AN APPROXIMATE ANALYSIS OF BALLOTING MOTION OF RAILGUN PROJECTILES

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This report deals with the approximate analysis of balloting motion. This model considers the effects of the propulsion force, the friction force of the projectile package (projectile and armature), air resistance, gravity, the elastic forces, and the projectile/barrel clearance. To simplify the modeling, a plane motion configuration is assumed. Though the projectile is moving with a varying yaw angle, the axis of the barrel and the projectile package, and the projectile center of gravity are always considered in a plane containing the centerlines of the rails. Equations of motion are derived and solved. A sample computation is performed and the results plotted to give a clearer understanding of projectile in-bore motion.
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This is the final of three basic reports dealing with the in-bore balloting motion of a projectile fired from an electromagnetic (EM) railgun. The first report titled "A Basic Simple Modeling of In-Bore Motion of Railgun Projectiles" (ref 1) of this series addressed axial projectile motion without cocking and included the derivation of basic equations for calculating projectile acceleration and velocity. The second report titled "A Basic Simple Modeling of Balloting Motion of Railgun Projectiles" (ref 2) addressed the cocking (yaw) angle of the projectile package in the railgun and its impact on projectile balloting. In order to make this presentation easier to understand and more complete, some statements which were discussed in these previous reports are repeated in this report.

In-bore projectile motion is the start of its subsequent motion. The lateral forces and lateral projectile/barrel impact affects muzzle jump, intermediate and terminal ballistics and, consequently, weapon system accuracy. The lateral projectile/barrel impacts during in-bore motion also would affect the more sensitive components of some projectiles such as those containing electronics. The force structure and in-bore projectile dynamics are an important concern in the development of an armament system for an EM launcher since the average accelerations are much larger and the length of the barrel may be longer. In addition, unlike for a conventional gun (ref 3), the circumferential construction of the barrel is not uniform, complicating the analytical work.

The balloting motion of the projectile package is a complicated problem since many parameters are involved and it is not easy to determine the interacting relationships between them. To make the in-bore motion problem easier to understand it was analyzed on several levels. Beginning from the basic simple model which analyzed only axial motion, more complicated models will be introduced in upper levels that include many of the lateral forces and gun tube vibration effects.

This report provides an approximate analysis of balloting motion. The model considers only the effect of the propulsion (Lorentz) force, the friction force of the projectile package (projectile and armature), air resistance, gravity, the elastic forces, and the clearance between the projectile and the barrel. Barrel expansion is assumed to be constant. Vibration of the gun, the thermal effect and other electromagnetic interferences are ignored. The propulsion force is assumed to be of known quantity. To simplify the modeling a plane motion configuration is assumed. Though the projectile is moving with a varying cocking (yaw) angle, the axes of the barrel, the projectile package and the projectile center of gravity (c. g.) are considered to be always in a plane containing the centerlines of the rails. Equations of motion are derived and solved.

The solutions to the derived equations are obtained by either closed form (for the simpler cases) or numerical methods. The results are expected to provide a basic understanding of the projectile's in-bore balloting condition.
A sample calculation is shown with available required data. Figures are provided to show some of the computed results with respect to time and projectile displacement.

**DISCUSSION**

**Assumptions**

The following assumptions are made to simplify this basic analysis of the projectile balloting motion:

The projectile and the armature are assumed to be integrated into one projectile package; hereafter the term projectile means projectile package. The contact of the projectile with the barrel is taken to be on the armature and the bourrelet portion only and they are considered as point contacts. The center of the armature base is assumed to always move along the barrel centerline. Thus, there is uniform rail and insulation pressure and a normal reaction force acting along and at the armature circumference. These two forces produce friction along the circumference and at the contact points. The propulsion force is uniformly applied to the rear face of the armature so that the resultant propulsion force is acting on the armature base center. It is coinciding with and directed along the barrel centerline. The mass center of the projectile may have an offset, \( \varepsilon \), from its geometrical centerline. All components such as the barrel, projectile, and armature are considered to be rigid except for that portion of the bourrelet contacting the barrel. The bursting effect of the rail is considered constant. It increases the barrel radius to a small amount which is taken care of by the effective barrel radius.

Although there is no leakage around the armature and the center of the armature is always at the centerline of the barrel, the diameter of the bourrelet may be smaller than that of the bore. Thus, there is some clearance between the bourrelet and the rail and the projectile may yaw inside the barrel even though there is no compression at the bourrelet. The yaw angle is assumed to be small. The yaw or normal motion which was ignored in the previous two reports is now included.

To simplify the analysis further all forces or their resultants are assumed to be in one plane containing the center of mass of the projectile, the centerlines of the rails, projectile, and barrel, and the bourrelet-barrel contact point. Consequently a plane motion is analyzed. The reaction of forces normal to the barrel centerline are computed in addition to the frictional forces and the Lorentz force.
Coordinate System

In this analysis, a Cartesian right-handed coordinate system is employed as shown in figure 1. The x-y plane contains the center of mass of the projectile package, the centerlines of rails, projectile and barrel, and the armature and bourrelet contact points. The x-axis is taken to the centerline of the barrel and the y-axis is normal to the barrel and pointing upward in a vertical plane. The origin of the coordinate system is located at the breech. The x-axis or the barrel may have an inclination angle, α, with respect to the horizon (known as the angle of elevation) as shown in the figure.

![Figure 1. Coordinate system](image)

Governing Equations

The basic equations of motion are similar to those for a conventional gun with a smooth gun tube (ref 3). However, the special arrangement of the rails and the insulation inside the barrel must be considered to compute the associated contact conditions and forces.

From the above-mentioned assumptions and the coordinate system, the main applied propulsion, reaction and interacting forces are shown in figure 2. For clarity the aerodynamic drag and lift forces and the turning moments are not shown. The gravitational force is also omitted from the figure. Projectile deformation and dimensions in the x-y plane are shown in figure 3. Note that the fore and aft bore rider of saboted projectiles corresponds to the bourrelet and the armature portion of the projectile without sabot.
Figure 2. Barrel and projectile package configuration showing propulsion and interacting forces

Figure 3. Projectile deformation and dimensions

According to the geometrical conditions and the dynamic equilibrium of forces in a plane, equations of motion are derived as follows.

For the translational motion, the $x$- and $y$-axis equations of motion are

$$ma = F - f_a - f_b - f_{ar} - f_{al} - D - mgsin\alpha$$

$$m\ddot{y} = N_a - N_b + L - mgcos\alpha$$

where

$m$ = mass of projectile package or sum of masses of armature and projectile

$a$ = axial or $x$-direction acceleration of projectile package

$F$ = total propulsion or Lorentz force

$f_a$ = friction force between armature and rail due to normal reaction force
\[ f_b = \text{friction force between bourrelet and rail due to normal reaction force} \]

\[ f_{ar} = \text{resultant friction force between armature and rail due to uniform circumferential compression} \]

\[ f_{al} = \text{resultant friction force between armature and insulator due to uniform circumferential compression} \]

\[ D = \text{aerodynamic drag} \]

\[ g = \text{gravitational constant} = 9.81 \text{ m/sec/sec} \]

\[ \alpha = \text{inclination of x-axis or barrel with respect to the horizon} \]

\[ \ddot{y} = \text{normal or y acceleration of projectile c. g.} \]

\[ N_a = \text{normal reaction force at the armature} \]

\[ N_b = \text{normal reaction force at the bourrelet} \]

\[ L = \text{aerodynamic lift} \]

For the yaw or transverse rotational motion, the equation of motion is,

\[
I_t \ddot{\theta} = y(F - f_{ar} - f_{al}) - N_a (\theta \cos \theta - R \tan |\theta| + \varepsilon \sin \theta) \\
- N_b [h \cos \theta + \varepsilon \sin \theta - (r - \delta_b) \sin |\theta|] \\
- f_a \left( \frac{\theta}{|\theta|} R + y \right) + f_b \left( \frac{\theta}{|\theta|} R - y \right) + M \tag{3}
\]

where

\[ I_t = \text{transverse moment of inertia of projectile} \]

\[ \theta = \text{yaw or cock angle of projectile} \]

\[ \dot{\theta} = \text{yaw acceleration} \]

\[ y = \text{normal or y coordinate of projectile c. g.} \]

\[ L = \text{distance between c. g. and base of armature} \]

\[ h = \text{distance between bourrelet and c. g.} \]

\[ R = \text{effective bore radius. It is equal to the original radius plus the expansion of the barrel due to the average rail bursting force} \]

\[ \varepsilon = \text{projectile c. g. eccentricity} \]

\[ r = \text{bourrelet radius} \]

\[ \delta_b = \text{contact point deformation at the bourrelet, normal to bourrelet} \]

\[ M = \text{aerodynamic moment} \]

From the system geometry

\[ y = L \sin \theta + \varepsilon \cos \theta \tag{4} \]
Friction forces will be determined from the friction coefficients and the design or actual contact pressure at the armature-rail, armature-insulation, bourrelet-rail, and bourrelet-insulation interfaces. They are difficult to determine and simplified approximations from experiments are recommended. The equations are derived from geometrical conditions, force reactions, bourrelet deformation, and the friction laws as follow:

\[ f_{ar} = 2\mu_{ar} Rpb \beta \]  
\[ f_{ai} = 2\mu_{ai} Rpb (\pi - \beta) \]  
\[ f_a = \mu_{ar} |N_a| \]  
\[ f_b = \mu_{b} |N_b| \]

where

- \( \mu_{ar} \) = friction coefficient of armature on rail
- \( \mu_{ai} \) = friction coefficient of armature on insulation
- \( \mu_{b} \) = friction coefficient of bourrelet on rail
- \( b \) = width of armature circumferential contact
- \( p_r \) = contact pressure between armature and rail
- \( p_i \) = contact pressure between armature and insulation
- \( \beta \) = angle subtended by rail with respect to barrel center
- \( \pi \) = 3.141593
- \( |N_a|, |N_b| \) = absolute value of \( N_a, N_b \)

The air resistance force and moment components may be computed according to aerodynamic force equations if appropriate coefficients are known or measured from experiments. For simplicity in deriving the equations the aerodynamic center in the above-derived equations is taken to be at the projectile c. g.. However, these resistance and frictions may be ignored if the coefficients of air resistance and frictions are low, which they normally are except for the drag force. The aerodynamic equations of air resistance are as follows:

\[ L = 0.5 \rho A C_L v^2 \]  
\[ D = 0.5 \rho A C_D v^2 \]  
\[ M = 0.5 \rho A C_M v^2 \]
\[
\begin{align*}
p & = \text{air density} \\
A & = \text{sectional area for aerodynamic force computation. It is taken to be bore cross-sectional area} \\
C_L & = \text{coefficient of normal or lift force} \\
C_D & = \text{coefficient of axial or drag force} \\
C_M & = \text{coefficient of yaw or turning moment} \\
c & = \text{reference length used to compute air resistance moment} \\
v & = \text{axial or x-direction velocity of projectile package}
\end{align*}
\]

The deformation at the bourrelet depends on the bourrelet geometry and the spring constant. It is considered as positive when the deformation occurs from compression. The normal reaction forces are considered as positive or negative as shown in figure 2 when the yaw angle is positive. They are in the opposite direction when the yaw angle is negative. The deformation and the bourrelet force are computed from the following equations

\[
\begin{align*}
N_b & = \frac{\theta}{|\theta|} \delta_b k & (7a) \\
\delta_b & = 0 \quad \text{when } |\theta| \leq \theta_0 & (7b) \\
\delta_b & = [(\lambda + k) |\theta| - R]/\cos \theta + r \quad \text{when } |\theta| \geq \theta_0 & (7c)
\end{align*}
\]

where

\[
\begin{align*}
k & = \text{spring constant of the bourrelet-barrel contact point} \\
|\theta| & = \text{absolute value of } \theta \\
\theta_0 & = \text{yaw angle when the bourrelet just touches the rail}
\end{align*}
\]

The Lorentz force may be computed from special formula using rail current and inductance values, such as the following simple equation

\[
F = .5L' I^2 & (8)
\]

where

\[
\begin{align*}
L' & = \text{rail inductance per unit length} \\
I & = \text{rail current}
\end{align*}
\]
However, more complicated Lorenz force formulations may be used when they are available.

The velocity, \( v \), and the travel or displacement, \( x \), are the first and second integrations of axial acceleration with respect to time as follow

\[
\begin{align*}
  v &= \int_{0}^{t_f} a \, dt \quad \text{(9)} \\
  x &= \int_{0}^{t_f} v \, dt \quad \text{(10)}
\end{align*}
\]

Similar integration equations also may be applied to y-direction motion and rotation.

**Solutions of Governing Equations**

The above-derived governing equations are, in general, solved with numerical methods. A closed form solution is available only in simple or simplified cases which are not discussed here.

Substituting the friction equations 5a-5d and the air resistance drag force equation 6b into equation 1, the equation becomes

\[
a = \left[ F - \mu_a |N_a| - \mu_b |N_b| - 2\mu_a Rb_p \beta - 2\mu_a Rb_p (\pi - \beta) \\
- .5\rho C_D v^2 - \frac{mg \sin \alpha}{m} \right] / m
\]

If the \( \alpha \) angle, coefficients of friction, coefficients of air resistance or the projectile mass are small, then the corresponding terms in equation 11 may be further ignored to simplify the computation. Hence the upper bound of the axial acceleration is

\[
a = \frac{F}{m}
\]

The deformation at the bourrelet is computed from equations 7b and 7c. After it is computed, the normal reaction force at the bourrelet, equation 7a, becomes

\[
N_b = \frac{\theta}{|\theta|} \delta_b k
\]
Substituting equation 4 and air resistance lift force equation 6a into equation 2 and solving for the reaction force at the armature, the equation becomes

\[ N_a = N_b - 0.5pA_Lv^2 + mg\cos\alpha + m[(\ell \cos\theta - \epsilon \sin\theta)\dot{\theta}
- (\ell \sin\theta + \epsilon \cos\theta)\dot{\theta}^2] \]  

(14)

Similar substitution may be done on equation 3 if desired. The final solution is obtained using general numerical methods.

**Sample of Computation**

A simple example is presented here to show the general state of the projectile balloting. All data are assumed values and are not from an actual design. The frictions and the air resistance are ignored because they are very small. Therefore, the corresponding data are all zero, and are not shown below.

The given data of this example are:

- Barrel length = 4.0 m
- Gun inclination = 0
- Effective barrel radius = 2.5 cm
- Bourrelet radius = 2.4 cm
- Width of armature circumferential contact = 0.5 cm
- Angle subtended by rail with respect to barrel center = 90 deg
- Bourrelet-projectile c. g. distance = 2.5 cm
- Armature-projectile c. g. distance = 2.5 cm
- Projectile c. g. eccentricity = 0.1 cm
- Mass of projectile package = 0.005 kg
- Moment of inertia of projectile package = 5 E-6 kgm²
- Bourrelet spring constant = 8 E+6 N/m
- Rail inductance gradient = 0.35 μH/m
- Initial projectile axial displacement = 0
- Initial projectile axial velocity = 0
- Initial yaw or cock angle = 1.1515 deg
- Initial projectile normal velocity = 0
- Rail current versus time curve is as shown in figure 4
Using these values and the derived equations, the balloting motion of the projectile and the normal reaction forces are computed and shown in figures 5 through 19. Other quantities may be computed from the results, such as the bourrelet deformation which is computed by dividing the bourrelet normal reaction force by the spring constant.
Figure 6. Acceleration versus projectile displacement

Figure 7. Velocity versus time
Figure 8. Velocity versus projectile displacement

Figure 9. Projectile displacement versus time
Figure 10. Yaw acceleration versus time

Figure 11. Yaw acceleration versus projectile displacement
Figure 12. Yaw velocity versus time

Figure 13. Yaw velocity versus projectile displacement
Figure 14. Yaw angle versus time

Figure 15. Yaw angle versus projectile displacement
Figure 16. Armature normal reaction force versus time

Figure 17. Armature normal reaction force versus projectile displacement
Figure 18. Bourrelet normal reaction force versus time

Figure 19. Bourrelet normal reaction force versus projectile displacement
RESULTS

The computation results of this analysis give a clear idea of the projectile balloting inside an electromagnetic railgun. However, for more accurate analysis, the following extra effects must also be considered.

In this analysis the rail bursting effect has been approximated by adding the bursting expansion to the bore radius and, consequently, an effective barrel radius has been used. This approximation does not account for the complex vibration of the rail from the bursting force. It will be formulated in a more advanced analysis.

The center of the armature base has been assumed to be always on the centerline of the barrel. This is generally true when the armature is squeezed inside the barrel and it is rather stiff. The projectile base resistance moment against the transverse rotation is not included in this analysis. It will be included in the advanced computation as well as the normal motion of the center of the armature base.

The pressure at the interface between the rail and the insulation or the armature depends on the geometrical condition and the physical properties of the contacting materials. This may be obtained by experimental measurement or computed by an available finite element computer program. However, if coefficients of friction are small, the frictions and pressure computation may be ignored.

The success of the analysis depends very much upon the determination of the spring constant which is not a linear constant. For accurate computation a force-deformation curve, or data obtained from experimental measurement, is recommended.

CONCLUSIONS

A set of approximate equations has been derived to compute the balloting motion of the projectile in addition to the axial acceleration, the velocity, and displacement. Reaction forces at the armature and the bourrelet contact regions, and the bourrelet deformation are also computed. Consequently, the associated curves with respect to time and projectile displacement may give some basic idea of the balloting motion of the projectile inside an electromagnetic railgun.
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