# Final Report

**Theoretical and Numerical Study of Anomalous Turbulent Transport in Plasmas (U)**

**Osamu Ishihara, Principal Investigator**

This final technical report describes research work performed under AFOSR contract 86-0156 during the period of June 15, 1986 to December 14, 1990. Theoretical and computational analysis of particle transport in a plasma turbulence was carried out. Particle behavior was investigated by test particle simulations in different types of plasma turbulence, including drift wave turbulence, ion acoustic turbulence and Langmuir turbulence. Resonance broadening in real space around mode rational surface was confirmed in drift wave turbulence. Nonresonant interaction of waves and particles were studied in ion acoustic turbulence. Resonance broadening in velocity space was found as a result of non-Markovian process in Langmuir turbulence. A semiclassical quantum plasma approach was also applied to plasma turbulence. Some instabilities in toroidal geometry were studied as eigen value problems. Undergraduate research assistantship program associated with this contract is also described.

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FINAL TECHNICAL REPORT

on

Contract AFOSR 86-0156

THEORETICAL AND NUMERICAL STUDY
OF ANOMALOUS TURBULENT TRANSPORT IN PLASMAS

June 15, 1986 to December 14, 1990

Osamu Ishihara

Principal Investigator
Professor
Department of Electrical Engineering
Texas Tech University
FINAL TECHNICAL REPORT

on

Contract AFOSR 86-0156

THEORETICAL AND NUMERICAL STUDY
OF ANOMALOUS TURBULENT TRANSPORT IN PLASMAS

for the period
June 15, 1986 to December 14, 1990

Submitted to

The Air Force Office of Scientific Research

by

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for

Dr. Robert J. Barker
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Air Force Office of Scientific Research
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February 5, 1991

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Summary</td>
<td>2</td>
</tr>
<tr>
<td>II. Research Accomplishments</td>
<td>3</td>
</tr>
<tr>
<td>III. Undergraduate Fellowship Program</td>
<td>5</td>
</tr>
<tr>
<td>IV. Listing of Conference Presentations</td>
<td>6</td>
</tr>
<tr>
<td>V. Listing of Publications</td>
<td>8</td>
</tr>
<tr>
<td>VI. Listing of Theses and Dissertations</td>
<td>9</td>
</tr>
<tr>
<td>VII. Staffs</td>
<td>10</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>11</td>
</tr>
</tbody>
</table>

Conference Abstracts Supported by Grant AFOSR 86-0156
Publication Abstracts Supported by Grant AFOSR 86-0156
I. SUMMARY

This final technical report describes research work "Theoretical and Numerical Study of Anomalous Turbulent Transport in Plasmas" performed under AFOSR contract 86-0156 during the period of June 15, 1986 to December 14, 1990.

Theoretical and computational analysis of particle transport in a plasma turbulence was carried out. Particle behavior was investigated by test particle simulations in plasma turbulences, including drift wave turbulence, ion acoustic turbulence and Langmuir turbulence. A semiclassical approach to analyze plasma turbulence was taken to understand particle transport.

The report includes a brief description of research accomplishments under the support of this grant in Sec. II. Section III describes the undergraduate fellowship program associated with this grant. Section IV lists conference presentations, while Sec. V lists papers published in professional journals. Section VI lists the titles of master theses and PhD dissertations supported by this grant. The report concludes in Sec. VII by listing staffs involved in this contract.
II. RESEARCH ACCOMPLISHMENTS

Drift wave turbulence ---- Anomalous transport of electrons in the low frequency electrostatic turbulence (drift wave turbulence) has been studied extensively. Electrons were known to diffuse across a magnetic field by $E \times B$ drift, where $E$ is a turbulent electric field fluctuation and $B$ is a background magnetic field. We consider the transport of electrons in the presence of shear structure in the magnetic field. We have formulated the transport of electrons in a moderately strong turbulence based on the drift kinetic equation. Electrons are in resonance with waves at a mode rational surface, which is determined by the rational numbers associated with Fourier modes of drift waves. We have shown that the resonance region broadens as the turbulence becomes stronger and that the broadening develops with time. Our new formulation of resonance broadening explains our observation of particle transport in the test particle numerical experiment. The resulting transport coefficient can describe nonlinear aspect of wave-particle interaction in the drift wave turbulence. This principal investigator was invited to give a talk on this subject at the 1989 IEEE International Conference on Plasma Science at Buffalo, New York in May 1989. The full paper is published in Physics of Fluids (1990).

Langmuir Turbulence --- Time dependence of velocity-diffusion coefficient of plasma particles in Langmuir turbulence has been studied based on the generalized Langevin equation and the derived Fokker-Planck-type equation. The time dependent nature of the diffusion coefficient is caused by the retarded effect of the turbulent collisions, which is responsible for the non-Markovian stochastic process in a Gaussian (or non-Gaussian) system. The projection operator method has been used to derive the evolution equation known as the generalized Langevin equation for the particle velocity in an electrostatic plasma turbulence. Upon completion of hundreds of realizations in a test particle simulation, we obtained numerical results in agreement with analytical results. Our results were reported by a series of papers at IEEE International Conference on Plasma Science and at the Annual Meeting of Division of Plasma Physics of the American Physical Society.

Ion Acoustic Turbulence --- Nonresonant interaction of plasma particles and plasma waves has been studied in the presence of ion acoustic turbulence. We have confirmed by test particle computer experiments that particles away from the resonance layer could indeed diffuse as was predicted by our analytical work. The results were reported in the Annual Meeting of Division of Plasma Physics of the APS (1987).
Relativistic particle acceleration --- Relativistic particle acceleration by plasma turbulence has been studied in a quasilinear regime. Relativistic charged particles can be accelerated by taking energy from plasma turbulence. We have formulated the problem classically and quantum mechanically. The theory describes the interaction between charged particles and plasmons. The self-field created by the particle itself is also considered. Our equation reduces to the Balescu-Lenard equation in the classical limit. This result was presented at the APS Topical Conference on Plasma Astrophysics at Santa Fe in September, 1988.

Turbulence in toroidal geometry --- Analytical study of turbulent fluctuations in toroidal geometry has been carried out in collaboration with Professor A. Hirose of University of Saskatchewan, Canada. In a collisionless plasma we have generalized the ideal MHD mode equation to describe drift-Alfven mode and examined numerically the stability problem (published in Nuclear Fusion (1987)). In a collisional plasma, the resistivity could be responsible for the instability. We have examined the resistive MHD balloning mode by solving the second order mode equation (published in the Canadian Journal of Physics (1988) and Nuclear Fusion (1989)). The mode equation is extended to incorporate the plasma inhomogeneity like ion-temperature-gradient. The numerical study of the ion-temperature-gradient instability is published in Nuclear Fusion (1987).

Semiclassical approach to plasma turbulence --- We have applied the method of quantum electrodynamics to the analysis of plasma turbulence. The wave-particle interaction is viewed as a scattering process between plasma particles and quasiparticles(plasmons, phonons). The particle transport in plasma turbulence is then closely related to the transition probability of absorption and emission processes. Successful application of the method resulted in clarification of the nonresonant wave-particle interaction in plasma turbulence (published in Physical Review A (1987)). On the other hand, such a viewpoint is applied to the interaction between photons and plasmons, and a novel scheme of photon acceleration (or frequency up-shifting of electromagnetic wave) is proposed (accepted for publication in Physical Review A).
The undergraduate research fellowship program was initiated by Dr. Robert Barker of AFOSR and our plasma laboratory was involved in the program during the period of the contract. The selected undergraduate students were involved in the related research. They participated in a weekly plasma seminar with graduate students and presented their assigned topics at the seminar. The names of undergraduate students and the titles of their final reports are listed here.

From May 16 to August 31, 1987:

1. Theodore Christopher Grabowski
   "Anomalous Particle Transport in Drift Wave Turbulence in a Plasma"
2. Christopher James Koath
   "Study of Diffusion and Friction in an Electrostatic Turbulent Plasma"
3. David Edward Stahlke
   "Ion Acoustic Plasma Turbulence and Data Processing"

From June 15, 1988 to May 31, 1989:

1. Hok Tung Wong
   "Application of MAGIC Code to the Study of Plasma Turbulence"
2. David Stahlke
   "A Computational Study of Various Two-Stream Instabilities"
3. Mark Thompson
   "A Numerical Experiment of Relativistic Particle Diffusion in a Plasma Turbulence"
Conference presentations of the research results obtained under the contract AFOSR 86-0156 are listed here.


V. LISTING OF PUBLICATIONS

Publications of the research results obtained under the contract AFOSR 86-0156 are listed here.

VI. LISTING OF THESES AND DISSERTATIONS

Master theses and doctoral dissertations supported by the contract AFOSR 86-0156 are listed here.

Maser Theses


PhD Dissertations

1. Wai H. Ho, "Wave-Particle Interaction in Plasma Turbulence" (final version is in preparation).

VI. STAFFS

Faculty Investigator:

Professor Osamu Ishihara of Department of Electrical Engineering and Department of Physics at Texas Tech University is a principal investigator.

Graduate Research Assistants:

During the period of the contract, two doctoral students and two master students were supported.

Undergraduate Research Assistants:

Five undergraduate students were supported under the program of undergraduate research fellowship.

Accountants:

Financial record was kept by Ms. Sandra Branch of the Department of Electrical Engineering. She was partially supported by this contract.
APPENDIX
**TP 13**

**Nonresonant Wave-Particle Interaction from the Quantum Mechanical Viewpoint.** Osamu Ishihara, Texas Tech University.

The interaction of particles and quasiparticles (plasmons, phonons, etc.) is described by a scattering process which is characterized by a transition probability. A proper treatment of the time-dependent interaction Hamiltonian reveals that both the resonant and nonresonant nature of particle-quasiparticle interaction contributes to transition probability. The nonresonant transition probability known as Fermi’s golden rule, is now supplemented by a nonresonant part, which guarantees the conservation of energy and momentum in the system as is predicted by the classical quasilinear theory.

* Work supported by AFOSR under grant AFOSR-86-0156.


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**FT 26**

**Numerical Study of Test Particle Transport in Drift Wave Turbulence.** E. Robinson and O. Ishihara, Texas Tech University.

In drift wave turbulence where the magnetic field is expressed as $B = B_0(x/L_0, t)$, a real space diffusion coefficient has been formulated and is found to have similar characteristics as the time dependent diffusion coefficient derived for velocity space in Langmuir turbulence.

A numerical experiment reveals the particle diffusion in real space in the $x$ direction. Test particles are followed in their trajectories in the prescribed random electrostatic fields, $f(x,t) = \sum \phi_{nm}(t) \exp(ik_n x + k_m z - \omega_m t)$, where $\phi_{nm}$ is the electric potential of the $(m,n)$ mode. A preliminary result of the observed diffusion coefficient demonstrates the time dependent structure of the diffusion coefficient as predicted by the analytical study.

* Work supported by AFOSR under grant AFOSR-86-0156.

2X5
Real Space Diffusion in Drift Wave Turbulence

E. Robinson and O. Ishihara
Department of Electrical Engineering
Texas Tech University, Lubbock, Texas 79409

In drift wave turbulence characterized by a sheared magnetic
field \( \mathbf{B} = B_y(\theta_z + x/L_y \theta_y) \), where \( L_y \) is the shear
length, and random electrostatic fields, \( \Phi(x,t) = \sum_{mn} \phi_{mn} \)
\( x e^{ik_m y + k_n z - \omega_{mn} t} \), where \( \phi_{mn} \) is the electric potential
of the \( (m,n) \) mode, a numerical experiment has been carried
out to examine the particle diffusion process in real space. Two
cases are examined, one with an infinite number of \( n \)-modes
and the other with a single \( n \)-mode. 100 \( n \)-modes are used for
both cases. For infinite \( n \)-modes, diffusion coefficients found
are in agreement with the value \( D = \pi \sum_{m} \langle |m \phi_m|^2 \rangle \) as was
predicted\(^1\) for a low turbulence level, while for a single
\( n \)-mode we observe that diffusion greatly depends on the choice
of \( m \) and \( n \).

An analytical expression for the space diffusion in the
\( x \)-direction is given by

\[
D(X,t) = \frac{1}{2} \frac{d}{dt} \sum_{mn} \langle |m \phi_{mn}|^2 \rangle \int_0^T \int_{S-T}^T ds dt \ e^{imX + n t} \ e^{\frac{-1}{2} m^2 \omega^2 (3s - 2t)} x e \ , \quad (1)
\]

where \( X \) is the initial position of a test particle, exhibits
analogous characteristics to the quasilinear velocity space
diffusion coefficient.\(^2\) For low turbulence levels, Eq. (1)
shows quasilinear-like behavior,

\[
D(X) = \pi \sum_{m} \langle |m \phi_{m}|^2 \rangle \delta(mX + n) . \quad (2)
\]

at the resonant locations, \( X = -(m/n) \). From further
examination of the single \( n \)-mode at a rather high turbulence
level, we have observed deviation of the diffusion coefficient
from that of Eq. (2), i.e. high turbulence levels cause diffusion
in the space surrounding the resonant locations as is predicted
by Eq. (1). A comparative study between the real space
diffusion in drift wave turbulence and the velocity space
diffusion in Langmuir turbulence, \( \omega = \omega_{pe} \) at a rather strong
turbulence level demonstrates the time dependent nature of the
diffusion coefficient as was described by Eq. (1).

\(^*\) Work supported by AFOSR under grant AFOSR-66-0156.
Analytical and Numerical Study of Anomalous Friction in a Plasma Turbulence.

Wai Ho and Osamu Ishihara, Texas Tech University, Lubbock, TX 79409-4439. - - - The anomalous dragging effect on plasma particles caused by the electrostatic plasma turbulence in the absence of magnetic field is studied numerically and analytically. It has been known that the frictional effect together with the diffusion effect appear in the presence of stochastic electric fields. In the turbulent electric fields, the distortion of particle trajectories can be given by

\[ \Delta \mathbf{v}(t) = \frac{\delta \mathbf{F}(t)}{m} \Delta t, \]

where \( \delta \mathbf{F}(t) = \frac{1}{2} \int_0^t \mathbf{E}(t', t') \, dt' \).

We thus define the friction coefficient as \( \mathbf{F}(\mathbf{v}, 0) = \delta \mathbf{F}(t)/\delta \mathbf{v} \). It is straightforward to show that the friction coefficient is given by

\[ \mathbf{F}(\mathbf{v}, 0) = \frac{3 \chi_3}{2} \int_0^1 dv \int_0^1 ds \int_0^d x \exp \left( \frac{1}{3} \frac{\chi_3^2}{\delta v} \right) < E(\mathbf{v}, x) E(0, 0) > P(\mathbf{v}, \mathbf{v}, x) \]

(1)

where \( \chi_3(\mathbf{v}, x) = \frac{1}{3} \int_0^1 dv \int_0^1 ds \int_0^d x \exp \left( \frac{1}{3} \frac{\chi_3^2}{\delta v} \right) < E(\mathbf{v}, x) E(0, 0) > P(\mathbf{v}, \mathbf{v}, x) \)

and \( P(\mathbf{v}, \mathbf{v}, x) = \frac{1}{2} \int_0^1 dv \int_0^1 ds \int_0^d x \exp \left( \frac{1}{3} \frac{\chi_3^2}{\delta v} \right) < E(\mathbf{v}, x) E(0, 0) > P(\mathbf{v}, \mathbf{v}, x) \).

Alternatively, we can write \( \mathbf{F}(\mathbf{v}, 0) = \frac{3 \chi_3}{2} \int_0^1 dv \int_0^1 ds \int_0^d x \exp \left( \frac{1}{3} \frac{\chi_3^2}{\delta v} \right) < E(\mathbf{v}, x) E(0, 0) > P(\mathbf{v}, \mathbf{v}, x) \)

(2)

where \( P(\mathbf{v}, \mathbf{v}, x) = \frac{1}{2} \int_0^1 dv \int_0^1 ds \int_0^d x \exp \left( \frac{1}{3} \frac{\chi_3^2}{\delta v} \right) < E(\mathbf{v}, x) E(0, 0) > P(\mathbf{v}, \mathbf{v}, x) \).

A numerical experiment has been carried out, in which one dimensional random electric fields are modeled for Langmuir turbulence and are described by

\[ <E(\mathbf{v}, x)E(0, 0)> = -\frac{3}{2} \left( \frac{\omega P}{q_0} \right)^2 \frac{d_x^2}{\delta v} \left[ \exp \left( \frac{1}{2} \frac{\chi_3^2}{\delta v} \right) \cos (q_0 x - \omega t) \right] \]

where \( \omega P \) is the plasma frequency, \( k_0 \) is the center wavenumber and \( \chi \) is the normalized rms value of the turbulent potential. In the limiting case of weak turbulence (\( \chi \approx 0.01 \)), we have observed a well-defined friction coefficient as given by the limit \( \delta v \to 0 \) of Eq. (1);

\[ \mathbf{F}(\mathbf{v}) = \frac{3 \chi_3}{2} \int_0^1 dv \int_0^1 ds \int_0^d x \exp \left( \frac{1}{3} \frac{\chi_3^2}{\delta v} \right) < E(\mathbf{v}, x) E(0, 0) > P(\mathbf{v}, \mathbf{v}, x) \]

(3)

In the moderate turbulence level (\( \chi = 0.01 \)), the friction coefficient is observed to deviate from the value given by Eq. (3), and is observed to show some broadening effect as represented by \( \Delta \mathbf{x} \) term in Eq. (2).

The formulation of the friction coefficient is also studied by a renormalized method with Green's function based on the Vlasov equation. With the help of diagrammatical techniques, we formulate the turbulence friction by taking secular terms in the Green's function which agrees with Eq. (1) in some limiting situations.

* Work supported by AFOSR under grant AFOSR-86-0156.

3R 15  Resonant Modes in Tokamaks*  A. HIRAO,
T. L. KROMER, UNIV. OF SASK., O. ISHIHARA, TEXAS TECH
UNIV. --The resonant g and rippling modes in tokamak
geometry have been reexamined in terms of the generalized
moments free from any ordering among frequencies, ,
, and (the magnetic
drift frequency in the ballooning space). Such general-
ization is necessary for resonant modes because of
strong radial localization about a rational surface and
corresponding large mode width in q-space, .
For typical tokamak edge parameters, a numeri-
cal search for unstable, bounded eigenfunctions of
resonant modes has failed. Only the stable (stabilized
by shear and toroidicity) drift mode (predicted analyti-
cally) has been found.

*Supported by NERSC (Canada) and USAF-OSR

6T 11
Study of Velocity Diffusion of Test Particles in Ion Acoustic
Turbulence.  K.M. Yuen and O. Ishihara, Texas Tech Univ.,
and A. Hirose, UNIV. OF SASKATCHEWAN.--In order to study
velocity diffusion of ions in ion acoustic turbulence, we set
the test particles in prescribed turbulent electric fields. The
numerical experiments are carried out with particles placed
randomly in the system, or alternatively we followed the
test particles with the same initial position for different
realization of electric fields. The ion acoustic turbulence is
modeled by the dispersion relation which is characterized by
either nondispersive or dispersive nature. The results of the
numerical experiments indicate that in both nondispersive
and dispersive cases, ions which do not satisfy the resonant
condition do diffuse in velocity space. The analytical study
also shows a possibility of diffusion of nonresonant particles
if the turbulent spectrum has a finite correlation time.
The numerical results show fair agreement with theoretical
predictions provided that the initial velocities are set within
critical values, which are located well above and below the
phase velocity of the waves. It has also been observed that
there is no stochastic heating of particles if their ini-
tial velocities are set outside the critical range.
*Supported by AFOSR under Grant AFOSR-86-0156A.
Quasilinear Effects on Turbulent Bremsstrahlung. O. Ishihara, Texas Tech Univ., and A. Hirose, Univ. of Saskatchewan—The upconversion of low frequency ion acoustic turbulence into high frequency Langmuir waves is re-examined in the context of weak turbulence theory. Contrary to the earlier observations\textsuperscript{1,2} that there exists turbulent bremsstrahlung of Langmuir waves with a background of ion-acoustic turbulence, it is shown that quasilinear effects on the electron distribution function cause stabilization of Langmuir waves. The quasilinear effects are found to cancel the destabilizing terms originated from the direct turbulent corrections to the linear response function. Together with the earlier findings\textsuperscript{2–3} that polarization effects do not cause destabilization of Langmuir waves, it is now concluded that there should be no upconversion of ion acoustic turbulence into Langmuir turbulence, which agrees with the conclusion derived by the symmetry argument.\textsuperscript{4}

\textsuperscript{5}Supported by AFOSR under grant AFOSR-86-0156A (O.I.) and NSERC, Canada (A.H.).

PROGRAM OF THE JOINT SPRING MEETING
of the
Texas Section of the American Physical Society
Texas Section of the American Association of Physics Teachers
Society of Physics Students, Zone 10
The University of Texas
Austin, Texas
March 4 & 5, 1988

9:12
AD2
Stochastic Heating of Plasma Particles in Ion Acoustic Turbulence.* K.H. Yuen and O. Ishihara, Texas Tech Univ. Stochastic process of plasma ions placed in the background ion acoustic turbulence is studied analytically and numerically. The dispersion relation of the ion acoustic wave is given by $\omega - k_v C_i$, where $C_i$ is the ion acoustic velocity. The random nature of the turbulence is characterized by the correlation function of electric fields. We model the turbulence by discrete Fourier modes and the envelope of its correlation function forms a Gaussian spectrum. By controlling the width of the spectrum, the model is applicable to a single mode as a limit and to the wide-band turbulence. It is shown that when the spectrum width becomes wider the heating rate of plasma particles becomes larger. Stochastic heating is found to take place for ions in resonance and also not in resonance with ion acoustic waves.

*Supported by AFOSR under grant AFOSR-86-0156A.

11:12
AD12
Plasma Particle Diffusion in Drift Wave Turbulence. C. Grabowski and O. Ishihara, Texas Tech U.* Motions of plasma particles have been studied numerically and analytically when the particles are subjected to both a background magnetic field and electric fields created by drift wave turbulence. For these studies, test particles are placed in a field model in slab geometry. Both the random electric fields and the background magnetic field, with increasing shear in the x direction, are set to be confined in the y-z plane. Numerical and analytical results indicate that particle motions increase along the x axis as the magnitude of the electric fields increases and as the density of the mode rational surfaces increases along the x axis, where the mode rational surface is defined by $k = \pm n/m$ with $n$ and $m$ being mode numbers of the electric fields.

*Supported by AFOSR under grant AFOSR-86-0156A.
Stochastic Diffusion in the Absence of Resonances

O. Ishihara, V. Ho and K.M. Yuen
Department of Electrical Engineering
Texas Tech University, Lubbock, Texas
and
A. Hirose
Department of Physics
University of Saskatchewan, Saskatoon, Canada

The nonresonant nature of charged particle motion in the electrostatic plasma turbulence is studied numerically and analytically by artificially suppressing the resonant conditions of wave-particle interaction. Nonresonant structure in the phase space is realized in the test particle numerical experiments by two different methods. One approach is to make use of discreteness of the spectrum of plasma turbulence. Test particles placed in velocity space may not reach any of two adjacent modes in the extremely weak turbulence. This situation is modeled for electrons placed in the Langmuir turbulence. Another approach is to place test particles completely out of the resonant region in velocity space. This situation is modeled for ions placed in the ion acoustic turbulence.

Numerical experiments are carried out for test particles placed in prescribed random electric fields, whose second order correlation functions are characterized by sinusoidal structure with the exponential damping in real space, described by the summation of Fourier modes as

$$E(x,t) = \sum_j f(k_j) \cos(\omega(k_j)x_j + \delta_j).$$

where $f(k_j)$ is a shaping factor with Gaussian form, $\delta_j$ is a random phase, and $\omega(k_j) = 1$ for Langmuir waves and $\omega(k_j) = k_j/(k^2 + k_j^2)^{1/2}$ for ion acoustic waves. Test particles, ranging from 20 to 2000 in number, are followed in their trajectories in the discrete fluctuation spectrum with mode numbers 20 to 400. Diffusion coefficients are measured in velocity space by taking mean square variation as a function of time and compared with the analytic predictions evaluated by

$$D(v) = \left(\frac{\sigma}{m}\right)^2 \int_0^\infty \int \langle 6(x,v,t)E(x,v',t')E(x+x',v'+v,t+t') \rangle dx dv dt,$$

where $e/m$ is a charge/mass ratio and set to be 1.

Stochastic diffusion observed numerically both in Langmuir turbulence and in ion acoustic turbulence is in good agreement with the theory in the scale of Kolmogorov time defined by $t_K = (k^2 D)^{-1/3}$. In the Langmuir turbulence, diffusion is observed even well below the critical turbulence level given by the criterion for the resonance overlapping. Our observation suggests the possible stochastic acceleration of charged particles in the nonresonant nature in the plasma turbulence.


- Work supported by AFOSR under Grant AFOSR-86-0156A (O.I., K.Y.) and by NSERC of Canada (A.H.).
RESISTIVE MHD BALLOONING MODE IN TOKAMAKS

A. Hirose
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Saskatoon, Saskatchewan, S7N 0W0, Canada

O. Ishihara
Dept. of Elect. Eng., Texas Tech Univ.
Lubbock, TX 79409

A mode equation for the resistive MHD ballooning mode in low β tokamak magnetic geometry has been derived and analyzed both analytically and numerically. Any resistive modes in a high temperature tokamak require strong radial localization about a mode rational surface in order to satisfy ω_e ≈ k_θ(r)ω_0. Here, ω_e is the electron-ion collision frequency and k_θ(r) = k_θ(r)/r_0 is the parallel gradient with r being the radial distance from the rational surface, and L_θ the shear length. Strong radial localization makes the magnetic drift frequency operator

$$u_0 = 2π e_z \left[ \cos \theta - \frac{\text{Im} \left( \hat{a} \right)}{k_\theta} \right]$$

secular, and the conventional ordering

$$u_c \approx u_0$$

totally breaks down. The formulation in the present study is free from any ordering among the frequencies

$$u_c, u_e, u_0$$

and thus offers a generalization of the conventional mode equation for arbitrarily strong radial localization. The mode equation, based on electrostatic ion response and fully electromagnetic electron response, is given in Fourier space by

$$\frac{d}{dk} \left[ \frac{d}{d\kappa} \frac{e^{-\kappa}}{A}\frac{B}{C} \right] G(k) = 0$$

where

$$A = \omega - \omega_{ce} + \omega_0 e^{-\kappa} \left( \frac{\kappa c}{\omega_{pe}} \right)^2 \kappa e^{-\kappa^2}$$
$$B = \omega - \omega_{ce} + \omega_0 e^{-\kappa} \left( \frac{\kappa c}{\omega_{pe}} \right)^2 \kappa e^{-\kappa^2} \left( 1 - \Gamma \right)$$
$$C = \left[ \omega_{ce} - \omega_{ce} e^{-\kappa} \left( \frac{\kappa c}{\omega_{pe}} \right)^2 \kappa e^{-\kappa^2} \right] \left[ \omega_{ce} - \omega_{ce} e^{-\kappa} \left( \frac{\kappa c}{\omega_{pe}} \right)^2 \kappa e^{-\kappa^2} \right] \left( 1 - \Gamma \right)$$
$$\Gamma = -\frac{\omega_{ce}}{\omega_{ce}} e^{-\kappa} \left( \frac{\kappa c}{\omega_{pe}} \right)^2 \left( \kappa e^{-\kappa^2} \right) \left( 1 - \Gamma \right)$$

No unstable modes have been predicted analytically and found numerically for the mode equation. The only bounded solution analytically predicted in the stable ion diamagnetic mode. The damping factor is proportional to the magnetic shear and \( \omega_e \), the toroidicity. This has been fully confirmed by numerical integration with parameters relevant to present-day tokamaks.

*Supported by NSERC, Canada (A.I.) and AFOSR (O.I.)
Relativistic Quasilinear Particle Acceleration in Plasma Turbulence.

O. Ishihara, Texas Tech U.

Relativistic quasilinear kinetic theory is developed to describe particle acceleration in the presence of plasma turbulence. Scattering between charged particles and plasmons is considered. It is assumed that each time a particle makes a transition the energy it gains (or loses) comes from (or goes into) the energy of turbulent fluctuations. The theory includes nonresonant interactions between charged particles and plasmons since plasmons are known to carry not only electric field energy but also oscillating energy of particles. The formulation takes into account the self fields created by the particle itself, which can be incorporated as the effect of renormalization of the particle mass. The derived equation reduces to the Balescu-Lenard equation in the classical limit. A possible mechanism of cosmic ray acceleration is discussed based on this theory.

*Supported by US AFOSR Grant AFOSR-86-0156.

Saturation of Nonlinear Drift Mode Induced by Toroidicity.* O. ISHIHARA, Texas Tech University, and A. HINOSE, University of Saskatchewan—Tokamaks are known to be unstable with respect to a drift mode because of their toroidicity. The turbulence level at saturation of the drift mode is studied with the assumption of cold ions and the electron temperature gradient as a principal nonlinearity. A positive nonlinear frequency shift is given by $\delta \omega = (e_0/12k^2) \delta^2$, where $e_0$ is the electron temperature gradient relative to a density gradient, $k = k_{p/e} (k_t$ is a poloidal azimuthal wavenumber, $e_0$ is an ion Larmor radius with an electron temperature), and $\delta$ is the peak boundary value of the potential. The nonlinearity sensitively modifies the radial profile of the drift mode, which in turn induces nonlinear wave damping. The resulting turbulence level at saturation falls in between the strong turbulence limit due to ion mode coupling and the weak turbulence limit due to electron trapping.

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Time-Dependent Diffusion Coefficient for Non-Markovian Process in Plasma Turbulence.* H. XIA and O. ISHIHARA, Texas Tech University—Recent study of resonance broadening theory showed the time dependent nature of a diffusion coefficient of plasma particles in velocity space,\(^1,2\) where a particle orbit modification is properly treated. Such a time dependence of diffusion coefficient is studied based on the generalised Langevin equation, where a frictional force exerted by the effective collisions due to plasma turbulence is included with its retarded effect. This retarded effect is responsible for the non-Markovian process in a Gaussian system. It is shown that a derived Fokker-Planck-type equation has a diffusion coefficient which depends on time and reduces to the conventional quasi-linear diffusion coefficient in a short time limit of much less than the effective collision time.

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Study of Stochastic Heating Mechanism of Plasma Particles in the Presence of Ion Acoustic Turbulence. K.M. Yuen and O. Ishihara, Texas Tech. University. The velocity diffusion of ions in ion acoustic turbulence is studied numerically and analytically. Turbulence is modeled by random electric fields with discrete Fourier modes and the correlation function of the fields is assumed to be Gaussian. Test ions are placed in a background of ion acoustic turbulence and the statistics of particle motions are studied. We have observed that stochastic heating can take place for ions both in resonance and not in resonance with ion acoustic waves. Resonant particles keep absorbing energy from the background turbulence while the non-resonant particles undergo alternate heating and cooling periods with a net heating effect. For the ions which are located in velocity space closer to the phase velocity of the ion acoustic waves are observed to be heated more effectively than those which are located far from the phase velocity of the waves. The numerical results fairly agree with the analytical prediction.

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Velocity Diffusion Coefficient as a Function of Time. H. Xia, and O. Ishihara, Texas Tech. University. The time-dependent nature of the velocity diffusion coefficient in the plasma turbulence was presented by Weinstock and was developed as extended resonance broadening theories. The theories were developed on the assumption of Gaussian and Markovian stochastic process. Based on the Mori-Kubo generalized Langevin equation, the exact generalized Fokker-Planck equation corresponding to the Gaussian but otherwise arbitrary stochastic force is studied. It agrees asymptotically to the phenomenological Fokker-Planck equation for systems exhibiting short-time tail decay. And it shows a velocity diffusion coefficient as a function of time. These theories are examined by the numerical experiments where the velocity diffusion coefficients are measured in plasma turbulence.

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Particle Diffusion in Turbulent Fields: Transition from Quasilinear to Nonlinear Stage*

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Charged particles in random electric fields are known to diffuse over phase velocity resonant layer in velocity space [1,2], while charged particles placed in a sheared magnetic field diffuse over the mode rational surfaces due to \( \mathbf{E} \times \mathbf{B} \) random fluctuations [3]. We study in detail the transition of particle diffusion in turbulent fields from quasilinear (constant diffusion rate) to nonlinear regime (time dependent rate) by test particle numerical experiments.

We consider the equation of motion:

\[
d\mathbf{v}/dt = \mathbf{a}, \quad d\mathbf{c}/dt = R(\mathbf{v}),
\]

where \( R(\mathbf{v}) \) is a random electric field, \( (\mathbf{v}, \omega, qE/m) \) for particle motion in random fields, and \( (\mathbf{v}, \omega, qE/m) \) for guiding center motion in a sheared magnetic field, where \( L_s \) is the shear length, \( \omega \) is the particle velocity along the magnetic field, and \( c \) is the speed of light. Random fields are prescribed to have Gaussian structure in Fourier modes with the spectrum width \( \delta k \). Trajectories of test particles are followed and statistics of \( \zeta \) variance is examined. We define the diffusion coefficient by

\[
D_\zeta = \langle \partial^2 \zeta / \partial t^2 \rangle / \delta t,
\]

where the time slope of the \( \zeta \) variance is to be taken at \( t_{ac} < t < t_c \), \( t_{ac} \) is the autocorrelation time, \( 1/(\delta k \zeta) \), and \( t_c \) is the time at which particles reach the edge of the resonance layer. We have observed that particle diffusion rate is substantially deviated from the quasilinear value and depends on time when the amplitude of random field becomes larger. Such a transition from quasilinear to nonlinear diffusion is observed without particle loss from the resonance region. Time dependent diffusion coefficient may be explained by the effect of retarded friction caused by turbulent fluctuations [4].


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Generalized Langevin Equation for Plasma Turbulence

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The generalized Langevin equation, originally derived for the study of Brownian motion by Mori(1) and Kubo(2), has been applied to the study of plasma turbulence. The evolution equation for the particle velocity in an electrostatic plasma turbulence is derived by the projection operator method. The time integral appearing in the equation indicates that the wave-particle interaction in a plasma is not a Markovian process. Thus the wave-particle interaction cannot be described as instantaneous collisions between particles and wave quanta (quasi-particles) as is usually done in a conventional weak turbulence theory.

We consider plasma particles placed in an electrostatic plasma turbulence characterized by random electric fields (Gaussian process). Particles placed initially at \((x,v)\) move in the random fields to \((x_t,v_t)\) in phase space at time \(t\). Then the particle velocity can be expressed as

\[ v_t = \exp(-i\Omega t) v \]

where \(-i\Omega\) is the Liouville operator and is given by \(-i\Omega = \frac{1}{m} \nabla \cdot E(x) + \frac{e}{m} E(x) \frac{\partial}{\partial x}\). The evolution equation (the generalized Langevin equation) for \(v_t\) can be derived by the method of the projection operator as

\[ \frac{dv_t}{dt} = \exp(-i\Omega t) E(x) - \int_0^t \gamma_{t-t'}(x,v) v_t(x,v) \, dt \]

where \(-i\Omega = -i(1-P)\Omega\), \(P\) is a projection operator, and \(\gamma_{t-t'}\) is a friction function. The time integral of the right hand side demonstrates the memory effect in the wave-particle interaction. The cumulant expansion of the characteristic function of \(v_t\), \(\langle \exp(i\xi v_t) \rangle\), shows the time-dependent nature of the velocity variance. We find that the memory effect or the influence of the past on particle trajectories in phase space limits the particle diffusion in velocity space, in agreement with the earlier observations (3,4).


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PHOTON ACCELERATION: 
THREE-WAVE INTERACTION

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Recently a frequency upshifting of short pulses of laser light was studied by Dawson and others[1]. Photons, which propagate in a plasma with effective mass of \( \frac{m_0}{c^2} \), were shown to be accelerated (upshifted in \( \omega \)) by propagating down the density gradient of a longitudinal plasma wave.

We study the possibility of photon acceleration from the viewpoint of three-wave interaction[2]. We consider that a photon with momentum \( \hbar \omega_{pl} \) and energy \( \hbar \omega_{pl} \) will combine a plasmon with momentum \( \hbar \omega_{pl} \) and energy \( \hbar \omega_{pl} \) to give a photon with momentum \( \hbar \omega_{pl} \) and energy \( \hbar \omega_{pl} \). The waves will interact one another through the wave-particle interaction as shown below:

\[
\begin{array}{c}
\text{photon}(f) \\
\text{plasmon}
\end{array}
\]

\[
\begin{array}{c}
\text{photon}(f) \\
\text{plasmon}
\end{array}
\]

where the straight line indicates a plasma particle (charge \( e \), mass \( m \) and momentum \( p \)). The interaction vertex function for the plasmon-particle can be obtained by the use of the interaction Hamiltonian between particles and longitudinal quasiparticles

\[
H_c = \int d^3x \, \Psi^+(x) e\phi(x) \Psi(x),
\]

where \( \Psi(x) \) is a wave function of plasma particles, while the interaction vertex function for the photon-particle can be obtained from the interaction Hamiltonian between particles and transverse quasiparticles

\[
H_r = -\frac{e}{mc} \int d^3x \, \Psi^+(x) \left( p - \frac{eA_0}{c} \right) \cdot A_1(x) \Psi(x),
\]

where \( A_0 \) and \( A_1 \) are the zero-order and the first-order vector potential, respectively. Our calculation of the vertex function for this three-wave interaction process reveals that there will be no transition to occur if the plasma is cold and stationary. We have found that the transition probability becomes maximum when the plasma particles move in the direction of longitudinal plasma oscillations (plasmons) along with the direction of polarization of vector potential \( A \), resulting in the upshifting of frequency of the outgoing photon fields.

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EFFECT OF TURBULENT COLLISIONS ON DIFFUSION IN STATIONARY PLASMA TURBULENCE

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Recently the velocity diffusion process was studied by the generalized Langevin equation derived by the projection operator method(1). The further study shows that the retarded frictional function, \( \gamma(t) \), plays an important role in suppressing particle diffusion in the velocity space in stronger turbulence as much as the resonance broadening effect. The retarded frictional effect, produced by the effective collisions due to the plasma turbulence, is described as

\[
\gamma(v,t) = \frac{q^2}{m^2} \langle E_r(x,v)E_r(x,v) \rangle,
\]

where \( \langle \ldots \rangle \) indicates an average over real (x) and velocity (v) spaces and random fluctuation field \( E_r(x,v) \) is assumed to be a Gaussian, but non-Markovian and non-wide-sense stationary process. The velocity diffusion coefficient in a stationary plasma turbulence in the absence of magnetic field is given as

\[
D(v,t) = \frac{q^2}{2m^2} \frac{\partial}{\partial t} \int_0^t dt_1 \int_0^{t_1} dt_2 \chi(t-t_2) \beta(v,v,t_2),
\]

where

\[
\beta(v,v,t_2) = \langle E_r(x,v)E_r(x,v) \rangle = \sum_k E_k E_k^* \langle (v_k-v_0)(v_1-v_2) \rangle \langle e^{-i k(\Delta x_1+\Delta x_2)} \rangle
\]

with \( \langle \ldots \rangle \), a spatial average, is the resonance broadening term discussed in the reference 2. Here

\[
\chi(0) = -\frac{1}{2\pi} \int \frac{e^{-|w|}}{1 + \omega^2 \gamma(0)} dw
\]

introduces the effect of turbulent collisions on the velocity space diffusion (\( \gamma(0) \) is the Fourier transform of \( \gamma(0) \)). The diffusion rate becomes time-dependent because of the non-Markovian nature of the random process involved. The relation between the proposed formulation and the extended resonance broadening theory(2) is discussed. We also carry out test particle numerical experiment for Langmuir turbulence to test the theories. In a stronger turbulence a deviation of the diffusion rate from the one predicted by both the quasilinear and the extended resonance theories has been observed and is explained qualitatively by the present formulation.

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Photon Acceleration by the Plasmon-Photon Interaction. — O. ISHIHARA, Texas Tech University — Photon acceleration in a plasma will be observed as a frequency upshifting in the electromagnetic wave propagating in a plasma. We study photon acceleration from the viewpoint of photon emission by the interaction between plasmon and photon. Three waves, photon, plasmon and photon, interact one another through the interaction between the wave and plasma particles. The relativistic interaction Hamiltonian between particles of species $a$ and photons are given by $H = -e_p a^T d^4 x W_T + a^T W_T$, where $a$ is a $4\times4$ matrix expressed in terms of Pauli matrices and $a$ is a vector potential, while the interaction Hamiltonian between particles and plasmons is $H = e_p d^4 x W_T$, where $\phi$ is a scalar potential. The frequency upshifting is analyzed in terms of the energy gain of photon as a result of three wave interaction.

*Supported by AFOSR under grant 86-0156.

Analysis of the Non-Markovian Behavior in Velocity Diffusion Process. — H. XIA and O. ISHIHARA, Texas Tech University — The non-Markovian diffusion in velocity space was observed in the test-particle computer simulations of the plasma turbulence[1]. The intrinsic non-Markovian effects such as resonance broadening and retarded turbulent collisions were found to be responsible for such behavior[2]. The numerical experiment in the moderately strong Langmuir turbulence, where the resonance region is bounded by the lower boundary, shows that particles diffuse toward the upper resonance layer after they hit the lower boundary. It is found that the non-Markovian behavior due to the boundary effect further deviates the time-dependent diffusion coefficient predicted by the intrinsic non-Markovian effects.

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PHOTON ACCELERATION BY PLASMA TURBULENCE

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Recently, we proposed a novel method of photon acceleration in which electromagnetic waves injected into a current carrying plasma are scattered by the plasma turbulence and the frequency of the electromagnetic waves is upshifted by absorbing energy of the plasma turbulence.

We examine the conditions of effective photon acceleration for the above mentioned scheme for the case either plasma particles are non-relativistic or relativistic. We start with Hamiltonian for a particle of species s in an electromagnetic field:

\[ S^\text{rel}_s = \frac{1}{2m_s} \left| p - \frac{e_s}{c} A(x) \right|^2 + e_s \Phi(x) \]

\[ S^\text{uni}_s = \alpha \left( p - \frac{e_s}{c} A(x) \right) + \beta m_s c^2 + e_s \Phi(x) \]

where \( A(x) \) and \( \Phi(x) \) are the vector and scalar potentials of the field. \( \alpha \) and \( \beta \) are the dimensionless Dirac \( 4 \times 4 \) matrices described as \( 2 \times 2 \) matrices. The three-wave interaction takes place among photon \( (\Omega_1, q_1) \), plasmon \( (\Omega_2, q_2) \) and photon \( (\Omega_5, q_5) \). The scattered photon \( (\Omega_3, q_3) \) will have the frequency of \( \Omega_5 = \Omega_1 + \Omega_2 \) governed by the conservation law. The vertex function for the photon-plasmon-photon interaction is given by:

\[ M = \sum \int d^4 \mathbf{r} f(\mathbf{r}) \frac{2 \pi \hbar c^2}{\gamma^2 m^2} \left( \frac{2 \pi \hbar c^2 \Omega_1}{m^2} \right)^{1/2} \times \left\{ \frac{2 u^*_s \cdot q_1 u^*_s \cdot q_1 u^*_s \cdot q_2 u^*_s \cdot q_2}{(\Omega_1 - \mathbf{v} \cdot \mathbf{q}_1)(\Omega_2 - \mathbf{v} \cdot \mathbf{q}_2)} \left( \frac{\mathbf{q}_3^2}{\Omega_1 - \mathbf{v} \cdot \mathbf{q}_3} + \frac{\mathbf{q}_3^2}{\Omega_2 - \mathbf{v} \cdot \mathbf{q}_3} \right) \right\} \]

where \( u^*_s \) and \( u^*_s \) are unit polarization vectors of photon fields. The terms with \( u^*_s \) are resulted from relativistic effect. This vertex function suggests that the frequency upshifting becomes maximum when the wave vector of the emitted photons is in the direction of electron streams in the plasma turbulence.

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on Plasma Science

Characteristics of Distribution Function
in a Time-Dependent Velocity Diffusion Process

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The time-dependency of the particle velocity diffusion in plasma turbulence has been discussed by various theoretical models and shown in the test-particle computer simulations [1]-[4]. The non-Markovian effects are responsible for the time-dependent nature of the diffusion coefficient that deviates far from the quasilinear value at a moderately strong turbulence level in Langmuir turbulence [5].

Under the assumption of Gaussian turbulent fields, the non-Markovian effects were discussed by treating the particle velocity as a Wiener process. The memory property appears in the spatial increment of particle trajectory as well as in the term of retarded effective turbulent friction in the generalized Langevin equation [5]. We further study such a non-Markovian behavior in velocity diffusion by introducing the non-Gaussian nature in turbulent fields. This modification leads to a deviation of probability distribution function of particle velocity from the Gaussian distribution. The effect of non-Gaussian turbulent fields on the particle diffusion coefficient is discussed in detail.

To examine the long time behavior of the diffusion process, we perform test-particle numerical experiments by a newly installed MasPar massively parallel computer at the Texas Tech University. In the test-particle simulation, the k-spectrum of stationary Langmuir turbulence is modeled by two different shapes; one is Gaussian-type, and the other is square-like. When particles hit the boundary of resonance region, a fictitious non-Markovian effect on the diffusion coefficient is observed. By varying the spectral shape, this fictitious non-Markovian behavior could be suppressed. The results of test-particle numerical experiments are presented and compared with the theory based on the assumption of non-Gaussian turbulent fields.

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Nonresonant wave-particle interaction in semiclassical quasilinear theory

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A nonresonant nature of wave-particle interaction is clarified from the viewpoint of quantum mechanics. The interaction of particles and quasiparticles can be described by the use of transition probability which is found to have both resonant and nonresonant contributions. The resonant transition probability is known as Fermi's golden rule, which is now supplemented by the nonresonant contribution, resulting in the proper conservation of energy and momentum in the particle-quasiparticle system.

DRIFT ALFVÉN EIGENMODE IN TOKAMAKS

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ABSTRACT. A drift Alfvén eigenmode having a phase velocity significantly larger than the electron diamagnetic velocity is shown to exist in the tokamak magnetic geometry. For typical ohmically heated tokamak discharges the approximate dispersion relation is \( \omega \approx \beta c \omega_A \). Such a fast drift mode has recently been observed in the TEXT tokamak.
SEARCH FOR ION TEMPERATURE GRADIENT DRIVEN ELECTROSTATIC HYDRODYNAMIC INSTABILITY IN TOKAMAKS

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ABSTRACT. The ion temperature gradient driven electrostatic instability in tokamak magnetic geometry has been investigated numerically in terms of a generalized ion density moment. For typical tokamak parameters (ratio between the density gradient scale length and the major radius, $e_n = L_n/R$, about 0.1) and $T_i/T_e$ the search for rapidly growing instabilities of a hydrodynamic nature has been unsuccessful, even with the ion temperature gradient relative to the density gradient, $\eta_i = \frac{d\ln T_i}{d\ln n_i}$, as large as six.

Resistive ballooning mode in tokamaks

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The resistive ballooning mode in tokamak geometry has been re-investigated without imposing any ordering among characteristic frequencies, $\omega_i$ (growth rate), $\omega_d$ (diamagnetic frequency), and $\omega_m$ (magnetic-drift frequency in the ballooning space). An extensive numerical search for unstable eigenfunctions has failed. Only a stable-ion diamagnetic mode (analytically predicted) has been found.
RESISTIVE MHD BALLOONING MODE IN TOKAMAKS

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ABSTRACT. A mode equation for the resistive MHD ballooning mode in low beta tokamak magnetic geometry has been derived and analyzed without imposing any ordering among the frequencies $\omega$, $\alpha$, (diamagnetic frequency) and $\omega_0$ (magnetic drift frequency operator). Only a stable, ion diamagnetic mode has been found. The resistive ballooning mode is strongly stabilized by magnetic shear and toroidal effects.

Resonance broadening in drift wave turbulence

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Turbulent diffusion of electrons in drift modes in a sheared magnetic field is studied by using a test particle numerical experiment. Electrons diffuse across the magnetic field over mode rational surfaces, where electrons interact with waves in resonance. A spatial diffusion coefficient, which describes resonance broadening, is found to be time dependent and departs from quasilinear predictions in stronger turbulence even well before the time when particles hit the resonance boundary of the rational surfaces.
Photon-Plasmon-Photon Interaction

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ABSTRACT

A theory of photon-plasmon-photon three-wave interaction is presented. An electromagnetic wave launched into the plasma turbulence interacts each other through the wave-particle interaction resulting in the emission of an electromagnetic wave with frequency upshifted. The theory is based on the Dirac relativistic wave equation, where the interaction of plasma particles with electromagnetic waves are described. It is shown that the frequency upshifting of the electromagnetic wave can take place in a current carrying plasma.

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