PREDICTION OF FIBER/MATRIX INTERPHASE PROPERTIES AND THEIR INFLUENCE ON INTERFACE STRESS, DISPLACEMENT AND FRACTURE TOUGHNESS OF COMPOSITE MATERIAL

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Warminster, PA 18974-5000

3 JANUARY 1990

PHASE REPORT
Period Covering October 1988 to October 1989
Task No. R0220400A
Work Unit No. 108634
Program Element No. 0601153N

Approved for Public Release; Distribution is Unlimited

Prepared for
OFFICE OF NAVAL RESEARCH
Washington, DC 20350
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<th>OFFICE OR DEPARTMENT</th>
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</thead>
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<td>30</td>
<td>Warfare Systems Analysis Department</td>
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</tr>
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<td>Air Vehicle &amp; Crew Systems Technology Department</td>
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An elastic shear lag analysis, which includes an interphase region, has been developed and correlated with Mandell's micro-debonding test data [1] to determine the thickness and material properties of the interphase. The shear strength and the relationship between thickness and shear modulus of interphase were determined for both glass/epoxy and graphite/epoxy composites.

An axisymmetric finite element model, which includes the interphase properties obtained from the shear lag analysis, was also developed. The effects of the interphase on stresses and deformation at the interface and fracture toughness of the fibrous composites were investigated. The following conclusions were drawn from the study:

1. By including an interphase, both the shear lag analysis and finite element model provide very good predictions.
2. An interphase exists and is softer than the matrix.
3. The interphase has significant effect on stress concentration, displacement and fracture toughness of composite materials.

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<table>
<thead>
<tr>
<th>FIELD</th>
<th>GROUP</th>
<th>SUBGROUP</th>
</tr>
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<tbody>
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<td>11</td>
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</tbody>
</table>

**Subject Terms (Continue on reverse if necessary and identify by block number)**

- Fiber/Matrix Interface, Interphase, Interphase Properties, Fracture Toughness, Organic Composites
Block 19.

Fracture toughness of the fibrous composite.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>TABLES</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>SHEAR LAG THEORY WITH DISTINCT INTERPHASE</td>
<td>1</td>
</tr>
<tr>
<td>THEORY</td>
<td>2</td>
</tr>
<tr>
<td>CORRELATION WITH MANDELL'S MICRODEBONDING TESTS</td>
<td>10</td>
</tr>
<tr>
<td>Determination of Interphase Properties</td>
<td>10</td>
</tr>
<tr>
<td>Correlation With Microdebonding Test Results</td>
<td>11</td>
</tr>
<tr>
<td>INFLUENCE OF INTERPHASE PROPERTIES ON INTERPHASE SHEAR AND FRACTURE TOUGHNESS OF COMPOSITES</td>
<td>15</td>
</tr>
<tr>
<td>INTERPHASE SHEAR STRESS</td>
<td>15</td>
</tr>
<tr>
<td>MODE I FRACTURE TOUGHNESS</td>
<td>15</td>
</tr>
<tr>
<td>FINITE ELEMENT SIMULATION</td>
<td>19</td>
</tr>
<tr>
<td>MICRODEBONDING PROBLEM</td>
<td>19</td>
</tr>
<tr>
<td>INTERPHASE EFFECT ON OUT-OF-PLANE DISPLACEMENT</td>
<td>25</td>
</tr>
<tr>
<td>DISCUSSION AND CONCLUSION</td>
<td>25</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>33</td>
</tr>
<tr>
<td>APPENDIX A: DETERMINATION OF INTERPHASE PROPERTIES FOR S-GLASS/EPOXY AND GRAPHITE/EPOXY</td>
<td>A-1</td>
</tr>
<tr>
<td>APPENDIX B: CALCULATION OF INTERPHASE SHEAR STRESS CONCENTRATION FACTOR AND MODE I FRACTURE ENERGY OF UNIDIRECTIONAL COMPOSITES</td>
<td>B-1</td>
</tr>
</tbody>
</table>
# FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schematic of Loading for Microdebonding Tests</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Modeling of Fiber/Matrix Interface</td>
<td>3</td>
</tr>
<tr>
<td>3a</td>
<td>Free Body Diagram Involving Fiber and Interphase</td>
<td>5</td>
</tr>
<tr>
<td>3b</td>
<td>Free Body Diagram Involving Fiber, Interphase, and Matrix</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>Axial Displacement of Fiber, Interphase, and Matrix</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>Variation of Debond Stress vs. ( \frac{t_m}{d_f} ) for S-glass/Epoxy</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>Variation of Debond Stress vs. ( \frac{t_m}{d_f} ) for Graphite/Epoxy</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>Effect of Interphase Shear Modulus on Maximum Interfacial Shear Stress for ( \frac{t_m}{d_f} = 0.1 )</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>Effect of Interphase Shear Modulus on Maximum Interfacial Shear Stress for ( \frac{t_m}{d_f} = 1.0 )</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>Effect of Interphase on Opening Mode Fracture Energy of a Unidirectional Fiber Composite for ( \frac{t_m}{d_f} = 0.1 )</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>Effect of Interphase on Opening Mode Fracture Energy of a Unidirectional Fiber Composite for ( \frac{t_m}{d_f} = 1.0 )</td>
<td>21</td>
</tr>
<tr>
<td>11</td>
<td>Finite Element Model for Axisymmetric Solid</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td>Axisymmetric Finite Element Model for Microdebonding Problem</td>
<td>23</td>
</tr>
<tr>
<td>13</td>
<td>Interphase Effect on Interfacial Shear (( \frac{t_m}{d_f} = 0.36, V_f = 0.4, t_i = 0.1 \mu m ))</td>
<td>24</td>
</tr>
<tr>
<td>14</td>
<td>Variation of ( \sigma_c ) vs. ( \frac{t_m}{d_f} ) as Predicted by F.E.A. With and Without Interphase for Graphite/Epoxy</td>
<td>27</td>
</tr>
<tr>
<td>15</td>
<td>Axial Displacement of Fiber End Unser 1 N Load Applied at Fiber End</td>
<td>28</td>
</tr>
<tr>
<td>16</td>
<td>Modeling of Matrix Deformation Near Loaded Fiber</td>
<td>29</td>
</tr>
<tr>
<td>17</td>
<td>Interphase Effect on Displacement at Top Surface (( P = 0.017N, t_i = 0.1 \mu m, G_m/G_i = 25 ))</td>
<td>30</td>
</tr>
<tr>
<td>18</td>
<td>Interphase Effect on Displacement at Top Surface (( P = 0.017N, t_i = 0.1 \mu m, G_m/G_i = 74 ))</td>
<td>31</td>
</tr>
<tr>
<td>19</td>
<td>Correlation of Finite Element Analysis and Out-of-Plane Displacement Test Data for 0.4% Fiber Strain Load With ( t_i = 0.1 \mu m )</td>
<td>32</td>
</tr>
</tbody>
</table>
TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Interphase Properties of S-Glass/Epoxy</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Interphase Properties of Graphite/Epoxy</td>
<td>12</td>
</tr>
<tr>
<td>A-1</td>
<td>( \bar{a} ) vs. ( \frac{t_m}{d_f} ) for Graphite/Epoxy</td>
<td>A-4</td>
</tr>
<tr>
<td>A-2</td>
<td>( \bar{a} ) vs. ( \frac{t_m}{d_f} ) for S-Glass/Epoxy</td>
<td>A-4</td>
</tr>
<tr>
<td>A-3</td>
<td>Finite Element Results for ( P=1 ) N (( t_i = 0.1 ) ( \mu )m)</td>
<td>A-5</td>
</tr>
<tr>
<td>A-4</td>
<td>Comparison of Debonding Stress Between Test and Finite Elements Analysis</td>
<td>A-5</td>
</tr>
<tr>
<td>A-5</td>
<td>Comparison of Out-of-Plane Displacement Between Shear Lag Theory and Finite Element Analysis</td>
<td>A-6</td>
</tr>
<tr>
<td>B-1</td>
<td>( K_s ) vs. ( \frac{G_m}{G_i} ) for Various ( t_i/d_f ) (( t_m/d_f = 0.1 ))</td>
<td>B-1</td>
</tr>
<tr>
<td>B-2</td>
<td>( K_s ) vs. ( \frac{G_m}{G_i} ) for Various ( t_i/d_f ) (( t_m/d_f = 1.0 ))</td>
<td>B-1</td>
</tr>
<tr>
<td>B-3</td>
<td>( \frac{G_{inf}}{C_{inf}} ) vs. ( \frac{G_m}{G} ) for Various ( t_i/d_f ) (( t_m/d_f = 0.1 ))</td>
<td>B-2</td>
</tr>
<tr>
<td>B-4</td>
<td>( \frac{G_{inf}}{C_{inf}} ) vs. ( \frac{G_m}{G} ) for Various ( t_i/d_f ) (( t_m/d_f = 1.0 ))</td>
<td>B-3</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The work described here was performed at the Naval Air Development Center under the sponsorship of the Office of Naval Research. We are grateful to Dr. A.K. Vasudevan of the Office of Naval Research for his encouragement and cooperation.
INTRODUCTION

The fiber/matrix interface has a significant influence on the structural integrity of a fibrous composite. For example, the stronger the interface bond, the higher the composite's static strength. However, a strong interface bond also yields a brittle and notch-sensitive composite and, on the other hand, a weaker interface results in a higher fracture toughness composite. Conventional methods for predicting the stress at the fiber/matrix interface can be divided into three categories. These are: shear lag theory, two-dimensional, finite-element analysis, and three-dimensional, finite-element analysis. These methods all assume zero interface thickness and uniform, homogeneous matrix properties.

Recently, several researchers have suggested that the volume of material immediately surrounding the fiber is significantly different from the bulk matrix. This volume of material is commonly referred to as the interphase. Drzal has suggested that this material may be more rigid than the bulk matrix. Piggott has found that to explain the Young's moduli of short-fiber composites, the interphase must have a very low modulus, i.e., much softer than the bulk matrix.

No direct evidence has been found for the presence of an interphase in organic matrix composites nor are its properties and dimensions known. The objective of this study is to prove the existence of an interphase and to determine interphase elastic properties, if possible. Included in this work is an investigation to determine the influence of interphases on stress at the fiber/matrix interfaces on displacements and on fracture toughnesses of fibrous composites.

SHEAR LAG THEORY WITH DISTINCT INTERPHASE

To include the interphase into a composite-material structural analysis model, material properties and the thickness of the interphase must be known in advance. Experimental data on interphase properties does not exist. One way to deduce the interphase characteristics is to assume initial values for the interphase and then iterate the properties and thickness until analytical results converge to match the corresponding experimentally observable results.

Since the thickness of interphase can be very small compared with the fiber diameter, a very large number of finite elements would be needed if the finite element technique was used for the iteration procedure. This would be time-consuming and costly. A closed-form solution, based upon shear-lag theory, was employed instead.

In the following, a shear lag analysis, which includes and interphase region between the fiber and bulk matrix, has been developed. Mandell's microdebonding test data was used to determine the interphase material properties and thickness.

THEORY

Mandell’s microdebonding test, Figure 1, was modeled as shown in Figure 2. Four types of materials were included in this model; namely, fiber, interphase, matrix, and composite. Note that a load is applied at the end of a fiber, and that the material outside the matrix was modeled as equivalent smeared composite material.

By employing the shear lag theory, the following assumptions were made:

1. Axial load is carried by the fiber alone, while the interphase and matrix carry the shear load only.
Figure 1. Schematic of Loading for Microdebonding Test.
Figure 2. Modeling of Fiber/Matrix Interface.
(2) Fixed boundary conditions apply at the boundary between the matrix and equivalent composite.

The freebody diagrams of the loaded fiber and adjacent interphase and matrix are shown in Figures 3a and 3b. The equilibrium equations are as follows:

\[
\frac{dF}{dz} + 2\pi r_i \tau_i = 0 \quad (1)
\]

\[
\frac{dF}{dz} + 2\pi r_m \tau_m = 0 \quad (2)
\]

From equations (1) and (2), we have:

\[
\tau_i r_i = r_m \tau_m = r \tau \quad (3)
\]

where

- \( F \) = fiber force at \( z \)
- \( r_f, r_i \) = radius of fiber and interphase, respectively
- \( \tau_i, \tau_m \) = shear stress of interphase and matrix, respectively.

From the theory of elasticity and Figure 4:

\[
\tau = \frac{dW}{dr} G
\]

\[
r_F \leq r \leq r_i
\]

\[
\frac{dW}{dr} = \frac{\tau}{G_i}
\]

where \( \tau \) is the shear stress in the interphase, and \( G_i \) is the shear modulus of interphase.
Figure 3a. Free Body Diagram Involving Fiber and Interphase.

Figure 3b. Free Body Diagram Involving Fiber, Interphase, and Matrix.
Figure 4. Axial Displacement of Fiber, Interphase, and Matrix.
From equation (3):

\[ \tau = \frac{\tau_i r_i}{r} \]  \hspace{1cm} (5)

substituting equation (5) into (4) and integrating, we have:

\[ w = \frac{\tau_i r_i}{G_i} \ln(r) + C \]  \hspace{1cm} (6)

but,

at \hspace{1cm} r = r_f, \hspace{0.2cm} w = w_1

\[ r = r_i, \hspace{0.2cm} w = w_2 \]

Applying these boundary conditions to equation (6), we have:

\[ (w_2 - w_1)G_i = \frac{\tau_i r_i}{r_i \ln \left( \frac{r_i}{r_f} \right)} r_f \]

By the same procedure and noting that at \( r = r_m, w = 0 \) we have:

\[ \tau_m = \frac{-w_2 G_m}{r_m \ln \left( \frac{r_m}{r_i} \right)} \]  \hspace{1cm} (8)

Also,

\[ F = E_f \pi r_f^2 \frac{dw_1}{dz} \]  \hspace{1cm} (9)
substituting equations (7) through (9) into equation (1) and (2) we have:

\[
\frac{d^2 w_1}{dz^2} - \frac{2G_i}{E_f r_f^2 \ln \left( \frac{r_i}{r_f} \right)} (w_1 - w_2) = 0
\] (10)

\[
\frac{d^2 w_2}{dz^2} - \frac{2G_m}{r_f^2 E_f \ln \left( \frac{r_m}{r_i} \right)} w_2 = 0
\] (11)

After some algebraic manipulations of equations (10) and (11), we have:

\[
\frac{d^2 w_1}{dz^2} - \alpha^2 w_1 = 0
\] (12)

where

\[
\alpha^2 = \frac{\frac{2G_i}{E_f r_f^2 \ln \left( \frac{r_i}{r_f} \right)}}{G_i \ln \left( \frac{r_m}{r_i} \right)} \frac{1}{1 + \frac{G_m \ln \left( \frac{r_i}{r_f} \right)}{G_m \ln \left( \frac{r_m}{r_i} \right)}}
\]

The solution of equation (12) is:

\[
w_1 = C_1 \cosh(\alpha z) + C_2 \sinh(\alpha z)
\] (13)
since
\[ z = 0, \quad F = -P_0 = -\pi r_f^2 \bar{\sigma} = E_f \pi r_f^2 \frac{d\omega_1}{dz} \]
\[ z = l, \quad w_1 = 0 \] (14)

where \( \bar{\sigma} \) is the average applied stress at the fiber end.

Applying the boundary conditions (i.e., equation (14)) to equation (13), we have:

\[ w_1 = -\frac{\bar{\sigma}}{E_f \alpha} (\tanh(\alpha l) \cosh(\alpha z) + \sinh(\alpha z)) \] (15)

\( w_2 \) is obtained from equations (11) and (15) such that:

\[ w_2 = \frac{r_f^2 E_f \alpha^2 \ln \left( \frac{r_m}{r_i} \right)}{2G_m} w_1 \] (16)

From equations (7), (15), and (16), the interphase shear stress at \( r=r_i \) is written as:

\[ \tau_i = \frac{\bar{\sigma} \sqrt{G_m}}{\sqrt{2E_f}} \frac{r_f}{r_i} \left( \frac{r_f}{r_i} \right) \left[ \frac{G_m}{G_i} \ln \left( \frac{r_i}{r_f} \right) + \ln \left( \frac{r_m}{r_i} \right) \right] \] (17)

From equation (3), the interphase shear stress at \( r=r_f \) is written as:

\[ \overline{\tau}_i = \frac{\bar{\sigma}}{\sqrt{2}} \sqrt{\frac{G_m}{E_f}} \left( \frac{r_f}{r_i} \right) \left[ \frac{G_m}{G_i} \ln \left( \frac{r_i}{r_f} \right) + \ln \left( \frac{r_m}{r_i} \right) \right] \] (18)

The maximum interphase shear stress occurs at:

\[ \tau_{\text{max}} = \overline{\tau}_i |_{z=0} = -\frac{\bar{\sigma}}{\sqrt{2}} \sqrt{\frac{G_m}{E_f}} \frac{\tanh(\alpha l)}{\sqrt{\frac{G_m}{G_i} \ln \left( \frac{r_i}{r_f} \right) + \ln \left( \frac{r_m}{r_i} \right)}} \] (19)

Note that equation (19) contains two unknowns to be determined (namely, \( G_m \) and \( r_i \)).
CORRELATION WITH MANDELL'S MICRODEBONDING TESTS

DETERMINATION OF INTERPHASE PROPERTIES

We now postulate that debonding of fiber from the matrix is due to interphase shear failure. Based on this assumption, the debonding criterion for the fiber/matrix interface is written as follows:

\[ \tau_0 = \frac{\bar{\sigma}_0}{\sqrt{2}} \sqrt{\frac{G_m}{E_f}} \frac{\tanh(\alpha l)}{\sqrt{\left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{2t_i}{d_f} \right) + \ln \left( 1 + \frac{2t_m}{d_f} \right)}} \]  \hspace{1cm} (20)

where

- \( \tau_0 \) = interphase shear strength
- \( \bar{\sigma}_0 \) = average applied stress at the fiber end which causes fiber-matrix debonding (referred to as the debonding stress).

Let

\[ r_i = r_f + t_i \]

\[ r_m = r_f + t_m \]

where \( t_i \) is interphase thickness and \( t_m \) is the thickness combining interphase and matrix.

Equation (20) becomes:

\[ \tau_0 = \frac{\bar{\sigma}_0}{\sqrt{2}} \sqrt{\frac{G_m}{E_f}} \frac{\tanh(\alpha l)}{\sqrt{\left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{2t_i}{d_f} \right) + \ln \left( 1 + \frac{2t_m}{d_f} \right)}} \]  \hspace{1cm} (21)

From equation (21) we conclude that:

\[ \bar{\sigma}_0 \tanh(\alpha l) \sqrt{\left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{2t_i}{d_f} \right) + \ln \left( 1 + \frac{2t_m}{d_f} \right)} = \text{constant} \]  \hspace{1cm} (22)

By selecting \( t_m/d_f = 0.4 \) and \( t_m/d_f = 1.0 \) to correlate the test data, the following results were obtained (for detailed calculations, refer to Appendix A).
For S-glass/epoxy,

$$\left(\frac{G_m}{G_i} - 1\right) \ln \left(1 + \frac{2t_i}{d_f}\right) = 1.8446$$

$$\tau_0 = 39 \text{ MPa}$$

For graphite/epoxy

$$\left(\frac{G_m}{G_i} - 1\right) \ln \left(1 + \frac{2t_i}{d_f}\right) = 2.0672$$

$$\tau_0 = 27 \text{ MPa}$$

Note that $G_i$ and $t_i$ cannot be determined separately from equations (23 and 24), but if $t_i$ can be determined from an experiment, then $G_i$ can be determined. Tables 1 and 2 list the interphase properties of S-glass/epoxy and graphite/epoxy, respectively, for various interphase thickness values and show the interphase shear modulus to be softer than bulk matrix for both S-glass/epoxy and graphite/epoxy composite materials.

**CORRELATION WITH MICRODEBONDING TEST RESULTS**

After the interphase shear strength has been determined, the debonding stress $\overline{\sigma}_0$ is determined as follows:

$$\overline{\sigma}_0 = \sqrt{2\tau_0} \sqrt{\frac{E_f}{G_m}} \sqrt{\left(\frac{G_m}{G_i} - 1\right) \ln \left(1 + \frac{2t_i}{d_f}\right) + \ln \left(1 + \frac{2t_m}{d_f}\right)}$$

(25)

Making use of equations (23) and (24), and noted that we have for S-glass/epoxy:

$$\overline{\sigma}_0 = \tau_0 \sqrt{2\frac{E_f}{G_m}} \sqrt{1.8446 + \ln \left(1 + \frac{2t_m}{d_f}\right)}$$

(26)

and for graphite/epoxy:

$$\overline{\sigma}_0 = \tau_0 \sqrt{2\frac{E_f}{G_m}} \sqrt{2.0672 + \ln \left(1 + \frac{2t_m}{d_f}\right)}$$

(27)

Equations (26) and (27) are plotted in Figures 5 and 6. The correlations with the microdebonding test results are excellent.
### Table 1. Interphase Properties of S-glass Epoxy

**Interphase Shear Strength = 39 MPA**

<table>
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<td>.1</td>
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<td>9.0</td>
</tr>
</tbody>
</table>

### Table 2. Interphase Properties of Graphite/Epoxy

**Interphase Shear Strength = 27 MPA**

<table>
<thead>
<tr>
<th>$t_1/d_f$</th>
<th>$t_1$ (nm)</th>
<th>$G_m/G_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0014</td>
<td>10</td>
<td>724</td>
</tr>
<tr>
<td>.0071</td>
<td>50</td>
<td>145</td>
</tr>
<tr>
<td>.0143</td>
<td>100</td>
<td>74</td>
</tr>
<tr>
<td>.0714</td>
<td>500</td>
<td>15</td>
</tr>
<tr>
<td>.143</td>
<td>1000</td>
<td>7</td>
</tr>
</tbody>
</table>
Figure 5. Variation of Debond Stress vs. $t_m/d_f$ for S-glass/Epoxy.
Figure 6. Variation of Debond Stress vs. $t_m/d_f$ for Graphite/Epoxy.
INTERPHASE SHEAR STRESS

Equation (19) can be rewritten as:

\[
\frac{\tau_{\text{max}}}{\sigma} = -\frac{1}{\sqrt{2}} \sqrt{\frac{G_m}{E_f}} \frac{1}{\sqrt{\frac{G_m}{G_i} - 1} \ln \left(1 + \frac{2t_i}{d_f}\right) + \ln \left(1 + \frac{2t_m}{d_f}\right)} \tag{28}
\]

Let

\[
K_s = (\text{stress concentration factor}) = \frac{1}{\sqrt{2}} \frac{\tau_{\text{max}}}{\sigma} \sqrt{\frac{E_f}{G_m}} \quad \tag{29}
\]

Thus

\[
K_s = \frac{1}{\sqrt{2}} \sqrt{\frac{G_m}{G_i} - 1} \ln \left(1 + \frac{2t_i}{d_f}\right) + \ln \left(1 + \frac{2t_m}{d_f}\right) \quad \tag{30}
\]

Equation (30) is plotted in Figures 7 and 8 for \(K_s\) vs. \(G_m/G_i\) for various ratios. We can conclude that:

1. The lower the \(G_m/G_i\) ratio, the higher the stress concentration and the more likely debonding.
2. The thinner the interphase thickness, \(t_i\), the higher the stress concentration, again making debonding easier.

MODE I FRACTURE TOUGHNESS

Murphy,\textsuperscript{3} etc., showed that the Mode I fracture energy of unidirectional composites can be expressed as follows:

\[
(G_f)_c = (1 - v_f)G_M + v_f(G_F + G_{INF}) \quad \tag{31}
\]

where

\[
(G_f)_c = \text{Mode I fracture energy of composite}
\]
Figure 7. Effect of Interphase Shear Modulus on Maximum Stress for $t_m/d_f = 0.1$. 

\[ K_S = \frac{T_{\text{max}}}{\sigma} \left( \frac{E_f}{G_m} \right)^{1/2} \] 

\[ t_{m}/d_f = 0.1 \] 

\[ t_{l}/d_f = 0.01 \] 

\[ t_{l}/d_f = 0.005 \]
Figure 8. Effect of Interphase Shear Modulus on Maximum Interfacial Shear Stress for $t_m/d_f = 1.0$. 

\[ K_S = \left( \frac{t_m/d_f}{\theta} \right)^{1/2} \]
\( G_M \) = Mode I fracture energy of matrix
\( G_F \) = Mode I fracture energy of fiber
\( \nu_f \) = volume fraction of fiber
\( G_{INF} \) = Mode I fracture energy associated with the decohesion of the fiber matrix interface.

Murphy further showed that:

\[
G_{INF} = \frac{d_f \sigma_f^3}{4\tau_f E_f} (1-g+g^2-g^3+\frac{1}{2}(\frac{l_p}{l_d})^2(1-g)(1-g^2))
\]  

(32)

where

\( \sigma_f \) = ultimate stress of fiber
\( l_d \) = \( \frac{(d_f/4\sigma_f)(\sigma_f - \sigma_0)}{10} \)
\( \sigma_0 \) = debonding stress
\( l_p \) = fiber pull-out length
\( \tau_f \) = post debonding friction shear stress
\( g = \frac{\sigma_0}{\sigma_f} \) = debonding fracture ratio

Examining equations (31) and (32), it can be seen that the smaller the \( g \) (i.e., weaker interface), the larger the \( G_{INF} \) and hence, the tougher the composite. On the other hand, the larger, the \( g \) (i.e., stronger interface), the smaller the \( G_{INF} \) yielding a more brittle composite. If \( g=1 \), (i.e., the fiber breaks with no debonding between fiber and matrix), equation (31) reduces to:

\[
(G_1)_c = (1-\nu_f) G_M - \nu_f G_F
\]

Murphy\(^2\) showed that for high-volume fractions the Mode I fracture energy, \((G_1)_c\), of composites can range over five orders of magnitude, while the debonding fracture ratio, \( g \), ranges from zero to one.

From equations (29) and (33), \( g \) can be expressed as follows:

\[
g = \frac{\tau_0 1}{\sigma_f K_s \sqrt{\frac{E_f}{G_m}}}
\]  

(34)
To demonstrate how interphase properties will effect the Mode I fracture toughness of the composite, equation (32) is rewritten as follows:

\[
\frac{G_{INF}}{C_{INF}} = 1 - g + g^2 - g^3 + \frac{1}{2} \left( \frac{I_p}{l_d} \right)^2 (1 - g)^2
\]  

(35)

where

\[
C_{INF} = \frac{d_f \sigma_f^3 u}{4 \tau_f E_f}
\]

For simplicity, assume \( l_p = 0 \) (i.e., no fiber pull-out occurs during fracture of composite). Equation (35) then becomes:

\[
\frac{G_{INF}}{C_{INF}} = 1 - g + g^2 - g^3
\]  

(36)

Assuming that the interphase strength, \( \tau_o \), is constant for various interphase thicknesses and making use of equation (34), equation (36) is plotted as shown in Figures 9 and 10. The detailed calculation is shown in Appendix B.

From the plots, it can be seen that when the interphase property, \( G_o \), is closer to the bulk matrix, \( G_m \), the composite material will have higher fracture toughness. Note that the thinner the interphase thickness, \( t \), the tougher the composite material.

**FINITE ELEMENT SIMULATION**

**MICRODEBONDING PROBLEM**

An axisymmetric finite element model that includes the interphase was developed. Interphase properties were obtained by either using the values contained in Tables 1 and 2 or those derived from using equation (23) for S-glass/epoxy and equation (24) for graphite/epoxy for various interphase thicknesses. Only those results for graphite/epoxy are presented. Figures 11 and 12 are typical models used to analyze the microdebonding problem. The NASTRAN computer code was used to perform the analysis. Note that the top surface and side surface corresponding to outside radius were assumed fixed. A 1 N force was applied at the center of the cylindrical composite disc.

Figure 13 shows the typical interphase shear distribution along the axis. The maximum shear stress occurs at approximately 0.4 fiber diameter below the surface. Shear lag theory positions the maximum shear at the surface. Note that the magnitude of the interface shear for composite without an interphase (i.e., \( G_m/G_o = 1.0 \)) is much higher than for a composite with an interphase. This result agrees with the conclusion from the shear lag analysis in the foregoing section.
Figure 9. Effect of Interphase on Opening Mode Fracture Energy of a Unidirectional Fiber Composite for \( \frac{t_m}{d_f} = 0.1 \).
Figure 10. Effect of Interphase on Opening Mode Fracture Energy of a Unidirectional Fiber Composite for $t_m/d_f = 1.0$
Figure 11. Finite Element Model for Axisymmetric Solid.

TOTAL NO. OF NODES = 2367
TOTAL NO. OF ELEMENTS = 2500

INTERPHASE

Z

INTERPHASE

N

M (MATRIX)

F (FIBER)

500 μm

UNITS: FORCE N μm

200 μm

R

P
Figure 12: Axisymmetric Finite Element Model for Microdebonding Problem.
Figure 14 shows the prediction of debonding stress $\sigma_o$, for $t_m/d_f=0.1$ and $t_m/d_f=1.0$, where data for $t_m/d_f=0.4$ was used to determine the interphase shear strength. We see that the model, which includes the interphase, provides a better solution than those without the interphase.

Also, note that interphase shear strength predicted by finite element analysis, 23.9 mpa, is close to that predicted by the shear lag theory. (For detailed calculation, refer to Appendix A. This interphase shear strength value is different from the values of interfacial shear strength obtained by other investigators).

Figure 15 shows the fiber end displacement in the axial direction under a 1.0 N axial load applied at the fiber end. The correlation between the finite element analysis and new shear lag theory is very good, validating the adequacy of the shear lag theory.

INTERPHASE EFFECT ON OUT-OF-PLANE DISPLACEMENT

Finite element calculations were performed to determine the effect of the interphase on the out-of-plane displacement using the model shown in Figure 16. This model corresponds to the experimental geometry used for tensioned fiber tests in reference 14. Figures 17 and 18 show the predicted out-of-plane displacements with and without interphase for $G_m/G_i=25$ and $G_m/G_i=74$ respectively. It can be seen that a significant displacement gradient occurs in the vicinity of interphase region. Thus, the interphase region may be defined as a region with a very high displacement gradient allowing the dimension of interphase to be determined through direct measurement of the length of this high displacement gradient region.

The results shown in Figures 17 and 18 were adjusted for a 0.4% fiber strain load and correlated with test results as shown in Figure 19. Interphase properties with $G_m/G_i=25$ and $t_i=0.1\mu m$ match the test data. However, if this method is not believed adequate to determine the exact interphase thickness, then the radial displacement field must also be measured in addition to the axial displacement field so that the test results can be correlated to determine separate values for the interphase thickness and modulus. Note that the softer the interphase, the greater the displacement gradient in the interphase region.

DISCUSSION AND CONCLUSION

The ratio between interphase shear modulus and thickness can be established from shear lag analysis, but not specific values for $G_i$ or $T_i$ separately. If the interphase thickness can be determined from an experimental measurement, then equation (23) or equation (24) can be used to calculate the interphase shear modulus for S-glass/epoxy and graphite/epoxy, respectively. If the interphase thickness is not available experimentally, then an iteration procedure must be performed using equations (23) and (24) as starting values for interphase shear modulus and thickness. Iterate $G_i$ and $T_i$ until the analytical results (i.e., out-of-plane displacement and radial displacement) converge to the test results. More studies need to be pursued of this approach to compute $G_i$ or $T_i$.

In this report it is assumed that the interphase is an axisymmetric region of uniform modulus and fixed radial dimension. We recognize that the true interphase region has radially varying properties and can also exhibit circumferential property gradients.

For a tougher composite, equation (32) and Figures 9 and 10 suggest that the composite material must possess:
1. Larger fiber diameter and fiber ultimate stress

2. Smaller post debond frictional shear stress and fiber Young's modulus

3. Smaller interphase shear strength

4. Smaller interphase thickness and $G_m/G_i$ ratio.

Although interphase shear strength, $r_o$, and post debond frictional shear stress vary with interphase thickness, $t$, Figures 9 and 10 give approximate guidelines for obtaining tougher composite materials. Note that the weaker interface (i.e., small interphase shear strength) provides tougher fracture resistance, but may affect the strength of composite, especially short fiber composites.

The following conclusions are drawn from this study:

1. By including the interphase, both shear lag analysis and finite element analysis provide much better predictions of material response.

2. The interphase exists and is softer than the matrix for the composites used by Mandell in his microdebonding tests (i.e., S-glass/epoxy and graphite/epoxy).

3. The properties and thickness of the interphase have significant influence on the interface stress, displacement, and fracture toughness of fibrous composites.
Figure 14. Variation of $\sigma_0$ vs. $t_m/d_f$ as Predicted by FFEA With and Without Interphase for Graphite/Epoxy.
Figure 15. Axial Displacement of Fiber End Under 1 N Load Applied at Fiber End.
Figure 16. Modeling of Matrix Deformation Near Loaded Fiber.
Figure 17. Interphase Effect on Displacement at Top Surface (P=0.017N, t_i = 0.1 μm, G_m/G_i = 25).

R (micro m) + W/O INTERPHASE

TRANVERSE DISPLACEMENT (μm)
Figure 19. Correlation of Finite Element Analysis and Out-of-Plane Displacement Test Data for 0.4% Fiber Strain Load With $t_i=0.1\mu\text{m}$. 

Vertical Displacement ($\xi$) vs Radial Position ($\mu\text{m}$)
REFERENCES


12. Garton, Andrew; Daly; John H. Polymer Composites 6 (4), 1985.


APPENDIX A: DETERMINATION OF INTERPHASE PROPERTIES
FOR S-GLASS/EPOXY AND GRAPHITE/EPOXY

NEW SHEAR LAG THEORY

INTERPHASE PROPERTIES

From equation (22), we have:

\[
\frac{\bar{\sigma} \tanh (\alpha l)}{\left\{ \left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{2t_i}{d_i} \right) + \ln \left( 1 + \frac{2t_m}{d_i} \right) \right\}^{1/2}} = \text{Const} \tag{A-1}
\]

Substituting \( t_m/d_f = 1.0 \) and \( t_m/d_f = 0.4 \) into equation (A-1), and note that \( \tanh (\alpha l) = 1.0 \), we have:

\[
\frac{\bar{\sigma}_{1.0}}{(A + 1.0986)^{1/2}} = \frac{\bar{\sigma}_{0.4}}{(A + 0.5878)^{1/2}} \tag{A-2}
\]

where

\( \bar{\sigma}_{0.4}, \bar{\sigma}_{1.0} = \) debonding stress of fiber/matrix interface corresponding to \( t_m/d_f = 1.0 \) and \( t_m/d_f = 0.4 \), respectively

\[
A = \left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{t_i}{r_i} \right) \tag{A-3}
\]

Equation (A-2) can be rearranged as follows:

\[
\left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{t_i}{r_i} \right) = \frac{(1.0986 - 0.5878 B)}{(B - 1)} \tag{A-4}
\]

where

\[
B = \left( \frac{\bar{\sigma}_{1.0}}{\bar{\sigma}_{0.4}} \right)^2 \tag{A-5}
\]

For S-glass/epoxy, from reference 1:

\[
\frac{\bar{\sigma}_{1.0}}{\bar{\sigma}_{0.4}} = 1.1 \quad B = 1.21
\]
equation (A-4) becomes:

\[ \left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{t_f}{r_f} \right) = 1.8446 \]

For graphite/epoxy, from reference 1:

\[ \frac{\bar{\sigma}_{1.0}}{\bar{\sigma}_{0.4}} = 1.092 \]

\[ B-1.1924 \]

equation (A-4) becomes:

\[ \left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{t_f}{r_f} \right) = 2.0672 \]

INTERPHASE-SHEAR STRENGTH

From equation (28), the maximum interfacial shear stress can be written as follows:

\[ \tau_{\text{max}} = \frac{\bar{\sigma}}{\sqrt{2}} \sqrt{\frac{G_m}{E_f} \left[ \left( \frac{G_m}{G_0} - 1 \right) \ln \left( 1 + \frac{t_f}{r_f} \right) + \ln \left( 1 + \frac{2t_m}{d_f} \right) \right]} \]  \hspace{1cm} (A-6)

For S-glass fiber, \( G_m = 1.07 \) GPA, \( E_f = 86 \) GPA from reference 1, for \( t_m/d_f = 0.4 \), \( \bar{\sigma} = 777 \) MPA, equation (A-6) becomes:

\[ \tau_{\text{c}} = 39 \text{ MPA} \]

For \( G_i/G_\phi \), \( G_m = 1.21 \) GPA, \( E_f = 241 \) GPA, from reference 1, for \( t_m/d_f = 0.4 \), \( \bar{\sigma} = 883 \) MPA, equation (A-6) becomes:

\[ \tau_{\text{c}} = 27 \text{ MPA} \]

OUT-OF-DISPLACEMENT OF FIBER

From shear lag theory, the fiber axial displacement at \( z = 0 \) can be written as:

\[ w = \frac{\bar{\sigma}}{E_f \bar{\alpha}} \]  \hspace{1cm} (A-7)
where

\[
\alpha = \frac{\sqrt{2}}{r_f} \sqrt{\frac{G_m}{E_f} \left( \frac{G_m}{G_f} - 1 \right) \ln \left( \frac{r_f}{r_i} \right) + \ln \left( \frac{r_m}{r_f} \right)}^{-1/2}
\]  

(A-8)

For graphite/epoxy an \( P = 1 \) N, equation (A-7) becomes:

\[
W - \frac{\bar{\sigma}}{E_f \alpha} = \frac{25.984}{241} \cdot \frac{1}{\alpha} - \frac{0.1078}{\alpha}
\]  

(A-9)

\[
\alpha = 0.0286 \left( 2.0672 + \ln \left( 1 + \frac{2r_m}{d_f} \right) \right)^{-1/2}
\]  

(A-10)

DEBONDING STRESSES

Graphite/Epoxy

From equation (A-1), we have:

\[
\frac{\bar{\sigma}}{\sigma_{1,0}} = \frac{\left( 2.0672 + \ln \left( 1 + \frac{2r_m}{d_f} \right) \right)^{-1/2}}{(2.0672 + 1.0986)^{1/2}}
\]

After simplification, we have:

\[
\bar{\sigma} = 542.36 \left( 2.0672 + \ln \left( 1 + \frac{2r_m}{d_f} \right) \right)^{1/2}
\]

\( \sigma \) vs. \( t_m/d_f \) is tabulated as shown in Table A-1.

S-glass/Epoxy

From equation (A-1) we have:

\[
\frac{\bar{\sigma}}{\sigma_{1,0}} = \frac{\left( 1.8446 + \ln \left( 1 + \frac{2r_m}{d_f} \right) \right)^{1/2}}{(1.8446 + 1.0986)^{1/2}}
\]

After simplification we have:

\[
\bar{\sigma} = 498.375 \left( 1.8446 + \ln \left( 1 + \frac{2r_m}{d_f} \right) \right)^{1/2}
\]
\( \sigma \) VS. \( t_m/d_f \) is tabulated as shown in Table A-2.

### Table A-1. \( \bar{\sigma} \) vs. \( t_m/d_f \) for Graphite/Epoxy

<table>
<thead>
<tr>
<th>( t_m/d_f )</th>
<th>( \bar{\sigma}_{\text{test}} ) (MPA)</th>
<th>( \bar{\sigma} ) (MPA)</th>
<th>( E_{%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>773.6</td>
<td>813.45</td>
<td>5.2</td>
</tr>
<tr>
<td>0.2</td>
<td>826.8</td>
<td>840.86</td>
<td>1.7</td>
</tr>
<tr>
<td>0.3</td>
<td>859.7</td>
<td>863.9</td>
<td>0.49</td>
</tr>
<tr>
<td>0.4</td>
<td>883.0</td>
<td>883.0</td>
<td>0.</td>
</tr>
<tr>
<td>0.5</td>
<td>902.9</td>
<td>901.09</td>
<td>-0.2</td>
</tr>
<tr>
<td>0.6</td>
<td>918.8</td>
<td>916.51</td>
<td>-0.25</td>
</tr>
<tr>
<td>0.7</td>
<td>932.5</td>
<td>930.37</td>
<td>-0.23</td>
</tr>
<tr>
<td>0.8</td>
<td>944.5</td>
<td>942.94</td>
<td>-0.17</td>
</tr>
<tr>
<td>0.9</td>
<td>955.3</td>
<td>954.4</td>
<td>-0.09</td>
</tr>
<tr>
<td>1.0</td>
<td>965.0</td>
<td>965.0</td>
<td>0.</td>
</tr>
</tbody>
</table>

### Table A-2. \( \bar{\sigma} \) vs. \( t_m/d_f \) for S-glass/Epoxy.

<table>
<thead>
<tr>
<th>( t_m/d_f )</th>
<th>( \bar{\sigma}_{\text{test}} ) (MPA)</th>
<th>( \bar{\sigma} ) (MPA)</th>
<th>( E_{%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>673.0</td>
<td>709.5</td>
<td>5.4</td>
</tr>
<tr>
<td>0.2</td>
<td>723.0</td>
<td>736.0</td>
<td>1.8</td>
</tr>
<tr>
<td>0.3</td>
<td>754.0</td>
<td>758.2</td>
<td>0.56</td>
</tr>
<tr>
<td>0.4</td>
<td>777.0</td>
<td>777.0</td>
<td>0.</td>
</tr>
<tr>
<td>0.5</td>
<td>795.5</td>
<td>793.92</td>
<td>-0.2</td>
</tr>
<tr>
<td>0.6</td>
<td>810.8</td>
<td>808.7</td>
<td>-0.26</td>
</tr>
<tr>
<td>0.7</td>
<td>823.9</td>
<td>821.95</td>
<td>-0.24</td>
</tr>
<tr>
<td>0.8</td>
<td>835.4</td>
<td>833.95</td>
<td>-0.17</td>
</tr>
<tr>
<td>0.9</td>
<td>845.7</td>
<td>844.92</td>
<td>-0.09</td>
</tr>
<tr>
<td>1.0</td>
<td>855.0</td>
<td>855.0</td>
<td>0.</td>
</tr>
</tbody>
</table>
NADC-90001-60

FINITE ELEMENT ANALYSIS

From finite element analysis, for center fiber subjected to $\bar{\sigma} = 25,984.0$ MPA (i.e., $P = 1$ N), the computer outputs for interfacial shear, $\tau$, and out-of-plane displacement, $w$, are shown in Table A-3.

Table A-3. Finite Element Results for $P = 1$ N ($t_i = .1 \mu$m).

<table>
<thead>
<tr>
<th>$t_m/d_f$</th>
<th>$w$ (\mu m)</th>
<th>$\tau$ (MPA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>5.62</td>
<td>775.0</td>
</tr>
<tr>
<td>0.35</td>
<td>6.23</td>
<td>710.7</td>
</tr>
<tr>
<td>1.0</td>
<td>6.86</td>
<td>641.0</td>
</tr>
</tbody>
</table>

INTERPHASE SHEAR STRENGTH

Define the interphase shear stress at $\bar{\sigma} = 883$ MPA, which corresponds to the debonding stress of composite with $t_m/d_f = 0.4$ as interphase shear strength, then $\tau = 704/25984 \times 883 = 23.9$ MPA.

DEBONDING STRESS $\bar{\sigma}$ vs. $t_m/d_f$

The debonding stress of composite for various $t_m/d_f$ can be written as follows:

$$\bar{\sigma} = \frac{25984}{\tau^*} \times 23.9 \quad (A-11)$$

where

$\tau^* = \text{computer output of interphase shear stress corresponding to } \bar{\sigma} = 25984 \text{ MPA}$.

Making use of data from Table A-3, we have the comparison of finite element results with the test results as shown in Table A-4.

Table A-4. Comparison of Debonding Stress Between Test and Finite Element Analysis.

<table>
<thead>
<tr>
<th>$t_m/d_f$</th>
<th>$\bar{\sigma}_{test}$ (MPA)</th>
<th>$\tau^*$ (MPA)</th>
<th>$\bar{\sigma}$ (MPA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>773.6</td>
<td>775.0</td>
<td>801.3</td>
</tr>
<tr>
<td>0.35</td>
<td>874.2</td>
<td>710.7</td>
<td>874.2</td>
</tr>
<tr>
<td>1.0</td>
<td>965.0</td>
<td>641.0</td>
<td>969.2</td>
</tr>
</tbody>
</table>
The comparison of out-of-phase displacement for shear lag theory and finite element analysis is shown in Table A-5. Shear lag solution is based on equations (A-9) and (A-10).


<table>
<thead>
<tr>
<th>(t_m/d_h)</th>
<th>(w(\mu m)) (\text{SHEAR})</th>
<th>(w(\mu m)) (\text{LAG})</th>
<th>(w(\mu m)) (\text{F.E.A.})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>5.653</td>
<td>5.62</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>5.84</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>6.004</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>6.075</td>
<td>6.23</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>6.2623</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>6.37</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>6.47</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>6.55</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>6.63</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>6.71</td>
<td>6.86</td>
<td></td>
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</tbody>
</table>
NADC-90001-60

APPENDIX B: CALCULATION OF INTERPHASE SHEAR STRESS CONCENTRATION FACTOR AND MODE I FRACTURE ENERGY OF UNIDIRECTIONAL COMPOSITES

INTERPHASE SHEAR STRESS CONCENTRATION FACTOR, $K_s$

From equation (30) we have:

$$K_s = \frac{1}{\sqrt{2}} \frac{1}{\left( \frac{G_m}{G_i} - 1 \right) \ln \left( 1 + \frac{2t}{d_i} \right) + \ln \left( 1 + \frac{2t_m}{d_i} \right)}$$

Equation (B-1) is used to calculate $K_s$ for $t_m/d_i=0.1$ and $t_m/d_i=1.0$, respectively, as tabulated in Tables B-1 and B-2.

Table B-1. $K_s$ vs. $G_m/G_i$ for Various $t/d_i$ ($t_m/d_i=0.1$).

<table>
<thead>
<tr>
<th>$G_m/G_i$</th>
<th>$t/d_i=0.001$</th>
<th>$t/d_i=0.005$</th>
<th>$t/d_i=0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.66</td>
<td>1.66</td>
<td>1.66</td>
</tr>
<tr>
<td>10</td>
<td>1.58</td>
<td>1.36</td>
<td>1.17</td>
</tr>
<tr>
<td>$10^2$</td>
<td>1.15</td>
<td>0.65</td>
<td>0.48</td>
</tr>
<tr>
<td>$10^3$</td>
<td>0.48</td>
<td>0.22</td>
<td>0.16</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.16</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table B-2. $K_s$ vs. $G_m/G_i$ for Various $t/d_i$ ($t_m/d_i=1.0$).

<table>
<thead>
<tr>
<th>$G_m/G_i$</th>
<th>$t/d_i=0.001$</th>
<th>$t/d_i=0.005$</th>
<th>$t/d_i=0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.675</td>
<td>0.675</td>
<td>0.675</td>
</tr>
<tr>
<td>10</td>
<td>0.669</td>
<td>0.649</td>
<td>0.626</td>
</tr>
<tr>
<td>$10^2$</td>
<td>0.621</td>
<td>0.49</td>
<td>0.404</td>
</tr>
<tr>
<td>$10^3$</td>
<td>0.402</td>
<td>0.21</td>
<td>0.155</td>
</tr>
<tr>
<td>$10^4$</td>
<td>0.15</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>
MODE 1 FRACTURE ENERGY OF UNIDIRECTIONAL COMPOSITE

From equation (29):

\[ \sigma = \frac{|\tau_{\text{max}}|}{K_{1}} \left( \frac{E_{f}}{G_{m}} \right)^{1/2} \]  \hspace{1cm} (B-2)

The debonding stress of fiber/matrix interface can be written as follows:

\[ \sigma = \frac{\tau_{0}}{K_{1}} \left( \frac{E_{f}}{G_{m}} \right)^{1/2} \]  \hspace{1cm} (B-3)

From equation (33):

\[ g = \frac{\sigma_{0}}{\sigma_{h}} \left( \frac{E_{f}}{K_{1}} \right)^{1/2} \]  \hspace{1cm} (B-4)

For graphite/epoxy:

\[ g = \frac{0.027 \left[ \frac{241}{1.21} \right]^{1/2}}{2.5 K_{1}} - \frac{0.1524}{K_{1}} \]  \hspace{1cm} (B-5)

where \( K_{1} \) can be obtained from equation (B-1) and Tables B-1 and B-2.

Table B-3. \( G_{\text{INF}}/C_{\text{INF}} \) vs. \( G_{m}/G_{t} \) for Various \( t/d_i \) (\( t_{m}/d_i = 0.1 \)).

<table>
<thead>
<tr>
<th>( G_{m}/G_{t} )</th>
<th>( t/d_i = 0.001 )</th>
<th>( t/d_i = 0.005 )</th>
<th>( t/d_i = 0.01 )</th>
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</thead>
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<tr>
<td>( g )</td>
<td>( 1 - g + g^2 - g^3 )</td>
<td>( 1 - g + g^2 - g^3 )</td>
<td>( 1 - g + g^2 - g^3 )</td>
</tr>
<tr>
<td>G_{m}/G_{t}</td>
<td>( G_{m}/G_{t} )</td>
<td>( G_{m}/G_{t} )</td>
<td>( G_{m}/G_{t} )</td>
</tr>
<tr>
<td>1</td>
<td>0.092</td>
<td>0.9157</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.0965</td>
<td>0.912</td>
<td>10</td>
</tr>
<tr>
<td>10^2</td>
<td>0.1325</td>
<td>0.883</td>
<td>10^2</td>
</tr>
<tr>
<td>10^3</td>
<td>0.3175</td>
<td>0.751</td>
<td>10^3</td>
</tr>
<tr>
<td>10^4</td>
<td>0.9526</td>
<td>0.09</td>
<td>2130</td>
</tr>
<tr>
<td>10^680</td>
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<td>0.09</td>
<td>1078</td>
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<td></td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.9526</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.09</td>
</tr>
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<td></td>
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<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0</td>
</tr>
</tbody>
</table>
Table B-4. $G_{\text{nf}}/C_{\text{nf}}$ vs. $G_{m}/G_{i}$ for Various $t_{i}/d_{i}$ ($t_{m}/d_{i} = 1.0$).

<table>
<thead>
<tr>
<th>$G_{m}/G_{i}$</th>
<th>$t/d_{i} = 0.001$</th>
<th>$G_{m}/G_{i}$</th>
<th>$t/d_{i} = 0.005$</th>
<th>$G_{m}/G_{i}$</th>
<th>$t/d_{i} = 0.01$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0.2258</td>
<td>0.8137</td>
<td>1</td>
<td>0.2258</td>
<td>0.8137</td>
</tr>
<tr>
<td>10</td>
<td>0.2278</td>
<td>0.8123</td>
<td>10</td>
<td>0.2348</td>
<td>0.8074</td>
</tr>
<tr>
<td>$10^{2}$</td>
<td>0.2454</td>
<td>0.8000</td>
<td>$10^{3}$</td>
<td>0.311</td>
<td>0.756</td>
</tr>
<tr>
<td>$10^{3}$</td>
<td>0.3791</td>
<td>0.7100</td>
<td>$10^{3}$</td>
<td>0.720</td>
<td>0.425</td>
</tr>
<tr>
<td>$10^{4}$</td>
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<td>2043</td>
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