MASS TRANSFER OF DECAYING PRODUCTS IN VORTEX PIPE FLOW WITH GRAVITY EFFECT

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Presented herein is the problem of steady-state diffusion of the decaying products resulting from the disintegration of an inert gas in a forced and diminishing vortex through a vertical cylindrical tube section under symmetrical gravitational influences. The disintegration products of the gas diffuse radically to the walls where they are completely annihilated. The flow properties of the fluid phase are determined by solving the complete Navier-Stokes equations, and the respective solutions for the concentration distributions are obtained. Also presented are the nondimensional bulk concentration values for the respective gravity and Peclet parameterization. These values are of importance in the determination of the diffusion coefficients of the disintegration products as well as the estimation of the losses of particles in connecting tubes (stationary/rotating) which arise from diffusion.

The results of these calculations indicate that increasing the inertial forces over the diffusive forces causes an increase in particle penetration; also, increasing the gravity field at a fixed Peclet number increases the penetration or inhibits wall deposition. Similarly, increasing the gravity field at a fixed Peclet number and increasing the Peclet number for a fixed gravity field diminished the concentration profile.
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1. INTRODUCTION

Formation in flight occurs when a radioactive rare gas (contained within an inert gas such as air), after entering a conduit through a high-efficiency filter, exhibits decay giving rise to a steady production of a certain number of daughter atoms per unit of volume. Unlike the radioactive gas, the daughter atoms adhere to the wall, and hence, are lost by diffusion. This dispersion of atoms may be considered to be an atomic aerosol (Tan and Hsu 1971) since it has the same property as submicron aerosols in that they can be collected at a surface. Formation in flight diffusion equations have been derived for cylindrical tubes (Tan and Hsu 1971; Berezhnoi and Kirichenko 1964) and for flat channels (Berezhnoi and Kirichenko 1964); all cases considered were for slug (uniform-velocity profile) or for Poiseuille pipe flow (parabolic-velocity profile).

The situation of a vortex or swirling flows involving just the fluid phase has been investigated by various authors (Fromm 1963; Pao 1967; Textor 1968; Munson 1980). A well-defined configuration of a confined axially degenerating vortex flow was introduced by Lavan, Nielsen, and Fejer (1969). More recently, Tung and Soo (1973) treated the case of swirling pipe flow of suspensions for the same defined geometry of Laven, Nielsen, and Fejer (1969).

The present study applies the results of the flow system of Laven, Nielsen, and Fejer (1969) and Tung and Soo (1973) (see Figure 1) to the case of a decaying radioactive gas under gravitational effects; the products of disintegration of the gas are filtered out at far upstream \( z = -\infty \), but are again generated by radioactive decay of a flowing inert gas along the entire length of the cylindrical tube. The radioelements diffuse radially to the tube walls where they decay into other radioelements.

The fluid phase is assumed incompressible and fully developed at both far upstream and downstream positions from the junction of the two pipes. A forced vortex is generated by the rotating pipe, and it degenerates or dissipates as the flow passes through the stationary duct due to friction. At very far downstream, the swirl vanishes and the axial velocity reverts back to the fully developed laminar parabolic profile. The solutions of the fluid phase velocities can be used to solve for the particulate distribution governed by the diffusion equation.

A vertical configuration will be assumed such that the axial flow direction is coincident with positive gravity having vertical symmetry.
Figure 1. Coordinate System and Boundary Values.
2. FORMULATION

The motion of the fluid phase is given by solving the continuity and the Navier-Stokes equations for a steady, laminar, incompressible, and axisymmetric swirling flow. The solution is given in terms of the radial, tangential, and axial components of the fluid velocity and pressure (Laven, Nielsen, and Fejer 1969; Tung and Soo 1973) such that,

\[
\begin{align*}
\frac{u}{r} \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (ru)}{\partial r} \right) + \frac{\partial^2 u}{\partial z^2} \right]; \\
\frac{u}{r} \frac{\partial v}{\partial r} + \frac{u v}{r} + w \frac{\partial v}{\partial z} &= \nu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (rv)}{\partial r} \right) + \frac{\partial^2 v}{\partial z^2} \right]; \\
\frac{u}{r} \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left[ \frac{1}{r} \frac{\partial (rw)}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right];
\end{align*}
\]

\[
\frac{\partial}{\partial r} (ru) + \frac{\partial}{\partial z} (rw) = 0.
\]

Equations 1 through 3 represent the momentum equations of the fluid phase in the radial, tangential, and axial directions, respectively. Equation 4 is the continuity equation for the fluid phase. The particle diffusion equation with gravity and generation can be written as:

\[
\frac{u}{r} \frac{\partial c}{\partial r} + w \frac{\partial c}{\partial z} = D_p \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{c}{r} \frac{\partial c}{\partial r} \right) \right] - \frac{\partial}{\partial z} \left( \frac{cg}{F} \right) + \dot{Q},
\]

where \(F\) in the inverse of relaxation time constant (Stokes' law [Soo 1969]).

The boundary conditions may be written as:

\[
at r = 0: u = v = \frac{\partial w}{\partial r} = \frac{\partial c}{\partial r} = 0,\]

(centerline)
at $r = R_o$: $u = w = c = \phi = 0$; $\nu = \begin{cases} R_0 \Omega, & -\infty \leq z \leq 0 \\ 0, & 0 < z \leq +\infty \end{cases}$; 
(at pipe wall)

at $z = -\infty$: $u = c = \phi = 0$; $\nu = r \Omega$; $w = 2\bar{w} \left(1 - \frac{r^2}{R_o^2}\right)$; 
(far upstream)

at $z = +\infty$: $u = v = 0$; $w = 2\bar{w} \left(1 - \frac{r^2}{R_o^2}\right)$; 
(far downstream)

where $\Omega$ is the constant angular velocity of the rotating pipe, $\bar{w}$ is the mean axial flow velocity, and $R_o$ is the radius of the pipe. The flow configuration together with the boundary conditions, is shown in Figure 1. The pipe section at the left of the $r$-axis rotates with constant angular velocity $\Omega$ and generates the swirl, while the pipe section at the right is held stationary. At $z = 0$, the two pipes join smoothly with no effect on the flow.

The results for the fluid phase are expressed in terms of nondimensional variables by defining:

$$R = r/R_o, \quad Z = z/R_o, \quad \Psi^* = \Psi/\Psi_w;$$

$$W = \frac{w}{\bar{w}/2}, \quad U = \frac{u}{\bar{w}/2}, \quad V = v/(R_o \Omega);$$

$$\zeta^* = \zeta R_o^3/\Psi_w, \quad P = p/(\rho R_o^2 \Omega^2);$$

where $\Psi_w$ is the stream function at the wall. Furthermore, the Reynolds number is given by

$$Rey = \frac{4\Psi_w}{\nu R_o} = \frac{2\bar{w} R_o}{\nu}.$$
and the swirl ratio is given by the ratio of the tangential velocity of the rotating pipe to the mean axial velocity, i.e.,

$$S = \frac{\Omega R_o^3}{2\Psi_w} = \frac{R_o \Omega}{w}$$  \hspace{1cm} (12)

The classical numerical approach which converts the velocity components into stream function $\Psi$, vorticity $\zeta$, and circulation $\Gamma$ was chosen because of its well-established stability. Similarity transformation (Kidd and Farris 1969) was not used because it is not always possible to obtain a physically realistic velocity distribution.

Numerical calculations were carried out for various Reynolds number $Rey$ and swirl ratio $S$ combinations; they were chosen to exhibit the various effects of the swirl on the flow field. In some cases, the axial velocity can reach zero or even negative velocity (reversed flow) at some downstream axial positions. Correctness of the numerical method and computer program was checked against the results of Laven, Nielsen, and Fejer (1969) and Tung and Soo (1973). The Reynolds numbers and swirl ratios were chosen to achieve rapid convergence and consistency with Laven, Nielsen, and Fejer (1969) and Tung and Soo (1973).

The diffusion equation, Equation 5, was also cast into nondimensional form by defining

$$C^* = D_p c/ (\Omega R_o^2)$$  \hspace{1cm} (13)

along with the aid of the diffusive Peclet number, $P_e = R_o \bar{w}/D_p$, and the gravity flow parameter, $\sigma = gR_o/(FD_p)$ such that,

$$\left(\frac{P_e}{2}\right) \left( U \frac{\partial C^*}{\partial R} + X' W \frac{\partial C^*}{\partial X} \right) = \frac{1}{R} \frac{\partial C^*}{\partial R} + \frac{\partial^2 C^*}{\partial R^2} - \sigma X' \frac{\partial C^*}{\partial X} + 1, \hspace{1cm} (14)$$
where the mapping function, given by Laven, Nielsen, and Fejer (1969), was used to transform the z-axis into the x-axis. Also, $F$ is the inverse of relaxation time constant defined as,

$$ F = \frac{9 \bar{\mu}}{2 a^2 \bar{\varepsilon}}, $$  \hspace{1cm} (15)

where $\bar{\mu}$ and $\bar{\varepsilon}$ are the viscosity of the fluid and material density of the particle phase, and $a$ is the radius of an atomic particle.

Since it was assumed that the fluid phase is not affected by the presence of the solid atomic particles (Soo 1969), the fluid velocities obtained from the fluid phase relations can be directly substituted into the particulate diffusion equation. After the particulate concentrations were found at each respective axial and radial position throughout the flow field, the bulk concentration was determined to estimate the deposition from far upstream to downstream axial positions.

Defining the $F(X)$ as the ratio of the total particle flux over a cross-section at distance $X$ from the far upstream position (at $z = -\infty$) to the rate of formation of the radioelements in the same element of the tube, we have

$$ F(X) = \frac{1}{X} \int_{0}^{1} W C^* R dR. $$  \hspace{1cm} (16)

The relations for the fluid phase were cast into explicit finite difference molecules (Southwell 1940; Ames 1972) and solved by numerical relaxation yielding results consistent with Laven, Nielsen, and Fejer (1969) and Tung and Soo (1973). As a result of the axial symmetry of the flow field, only the flow in the rectangular region,

$$ D = \{(R,X) \mid 0 \leq R \leq 1 \text{ and } 0 \leq X \leq 1\}, $$  \hspace{1cm} (17)

need be considered. On $D$, a network of parallel mesh points $(41 \times 41)$ was uniformly spaced by an amount of $\Delta$ in both $R$ and $X$ directions. With the initial assumed values, $\Gamma^*$, $\zeta^*$, and $\Psi^*$ are solved iteratively by a procedure consisting of making sweeps of the interior mesh points until a convergence criteria given by Forsythe and Wasow (1967) is satisfied.
Since Equation 14 is parabolic, and due to the universally stable implicit method (Hornbeck 1975), the particulate diffusion equation was cast into the implicit finite difference discretization form as,

\[
\beta_1 \cdot C_{i+1,j-1} + \beta_2 \cdot C_{i+1,j} + \beta_3 \cdot C_{i+1,j+1} = \alpha_{i,j},
\]

where

\[
\beta_1 = -\left(\frac{P_e}{2}\right) \frac{U_{i+1,j}}{2\Delta R} + \frac{1}{2R_j\Delta R} - \frac{1}{(\Delta R)^2};
\]

\[
\beta_2 = \left(\frac{P_e}{2}\right) \frac{X'_{i+1,j} W'_{i+1,j}}{\Delta X} + \frac{2}{(\Delta R)^2} + \frac{X'_{i+1} \sigma}{\Delta X};
\]

\[
\beta_3 = \left(\frac{P_e}{2}\right) \frac{U_{i+1}}{2\Delta R} - \frac{1.0}{2R_j(\Delta R)} - \frac{1}{(\Delta R)^2};
\]

\[
\alpha_{i,j} = \left(\frac{P_e}{2}\right) \frac{X'_{i+1,j} W'_{i+1,j} C_{i+1,j}}{\Delta X} + \frac{X'_{i+1} \sigma C_{i+1,j}}{\Delta X} + 1.
\]

Convergence was satisfied for all Peclet and gravity parameters considered in Equation 18 with respect to the given mesh sizes considered in the analysis. The sparse and tri-diagonal system resulting from the finite difference expressions for the diffusion equation was solved by Gaussian elimination with full and partial pivoting aided by the Crout reduction technique.

3. RESULTS AND DISCUSSION

The results of the fluid phase are virtually the same as Laven, Nielsen, and Fejer (1969) and Tung and Soo (1973), the only difference being with respect to precision which was within 0.01% for most of the comparable calculations.
Figure 2 depicts the axial variation (from \( z = -\infty \) equivalent to \( x = 0 \) to \( 1 \)) of the bulk concentration of the disintegration product formation of particles in varying gravity fields at low and moderate Peclet numbers. At the same gravitational influence, increasing the inertial forces over the diffusive forces causes an increase in penetration; also, increasing the gravity field at a fixed Peclet number increases the penetration or inhibits wall deposition.

As seen in Figure 3, increasing the gravity field at a fixed inertial to diffusive force ratio caused a smaller concentration profile; whereas, at a fixed gravity field, increasing the Peclet number diminishes the axial distribution of the concentration profile.

The effects of varying the Reynolds number and swirl ratio at a given gravity flow and Peclet parameter (viz., \( \text{Re}_e \{1, 4, 10, 20\} \), Swirl Number \( e\{12, 7, 8\} \), respectively) did not significantly alter the deposition, perhaps due to the restrictive range of the Reynolds and Swirl parameters. However, because of the need to be consistent with Laven, Nielsen, and Fejer (1969) and Tung Soo (1973), and for the desire to limit computer time, the above limits were set for this investigation.

The bulk concentration of particles,

\[
\bar{c}(z) = 2\pi \frac{\int_0^{\infty} w rdr}{w \pi R_o^2}
\]

is related to \( F(X) \) by:

\[
\bar{c}(z) = XF(X) \frac{Q R_o^2}{D_p} = XF(X) \frac{Q R_o}{w} P_e.
\]

This shows clearly that decreasing the particle diffusion coefficient increases the particle bulk concentration at each point in the tube if all other parameters are unchanged as exemplified in Figures 2 and 3; increasing the gravity parameter has a similar effect.
Figure 2. Axial Distribution of the Nondimensional Bulk Concentration at Different Fields (Rey = 20 and S = 8).
Figure 3. Axial Distribution of the Centerline Concentration at Different Gravity Fields (Re = 20 and S = 8).
4. REFERENCES


LIST OF SYMBOLS

\( a \) - Radius of particle
\( c \) - Particle concentration
\( C^* \) - Dimensionless particle concentration
\( \bar{c} \) - Material density of particle phase
\( D_p \) - Particle diffusivity
\( F \) - Inverse of relaxation time
\( g \) - Local acceleration of gravity
\( P \) - Dimensionless pressure
\( p \) - Fluid static pressure
\( P_e \) - Diffusive Peclet number
\( \dot{Q} \) - Rate of particle generation per unit of volume
\( r \) - Radial coordinate
\( R \) - Dimensionless radial coordinate, \( r/R_o \)
\( R_o \) - Pipe radius
\( Re \) - Reynolds number
\( S \) - Swirl ratio
\( u \) - Radial velocity
\( U \) - Dimensionless radial velocity, \( u \sqrt{w/2} \)
\( v \) - Tangential velocity
\( V \) - Dimensionless tangential velocity
\( w \) - Axial velocity
\( W \) - Dimensionless axial velocity
\( \bar{w} \) - Mean axial velocity
\( X \) - Transformation variable \([0,1]\)
\( X' \) - First derivative of \( X \) with respect to \( Z \)
\( z \) - Axial coordinate
\( Z \) - Dimensionless axial coordinate
\( \alpha \) - Electrostatic charge parameter
\( \beta_1, \beta_2, \beta_3 \) - Finite difference coefficients
\( \mu \) - Viscosity of the fluid
\( v \) - Fluid kinematic viscosity
\( \Delta R \) | Radial change
\( \Delta X \) | Axial change
\( \Gamma \) | Circulation
\( \zeta \) | Vorticity
\( \sigma \) | Gravity flow parameter
\( \Psi \) | Stream function
\( \Gamma^*, \zeta^*, \Psi^* \) | Dimensionless circulation, vorticity, and stream functions, respectively
\( \Omega \) | Constant angular velocity
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