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THEORETICAL ANALOGIES BETWEEN (GENERALIZED) LAMB AND RAYLEIGH WAVES ON INSONIFIED, SUBMERGED, ELASTIC, HOLLOW AND SOLID CURVED BODIES

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We analyze resonance features in the backscattering cross-sections (BSCS) of an air-filled steel spherical shell submerged in water and insonified by a plane c.w. sound wave. We generalize the concepts of Rayleigh (R), whispering gallery (WG), and Lamb waves in half spaces and plates in vacuo to curved solid and hollow bodies immersed in fluids. We study how each shell-wave manifests itself in the various frequency bands of the body's BSCS. We display dispersion plots for the phase velocities of the various waves in wide bands, and compare Lamb and R/WG waves as the shell becomes a solid sphere to extract their similarities. The fluid-loadings, the shell thickness, and the curvatures of the body generate novel waves in the shell and its BSCS that could have never emerged from earlier models that ignored these effects, and which we analyze here.

I. THEORETICAL BACKGROUND

The BSCS of an air-filled spherical steel shell of radii a and b immersed in water is (1)

\[ \frac{\alpha}{\pi a^2} = |f_n(\pi, x)|^2 = \sum_{n=0}^{\infty} f_n(\pi, x)^2 \]

\[ = \frac{2}{k_d^2} \sum_{n=0}^{\infty} (-1)^n (2n+1) \lambda_n(x)^2 \]

where: \( x = k_d a, k_d = \omega / c_s \), \( \omega \) = circular frequency, and \( c_s \) = sound speed in the outer fluid (medium #2). The coefficients \( \lambda_n(x) \) are determined from six boundary conditions at the two interfaces, \( r = a, b \). They come out to be ratios of two 6x6 determinants which we have spelled-out in earlier work. They all depend on \( x \). The present formulation is exact since the shell motions are described by 3-dimensional elastodynamics. Each partial-wave, \( f_n(\pi, x) \), contained within the sum in Equation (1) can be decomposed into backgrounds and resonances in the usual fashion and terminology of the resonance scattering theory (RST) (3), viz.,

\[ |f_n(\pi, x)|^2 = \left| \frac{2n+1}{x} \right|^2 \left| 2i^{n+1} \right|^2 \sin^2 \left( \frac{\pi}{2} n \right) \]

\[ + \frac{z^{-1} - z_1^{-1}}{F_n - R_{\pi} z_1^{-1} - i \pi z_1^{-1}} \]

(2)

where the quantities \( F_n, z, z_1, \ldots \) have all been defined before (4). The phase velocities can be found from the (real) roots of the real parts of the denominators of the fraction term of Equation (2) by means of the relation (1-4):

\[ \frac{c_{p}^2(x)}{c_1} = \frac{x_{nl}}{n+1/2} \]

(3)

where the \( x_{nl} \) are the real (resonance) roots. We note that for shells we have (spherically modified) Lamb modes and surface waves. In the solid sphere limit (b=0), a "corresponding" Rayleigh speed is obtained which is slightly higher than the one ordinarily found for flat interfaces, viz.,

\[ c_s = \frac{0.87 + 1.12 v}{1+v} c_s \]

(4)

II. NUMERICAL RESULTS

Figure 1 shows the isolated resonances of the \( n=0 \) mode of a fluid-loaded steel shell of increasing thickness. These resonances are isolated by the suppression of suitable (rigid) modal backgrounds. The thicknesses are as indicated in Figure 1. We see that thin shells support fewer modes and isolated resonances than thicker ones. Up to \( h = 20 \) only one or two resonances are visible in the displayed bands. For thicker shells we see as many as \( t=8 \) features in the same bands. The \( t=1 \) feature would be the spherical analogue of the Rayleigh resonance for a solid sphere, while all the others (\( t=2 \)) would correspond to the whispering gallery

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FIG. 1: Isolated resonances within the external acoustic band of an antisymmetric (i.e., $A_{0}$) Lamb wave in the steel spherical shell of increasing relative thickness $h/a = 1\%, 2.5\%, 5\%, 10\%, 20\%, 40\%, 60\%$, $80\%$, $90\%$, $95\%$, and $100\%$ (the solid line approach in all cases). The modal resonances of residual responses are labeled by the index $l$. 

FIG. 2: Dispersion plots for the phase velocity $c_{p}$ of the $A_{0}$ Lamb wave in the steel spherical shell. Within the frequency band: $0.4 < \omega/\omega_{R} < 100$. 

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FIG. 3: The phase velocity of any one of the shell modes can be plotted as a function of shell thickness $h'$. Here we display the situation for modes $A_{10}$ and $A_{15}$. The phase velocity of all the modes tends to a value near 3.2 km/s in the solid sphere limit (i.e., for $h' \rightarrow 1$). This value is already reached at (relative) thicknesses of about 40%, and tends to remain constant above it.

FIG. 4: Dispersion plots of the phase velocity $c_p$ of the $S_{10}$ Lamb mode (or wave) versus $x$ in the frequency band $0.4 \leq x \leq 190$ for a steel spherical shell in water of thickness $h' = 1\%, 2.5\%, 5\%, 10\%$, and $20\%$. All curves seem to approach the value of $c_p$ as $x \gg 1$. Note that this mode $S_{10}$ exists above and below the "coincidence" frequency $x_c$.

FIG. 5: Phase velocities of various Lamb modes (or waves) $A_{10}, S_{10}, A_1, S_1, S_2, \ldots$ versus $x$ for a steel shell in water of thickness $h' = 5\%$. The coincidence frequency here is $x_c \approx (h')^{-2}x_{20}$.

FIG. 6: $c_p$ versus $x$ for the $A_0$ Lamb wave (or mode) for a steel spherical shell in water of five thicknesses. Mode $A_0$ exists only for $x > x_c$ (i.e., for $c_p > c_1$ — solid lines). All curves approach $c_p$ for $x \gg 1$ and bend upwards for $x \rightarrow 0^+$. 
resonances. However, the t=1 feature, present here in all the modes, is due to the flexural (shell) Lamb wave \( \Lambda_t \). This is the spherical counterpart of the well-studied \( \Lambda_n \) surface wave for flat plates. This flexural or bending shell wave "corresponds" to the Rayleigh wave for solid spheres (for \( b=0+ \)), and ultimately to the Rayleigh wave in flat elastic half-spaces (for \( a=\infty \)). We exhibit the dispersion plots for the phase velocities of this (generalized) \( \Lambda_t \) (Lamb wave) in Fig. 2 for four shell thicknesses, indicated there. For \( h'<40\% \) the dispersion plot shows an upward bend at low frequencies due to the (double) curvature of the shell. This behavior differs from that in earlier works which use flat plate theories to generate the dispersion plots. The dispersion plots for any one of the shell modes can be (and have been) generated and plotted versus \( \rho \). Figure 3 displays them for the \( \Lambda_0 \) and \( \Lambda_1 \) modes. These plots are obtained from Equation (3). We note that the phase velocity \( c_\rho \) of all the modes approaches a value near 3.1 Km/s as \( h' \to 100\% \). This value is slightly above the value of \( c_\rho \) predicted by Equation (4). Figure 4 displays the phase velocities of the \( S_0 \) Lamb wave mode (or wave or branch) versus \( x \) in the band: \( 0<x<190 \), for five (indicated) shell thicknesses \( h' \). The \( S_0 \) mode exists for all frequencies above and below "coincidence." All the curves seem to approach the value of \( x=0 \) from above, or \( x>>1 \). At low frequencies all curves show an upward bend due to the shell's (non-zero) curvatures. For either broader bands or thicker shells, progressively more Lamb modes enter the picture. Figure 5 (upper), constructed in: 0x5x500, shows five such modes or branches when \( h'=5\% \). These are \( \Lambda_0 \), \( S_0 \), \( \Lambda_1 \), \( S_1 \), and \( S_2 \). They all have vertical asymptotes (cut-off frequencies) at various values of \( x \). The bottom part of Fig. 5 enlarges the band: 0x5x5120 in which only the two familiar modes \( \Lambda_0 \) and \( S_0 \) are present. Numbers along the various branches are values of \( n \) at the various frequencies \( x \). They are obtained from a partial-wave expansion of the residual responses \( f_n(x) = f_n(x) \) for higher values of \( n \), similar to that shown in Fig. 1 for \( n=0 \). Figure 6 shows the dispersion plots for the \( c_\rho \) of the single Lamb mode \( \Lambda_0 \) vs. \( x \) for \( h'=1\% \), 2.5\%, 5\%, 10\% and 20\%. This plot is analogous to that in Fig. 4, but now for \( \Lambda_0 \) rather than \( S_0 \). These are the two most important ones at (relatively) low frequencies. We again note the upward bend of all the curves in Figure 4 as \( x=0 \). Mode \( \Lambda_0 \) only exists above the coincidence frequency \( x=4(h') \). By means of plots of this type that we determine the coincidence frequency, via the coincidence condition: \( c_\rho = c_\rho \). The curves in Fig. 6 are drawn in solid lines for \( c_\rho > c_\rho \) and in dashed lines for \( c_\rho < c_\rho \). In this example, \( c_\rho = 1.5 \) Km/s. At higher frequencies (i.e., \( x>>1 \)) all branches approach the Rayleigh speed \( c_\rho \) found above. This high-frequency limit is approached faster the thicker the shell becomes. In the "subsonic" region \( c_\rho < c_\rho \), where \( \rho \) is dormant, there are other water-borne and curvature waves present, which we will discuss elsewhere. At the coincidence frequency \( x=x_\rho \), strong flexural vibrations are excited in the shell which are communicated to its BSCS. Further details will appear elsewhere, particularly the connection between (generalized) Lamb-poles for a shell and the Rayleigh poles for an elastic sphere in water. These later ones have already received some attention(1).

III. CONCLUSIONS

The \( t=1 \) antisymmetric flexural Lamb resonance (or leaky surface wave) present in the modes of a steel spherical shell in water is the analogue of the Rayleigh resonance (or leaky surface wave) of an submerged elastic sphere. Ploting the dispersion curves (viz., \( c_\rho \) vs. \( x \)) for a number of Lamb waves (or branches), including the \( \Lambda_0 \), we showed that \( c_\rho = c_\rho \) for \( x>>1 \). Further, the \( c_\rho \) of all the modes (i.e., \( n=1,2,\ldots,20,30,\ldots \)) approaches the value of \( c_\rho \) for that spherical mode in the solid-sphere limit (i.e., for \( h'=100\%) \). The key finding is the set of dispersion plots in Figure 5 for the first few modes/branches of a steel shell in water. These plots, displayed in very broad bands (0x5x500) differ from the corresponding set for flat-plates, particularly at low frequencies where the curvature effects are strongest. The phenomenon of "coincidence" seems to be responsible for the region of strong flexures that develops in the BSC of shells around the coincidence frequency, \( x=4(h') \). [Note: \( h'=a-b/a \).]

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