(2)

AD

TECHNICAL REPORT ARCCB-TR-91013

CORRELATION BETWEEN MACHINE TWIST ANGLE AND UNIT TWIST ANGLE IN CALCULATING SHEAR STRESSES FOR ELASTIC AND PLASTIC STRAINS IN TORSION

ROBERT R. FUJCZAK

APRIL 1991

US ARMY ARMAMENT RESEARCH,
DEVELOPMENT AND ENGINEERING CENTER
CLOSE COMBAT ARMAMENTS CENTER
BENÉT LABORATORIES
WATERVLIET, N.Y. 12189-4050

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The use of trade name(s) and/or manufacturer(s) does not constitute an official indorsement or approval.

DESTRUCTION NOTICE

For classified documents, follow the procedures in DoD 5200.22-M, Industrial Security Manual, Section II-19 or DoD 5200.1-R, Information Security Program Regulation, Chapter IX.

For unclassified, limited documents, destroy by any method that will prevent disclosure of contents or reconstruction of the document.

For unclassified, unlimited documents, destroy when the report is no longer needed. Do not return it to the originator.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARCCB-TR-91013	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CORRELATION BETWEEN MACHINE TWIST ANGLE AND UNIT TWIST ANGLE IN CALCULATING SHEAR STRESSES FOR ELASTIC AND PLASTIC STRAINS IN TORSION		5. TYPE OF REPORT & PERIOD COVERED Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Robert R. Fujczak		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army ARDEC Benet Laboratories, SMCAR-CCB-TL Watervliet, NY 12189-4050		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 6126.23.1BL0.0AR PRON No. 1A9AN9BM1AH3
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army ARDEC Close Combat Armaments Center Picatinny Arsenal, NJ 07806-5000		12. REPORT DATE April 1991 13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Torsion	Elastic
Twist Angle	Plastic
Shear Stress	Twist Moment
Shear Strain	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

For a solid torsion bar, the ratio between the unit twist angle and the machine twist angle remains linear, even when the twist shows plastic deformation. Therefore, the torsional stresses may be calculated from a torque-twist curve, whether the twist axis is unit twist angle or machine twist angle. This allows for stress calculations beyond the limit of the unit twist gage.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

	Page
INTRODUCTION	1
ELASTIC TORSION EQUATIONS	1
PLASTIC TORSION EQUATIONS	2
EXPERIMENTAL PROCEDURE	4
DATA ANALYSIS	5
CONCLUSIONS	7
REFERENCES	9

TABLES

I. TORQUE DATA FOR 4150H SPEC AA-5 6

LIST OF ILLUSTRATIONS

1. Torsion of a solid cylindrical bar 10
2. Method of calculating shear stress from torsion-twist curve 11
3. Torsion test specimen 12
4. Reproduction of experimental torsion-twist curve 13
5. Schematic of torsion-twist curve 14
6. Comparison of calculated torsional stress values 15
7. Ratio between unit twist angle and machine twist angle 16
8. Static torsion plot calculated from machine twist angle data 17

INTRODUCTION

In certain applications such as spring testing and twist testing of solid or hollow shafts, the torsion test has been valuable. However, it has not been as widely accepted and utilized as the tension test. Therefore, it has not been standardized to the same extent as the tension test. Material specifications rarely require torsion tests, therefore, the data are less common than tensile data (refs 1-3).

Torsion-testing equipment consists of a twisting head, with a chuck for gripping the specimen and for applying the twisting moment to the specimen, and a fixed end. Specimen deformation may be determined by unit twist angle over a specified gage length, but this method is limited to a small angular range, 4 to 5 degrees/inch gage length. Another measurement that may be determined during testing is the machine twist of the chuck applying the twist moment. This report shows the correlation between the machine twist angle and the limited unit twist angle and how they both can be used to calculate shear stress, even in the case of large plastic strains.

ELASTIC TORSION EQUATIONS

Figure 1 is a schematic of a torsional moment, M_T, on a cylindrical bar. The twisting moment is opposed by shear stresses in the bar cross section. The shear stress, τ, is zero at the center of the bar and increases linearly with the radius, r. The following is an equation of the twisting moment:

$$M_T = \int_{r=0}^{r=a} \tau r dA = \frac{\tau}{r} \int_0^a r^2 dA \qquad (1)$$

However, $\int r^2 dA$ is J, the polar moment of inertia of the area with respect to the axis of the bar. Therefore,

$$M_T = \frac{\tau J}{r}$$

or

$$\tau = \frac{M_T r}{J} \tag{2}$$

where τ = shear stress, psi

M_T = torsional moment, in.-lb

r = radial distance measured from center of bar, in.

J = polar moment of inertia, in.^{4}

Since the shear stress is a maximum at the outside surface of the bar, for a solid cylindrical specimen where $J = \pi D^4/32$, the maximum shear stress is

$$\tau_{max} = \frac{M_T D/2}{\pi D^4/32} = \frac{16 M_T}{\pi D^3} \tag{3}$$

The angle of twist, θ, is usually expressed in radians. If L is the test length of the specimen, from Figure 1 it can be seen that the shear strain, γ, is given by

$$\gamma = \tan \phi = \frac{r\theta}{L} \tag{4}$$

where ϕ = the torsional angular displacement across the gage length, L. This angle, γ, remains constant over any gage length, so twist angle θ is directly related to the gage length.

PLASTIC TORSION EQUATIONS

Once the torsional yield strength is surpassed, the shear stress over the cross section is no longer a linear function of the distance from the axis, and Eqs. (2) and (3) do not apply. Nadai (ref 4) presented a method to calculate the shear stress in the plastic range if the torque-twist curve is known. To simplify the analysis, the angle of twist per unit length may be defined as θ', where $\theta' = \theta/L$. Referring to Eq. (4), the shear strain is

$$\gamma = r\theta' \tag{5}$$

Equation (1), for the resisting torque in a cross section of the bar, can be expressed as follows:

$$M_T = 2\pi \int_0^a \tau r^2 dr \tag{6}$$

Now the shear stress is related to the shear strain by the stress-strain curve in shear. The shear stress τ can be related as a function of the shear strain γ, where $\tau = f(\gamma)$.

Introducing this equation into Eq. (6) and changing the variable from r to γ by means of Eq. (5) gives the following equation:

$$M_T = 2\pi \int_0^{\gamma_a} f(\gamma) \frac{(\gamma^2)}{(\theta')^2} \frac{d\gamma}{\theta'}$$

$$M_T(\theta')^3 = 2\pi \int_0^{\gamma_a} f(\gamma)\gamma^2 d\gamma \tag{7}$$

where $\gamma_a = a\theta'$ and $d\gamma = ad\theta'$. Differentiating Eq. (7) with respect to θ'

$$\frac{d}{d\theta'} (M_T\theta'^3) = 2\pi a[f(a\theta')a^2(\theta')^2] = 2\pi a^3(\theta')^2 f(a\theta')$$

However, the maximum value of shear stress in the bar at the outer fiber is $\tau_a = f(a\theta')$. Therefore, the equation becomes

$$\frac{d(M_T\theta'^3)}{d\theta'} = 2\pi a^3(\theta')^2\tau_a$$

$$3M_T(\theta')^2 + (\theta')^3 \frac{dM_T}{d\theta'} = 2\pi a^3(\theta')^2\tau_a$$

Rearranged, the equation becomes

$$\tau_a = \frac{1}{2\pi a^3}\left[\theta' \frac{dM_T}{d\theta'} + 3M_T\right] \tag{8}$$

The shear stress may be calculated with Eq. (8) if a torque-twist curve is available. Figure 2 shows a schematic of how this is done. Equation (8) can be rewritten in terms of geometry shown in Figure 2 as follows:

$$\tau_a = \frac{1}{2\pi a^3}\ (BC + 3CD) \tag{9}$$

It can also be noticed from Figure 2 that at the maximum value of torque the derivative $dM_T/d\theta' = 0$. Therefore, the ultimate torsional shear strength or modulus of rupture can be expressed by the following equation:

$$\tau_u = \frac{3M_{max}}{2\pi a^3} \tag{10}$$

If Eq. (9) is used at or before the elastic limit, then $M_T = BC = CD$, and since $a = D/2$, Eqs. (3) and (9) are equivalent up to this point. Thus, Eq. (9) may be used throughout the entire elastic-plastic range.

EXPERIMENTAL PROCEDURE

Figure 3 shows the torsional test specimen designed for use in an Instron Model No. 1323 Triaxial Test Machine, capable of static and fatigue testing in combinations of 100,000 lb tension-compression, 50,000 in.-lb torsion, and 50,000 psi pressure. In our case, only the static torsion capability was used.

The specimen was mounted in the twist chucks with instrumentation monitoring applied torque, unit twist angle, and machine twist angle at the chuck position. Figure 4 shows a reproduction of the original plot. The lower curve represents the unit twist angle from the biaxial extensometer, which had a maximum capacity of only about 5 degrees twist. So, this extensometer was removed before 5 degrees twist to prevent gage damage. Also, the upper curve represents the applied machine twist angle, but at over 40 degrees twist up to a value close to the maximum twist moment. The maximum twist moment also appears on the curve, but without any twist angle data, because the plasticity exceeded the machine's angle measurement capacity.

DATA ANALYSIS

Data points were taken from the curves in Figure 4, and Figure 5 represents a schematic of both machine angle and unit twist angle versus machine torque. Since calculation of the shear stresses along the torque-twist curve depends on derivatives, a cubic spline technique (ref 5) was used. The raw data and calculated values appear in Table I. Data for unit twist and machine twist angles were taken at equivalent torque values up to the limit of the unit twist values. After that, only machine twist values were recorded.

The first column in Table I is untitled and just identifies the data points as "pt 0," "pt 1," etc., taken from Figure 4. The next column entitled "Mach" contains values for the machine twist angle in total degrees. The column entitled "Unit" contains values for the unit twist angle in degrees/inch and is limited to less than 5 degrees/inch. The column entitled "M(T)" contains values of machine torque in in.-lb from minimum to maximum torque.

The column entitled "dy/dx" contains values of the derivatives calculated by the cubic spline approximation for the unit twist curve, and the column entitled "(dy/dx)M" contains values for the machine twist curve. "Tau-a" refers to the values of the shear stresses calculated from Eq. (9) using the unit twist derivative values. "Tau-M" refers to the values of the shear stresses using the machine twist derivative values.

TABLE I. TORQUE DATA FOR 4150H SPEC AA-51

	Mach	Unit	M(T)	(dy/dx)M	Tau-a	dy/dx	Tau-M
pt 0	0.00	0.00	0	4,626.80	0.00	16,868.75	0.00
pt 1	1.75	0.48	8,097	3,963.30	41,237.68	16,868.75	39,759.16
pt 2	3.00	0.85	12,218	2,894.50	57,381.58	9,898.10	57,725.50
pt 3	4.25	1.20	15,369	2,377.00	71,198.65	8,176.91	71,567.84
pt 4	5.55	1.59	18,278	1,869.50	83,192.97	6,607.30	83,027.60
pt 5	7.10	2.01	20,702	1,664.40	92,283.91	5,161.00	94,121.99
pt 6	8.45	2.53	23,126	1,616.70	101,669.05	4,139.40	105,728.68
pt 7	10.00	3.06	25,066	1,082.80	108,406.27	3,249.70	109,531.71
pt 8	11.85	3.73	27,005	945.80	115,405.34	2,580.20	117,421.63
pt 9	14.25	4.55	28,944	752.40	122,810.64	2,115.00	124,209.23
pt 10	16.25		30,399	655.40			129,675.95
pt 11	20.20		32,338	397.80			133,753.25
pt 12	25.00		34,035	291.40			139,279.67
pt 13	29.75		35,102	140.00			139,382.81
pt 14	34.70		35,732	68.00			139,490.52
pt 15	40.00		36,023	37.50			139,507.58
pt 16	60.00		36,556	0.00			139,633.63

By using Eq. (9) from the torque-twist diagram for unit twist and also machine twist angle, the torsion stress may be calculated either way. Figure 6 shows a comparison of torsional stress values calculated from both twist angles. Although the twist angles from each method are different with respect to twist moment, their corresponding derivative values yield comparable torsional stress values.

Figure 7 shows that the torsional stress calculated from unit twist is very closely related to the stress calculated from machine twist. Therefore, the machine twist angle may be used to calculate torsional stress with reasonable accuracy, using the limited unit twist angle to calculate torsional strain.

Figure 8 shows the static torsion plot calculated from the machine twist angle. Since the ratio between machine twist and unit twist is roughly 4 to 1, the machine twist data beyond about 16 degrees, or beyond the unit twist limit of about 4 degrees/inch, is the only means by which the torsional stress may be calculated up to maximum value.

CONCLUSIONS

1. For a solid torsion bar of 4150H steel, the ratio between the unit twist angle and the machine twist angle remains basically the same, even when the twist shows plastic deformation. For the configuration used herein, the ratio between machine angle and unit twist angle, θ/θ', is approximately 3.2. For every degree of unit twist, the machine twist is about 3.2 degrees.

2. Torsional stresses may be calculated by using the slopes from a torque-twist curve, whether the twist is unit twist per inch gage length or machine twist measured at the twisting head.

3. Since torsional stress may be calculated throughout the elastic-plastic range with the plastic equation, either twist may be used to calculate torsional stress up to the capability limit of the unit twist gage, but beyond that point the only method available to calculate torsional stress is the machine twist angle.

REFERENCES

1. G.E. Dieter, "The Torsion Test," in: Mechanical Metallurgy, Chapter 10, Third Edition, McGraw-Hill, Inc., New York, 1986.

2. J. Marin, Mechanical Behavior of Engineering Materials, Chapter 2, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1962.

3. Metals Handbook, "Torsion Testing," 9th Edition, American Society for Metals, Metals Park, OH, Vol. 8, 1985, pp. 137-184.

4. A. Nadai, Theory of Flow and Fracture of Solids, 2nd Edition, Vol. I, McGraw-Hill Book Company, New York, 1950, pp. 347-349.

5. A. Ralston and P. Rabinowitz, A First Course in Numerical Analysis, McGraw-Hill, Inc., New York, 1978.

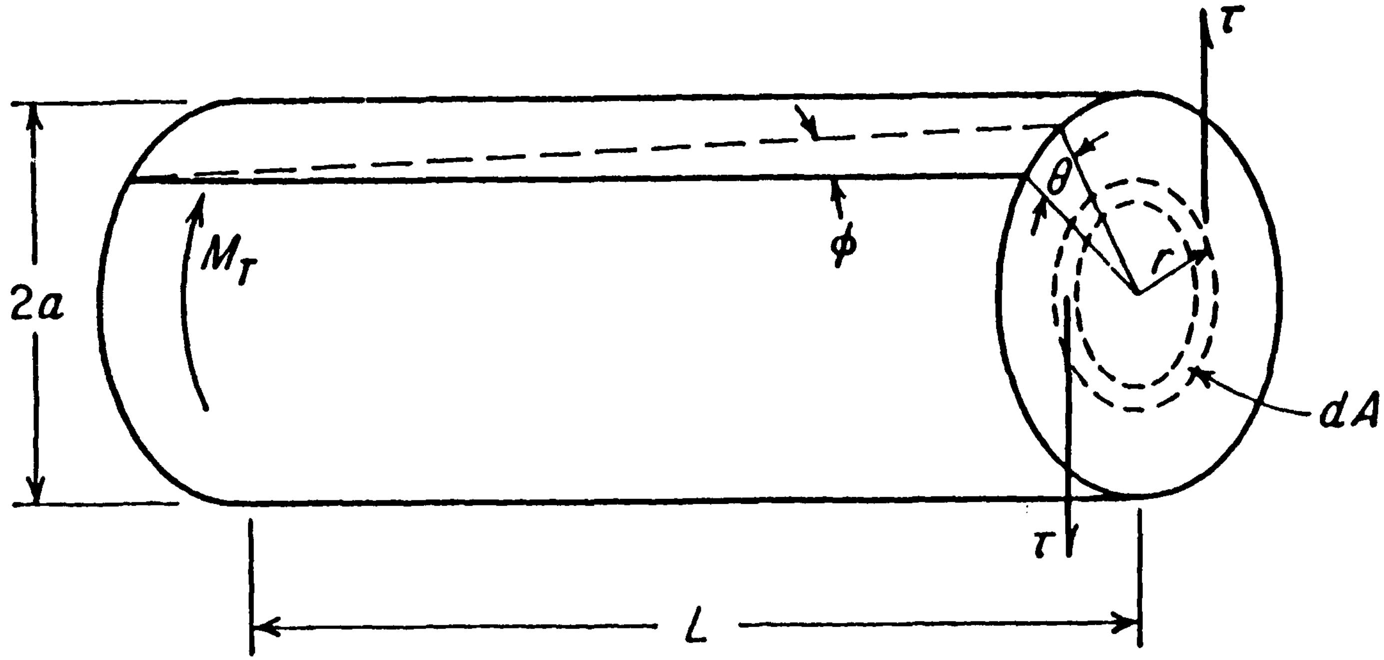

Figure 1. Torsion of a solid cylindrical bar.

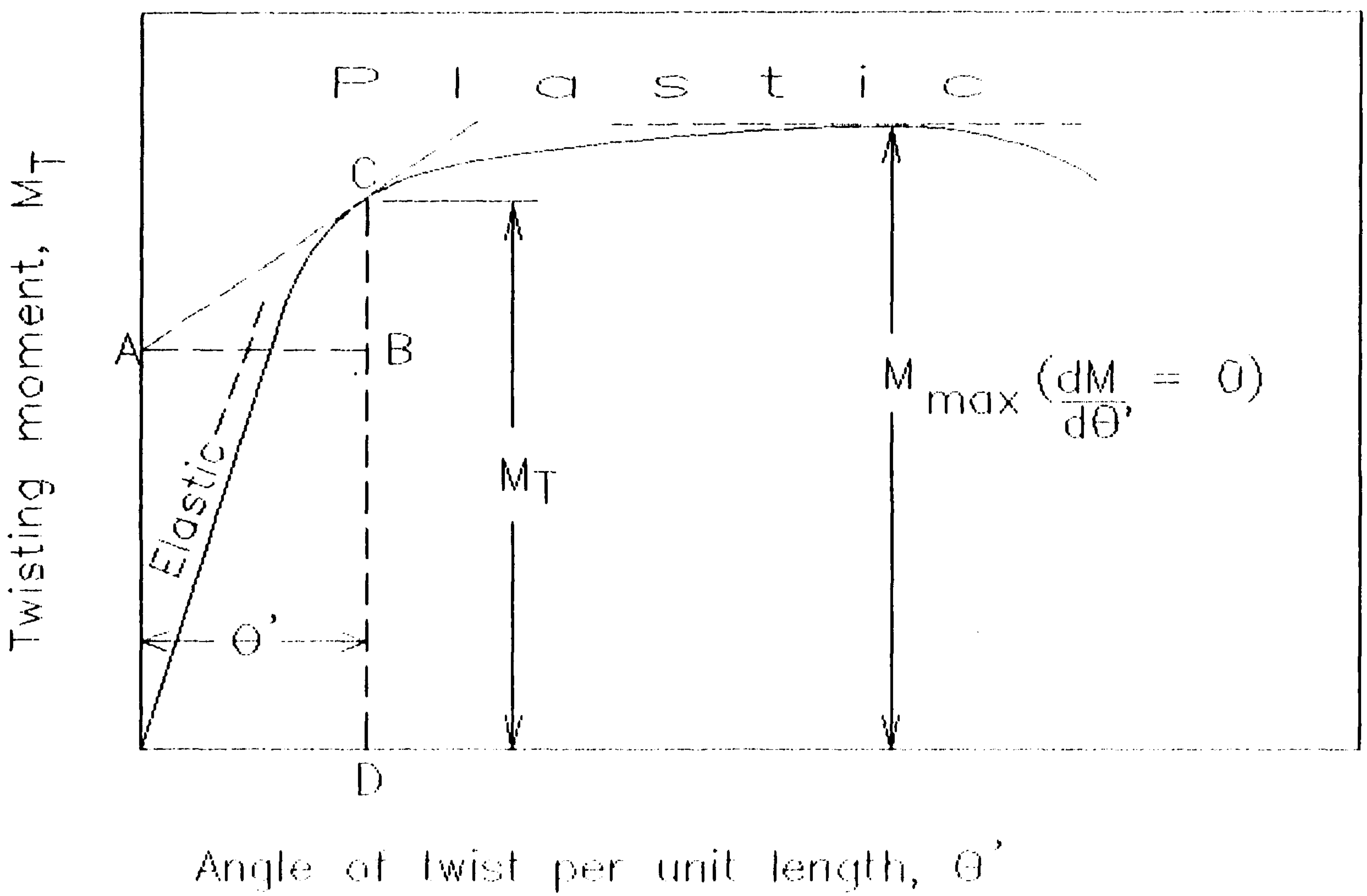

Figure 2. Method of calculating shear stress from torsion-twist curve.

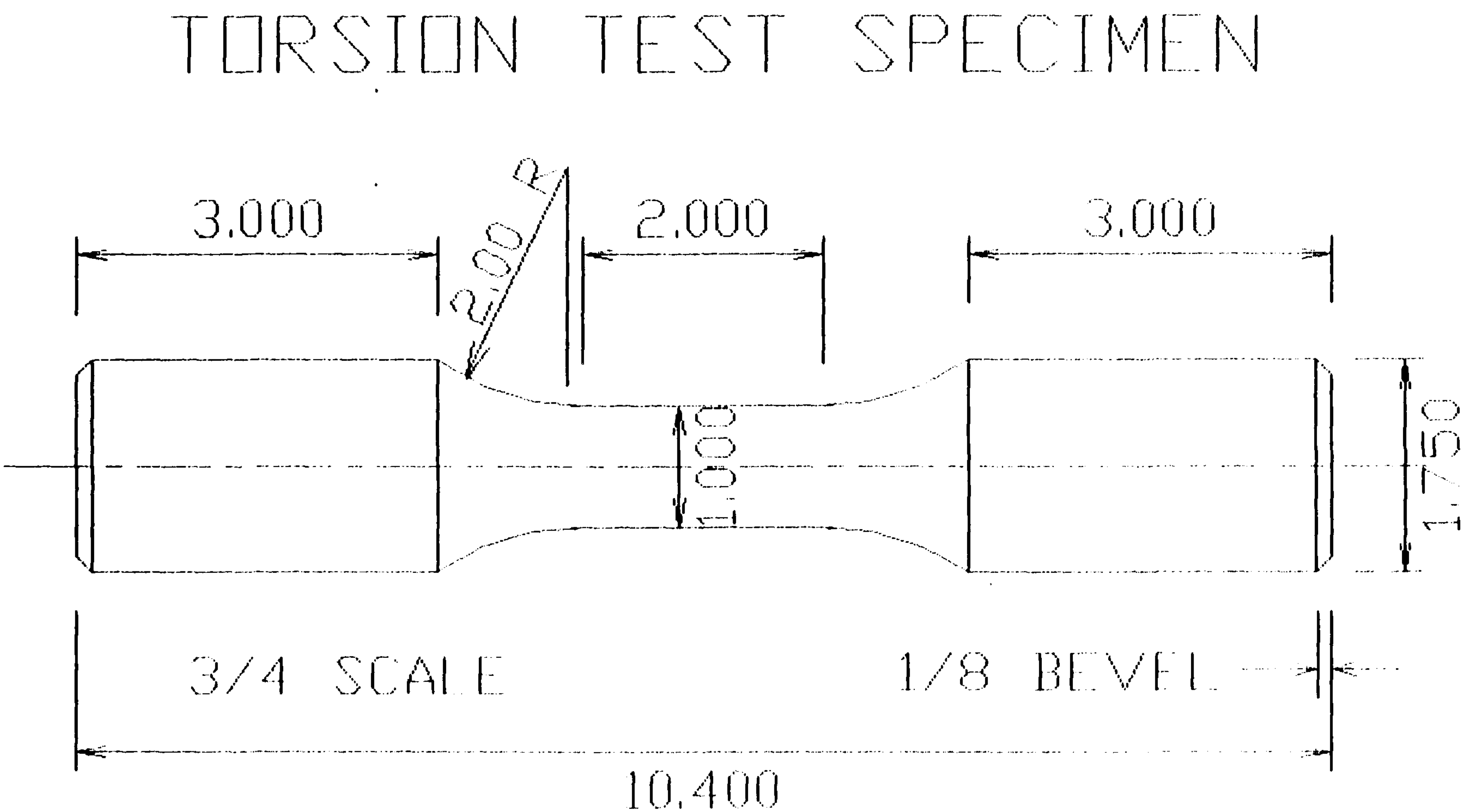

Figure 3. Torsion test specimen.

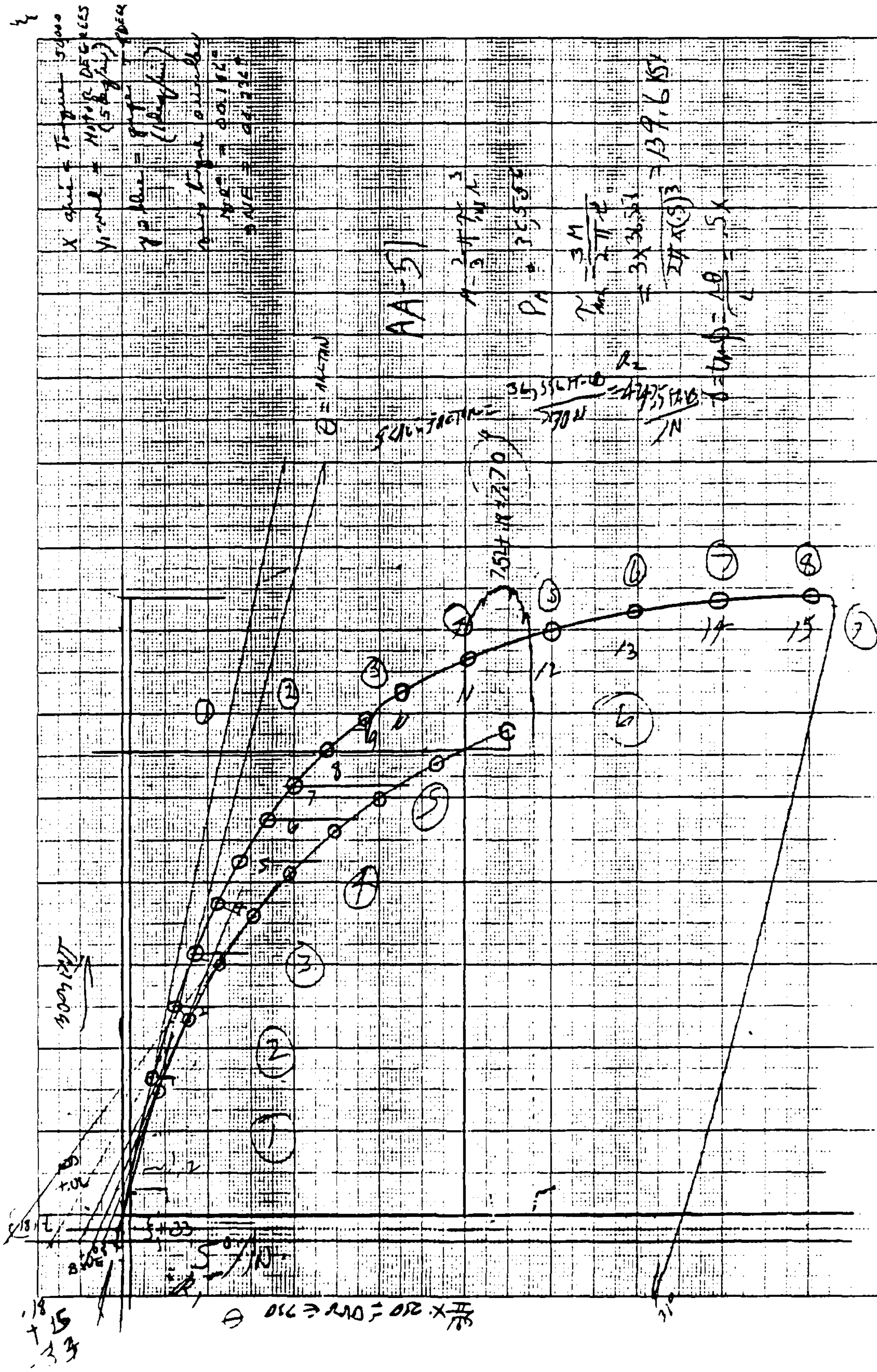

Figure 4. Reproduction of experimental torsion-twist curve.

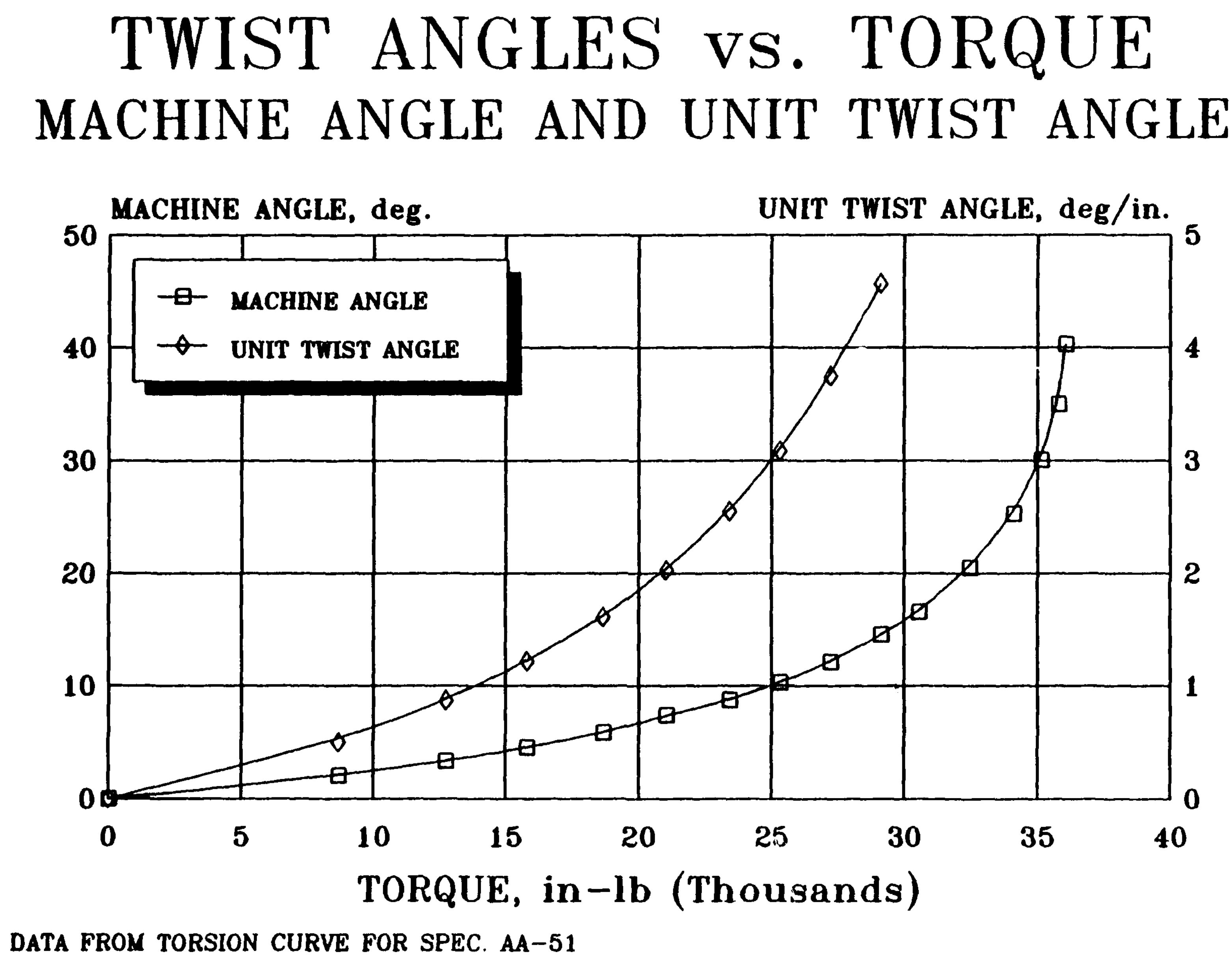

Figure 5. Schematic of torsion-twist curve.

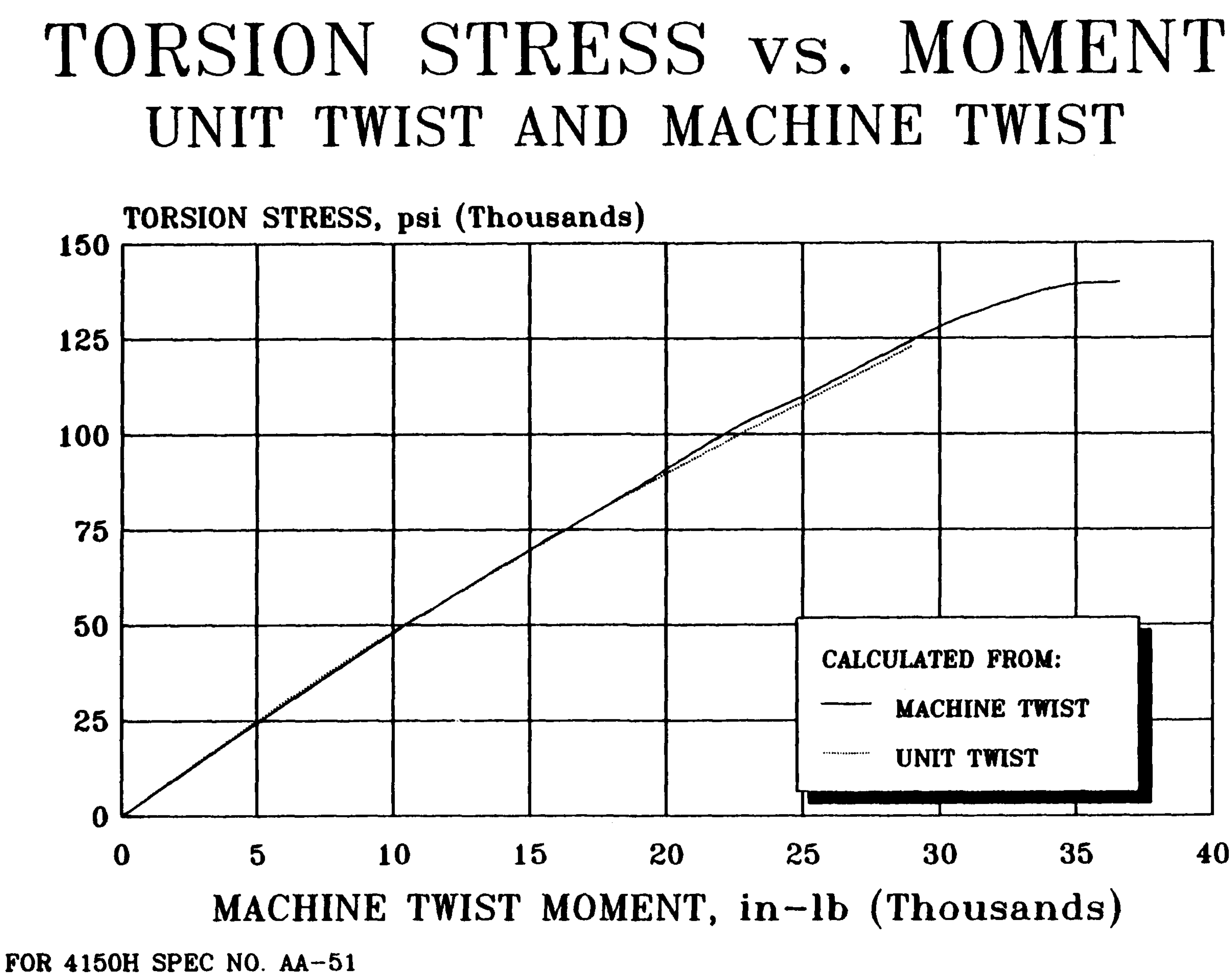

Figure 6. Comparison of calculated torsional stress values.

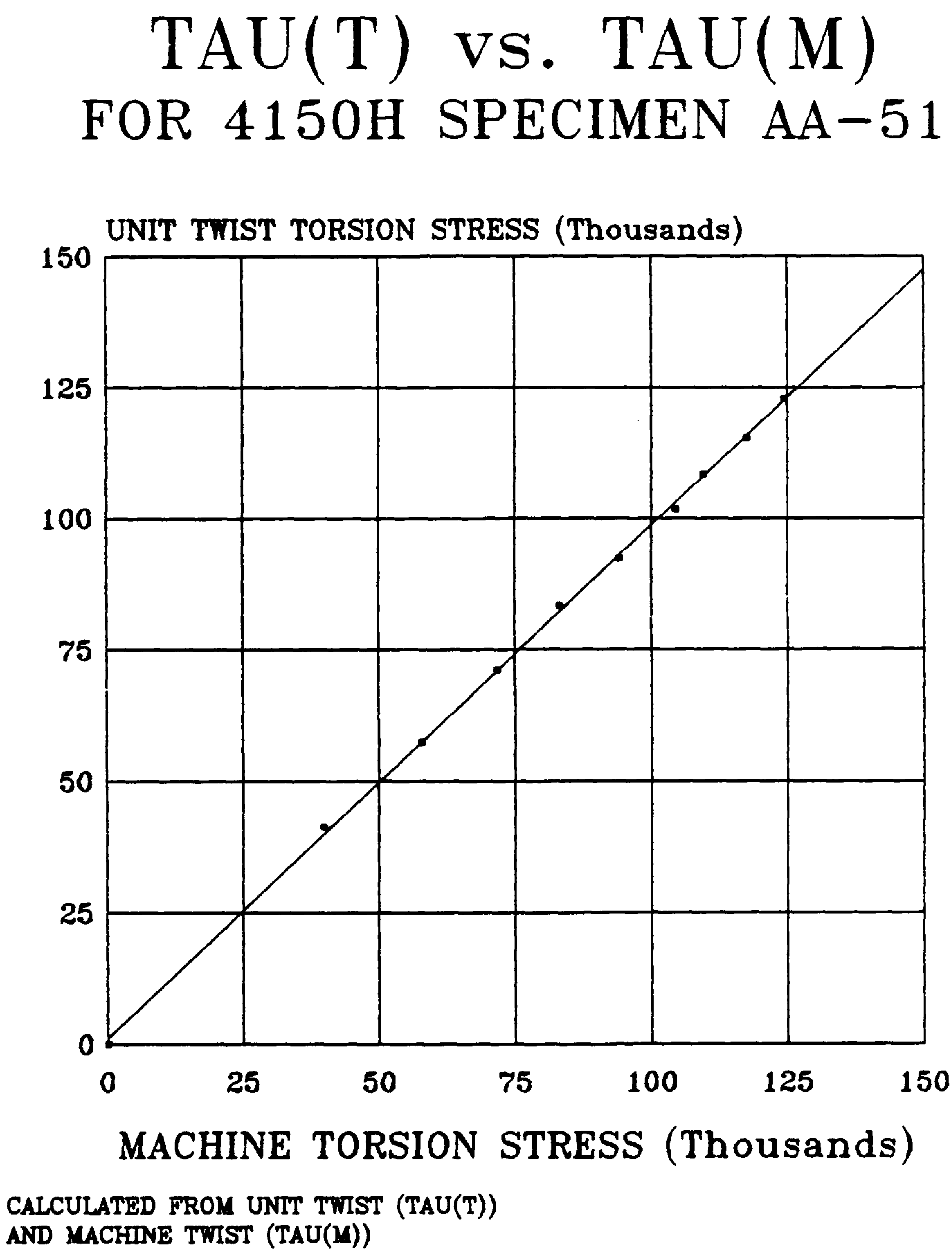

Figure 7. Ratio between unit twist angle and machine twist angle.

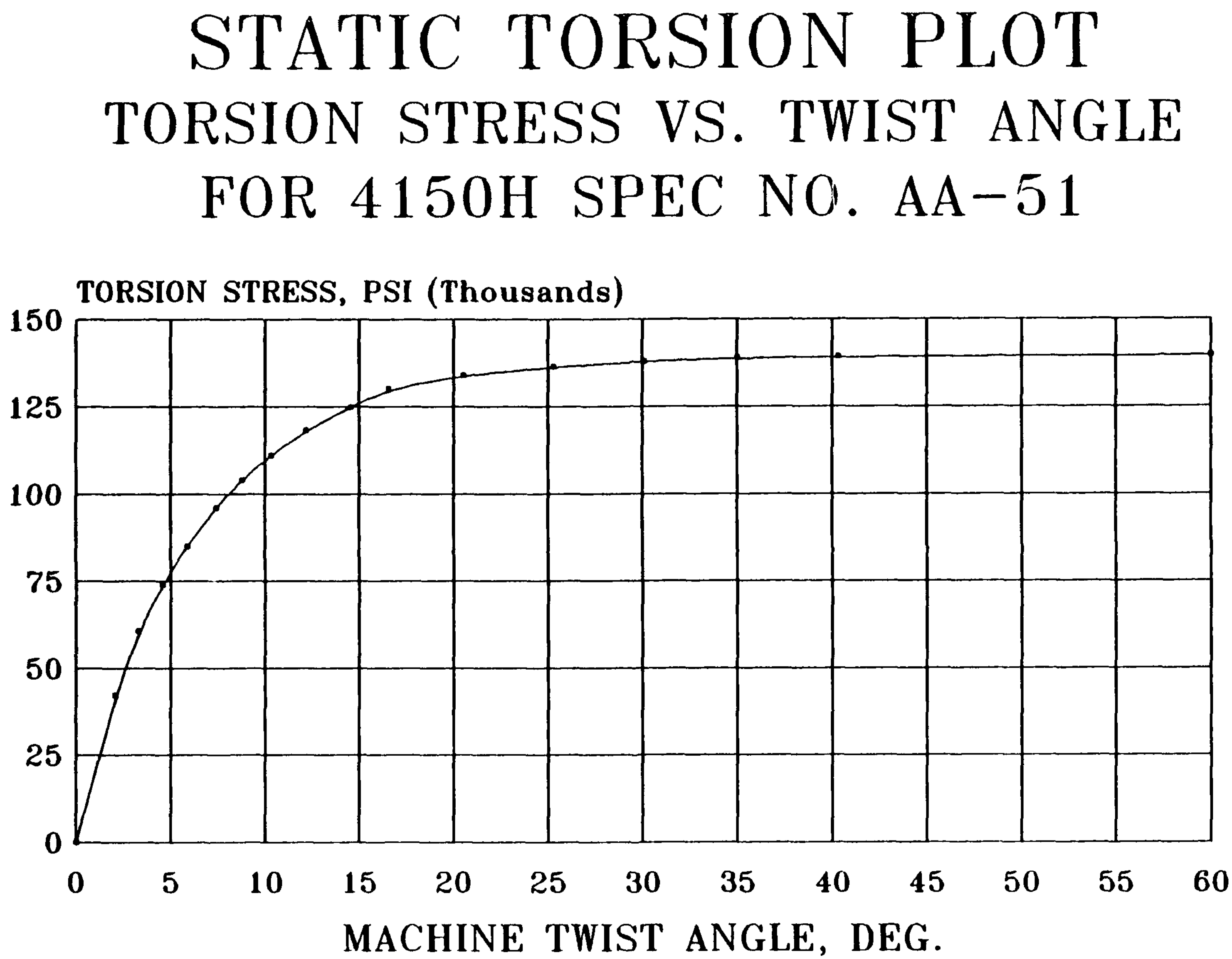

Figure 8. Static torsion plot calculated from machine twist angle data.

TECHNICAL REPORT INTERNAL DISTRIBUTION LIST

	NO. OF COPIES
CHIEF, DEVELOPMENT ENGINEERING DIVISION	
ATTN: SMCAR-CCB-D	1
-DA	1
-DC	1
-DI	1
-DP	1
-DR	1
-DS (SYSTEMS)	1
CHIEF, ENGINEERING SUPPORT DIVISION	
ATTN: SMCAR-CCB-S	1
-SE	1
CHIEF, RESEARCH DIVISION	
ATTN: SMCAR-CCB-R	2
-RA	1
-RE	1
-RM	1
-RP	1
-RT	1
TECHNICAL LIBRARY ATTN: SMCAR-CCB-TL	5
TECHNICAL PUBLICATIONS & EDITING SECTION ATTN: SMCAR-CCB-TL	3
DIRECTOR, OPERATIONS DIRECTORATE ATTN: SMCWV-OD	1
DIRECTOR, PROCUREMENT DIRECTORATE ATTN: SMCWV-PP	1
DIRECTOR, PRODUCT ASSURANCE DIRECTORATE ATTN: SMCWV-QA	1

NOTE: PLEASE NOTIFY DIRECTOR, BENET LABORATORIES, ATTN: SMCAR-CCB-TL, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST

	NO. OF COPIES
ASST SEC OF THE ARMY	
RESEARCH AND DEVELOPMENT	
ATTN: DEPT FOR SCI AND TECH	1
THE PENTAGON	
WASHINGTON, D.C. 20310-0103	
ADMINISTRATOR	
DEFENSE TECHNICAL INFO CENTER	
ATTN: DTIC-FDAC	12
CAMERON STATION	
ALEXANDRIA, VA 22304-6145	
COMMANDER	
US ARMY ARDEC	
ATTN: SMCAR-AEE	1
SMCAR-AES, BLDG. 321	1
SMCAR-AET-O, BLDG. 351N	1
SMCAR-CC	1
SMCAR-CCP-A	1
SMCAR-FSA	1
SMCAR-FSM-E	1
SMCAR-FSS-D, BLDG. 94	1
SMCAR-IMI-I (STINFO) BLDG. 59	2
PICATINNY ARSENAL, NJ 07806-5000	
DIRECTOR	
US ARMY BALLISTIC RESEARCH LABORATORY	
ATTN: SLCBR-DD-T, BLDG. 305	1
ABERDEEN PROVING GROUND, MD 21005-5066	
DIRECTOR	
US ARMY MATERIEL SYSTEMS ANALYSIS ACTV	
ATTN: AMXSY-MP	1
ABERDEEN PROVING GROUND, MD 21005-5071	
COMMANDER	
HQ, AMCCOM	
ATTN: AMSMC-IMP-L	1
ROCK ISLAND, IL 61299-6000	

	NO. OF COPIES
COMMANDER	
ROCK ISLAND ARSENAL	
ATTN: SMCRI-ENM	1
ROCK ISLAND, IL 61299-5000	
DIRECTOR	
US ARMY INDUSTRIAL BASE ENGR ACTV	
ATTN: AMXIB-P	1
ROCK ISLAND, IL 61299-7260	
COMMANDER	
US ARMY TANK-AUTMV R&D COMMAND	
ATTN: AMSTA-DDL (TECH LIB)	1
WARREN, MI 48397-5000	
COMMANDER	
US MILITARY ACADEMY	1
ATTN: DEPARTMENT OF MECHANICS	
WEST POINT, NY 10996-1792	
US ARMY MISSILE COMMAND	
REDSTONE SCIENTIFIC INFO CTR	2
ATTN: DOCUMENTS SECT, BLDG. 4484	
REDSTONE ARSENAL, AL 35898-5241	
COMMANDER	
US ARMY FGN SCIENCE AND TECH CTR	
ATTN: DRXST-SD	1
220 7TH STREET, N.E.	
CHARLOTTESVILLE, VA 22901	
COMMANDER	
US ARMY LABCOM	
MATERIALS TECHNOLOGY LAB	
ATTN: SLCMT-IML (TECH LIB)	2
WATERTOWN, MA 02172-0001	

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH, DEVELOPMENT, AND ENGINEERING CENTER, US ARMY AMCCOM, ATTN: BENET LABORATORIES, SMCAR-CCB-TL, WATERVLIET, NY 12189-4050, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST (CONT'D)

	NO. OF COPIES
COMMANDER US ARMY LABCOM, ISA ATTN: SLCIS-IM-TL 2800 POWDER MILL ROAD ADELPHI, MD 20783-1145	1
COMMANDER US ARMY RESEARCH OFFICE ATTN: CHIEF, IPO P.O. BOX 12211 RESEARCH TRIANGLE PARK, NC 27709-2211	1
DIRECTOR US NAVAL RESEARCH LAB ATTN: MATERIALS SCI & TECH DIVISION CODE 26-27 (DOC LIB) WASHINGTON, D.C. 20375	1 1
DIRECTOR US ARMY BALLISTIC RESEARCH LABORATORY ATTN: SLCBR-IB-M (DR. BRUCE BURNS) ABERDEEN PROVING GROUND, MD 21005-5066	1
COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/MN EGLIN AFB, FL 32542-5434	1
COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/MNF EGLIN AFB, FL 32542-5434	1
METALS AND CERAMICS INFC CTR BATTELLE COLUMBUS DIVISION 505 KING AVENUE COLUMBUS, OH 43201-2693	1

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH, DEVELOPMENT, AND ENGINEERING CENTER, US ARMY AMCCOM, ATTN: BENET LABORATORIES, SMCAR-CCB-TL, WATERVLIET, NY 12189-4050, OF ANY ADDRESS CHANGES.