry averages the target lobe pattern about an aspect angle. This phenomenon is the basis for 'windowing' of static RCS data. One preliminary analysis approach is to use a window size approximating the beamwidth of the threat radar. This window is continuously moved in azimuth along the raw data (moving average). The raw data inside the window are then averaged, medianized, and sorted into percentiles. Discrete sectors, moved incrementally, are also used. The actual windowing technique chosen depends on the intended use of the data.
Sorting the values inside the 'window' enables a PDF for the aspect angle of interest to be developed. When the window is positioned over an area of the raw data that is relatively free of scintillation, the mean and median are nearly equal. When the window is positioned over an aspect wedge that displays considerable scintillation, the mean and median separate and the percentiles spread. Windowing of raw static data provides a first estimation of the PDF and percentiles that would be obtained from a flying target. For those desiring more insight into the computation of RCS statistics, see Currie. 

To predict radar performance, an analyst requires raw (unprocessed) static RCS measurements. Medianized or otherwise processed data, which may suffice for signature control studies, is a form of data smoothing. Any data processing that modifies the azimuthal fluctuations of target RCS prohibits accurate computation of the PDF parameters actually sensed by a search radar.

IMPROVING THE SITUATION

Research to make static RCS data more useful for detection predictions has been a long-term effort. ITT Gilfillan studied the conversion of static BQM-34 drone RCS data to dynamic data in 1976. Stanford Research Institute (SRI) did considerable work in this area in the early '80s. As of this writing, the Air Force Institute of Technology (AFIT) continues the static-to-dynamic conversion study. AFIT efforts should ultimately produce improved static RCS measurements and a better understanding of dynamic RCS measurements. This is a matter of considerable importance. No dynamic RCS data for VHF and UHF currently exist (due to current equipment limitations and stringent siting requirements). The AFIT effort should make better VHF/UHF performance predictions possible until the DoD can develop an all-band dynamic RCS measurement facility.

In the meantime, should you require detection predictions from static or integrated-dynamic RCS data, find a good RCS analyst. With ample static measurements, an estimate of vehicle flight characteristics, threat radar parameters, and knowledge of target scattering centers, the skilled analyst can hypothesize an RCS value and distribution about the aspect(s) of interest; the target scattering centers can be determined from imaging RCS measurements (best), or from computer predictions (order of magnitude). Both the RATSCAT Advanced Measurements System (RAMS) and the RATSCAT Improved Measurement System (IRMS) make imaging and 0-D RCS measurements simultaneously.
IN CONCLUSION

Static RCS measurements are important in any RCS reduction effort and critical for the development of low observable aircraft and missiles. Static measurements yield less precise results than dynamic RCS measurements for predicting detection performance. This is because the required six degrees of freedom are only reflected in a dynamic measurement. When it comes to detection performance predictions, the RCS data statistics and the 'threat' radar detection process must be clearly understood.
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